Dynamic Shares: a novel mechanism for many-to-many allocation in system dynamics

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Abstract

The allocation problems are common in system dynamics models. They become many-to-many problems when products have to be allocated between many suppliers and many demanders. This paper describes a simple and effective way to solve many-to-many allocation problems based on a mechanism that we call Dynamic Shares. This mechanism has been successfully applied to several real-world problems of the models developed at GEEDS research group and enables flexible allocation of resources of many suppliers to many demanders in a simple and efficient way. Results of the allocation of agricultural area to several crops used in the WILIAM integrated assessment model and of the allocation of two final energy demands to three primary resources are shown.

Keywords

Allocation, distribution, share, markets, many-to-many

1. Introduction

When there is a mismatch between the amount of something available and the amount that is demanded some sort of allocation needs to be done. The allocation can be a one-to-may problem, when there is one supplier whose product has to be distributed among several demanders or a many-to-many allocation when there are many suppliers and many demanders. Markets are an example of many-to-may allocation when the price of the commodity traded is used to adjust the quantities between demanders and suppliers.

Vensim software [1], for example, provides two tools for one-to-many allocation: the ALLOCATE BY PRIORITY and the ALLOCATE AVAILABLE function, that solve the one-to-many allocation problem in a simple and effective way. The many-to-many allocation is addressed in Vensim with ALLOCATE AVAILABLE and FIND MARKET PRICE functions via a quite complex mechanism based on the definition of price-demand and price-supply curves that end up converging in a price of equilibrium that performs the optimal allocation between many demanders and many producers.

This mechanism of many-to-many allocation based on price curves is not easy to implement when the problem addressed is not a market and the price-demand and price-supply curves are not available.

This paper describes a simple and effective way to solve many-to-many allocation problems based on a mechanism that we call Dynamic Shares. The problem of many-to-many allocation and is explained using the example of the distribution of primary energies into final energies when there is a limited amount of primary sources available (in combination with a one-to-may allocation function) and Dynamic Shares are described using the example of the distribution of available cropland among several crops depending on the relative demand of each other and also the example of the distribution of three primary energies to two final demands. Some results of both examples are shown. The supplementary material includes a Vensim file with the model of the primary to final energy allocation.

2. The limits of one-to-many allocation

Let's think of a one-to-many allocation problem, for example: the demand electricity (final energy) in a country can be fulfilled with three primary sources: coal, gas and oil and we have to decide the amount of electricity generated with each primary source.

The amount assigned to a resource (oil, gas and coal) should not be greater than its supply (its maximum capacity). If the primary energy available is larger than the demand, all of the final demand shall be met by one primary source or another. These conditions are nicely solved by the algorithm of the Vensim ALLOCATE BY PRIORITY function [1]. In figure 1 we represent the way that the ALLOCATE BY PRIORITY function solves this problem. We give to the function the amount to distribute (1 dimension) and a vector of offers of the several providers (in this case the dimension is 3) and the priorities (dimension 3). Vensim function calculates the electricity allocated to the three primary resources (dimension 3).

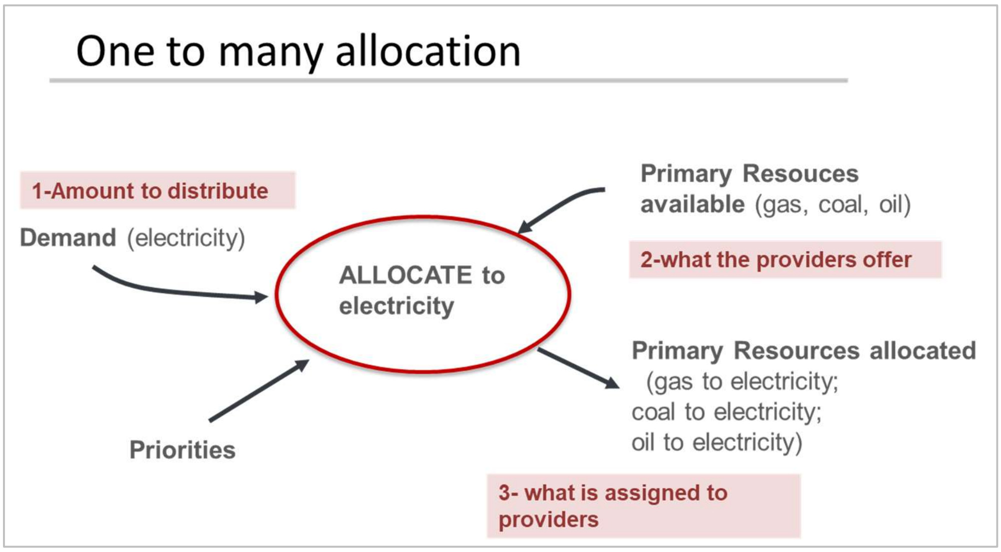

Figure 1: example of one-to-many allocation solved with ALLOCATE BY PRIORITY Vensim function.

Let us think now of a many to many allocation problem, for example: now we have two final demands, the demand of electricity and the demand of liquids that must be distributed among the same primary sources: coal, gas and oil (a similar problem appeared to us when we had to build the Energy module of the WILIAM model under the research project LOCOMOTION [2]). The most straightforward way of solve many-to-many allocation problems one would think of, is the use of several parallel one-to-many allocation functions.

In Figure 2 we tried to solve this problem with two one-to-many allocations. The demand of liquids is 1 and the demand of electricity 2. In order to distribute this demand among primary sources in a certain moment we might have a share of each final energy obtained with each one of the primary sources (variable *Shares from P to F*), for example, in Figure 1, 10% of the liquids come from gas, 0% from coal and 90% from oil, while 40%. Of the electricity, 40% comes from gas, 50% from coal and 10% from oil. If we use these shares to distribute the demand of final energy, we would need a total of 0.9 of gas, 1 of coal and 1.1 of oil.

If the demand of primary sources cannot be met, for example, in Figure 1 the oil available is only 0.5, which means that only 50% of the demand of oil can be met, an allocation between the primary resources available to the demanders must be done. We can do that via the ALLOCATE BY PRIORITY function that would assigning 0.9 to gas (the available is 1 which is more than the demand, therefore there is no shortage), to coal we assign 1 (the maximum is 2 and there is no shortage) but for oil the available, 0.5, is less than the demand, 1, and we have 50% shortage.

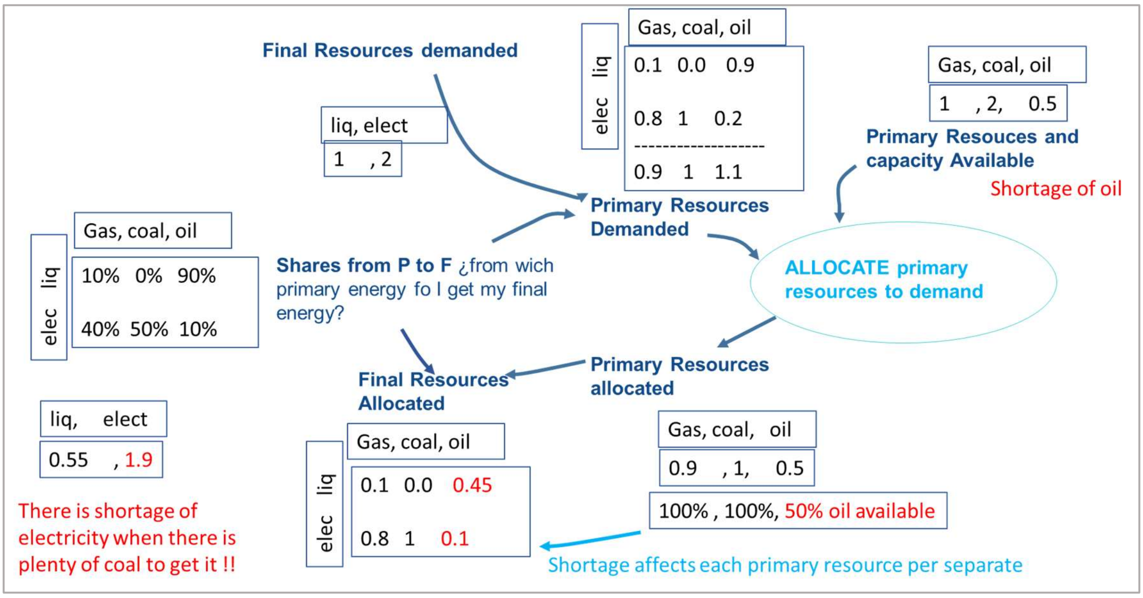

Figure 2: example of many-to-many allocation that is intended to be solved with allocate one-to-many function and static shares.

Once these primary resources are distributed, we must transmit the shortage to the final sources. If we do this one by one, we would have that only half of the oil demanded to generate electricity and only half of the oil demanded to generate liquids would be available. This would lower the available electricity to 1.9 (instead of the 2 demanded) and the available liquids to 0.55 (instead of the 1 demanded).

This distribution is far from being an optimal one: we have shortage of electricity while the available coal and gas are much higher than the one necessary to supply the electricity demanded. In the real world, these shortages of final energies should drive the systems to a different distribution of primary resources to final uses so that the demand is met (as long as the technical capacities of each fuel and the available infrastructure allow those changes).

The main drawback of this allocation of Figure 2 is the fact that the *Shares from P to F* that distribute the demand of final energies to primary sources are constant. If these shares are allowed to change and adapt to the relative shortage of each final and primary source, the problem is solved. This is what we get with the mechanism that we call Dynamic Shares and has been developed and used in models to solve several allocation problems in different parts of our system dynamics models.

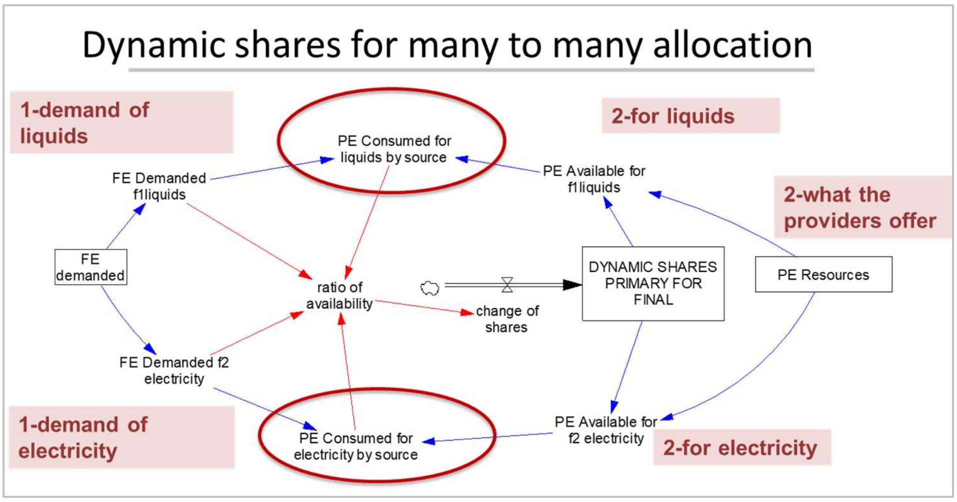

Figure 3: example of many-to-many allocation solved with two ALLOCATE BY PRIORITY Vensim functions and dynamic shares. The dynamic shares split the many-to-many problem into two one-to-many problems but the feedback of the signal of availability adapts the shares in order for them to adapt them and enable many-to-may allocation.

3. Dynamic Shares

The mechanism of Dynamics Shares developed at GEEDS is going to be described first based on the example of the land allocation between several crops that has been used in the WILIAM model. The Land module of WILIAM model has a submodule of crops production that allocates available cropland to 11 groups of crops (corn, rice, rest of cereals, tubers, soy, pulses and nuts, oil crops, sugar crops, fruits and vegetables, crops for cellulosic biofuels and other crops). In order to do that, a vector of shares of crops $SHCrops_{\square}\ (Lp_k)$ is defined as the share of available cropland that is used for each crop Lp_k, therefore, the area used in each time step for each crop, $Area\ crop_{all}\ (Lp_k),$ is:

$$Area\ crop_{all}\ (Lp_k) = SHCrops_{\square}\ (Lp_k)\left(Area_{cropland}\ \right)$$

These shares change dynamically doing a dynamic interchange of land (land shares) between crops that (as illustrated in Figure 4) where all the cops receive and give at the same time from each other.

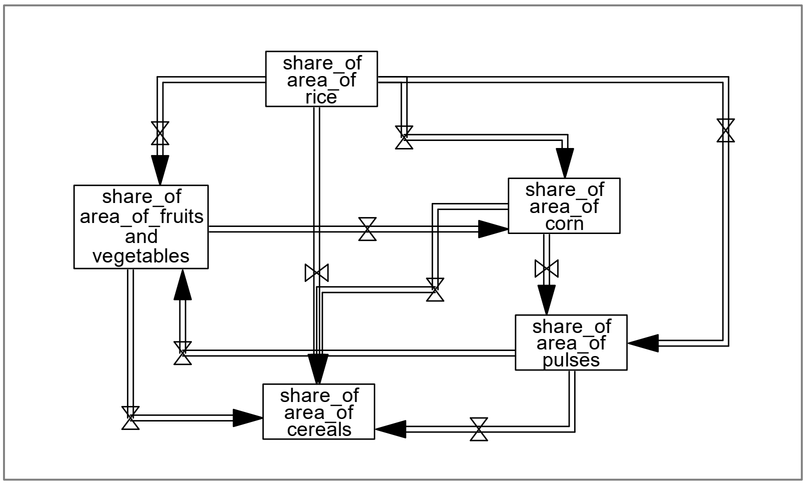

Figure 4: illustration of the dynamic shares of the crops example.

The demand of crops $C_{dem}(Lp_k)$ comes from other submodules of the model and the crops production is calculated as:

$$C_{avail}(Lp_k) = Area\ crop_{all}\ (Lp_k) \cdot Y(Lp_k)$$

where $Y(Lp_k)$ are the agricultural yields.

If demand and supply do not match we might define a signal of shortage $ratio(Lp_k)$:

$$ratio(Lp_k) = \frac{C_{dem}(Lp_k)}{C_{avail}(Lp_k)}$$

This ratio enables us to send a signal that modifies the matrix of shares $SH\ (Lp_k)$ via its derivative (making $SH\ (Lp_k)$ a stock in our model) and defining a matrix of interchanges many-to-many between shares of crops:

$$M_{SH}(i,j) =$$

$$\begin{cases} \beta(i) \cdot \big(ratio(j) - ratio(i)\big) \cdot f\_\min(i) \cdot f\_\max(j) & \text{if } ratio(j) > ratio(i) \\ 0 & \text{if } ratio(j) \le ratio(i) \end{cases}$$

where $\beta(i)$ is a constant factor that enables us to set priorities of several uses i, and · $f\_\min(i)$ is a function that becomes zero when $SH\ (i)$ approaches the minimum value it might have (in order to avoid shares to become negative) and $f\_\max(j)$ becomes zero when the share $SH\ (j)$ approaches the maximum value it might have (at least when they approach 1).

The matrix $M_{SH}(i,j)$ is used to change the Dynamic Shares by following the rationale shown in Figure 4: each share adds all the flows that others give to it and subtracts all the flows that gives to others:

$$\frac{d\ SH(j)}{dt} = \sum_i M_{SH}(i,j) - \sum_j M_{SH}(i,j)$$

If this mechanism is to be applied to the problem of the many-to-many allocation of two final energies demand (electricity and liquids) to three primary resources (oil, gas and coal) described in section 2, these equations can be used by assuming that the vector of shares becomes a matrix of shares $SH\ (i,k)$ for each k, primary resource and the matrix of shares becomes a set of matrices $M_{SH}(i,j,k)$. For further details on these equations see the Vensim program provided in the supplementary material.

4. Examples

This section shows the use of the mechanism of the Dynamics Shares for the allocation of available cropland to crops. The crops demand comes from other parts of the model. and growths smoothly until 2030 when we have added a sudden increase of 20% in some crops (corn, sugar crops and oil crops). In figure 5 we can see that the demand of crops adapts to this increasing demands much as it can, because the limitations of cropland availability do not allow to fulfil the demand as seen in figure 6.

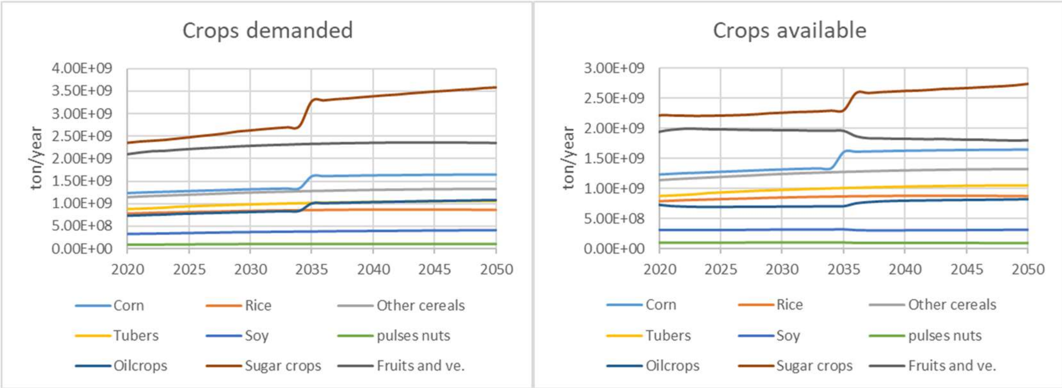

Figure 5: crops demanded (left) and available (right) in Run 2.

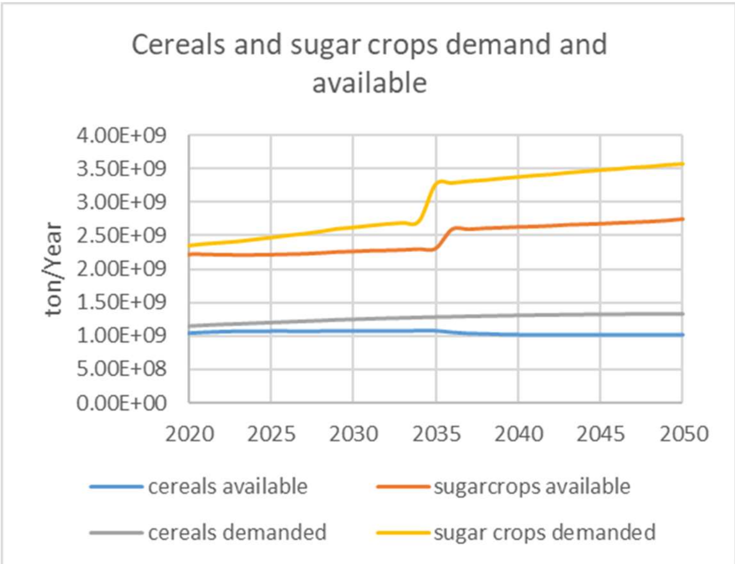

Figure 6: comparison of crops demand an availability for sugar crops and other cereals

Figure 7 shows the evolution of the shares of crops. We observe that the shares adapt to increase the oil crops, sugar crops and corn and, in turn, the shares of other crops decrees to maintain the consistency and keep the sum of all shares equal to 1.

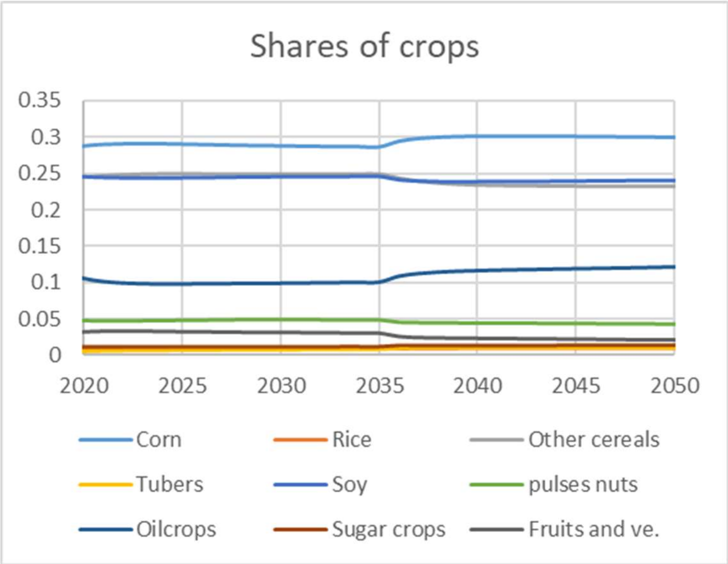

Figure 7: shares of crops.

All this is creating a stabilizing feedback loop as the one shown in Figure 8 that adapts crops production to crops demand as long as it is allowed by the rest of the limits of the model.

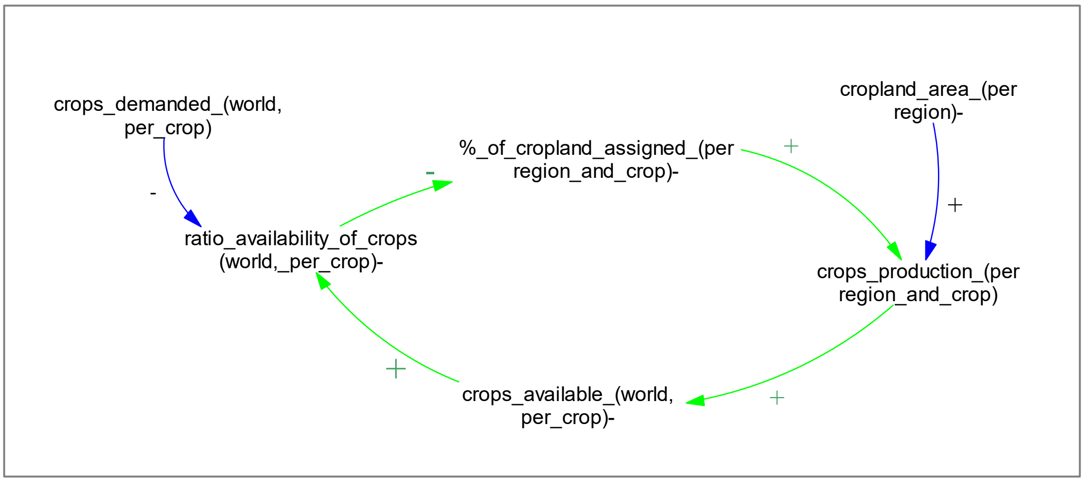

Figure 8: feedback of the crops allocation in WILIAM-TERRAE model. The available cropland (per region) is divided among several crops by the percent of cropland assigned per crop (Shares of crops), if the crops available is lower than the demanded, the ratio of availability changes the shares to correct the shortage.

In figures 9 and 10 the results of the many-to-many allocation of two final energies demands to three primary resources described in section 1 are shown (these results can be seen in the program provided in the supplementary material). The growing demand of liquids and electricity tries to be met by the three sources (coal, gas and oil) in step 41 the limits of all the primary resources are met (figure 10) and demand can no longer be fulfilled.

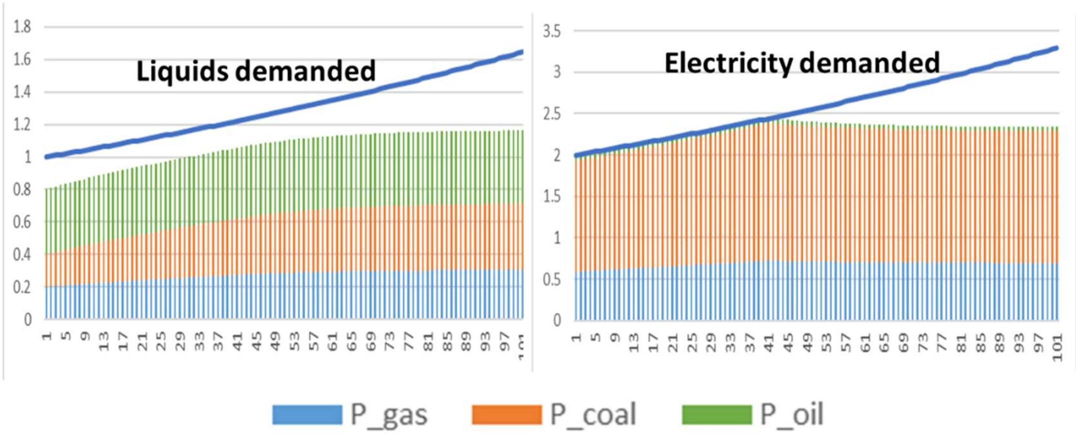

Figure 9: liquids demand (left) and electricity demand (right) assigned to with the three primary sources (gas, oil and coal).

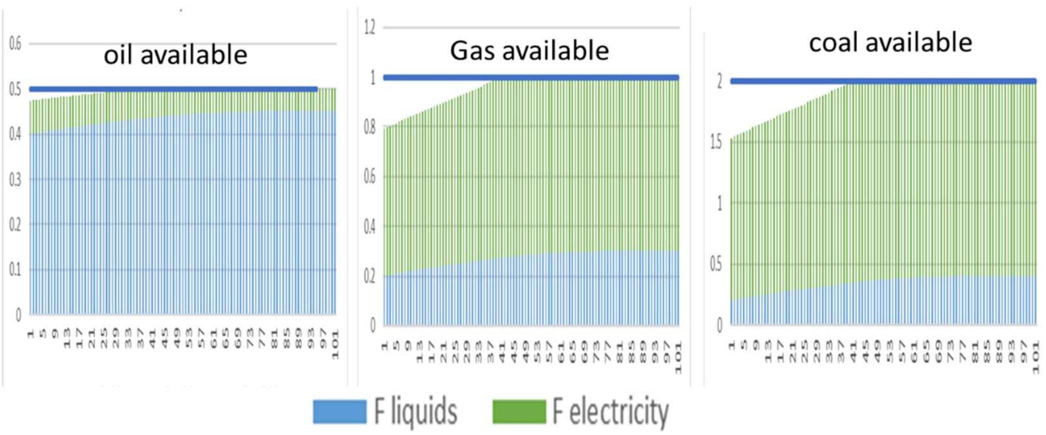

Figure 10: primary resources assigned to the two final demands (electricity and liquids).

5. Conclusions

In this paper a mechanism for many-to-many allocation has been presented and described. It has been successfully applied to several problems and enables flexible allocation of resources of many suppliers to many demanders in a simple and efficient way. Results of the allocation of two final energy demands to three primary resources and of agricultural area to several crops are shown. The author encourages the use of this simple and effective mechanism to treat many-to-many allocation problems because of its good and robust performance in all the models where it has been used.

References

[1] https://www.vensim.com/documentation/users_guide.html

[2] LOCOMOTION https://www.locomotion-h2020.eu/