WHEN THE GRID LOSES IT'S GRIP

A Base Stock-and-Flow Model of Frequency Instability

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Abstract

Power system stability needs are increasing as renewable generation adoption increases, so as a starting point, this paper translates swing-equation and governor-control physics into a stock-and-flow model of power-system frequency response under declining inertia. Three stocks (frequency, supply, demand) and their associated control flows reproduce the essential transient behaviour—initial rate of change of frequency (RoCoF), nadir, arrest, and recovery—while remaining compact enough for later embedding in broader energy-system models. Scenario experiments varying disturbance size, inertia, and droop gain show that lower inertia worsens the initial excursion, narrows the operator response window, and alters oscillations in the recovery phase. A causal loop interpretation derived from the formal model reveals how balancing loops remain structurally present but become functionally weaker as inertial buffering declines. The study contributes a foundational bridge between engineering frequency dynamics and system dynamics modelling for resilience-oriented decision support.

Keywords: low-inertia power systems; frequency control; stock-and-flow modelling; resilience, stability

1. Introduction

Operating electrical power systems is becoming more challenging as variable renewable generation displaces synchronous machines, reducing the system's naturally available rotational inertia (He et al., 2024; Zhou et al., 2024). Declining inertia weakens the first physical buffer that resists frequency movement after a disturbance: the same event produces faster RoCoF, a deeper nadir, and a shorter corrective-action window (Tielens and Van Hertem, 2016). In practical terms, system operators must rely more heavily on fast reserves, improved inertia monitoring, and proactive security management.

A research gap persists between detailed engineering formulations of transient frequency response (Kundur, 1994; Machowski et al., 2020) and system dynamics representations that expose feedback structure and operator-relevant implications. Power engineering offers mature swing-equation descriptions while system dynamics has contributed substantially to electricity-sector planning, market behaviour, and energy-transition policy (Ahmad et al., 2016; Ford, 1997). Yet relatively little work translates reduced-order frequency-control physics into an explicit stock-and-flow form that preserves behavioural realism while revealing the balancing processes behind the operator's shrinking decision margin.

This paper addresses that gap. It develops a three-stock model of frequency stabilisation grounded in engineering physics, compact enough for later expansion into models of price formation, resilience, or transition dynamics. Three research questions guide the work: (1) Can system dynamics stocks and flows represent the complex behaviour of frequency stabilisation? (2) How does inertia affect system frequency during a load change? (3) What can be deduced from the balancing and reinforcing loops within the model? The paper's theme, managing instability under declining physical buffering, aligns directly with the 2026 conference call to navigate uncertainty, manage instability, and craft futures together.

2. Background

2.1 Power-system frequency dynamics

Conventional frequency dynamics are governed by the balance between mechanical input power and electrical output power (Machowski et al., 2020). Following a disturbance, the swing equation converts a power imbalance into rotor acceleration or deceleration, observable as a system-level frequency change seen in Figure 1. Inertia determines how strongly synchronous machines resist speed changes: higher inertia slows the initial frequency movement and buys time for controls to act. Primary control (governor droop) then arrests the decline, shaping the nadir; secondary control restores frequency toward nominal (Al Kez et al., 2023). Under reduced inertia the same balancing structure remains present, but the response window compresses and dependence on fast reserves increases (Zhou et al., 2024). Low-inertia operation is therefore not a smaller version of the traditional grid but a qualitatively different control environment.

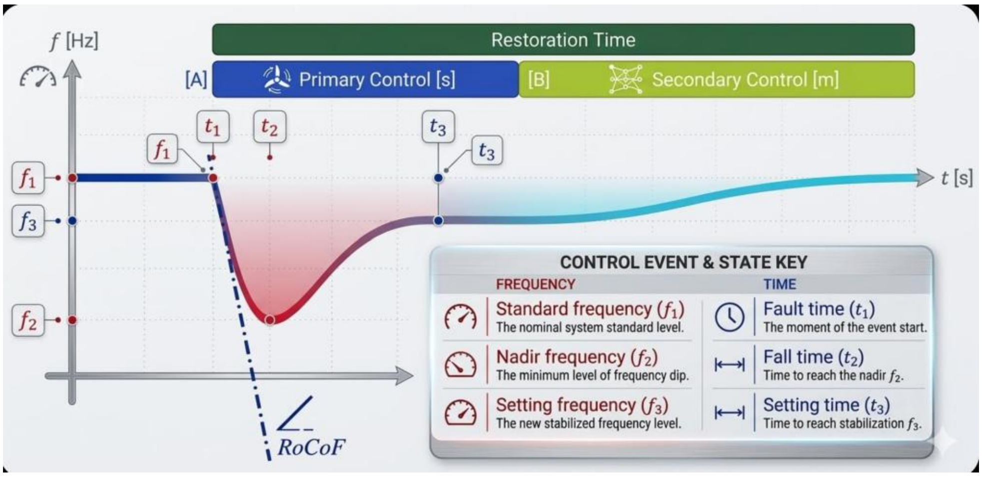


Figure 1: Power System frequency transients after contingency [Image generated via Nano Banana 2]

2.2 Research gap

System dynamics has been applied extensively to electricity-sector planning, capacity expansion, and policy analysis (Ahmad et al., 2016; Ford, 1997), but comparatively little attention has been paid to short-timescale frequency behaviour. Inertia remains largely absent as a central feedback variable in the SD literature on electricity, leaving a methodological gap between detailed engineering models and structural stock-and-flow representations suitable for embedding within broader energy-system analyses. This paper bridges that divide by translating established swing-equation physics into a reduced-order SD model whose feedback structure is explicit and interpretable.

3. Model structure

3.1 Purpose and boundary

The model reproduces the essential dynamics of frequency stabilisation during supply–demand disturbances using a small, transparent stock-and-flow structure. It is explanatory rather than fully predictive: its function is to establish a physically grounded foundation for later expansion. The boundary covers a conceptual four-generator system (150 MW capacity, 75 MW load) over a 25-second horizon at 0.01 s time steps. Network impedance, converter-specific dynamics, protection logic, and locational effects are excluded.

3.2 Dynamic hypothesis

A load increase creates an immediate power imbalance that causes frequency to fall. The initial decline is moderated by system inertia (kinetic energy stored in rotating electrical machines and turbines) acting as a physical buffer. As frequency deviates, primary and secondary controls increase generation and counteract the imbalance, producing a nadir and subsequent recovery. Under lower inertia, frequency changes faster before control can fully arrest the deviation since the balancing loops become functionally weaker.

3.3 Stock-and-flow derivation

The model is formulated around three stocks: **Frequency** (Hz), **Supply** (MW), and **Demand** (MW). The derivation proceeds in five steps from the swing equation.

Step 1: The swing equation

The classical swing equation for an equivalent single machine is:

$$\frac{2H}{f_0}\frac{df}{dt} = \frac{P_m - P_e}{S_{base}}$$

where H is the inertia constant (s), f_0 the nominal frequency (50 Hz), P_m mechanical power (MW), P_e electrical power (MW), and S_{base} the system power base (MW). Defining an inertia scaling factor $\mu = f_0 / (2H \cdot S\_base)$ [Hz/(MW·s)] and relabelling $P_m \equiv S$, $P_e \equiv D$:

$$\frac{df}{dt} = \mu(S - D)$$

This is the first stock equation. When D > S frequency falls; when D < S it rises. The initial RoCoF after a disturbance L (MW) is $\mathrm{RoCoF}|_{0+} = -\mu \cdot L$, confirming that initial RoCoF is proportional to disturbance size and inversely proportional to inertia.

Step 2: Primary control (governor droop)

Governor droop response adjusts mechanical power proportionally to the supply–demand gap (Undrill et al., 2018):

$$\frac{dS}{dt} = K_{droop}(D - S)$$

With only primary control, the system can arrest the decline and establish a nadir but cannot restore nominal frequency—a residual steady-state error persists (Al Kez et al., 2023).

Step 3: Secondary control (frequency restoration)

Secondary control, analogous to Automatic Generation Control, acts on the normalised frequency deviation $\varepsilon_f = (f - f_0)/f_0$:

$$\frac{dS}{dt} = -K_{sec}\epsilon_f$$

When frequency is below nominal ($\varepsilon_f < 0$), the term $-K_{sec} \cdot \varepsilon_f$ is positive, increasing supply and driving frequency back toward f_0. This integral-like action eliminates the steady-state error.

Step 4: Demand-side response

A demand-side control reduces demand when it exceeds supply:

$$\frac{dD}{dt} = -K_{cod}(D - S)$$

In the present model K_{cod} is set small (0.001) so that demand-side response does not dominate. Its presence preserves the structural possibility of potential load-shedding actions, such as applied during large incidents, and completes the causal loop structure.

Step 5: Stock-and-flow translation

Frequency is a stock with a single bi-flow whose rate is μ(S − D)—the SD equivalent of the swing equation. **Supply** is a stock with a bi-flow CS composed of primary droop and secondary control terms. **Demand** is a stock with a bi-flow CD representing demand-side curtailment. The auxiliaries μ, ε_f, and Δ_{SD} are converters computing intermediate quantities. The disturbance L enters as an exogenous pulse shifting the demand stock at a specified time.

Note on the primary-control formulation. Classical textbooks express governor droop as a function of frequency deviation rather than the power gap (Kundur, 1994). The present model uses (D − S) as the primary-control input—a simplification appropriate for the reduced-order setting that captures the essential balancing action. The frequency-responsive role is carried by the secondary-control term ($-K_{sec} \cdot \varepsilon_f$). Together, the two terms reproduce arrest-and-restore behaviour while keeping the model compact.

3.4 Equations and parameterization

Table 1 summarizes the model variables, equations, and parameter values. Parameters were selected to preserve plausible short-term frequency behavior and enable systematic variation across scenarios. Units were tracked explicitly.

Table 1. Model variables, equations, and parameters.

Variable	Equation / Definition	Units	Default	Role
f	df/dt = μ(S − D)	Hz	50	System frequency (stock)
S	dS/dt = CS	MW	1000	Aggregate supply (stock)
D	dD/dt = CD	MW	1000	Aggregate demand (stock)
μ	$f_0 / (2H \cdot S_{base})$	Hz/(MW·s)	—	Inertia scaling factor
ε_f	$(f - f_0) / f_0$	—	—	Normalised freq. deviation
CS	K_droop·(D−S) − K_sec·ε_f	MW/s	—	Supply control flow

CD	−K_cod·(D−S)	MW/s	—	Demand control flow
H	Parameter	s	3.0	Average inertia constant
K_droop	Parameter	1/s	0.03	Primary droop gain
K_sec	Parameter	MW/s	10	Secondary control gain
K_cod	Parameter	1/s	0.001	Demand-side control coeff.
L	Disturbance	MW	10	Generation loss / load step

3.5 Validation

Five tests were performed: (1) dimensional consistency of all variables; (2) structural adequacy relative to expected nonlinear transient behaviour; (3) a zero test confirming no behaviour change when supply equals demand with no disturbance; (4) expected-behaviour tests comparing simulated response to benchmark patterns in the literature; and (5) verification that changes in inertia and droop produce the behaviours predicted by the dynamic hypothesis.

3.6 Experimental design

Three scenario sets were analysed. Disturbance size L was varied from 0 to 25 MW in five steps; inertia H from 0.5 to 3.0 s in five steps; droop K_{droop} from 0.01 to 0.06 in five steps. Principal outcomes tracked: initial RoCoF, frequency nadir, and settling time.

4. Results

4.1 Base model and governor response

Without controls, the model confirms that a power imbalance causes frequency to fall linearly—isolating the role of inertia. Adding primary, secondary, and demand-side controls produces the expected stabilization pattern: initial deviation, arrest, nadir, and recovery toward nominal. This confirms that the stock-and-flow formulation reproduces both the physical buffering effect of inertia and the balancing role of control actions. In Figure 2 the demand and supply are both at 50 MW, keeping the frequency stock at equilibrium as expected.

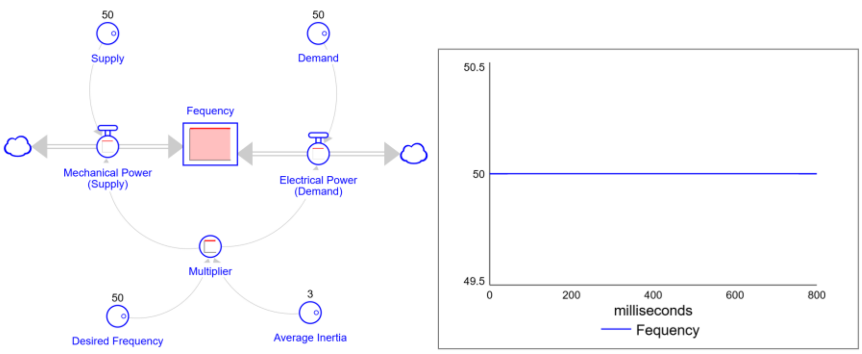


Figure 2: Swing equation as stock and flow in equilibrium

4.2 Behavioural validation

The model, seen in Figure 3, reproduces the literature's expected behaviours: larger disturbances generate larger excursions; lower inertia increases initial RoCoF and worsens the nadir; stronger control responsiveness improves arrest and recovery. Simulated trajectories are qualitatively consistent, visible in Figure 4, with the transient patterns reported by ENTSO-E for Nordic low-inertia scenarios (Ørum *et. al.,* 2015) shown in Figure 5.

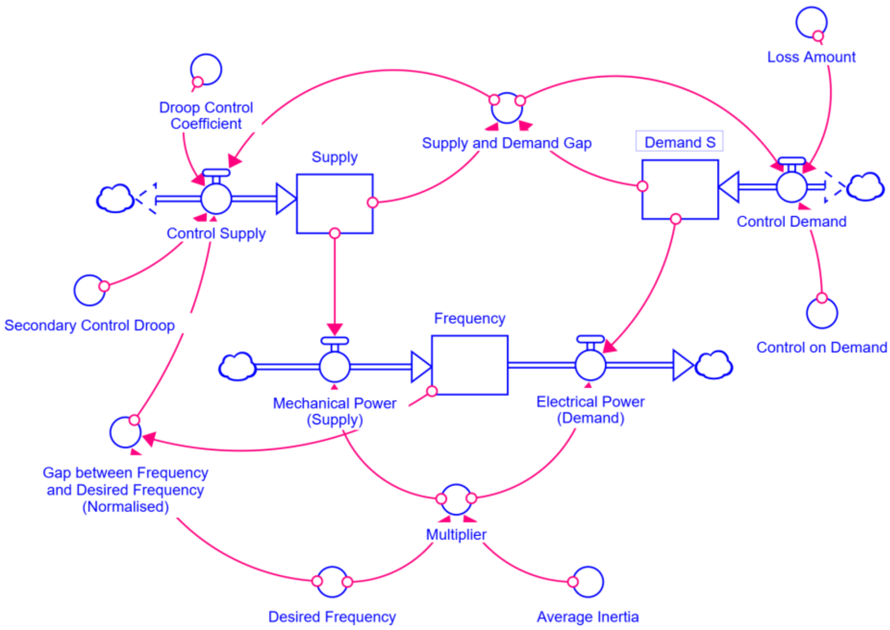


Figure 3: Full frequency simulation in stocks and flows

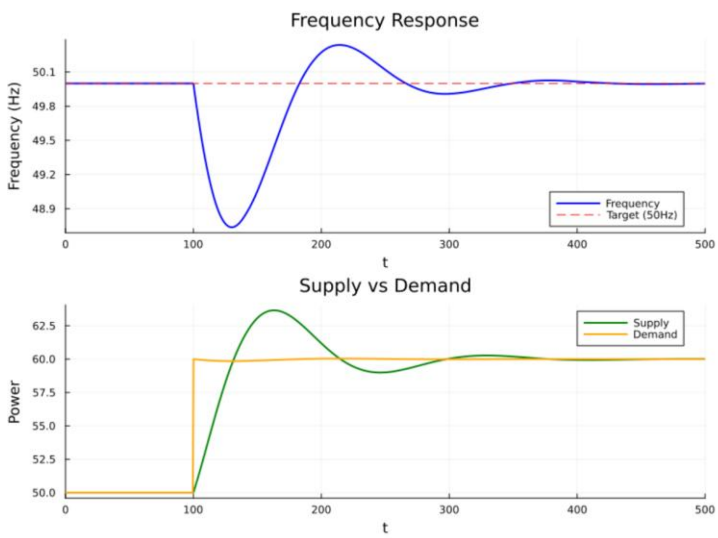


Figure 4: Simulation behavior after contingency

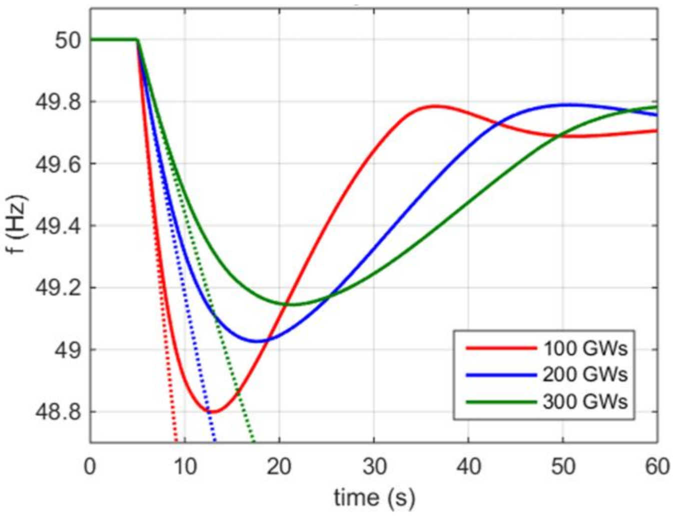
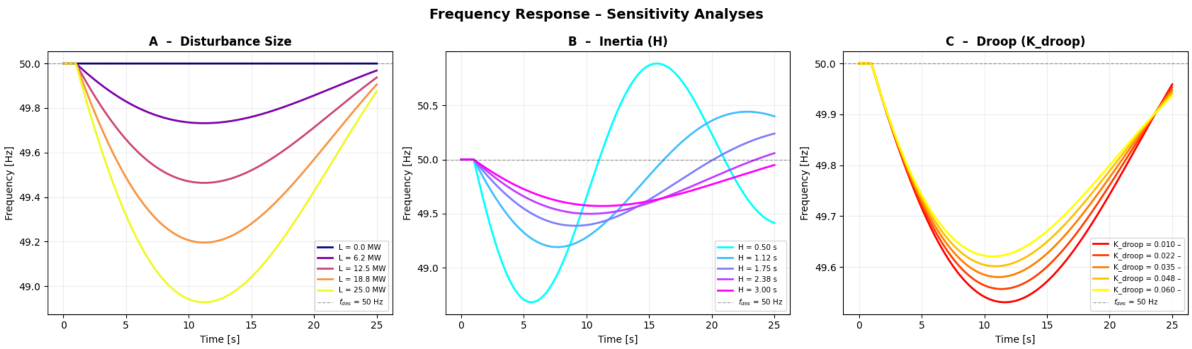


Figure 5: Results from ENTSO-E on inertial changes on frequency stabilization [From (Ørum *et. al.,* 2015)]

4.3 Scenario analysis

The effects of inertia are not merely proportional. As inertia declines, the system reaches more severe nadirs and exhibits more pronounced oscillations in the recovery phase (primary and secondary response). Larger load changes worsen both RoCoF and nadir, but their severity depends strongly on prevailing inertia. Faster droop response partially compensates for reduced inertia, although initial RoCoF is almost purely a function of inertia—droop control has negligible effect in the first moments after a disturbance. The overall pattern is one of shrinking resilience margins as inertia declines. These conclusions align with those drawn from ENTSO-E's Nordic system-inertia reports.



4.4 Structural interpretation

The causal loop analysis, visible in Figure 6, clarifies the control mechanisms. **B1** (primary control): falling frequency activates governor action, increasing generation. **B2** (secondary control): generation is further adjusted to restore nominal frequency. **B3** (load shedding): a least-preferred balancing action when the supply side cannot restore balance. **R1** (emergent reinforcing loop): if demand-side action were timed and sized similarly to primary control, the interaction between B1 and B3 could amplify rather than damp the mismatch. In the present model R1 does not dominate because load curtailment is intentionally weak, but its presence reveals a structurally possible instability that could become critical as converter-dominated systems with very low inertia emerge.

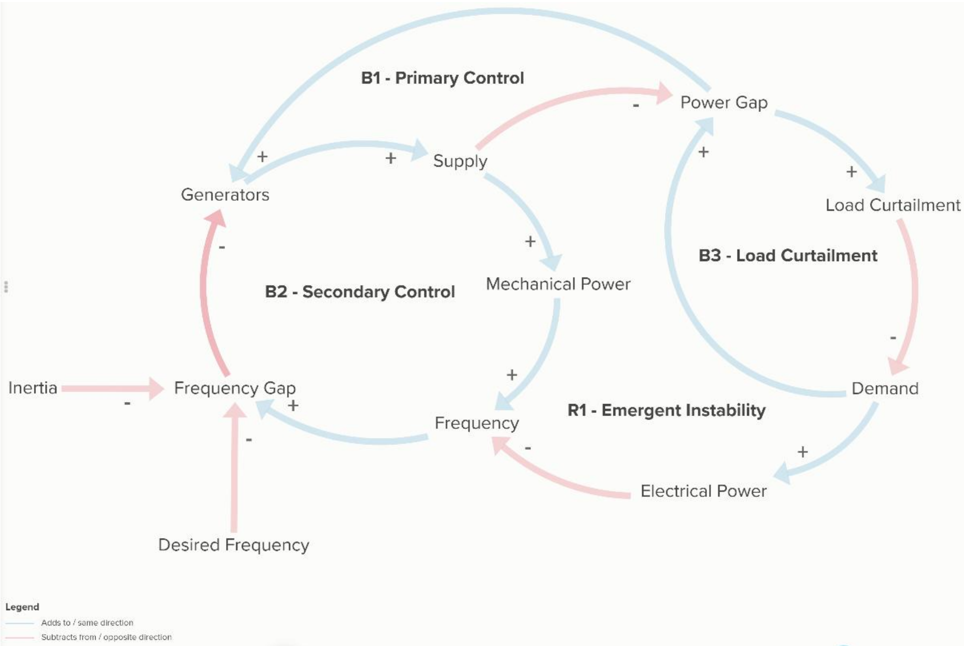


Figure 6: Causal Loop Diagram of frequency control

4.5 Limitations

The model does not represent individual generators with separate frequencies, reactive power, or locational characteristics. Line impedance and network topology are excluded. It should be interpreted as a foundational representation testing the viability of stock-and-flow modelling for inertia-related behaviour, not as a full transient-stability simulator.

5. Discussion

Three principal insights emerge from the model.

First, reduced inertia changes more than the initial RoCoF. It alters the Nadir non-linearly, speed and oscillatory character of the transient, while droop control has negligible effect in the initial moments. Operator difficulty is shaped by both how far frequency moves and how fast.

Second, declining inertia shrinks the operator's effective decision margin. Under lower inertia, smaller disturbances produce RoCoFs that previously would have been manageable but now require action, higher reserves, or tighter security margins. The recent Iberian blackout of April 2025, triggered after a system split, illustrates the operational stakes when inertial reserves are low.

Third, the SD formulation adds value beyond behavioural replication. The modular, interpretable stock-and-flow form can be embedded within broader models of market operations, investment, reserve procurement, or energy-transition planning. If system states and credible trip scenarios can be estimated in near-real-time, reduced-order models of this kind may help estimate likely RoCoF and nadir values before a severe event, pointing toward resilience-oriented decision-support tools.

A useful distinction emerges among four layers: physical buffering capacity (inertia), control response speed (droop/secondary gains), operator decision window (their interaction), and the economic burden of maintaining security. Declining inertia is therefore simultaneously a stability problem and a systems problem, linking physics, control, decision-making, and cost—precisely the kind of multi-loop, multi-timescale challenge for which system dynamics is well suited.

6. Conclusion

This paper developed a reduced-order system dynamics model of power-system frequency response grounded in the swing equation and governor/secondary control. The model shows that stock-and-flow structures can represent essential frequency-stabilisation behaviour, including the effects of inertia, disturbance magnitude, and control responsiveness on RoCoF, nadir, and recovery. Declining inertia weakens balancing loops by compressing the time available for corrective action.

The significance lies in the bridge between engineering physics and system dynamics structure. Transient frequency response can be understood not only as a set of differential equations but as a feedback system whose balancing and reinforcing mechanisms are explicit and interpretable. This provides a foundation for future work incorporating generator heterogeneity, converter-based resources, network effects, and broader market or planning dynamics. In Ford's (1997) spirit, once the feedback becomes visible, new opportunities to intervene also become visible.

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