Supplementary Information

Contents

1 Data

For the two experiments of Frey and van der Rijt (2021), we obtain the experiment data from their online replication files. Consistent with the data analysis performed in the original study, we removed participants with missing values.

For Becker et al. (2022), we obtain the data for the binary exchange experiment from the replication materials linked in the e-companion appendix.

For Mori et al. (2012), we use the data from experiments conducted at both universities reported in the paper. We obtain the data for the experiments from the paper's supplementary file, which is available on arXiv at https://arxiv.org/abs/1112.2816. We removed participants with missing values.

In the analysis shown in Figure 4 of the main text, after slicing the data to between 0.4 and 0.6 mean independent accuracy, we have 144 observations for panel A, 240 observations for panel B, and 368 observations for panel C, and 40 observations for panel D.

2 Parameter values

We use $\alpha = 2$ throughout all simulations of this paper, holding the same shape of the conformity function for all simulations and replications.

For both panels in Figure 3 of the main text, we use parameter values $I = 0.55$. For the agent-based model presented in panel A, we simulated 50 groups of 100 individuals. For panel B, we simulated the same number of groups of the same size after synchronously updating their opinions 100 times.

For the model prediction results presented in Figure 6, we use the same I values as analyzed in the experiments (which were filtered to be greater than 0.5, resulting in 25 questions). For the counterfactual predictions, we take one minus these I values, making the counterfactual I values all less than 0.5. In both simulations, we use $a = 0.33$, we run the agent-based model 30 times for 8 groups of 14 individuals (same as the original experiment), calculating the collective and individual accuracies for each.

For Figure 9, we evaluate the probability of a correct majority on a 51 by 51 grid of independent accuracy (I) and weight of social influence (w or a). For the sequential case, we simulate 400 groups of 21 individuals and then calculate the collective accuracies. For the synchronous case, we simulate 400 groups of 21 individuals each with 75 revisions.

3 Analytic solutions for synchronous updating bifurcation without functional form assumption for S(x)

While results of the main text rely on the assumption for a specific functional form of $S(x) = \frac{x^\alpha}{x^\alpha+(1-x)^\alpha}$, here, we show that the bifurcation appears for general $S(x)$ that satisfies a few specific conditions, when $I = 0.5$ for Equation 3. Note that by symmetry between the two options, $S(x)$ should satisfy $S(0.5) = 0.5$.

Equation 3 of the main text can be written as,

$$\frac{dx}{dt} = \frac{(1-w)I + wS(x) - x}{\tau} \tag{S1}$$

Let's define the right-hand side of the equation, to be $G(x) \equiv ((1-w)I + wS(x) - x)/\tau$. It is easy to see that $x = 0.5$ is an equilibrium of the differential equation since $G(x) = 0$ for $x = 0.5$ when $I = 0.5$.

The stability of this equilibrium depends on dG/dx, where a negative value indicates a stable equilibrium, and a positive value indicates an unstable one. The derivative at $x = 0.5$ is,

$$\left.\frac{dG}{dx}\right|_{x=0.5} = \frac{wS'(0.5) - 1}{\tau} \tag{S2}$$

The bifurcation happens when the stability of this equilibrium changes, that is $dG/dx = 0$, at the critical w value, w^*,

$$w^* = \frac{1}{S'(0.5)} \tag{S3}$$

Since w^* has a solution between 0 and 1 (which is the meaningful solution) as long as $S'(0.5) > 1$. So for a general conformity function, as long as the derivative at the midpoint is larger than 1, a bifurcation occurs. In the functional form we have chosen, this corresponds to every case where $\alpha > 1$. For $\alpha < 1$, we have an inverse S-shape and $S'(0.5) < 1$, so we do not expect a bifurcation. A later section will present the results for the $\alpha < 1$ case.

4 Analytic results for sequential updating through connections with Pólya's Urn

The sequential decision process described in this text is equivalent to a generalized Pólya's urn process, a classical model of path dependency. In this framework, the objects of interest——whether an individual selects option X or Y——are represented as colored balls in an urn (e.g., red and black). In each iteration, a ball is drawn, its color is observed, and it is returned to the urn. In the classical Pólya's urn model, an additional ball of the same color is added before the next draw. Generalized versions allow for the number of added balls to vary.

This equivalence arises because, in the sequential updating process, early choices influence later decisions: as an option gains more selections, it becomes increasingly likely to be chosen in subsequent rounds, mapping to the reinforcement dynamics of Pólya's urn.

Pólya's urn process has been extensively studied in mathematics, yielding some useful analytical results. In particular, Hill et al. (1980) analyzed a generalized Pólya urn process with a binary set of outcomes, where at each step, the probability of drawing a red ball is given by $f(x)$, and the probability of drawing a black ball is $1 - f(x)$. Here, x represents the proportion of previous draws that were red, and $f(x)$ can be any general continuous function mapping the interval $[0, 1]$ to itself. A key result from Hill et al. (1980) concerns the set of equilibrium points Q, defined as the set where $f(x)$ is equal to x, or $Q := \{x \in [0, 1] : f(x) = x\}$. The study proves that the urn process almost surely converges to points in Q where the derivative of $f(x)$ is less than one ($f'(x) < 1$).

To build intuition for this conclusion, consider the extreme condition when $w = 1$, where the probability of choosing correctly is given by the function $S(x)$, as defined in the main text and shown in Figure S1. For all values of α, the set Q consists of the points $\{0, 0.5, 1\}$, meaning that the generalized urn process will converge to one of these equilibrium points. Now, consider the derivative of $S(0.5)$ for different values of α (this can be visualized in Figure S1). The derivative of $S(x)$ at $x = 0.5$ is equal to one when $\alpha = 1$, greater than one for $\alpha > 1$, and less than one for $\alpha < 1$. This relationship is reversed for $x = 0$ and $x = 1$. Consequently, when $\alpha < 1$, the generalized urn process is expected to converge to $x = 0.5$, whereas for $\alpha > 1$ (assumed in our paper), it is expected to converge to either $x = 0$ or $x = 1$. Indeed, this result aligns with our simulations, as shown in Figure 3A of the main text.

For a general value of w, the set Q is, $Q := \{x \in [0, 1] : (1 - w)I + wS(x) = x\}$, and we expect the sequential decision-making process to converge to a point in the set $\{x \in Q : wS'(x) < 1\}$. This result is analogous to the analysis of the synchronous updating process presented in Section 3.

5 Results for inverse S-shaped conformity function ($\alpha < 1$)

The results in the main text consider S-shaped conformity function with $\alpha > 1$, to reflect the disproportionate adoption of majority behavior. Here we investigate how results would change if this over-adoption is not present. For $\alpha = 1$, the function reduces to $S(x) = x$, which is a line. The function takes an inverse-S shape for $0 < \alpha < 1$. These variations are shown in Figure S2.

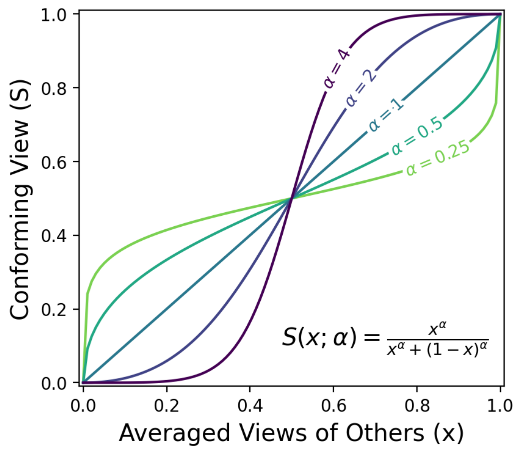

Figure S1: The conformity function for various values of α.

As we have shown the analytical results in the previous two sections, we expect the bifurcation conclusions to change when $\alpha < 1$. The equivalent results for Figure 3 of the main text, but with $\alpha = 0.5$, an instance of $\alpha < 1$, are shown in Figure S1. We see that in the sequential updating case, variance still increases with the strength of social influence. In both the synchronous and sequential cases, however, one sees that the bifurcation phenomena is no longer present.

Equivalent analysis for Figure 9 of the main text but with $\alpha = 0.5$ is shown in Figure S3. For the sequential updating condition, the results are qualitatively unchanged, while the results from the synchronous updating case now become similar to those of sequential updating. For easy questions with $I > 0.5$, in both cases, social influence hurts collective accuracy.

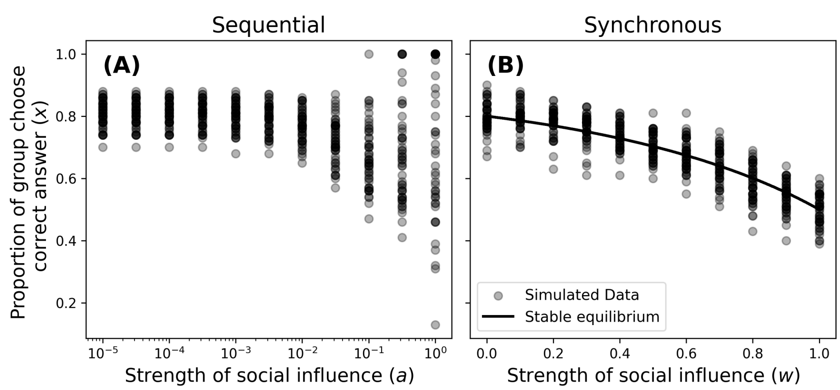

Figure S2: Predicted proportion of individuals choosing the correct outcome for $I = 0.8$ and $\alpha = 0.5$. The agent-based simulation uses 50 groups of 100 individuals for each level of social influence. The synchronous group updates their opinions 100 times.

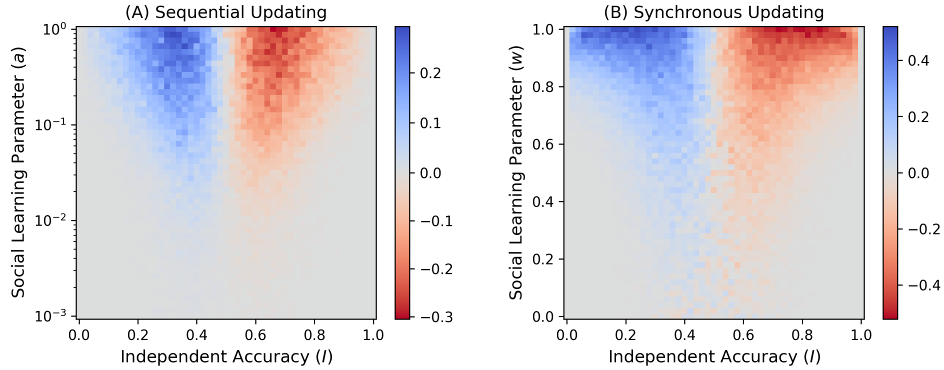

Figure S3: Contour plots showing the model's predicted difference in collective accuracy between social influence and independent conditions for (A) sequential updating experiments. (B) synchronous updating experiments, with $\alpha < 1$.

6 Figure 9 for varying group sizes

Here we consider the robustness of the result of Figure 9 of the main text to varying group sizes. Recall that the size of the groups simulated in Figure 9 was 21. In Figure S4, we show the equivalent results for groups of 11 individuals, and in Figure S5, for groups of 51 individuals.

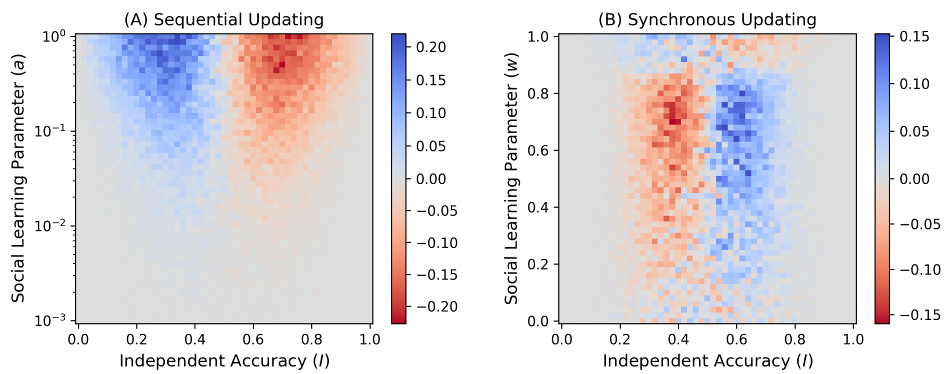

Figure S4: Simulation in Figure 9 repeated with group size of 11 individuals. Color shows the model's predicted difference in collective accuracy between social influence and independent conditions for (A) sequential updating experiments. (B) synchronous updating experiments.

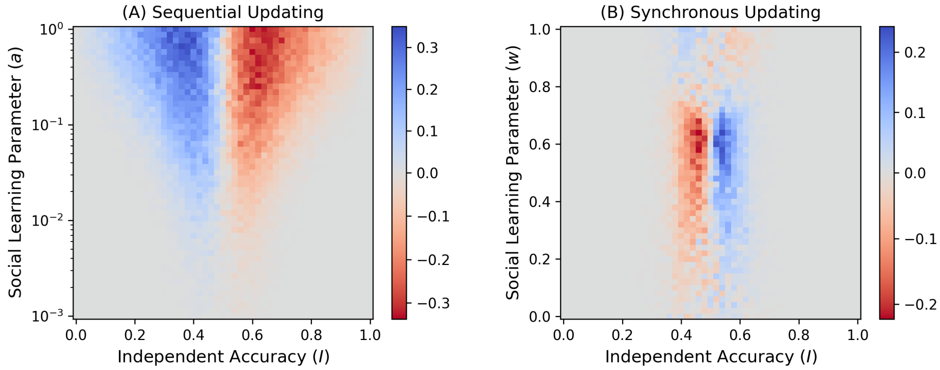

Figure S5: Simulation in Figure 9 repeated with group size of 51 individuals. Color shows the model's predicted difference in collective accuracy between social influence and independent conditions for (A) sequential updating experiments. (B) synchronous updating experiments.

While the qualitative results remain consistent across different group sizes, the range of parameter values in I where social influence affects performance narrows as group size increases in the synchronous updating condition. Additionally, larger group sizes amplify the magnitude of the difference in performance between the social influence and independent conditions for both updating processes.

References

Hill, B. M., Lane, D., & Sudderth, W. (1980). A strong law for some generalized urn processes. *The Annals of Probability*, 214–226.