A Comparison of Loop Dominance Methods: Measures and Meaning

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Abstract

Several methods of loop dominance analysis have been developed to quantify and analyse the relationships between feedback loops and system behaviour. The slow uptake of these methods by the system dynamics community may be partly explained by the lack of transparency of their conceptual interpretation. Our purpose in this paper is to address these conceptual hurdles by examining the well-known Workforce-Inventory model, while using a minimum of technical detail. For each method, we identify: the meanings of its loop measure; the number of such measures which are needed for each loop; and how much of the system behaviour is explained. We then apply the methods to the SI and SIR models and compare their results. It is our intention that this informal and comparative approach will encourage more system dynamicists to explore, use and develop loop dominance methods.

Introduction

The behaviour of a system dynamics model is determined by its structure expressed in its stocks, flows and feedback loops. The latter are particularly important as they represent endogeneity in the system and help explain complex behaviour in the variables through changes in dominance of different types of feedback.

To explore the relationship between structure and behaviour, consider the classic SI model. This model describes the spread of disease amongst a susceptible population by infected people who remain infected. There are two stocks, two feedback loops and one flow, Figure 1. That is the structure of the model.

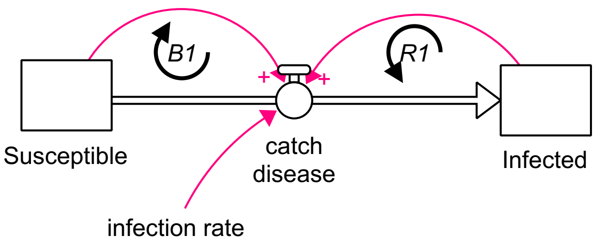

Figure 1: SI model of the spread of disease.

The loops determine the nature of the growth of the *Infected* stock, Figure 2, i.e. its behaviour. A dominance method indicates which loop has the most effect in different growth phases. In the accelerating phase, the reinforcing loop *R1* dominates. In the slowing down phase, dominance changes to *B1*. *Susceptible* has the same explanation, *R1* accelerates, and *B1* decelerates, Figure 2. This behaviour is classic shifting loop dominance, with the transition at the inflexion point. The explanation of behaviour in terms of structure looks convincing, but how can we prove it?

To answer this, I (John) would like to explain how I became interested in loop dominance. I taught system dynamics for 13 years to students on the final year of a mathematics degree. When I showed them this example, someone in the class would always ask, “Why?” “Prove it!” Each time I would point them to a paper by David Ford (1999) on “loop deactivation” – a method to determine dominance. Remove the reinforcing loop, Figure 3. The stocks are slowing down until they reach equilibrium. Thus, R1 must have dominated the region that was accelerating. Remove the balancing loop, Figure 4. Now the stocks accelerate until the susceptibles run out of people. Thus, B1 must have dominated the region that was slowing down.

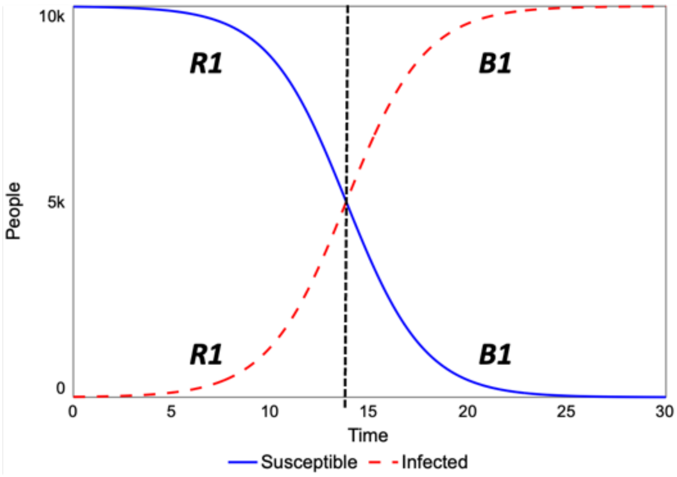

Figure 2: Stock behaviour of SI model.

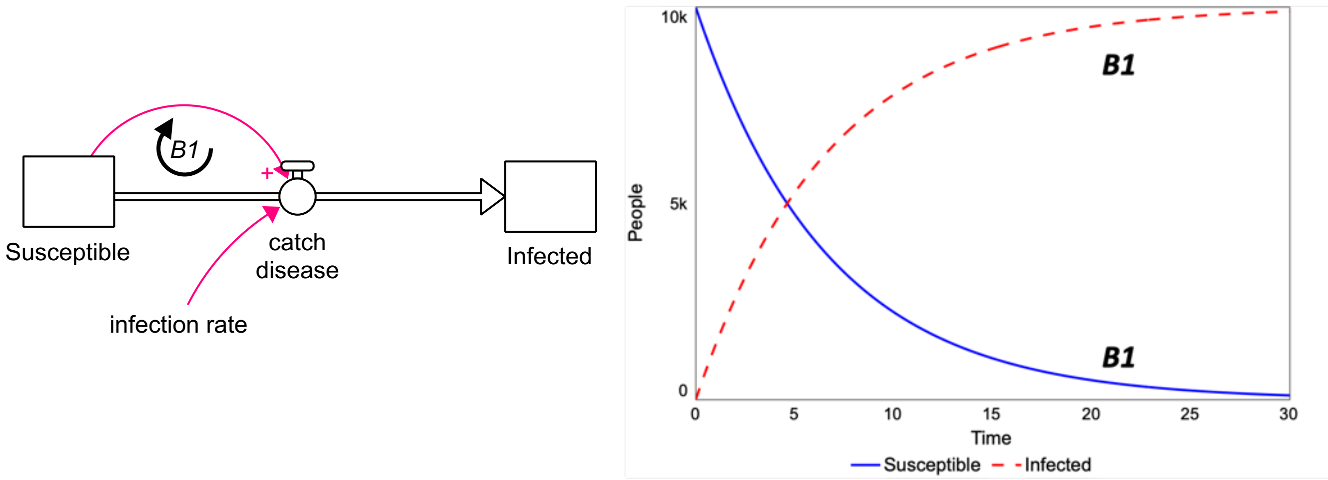

Figure 3: SI Model with reinforcing loop removed.

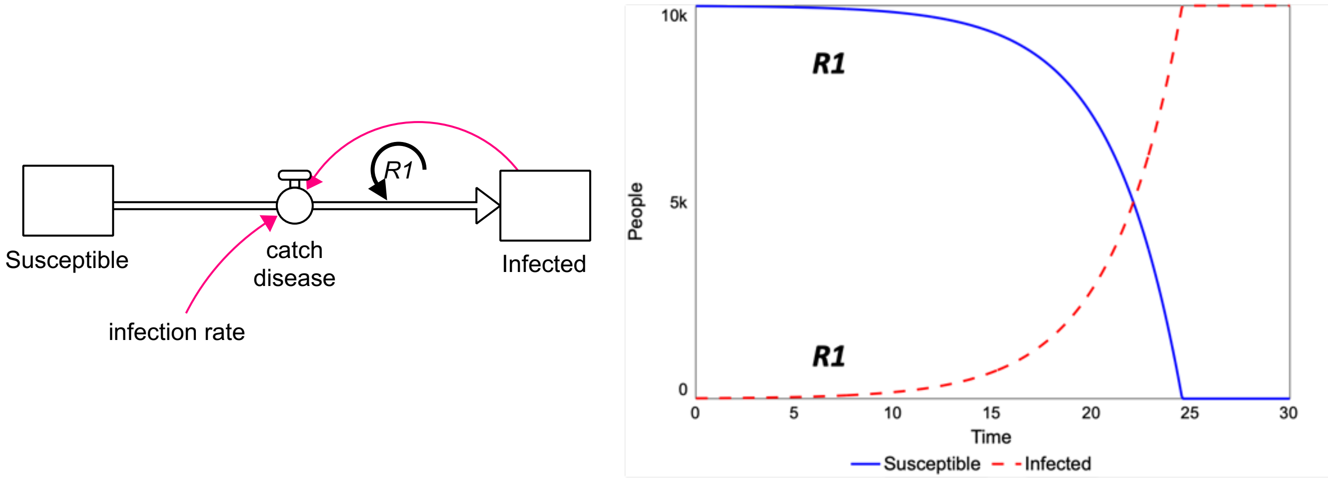

Figure 4: SI model with balancing loop removed.

Some of the students went further and asked me, "Can you measure how much a loop accelerates a stock?" Searching for answers to this question led me to a paper by Mohammad Mojtahedzadeh et al. (2004) on the Pathway Participation Metric (PPM). I distilled his work down to a process the students could follow and try out for themselves. I eventually published this work as the Loop Impact Method (Hayward and Boswell, 2014).

Loop impact and PPM are called *Pathway Methods* because they measure changes along pathways between stocks. Loop impact measures stock curvature. PPM apportions that by percentages. More recently, Schoenberg (2020) has developed Loops That Matter (LTM), integrating the method into the Stella SD simulator[1] (see also Schoenberg et al., 2020). Although developed independently of the other pathway methods, it effectively combines impacts on stocks into a single measure for each loop in a system.

A different approach to loop dominance, called Eigenvalue Elasticity Analysis (EEA), was proposed in 1996 by Kampmann (2012), based on the work of Forrester (1982), and further developed by Güneralp (2006); Kampmann and Oliva (2008); Gonçalves (2009); Oliva (2015). See Oliva (2020) for further details. This method breaks system behaviour into fundamental exponential and oscillating components and describes how the loops affect these components.

More recently, we extended the Loop Impact Method to measure the cumulative effect of loops on stock behaviour using the concept of energy from mechanics (Hayward and Roach, 2022).

There are three types of methods, pathway, eigenvalue and cumulative. Each method measures something. What do they measure, and what do they mean? That is what we will explain in this paper.

Workforce–Inventory Model

We use the well-known Workforce Inventory Model as our main case study, Figure 5. This model has been used to illustrate all loop dominance methods; EEA (Gonçalves et al., 2000; Gonçalves, 2009), PPM (Mojtahedzadeh et al., 2004), Loop Impact (Hayward and Boswell, 2014) and LTM

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[1] *Stella* software is available from isee Systems inc. https://www.iseesystems.com/

(Schoenberg et al., 2020). The model is sufficiently simple that most SD practitioners have some understanding of the behavioural role of its feedback loops without the use of analytical methods.

The model describes a simple production system that attempts to maintain desired inventory levels as demand changes by hiring and firing workers. If demand increases, the inventory falls below its target, and the desired production is increased to meet the shortfall and the expected demand. There are three stocks, *Inventory*, *Workforce* and *Expected Demand*; three feedback loops, workforce adjustment *B1*, inventory adjustment *B2*, and a first-order delay *B3*; and an exogenous demand that influences the system via *D1*, *D2* and (indirectly) *E1*, Figure 5.

The system starts in equilibrium, with a step increase in demand at 5 months, Figure 6. The system exhibits damped oscillations. There are no reinforcing loops, but there is an acceleration in both inventory and workforce numbers at different times during the oscillation. What structure has caused what behaviour? More specifically, how do the different dominance methods connect the loops and exogeneities to this behaviour?

To help answer this question, we note that there is no feedback through *Expected Demand* and the other two stocks. *Expected Demand* only has demand *D2* as input. Thus, we can split the system into two subsystems and concentrate on the two stock Workforce-Inventory model, *with Expected Demand* E1 as another exogenous input, Figure 7 (left). In the following sections, we apply the loop dominance methods to this Workforce-Inventory subsystem and relate its loops and exogeneities to its behaviour.

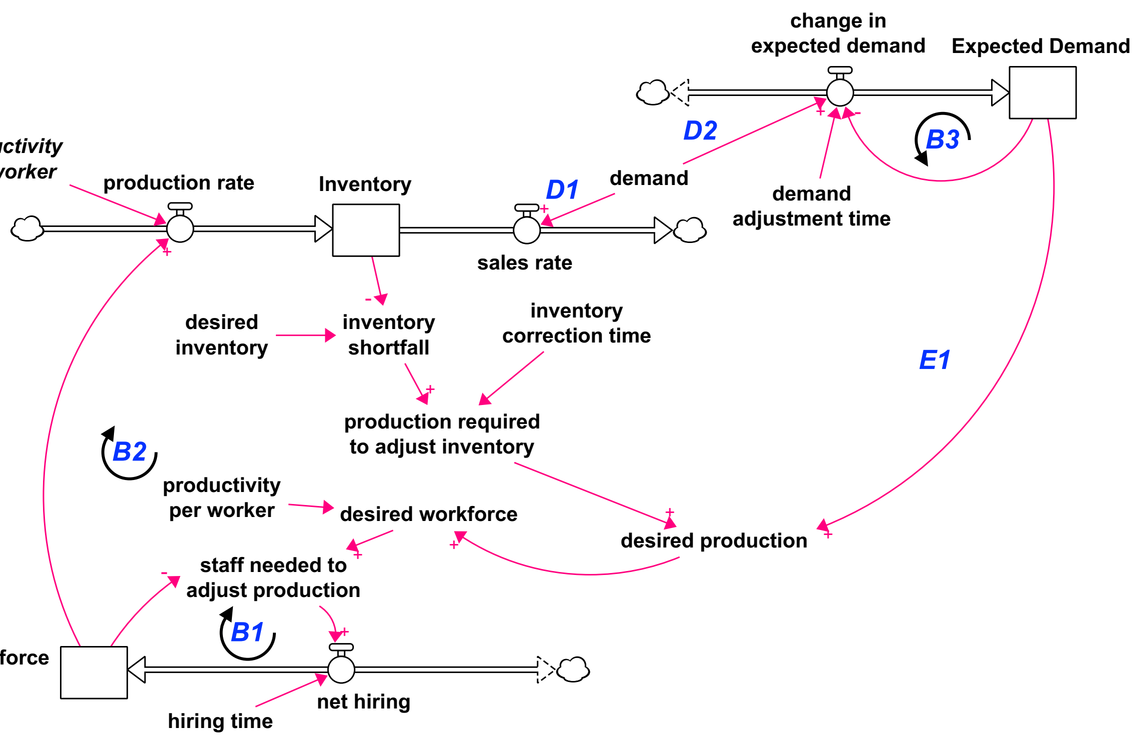

Figure 5: Workforce-Inventory Model

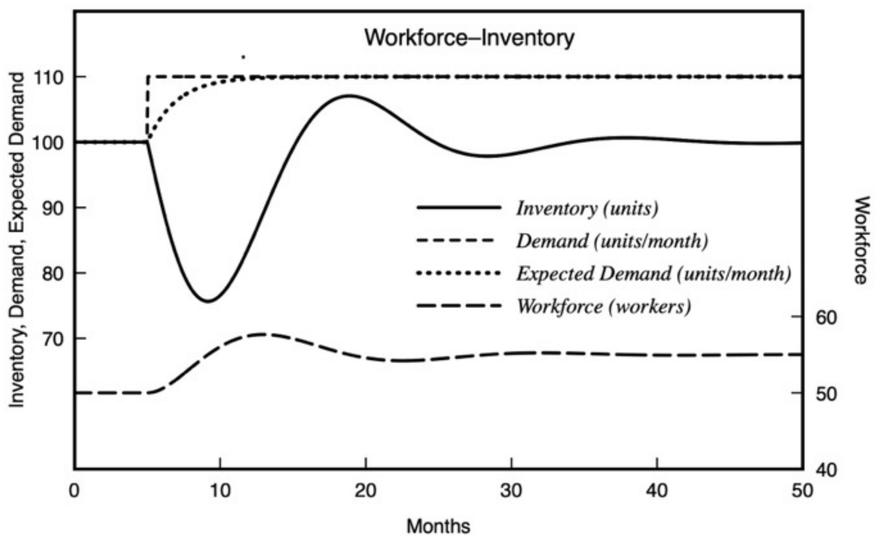

Figure 6: Stock behaviour of Workforce–Inventory Model. Productivity per worker = 2 units/worker/month, desired inventory = 100 units, hiring time = 4, inventory correction time = 2, demand adjustment time = 2 months, and demand increases from 100 to 110 units at 5 months.

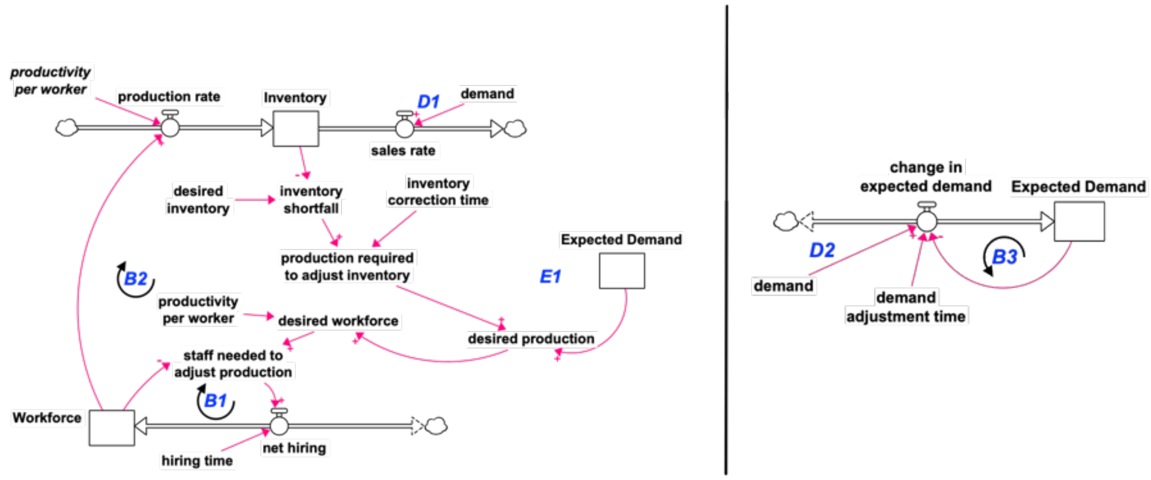

Figure 7: Workforce-Inventory subsystem (left) and the Expected Demand subsystem (right).

Pathway Methods

Dominance Analysis

Loop impact and PPM measure the effect of stocks on each other via loops. Loop Impact measures the curvature in stock behaviour. From 5 months, *Workforce* grows and accelerates. Loop impact and PPM attribute this acceleration to *B2*, Figure 8 (W1 and W2). *B2* is a second-order loop, thus, while *Workforce* accelerates, *Inventory* slows down, Figure 8 (I1). The workforce's behaviour is attributed to *B2* throughout. If *B2* accelerates one stock, it must decelerate the other.

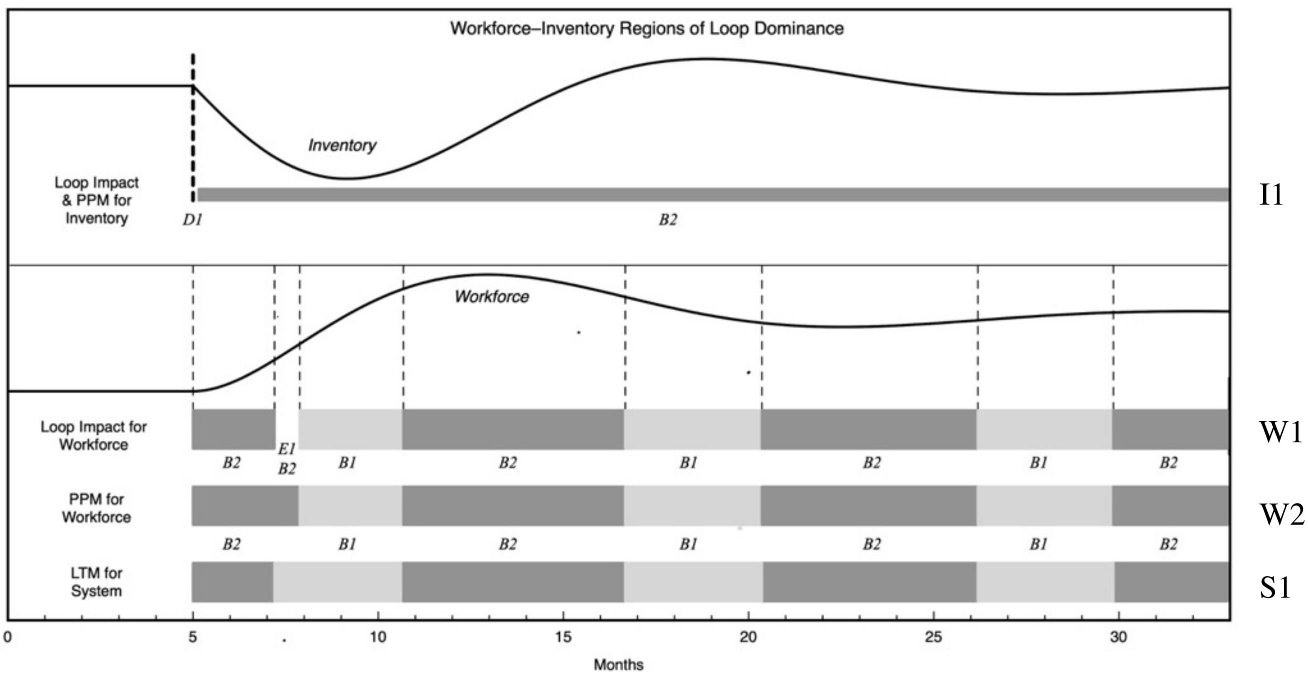

Figure 8: Loop dominance analysis for Workforce-Inventory model. Loop Impact and PPM have one analysis for each stock. By contrast, LTM has one analysis for both stocks.

For Loop Impact, *B1* dominates the slowing down phases of *Workforce* between the maxima and minima of the oscillation, Figure 8 (W1). However, it is always *B2* that dominates the change from growth to decline and vice versa. Thus, according to Loop Impact, the second-order loop *B2* explains the oscillations.

PPM apportions the Loop Impacts as a percentage of the total impacts. Thus, its dominance analysis is similar to Loop Impact but may differ when there are three or more influences on a stock. For example, between 7.2 and 7.9 months, Loop Impact determines that it takes both *B2* and *E1* to dominate *B1*, Figure 8 (W1). These two constitute a dominant loop set, the smallest number of loops to explain the curvature. By contrast, PPM regards the largest of that set, *B2*, as the dominant loop during this period, the largest loop consistent with the stock's behaviour, Figure 8 (W2). Thus, Loop Impact and PPM have slightly different mechanisms for comparing loops.

LTM combines the two stock patterns of Loop Impact, Figure 8 (I1 and W1), and produces one measure for the system, Figure 8 (S1). LTM does not include exogenous effects as it measures the endogenous influences on system behaviour, the "Loops that Matter". Because only one loop affects inventory, Figure 8 (I1), then the overall system result is similar to that of Loop Impact on Inventory, Figure 8 (W1). We will return to this lack of differentiation between LTM and the other methods later as it results from a peculiarity of this model.

Measures

We now examine the measures used in each of the three pathway methods that produced the dominance analysis in Figure 8. Figure 9a shows the loop and exogenous impacts on each stock in units of "per month". Loop impact measures the acceleration imparted by a loop divided by the net flow. This division ensures sign matches the loop polarity. As a result, has infinities when *Workforce* is briefly at rest. The impact of *B1* is constant as it is a first-order linear loop. Changes in dominance come from variability in the impacts of *B2* and *E1.*

PPM takes the loop impacts and presents them in percentage form, Figure 9b. This area graph illustrates the periods where one loop is larger than the others. Unlike Loop Impact, PPM does not have a constant measure for *B1* as the impacts of the other loops are changing.

Figure 9c shows the raw loop scores of LTM, dimensionless measures derived using the impacts, Figure 9a (Schoenberg et al., 2023). To construct these, each loop impact is divided by the total impacts on a stock – the raw *link* score. The constituent link scores in each loop are multiplied, giving the raw *loop* score of Figure 9c. In LTM, infinities in these scores occur at the inflexion points of *Workforce* because these are the points at which behaviour changes from acceleration to deceleration. The infinities disappear when the raw scores are turned into percentages, Figure 9d, and the behaviour pattern is the same as for PPM, Figure 9b, apart from the exogenous effect of E_1. Figure 9d is the result given by *Stella*.

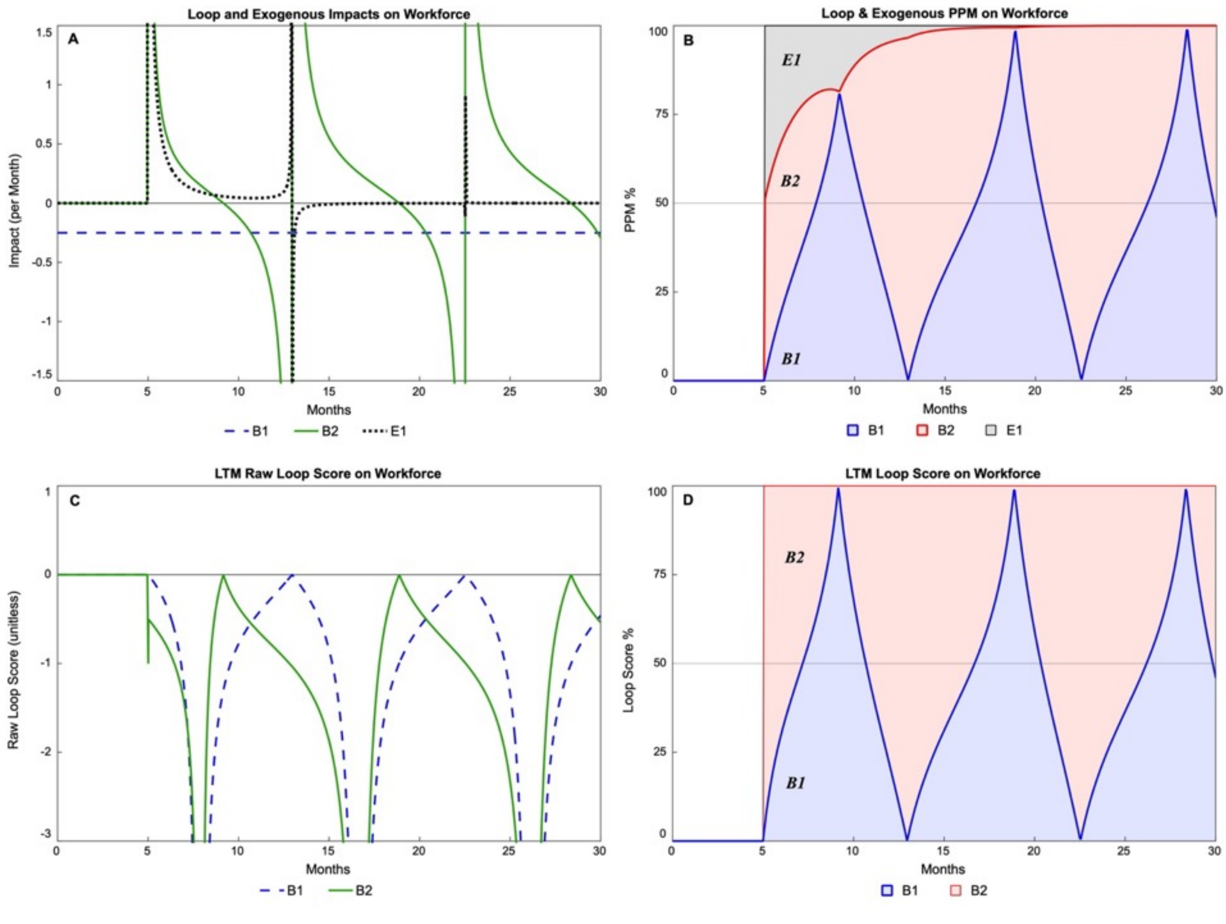

Figure 9: (a) Loop impact on Workforce; (b) Pathway participation metrics for Workforce; (c) Raw loop scores (LTM) for Workforce-Inventory system; (d) Percentage loop scores (LTM) for Workforce- Inventory system

Distinguishing LTM from Loop Impact and PPM

To distinguish LTM from the other two pathway methods, consider a model with two loops affecting both stocks, as in a second-order linear system, Figure 10. The system has been analysed previously for loop dominance (see Hayward and Roach, 2017). There are two stocks, *x* and *y*, two first-order loops *R1* and *B1* and one second-order loop *B2*.

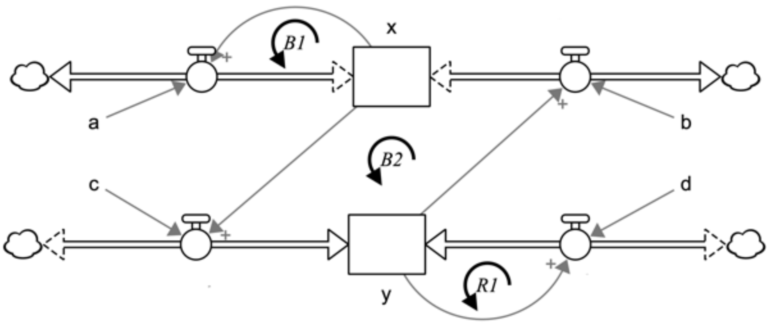

Figure 10: Second-order linear system.

Figure 11 shows that the dominance patterns given by Loop Impact and PPM have the same alternating pattern between two loops for both stocks, as seen in the Workforce-Inventory model, Figure 8. In each case, the turning points are explained by B2. Second-order loops cause stocks to change direction. LTM combines the loop impacts for *x* and *y*, producing one pattern for the system using all three loops, Figure 11, bottom bar. Dominance is a repeated pattern involving all three loops. LTM's dominance analysis no longer explains curvature in the behaviour of a given stock, and the connection between *B2* and a stock's change of direction is lost. However, it does say which loop has the most effect on all the stocks – the system – at a given time.

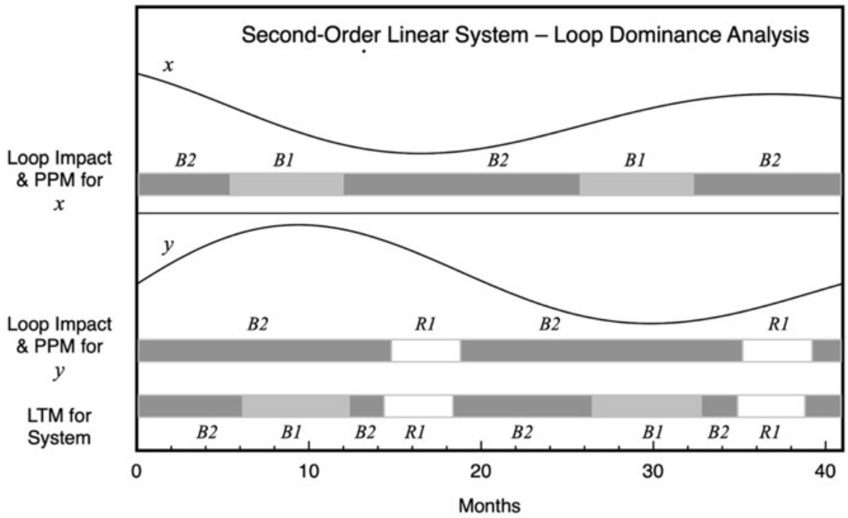

Figure 11: Loop dominance analysis of the second-order linear system of Figure 10, a = 0.1, b = 0.15, c=0.2, d=0.06, x0 =2, and y0 =0.

Eigenvalue Elasticity Analysis

The second type of dominance method we will look at is Eigenvalue Elasticity Analysis (EEA). This method analyses behaviour by linearising the system at each point in time and determining the exponential and oscillating modes of the endogenous component. Eigenvalues measure the growth and decay of the exponentials and the frequency of any oscillations. EEA describes the sensitivity of the eigenvalues to changes in the loop gains (Kampmann, 2012; Oliva, 2015; 2020). A toolset automates the EEA process (Naumov and Oliva, 2018)[2].

Using the Workforce-Inventory model, Figure 5, we first isolate the endogenous component of its behaviour. The solution for each stock can be split into endogenous and exogenous components. Consider the solution for Workforce in Figure 6, also shown in Figure 12, curve 1. This behaviour consists of the component derived from the exogenous demand, Figure 12, line 2, and the component from the exogenous expected demand curve 4. Subtracting these from curve 1 gives curve 3 the endogenous system reaction. This behaviour is what EEA measures.

The endogenous behaviours for the two stocks are given in Figure 13. The eigenvalues describe the damping and frequency of these waves. Following Oliva (2015), we use the *eigenvalue loop influence* as the most natural measure of the sensitivity of the eigenvalues to changes in the loops.

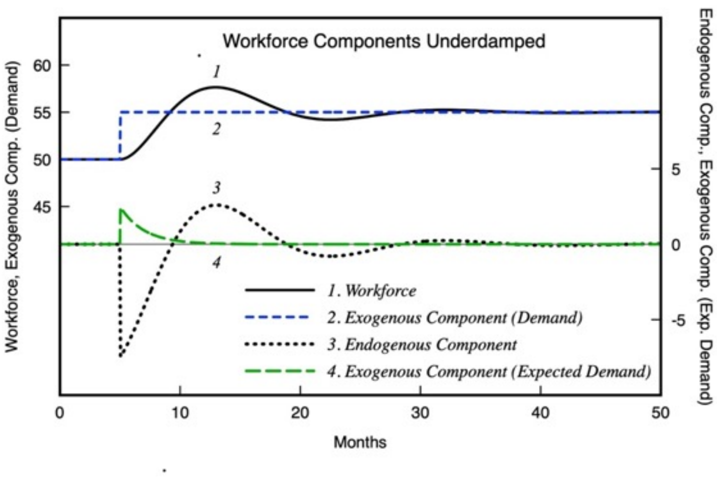

Figure 12: Workforce Behaviour decomposed into endogenous and exogenous components.

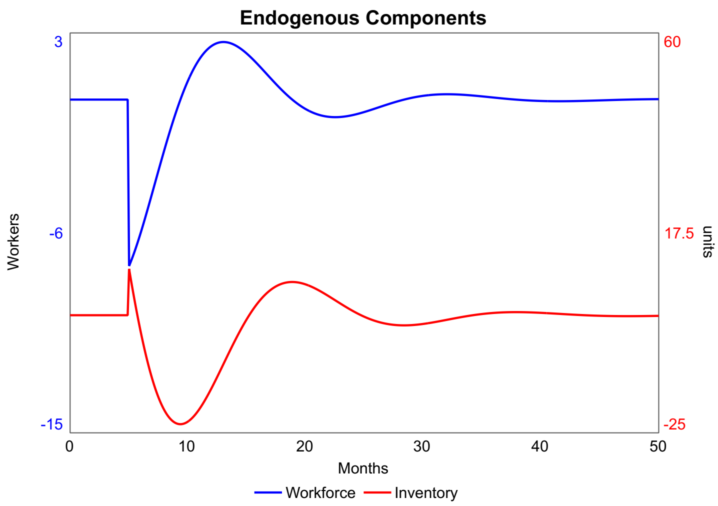

Figure 13: Endogenous components for Workforce and Inventory.

[2] The EEA Structural Dominance Analysis Toolset is available from http://people.tamu.edu/~roliva/research/sd/sda/ .

Damping, the exponential envelope, is explained by *B1* only, the workforce adjustment loop, Figure 14[3]. The frequency is affected by both loops, with *B2*, the inventory adjustment, having the most effect in this case. This *B2* dominance on the frequency of the oscillation is an artefact of the parameters, in particular, the lightly damped nature of this simulation.

It follows that EEA and the pathway methods all agree in attributing damping to *B1* and oscillations, where they occur, to *B2*. However, they differ in how they measure loops' influence on behaviour. For pathway methods, loops are measured by their direct influence on stock behaviour. By contrast, EEA measures the loop influence on the behaviour modes of the system rather than directly on the individual stocks.

Further developments in EEA allow for the examination of stock behaviour using weights determined by the system eigenvectors to connect behaviour modes to each stock (Saleh et al., 2010). These procedures are built into the EEA Structural Dominance Analysis Toolset (Naumov and Oliva, 2018). The reader is directed to Oliva (2015; 2016; 2020) for additional details.

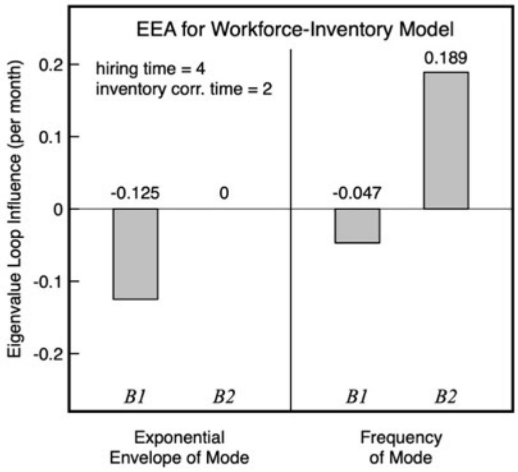

Figure 14: Eigenvalue loop influences of B1 and B2 on the Workforce-Inventory model.

Energy – Cumulative Approach

Hayward and Roach (2017; 2019) extended the loop impact method by drawing out the analogy between impact and mechanical force. In the loop impact method, each loop represents a force between stocks, causing acceleration in stock behaviour. Exogeneities also represent forces on stocks. In mechanics, forces do work. Reinforcing loops do work, injecting energy into stocks. Exogeneities also inject energy. First-order balancing loops remove energy, the only structure that does so. Second-order balancing loops exchange energy between stocks. Higher-order balancing loops also exchange energy but are not conserved, acting as a net source of energy. As energy enters the system, it is absorbed into the stocks in the form of kinetic energy – which measures how fast they change. The energy analogy is described by Hayward and Roach (2022).

The Workforce-Inventory subsystem has two energy sources, *D1* and *E1*, one exchange, *B2* and one sink, *B1*, Figure 7 (left). The system must stabilise once *demand* is switched off as all energy

[3] As the Workforce-Inventory system is linear, the eigenvalue loop influences are constant over time, Figure 14. In a non-linear system, these influences would change over time.

is transferred to *Workforce* as kinetic energy and dissipated through *B1*. The system's energy flows show that a percentage of all energy entering and leaving, 36.4% comes in through *D1* and 13.6% enters through *E1*, Figure 15 (top). These are balanced by 50% of the energy leaving through B1 once the system has stabilised.

From a stock viewpoint, 1.56 energy units enter inventory through *D1,* Figure 15 (bottom). This input energy is eventually all exchanged into *Workforce* by *B2*. 0.59 energy units enter the workforce through *E1*. The sum of these two, 2.15 energy units, leaves through the loop *B1*. These amounts are measured over a time horizon from zero months to system stability. If the time horizon was shorter and stability had not been achieved, the energy sink would be less than the sources, with the excess energy in the activity of the stocks measured as their kinetic energy.

We can express these energy flows on the SD diagram, Figure 16. The energy source on *Inventory*, *D1*, is passed through to *Workforce* by *B2*. This energy is added to the energy source on *Worforce*, *E1*, and the sum is removed by *B1*.

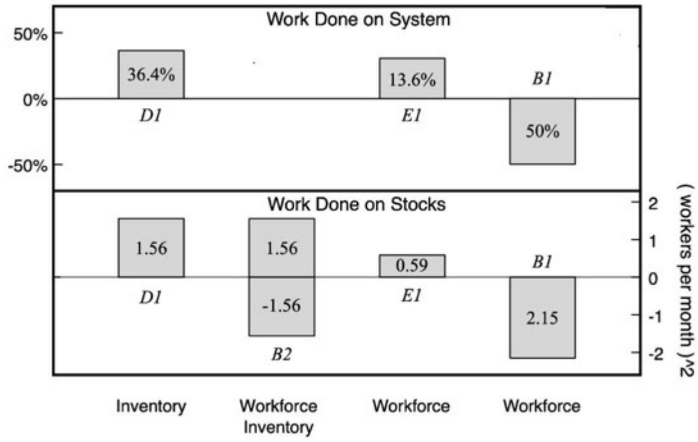

Figure 15: Work done by loops and exogeneities on the system (top of diagram, left axis), and the stocks (bottom of diagram, right axis), from 0 to 60 months (equilibrium); loop B2 has removed energy from Inventory and injected into Workforce.

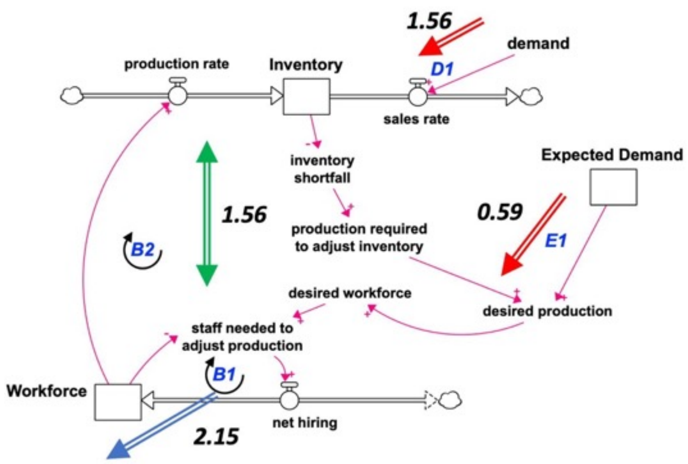

Figure 16: Energy flow diagram for the Workforce-Inventory model.
Red arrows, energy injection; blue arrow, energy removal; green arrow, conserved energy exchange.

Although energy can be used to give a loop dominance analysis (see Hayward and Roach, 2022), the method is less about dominance and more about providing a narrative description of each loop's role over time intervals. The energy perspective may give insights into system behaviour even without quantification, as we will show in our two applications.

Summary

Our study has identified four ways in which loop dominance methods differ. Firstly, the methods differ in their meaning. Loop impact measures the acceleration imparted on one stock by changes in another, whereas PPM and LTM measure the percentage influences of loops on stocks and the system, respectively. EEA measures the sensitivities of behaviour modes to changes in loop gain. Energy measures the accumulated effect of the loops.

Secondly, the methods have differing numbers of measures for loops. For example, in Loop Impact and PPM, a loop has the same number of measures as there are stocks in the loop. By contrast, LTM combines these and has one measure per loop. In EEA, a loop has a measure for each behaviour mode, normally the same as the number of stocks in the system. For energy, each loop has one measure for each of its stocks, but these may be combined to have one measure in the system.

Thirdly, the methods differ in their use of exogeneities in dominance descriptions. Loop Impact, PPM and the energy approach explain behaviour using both endogenous and exogenous influences. LTM concentrates on endogenous influences alone. Likewise, EEA only considers endogenous influences and examines the endogenous component of the system solution.

Fourthly, the methods have different ways of comparing loops and deciding which one or more are dominant. PPM and EEA generally consider the largest loop as the dominant one. For PPM, that loop is must be consistent with stock behaviour. By contrast, Loop Impact uses the dominant loop set. That is, the smallest number of loops whose behaviour matches that of the stock and whose sum exceeds the sum of all loops of opposite polarity. For LTM, Schoenberg et al. (2020) state "the loop (or set of loops) that describe at least 50% of the observed change in behaviour across all stocks in the model over the selected time period". For this, they use the total loop score over the interval. For loop dominance at each point in time, the dominant loop or loops is determined by visual inspection, for example, Figure 9d. Dominance in the energy method follows that of loop impact but also provides a narrative description of the role of each loop in the dynamics rather than the size of their effect.

This fourth difference is less significant than the others as any of the mechanisms of loop comparison may be used with a given dominance method. The differences are often minor, as in pathway analyses of months 5-8 of Workforce-Inventory, Figure 8. When differences are larger, we suggest that, for a given loop dominance method, more than one mechanism of comparison is reported to give a richer view of the loop dominance phases.

Applications

SI Model

How do these methods deal with the SI model, Figure 1? As this model has a conserved flow, it is effectively a one-variable model. Thus, it is sufficient to consider the infected's behaviour only, Figure 17.

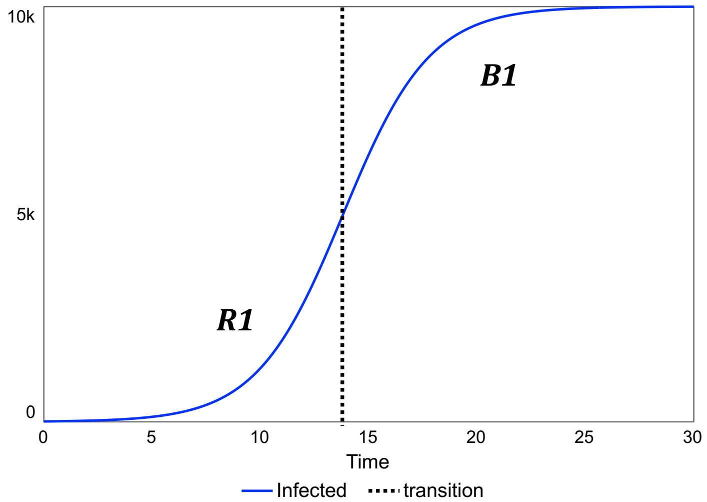

Figure 17: Behaviour of the stock Infected in the SI model, Figure 1.

Loop impact measures the effect of *R1* as 0.5 per unit time at time zero, dropping to zero at time 30, Figure 18. By contrast, *B1* has near zero impact at the beginning dropping to -0.5 at the end. The two are numerically equal, 0.25, at the transition point, confirming the transition of dominance in Figure 17. PPM would express these impacts as percentages.

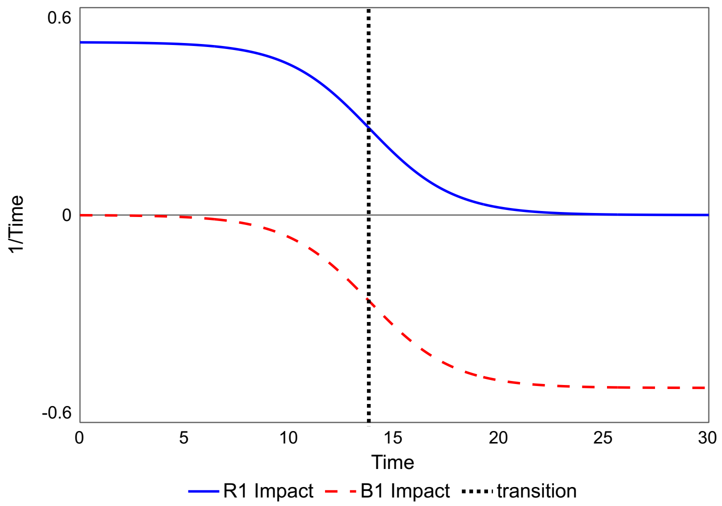

Figure 18: Loop impacts on Infected in the SI model.

As the SI model is first-order, LTM reproduces the same percentages as PPM, the result produced in Stella, Figure 19. LTM's raw loop scores of the two loops are quite different to loop impacts with infinities at the inflexion point, Figure 20. However, both these raw scores tend to infinity such that, in the limit, their ratio is numerically one, as shown in the transition (inflexion) point in Figure 19.

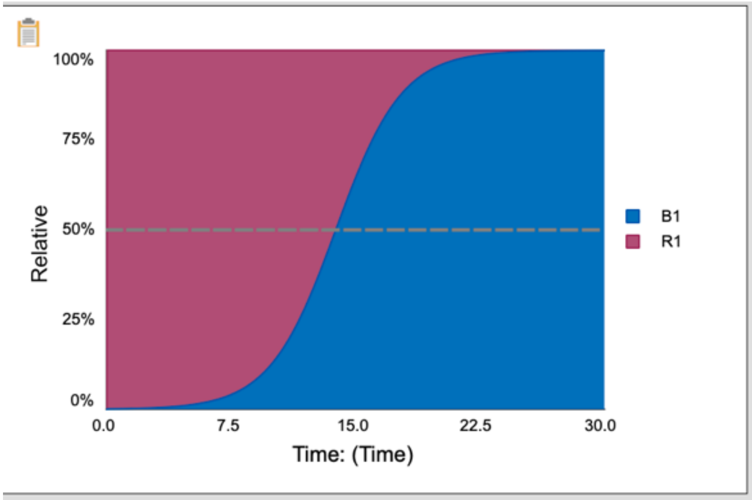

Figure 19: Loop scores in the SI model using LTM. Output from Stella's Loops That Matter display.

EEA describes *Infected's* behaviour as a single exponential whose exponent, the eigenvalue, is falling from 0.5, through zero at the transition to -0.5, Figure 21. Thus, *Infected* follows a growing exponential to the left of the inflexion point and a decaying one to the right. The influences of the two loops on this eigenvalue have *R1* larger to the left of the inflexion point and B1 numerically larger to the right, confirming this point as the transition of loop dominance, Figure 22.

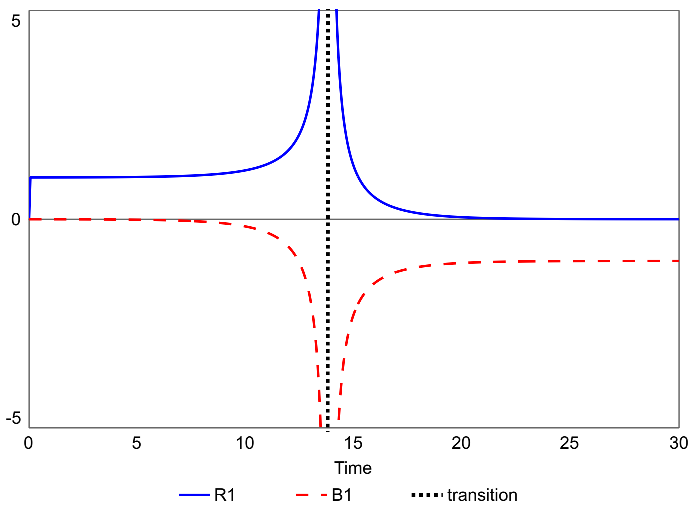

Figure 20: Raw loop scores of the SI model using LTM.

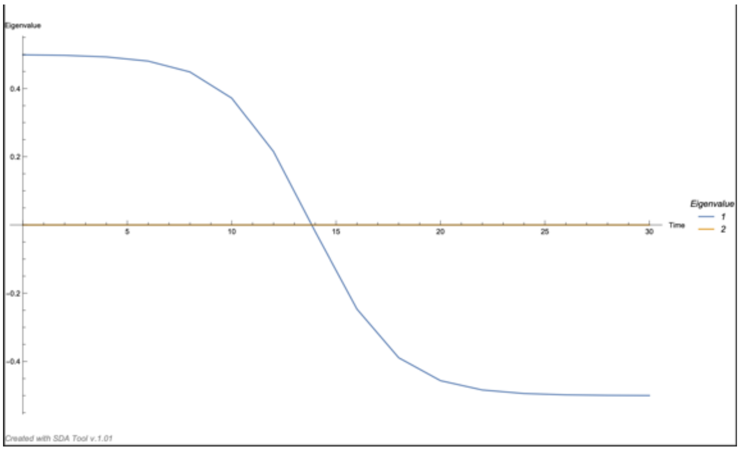

Figure 21: Eigenvalue of the SI model using EEA. Output from the Structural Dominance Analysis Toolset (Naumov and Oliva, 2018).

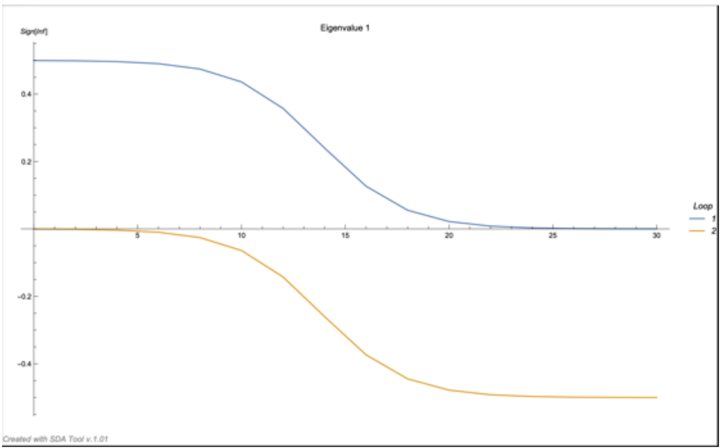

Figure 22: Eigenvalue loop influence for the SI model using EEA. Output from the Structural Dominance Analysis Toolset, (Naumov and Oliva, 2018).

The energy method indicates the system has one energy source, *R1*, with one sink, *B1*, Figure 23. All the energy is dissipated from the susceptibles through *B1*. Thus, the system stabilises with infected stock depleted. That is, all susceptibles catch the disease.

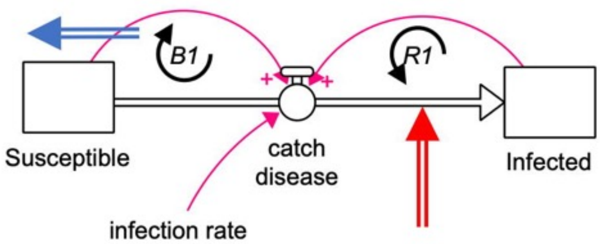

Figure 23: Energy flows for the SI model.

At the transition point, 69% of energy has entered through *R1*, with only 31% removed through *B1*, Figure 24. The excess energy is in the kinetic energy of the stocks, which are changing at their fastest at that point in time. By time 30, the system has stabilised with zero kinetic energy and all the energy injected by *R1* has been removed by *B1*.

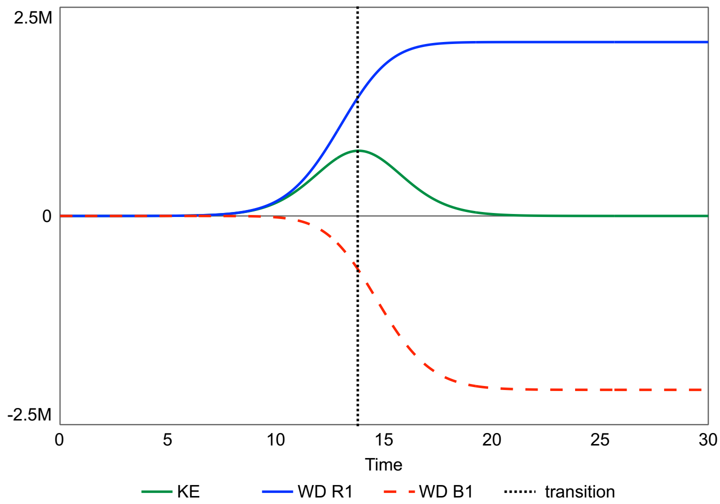

Figure 24: Total change in work done by the SI model loops and kinetic energy of Infected over time.

Figure 25 displays the *rate* of change of energy of each loop at each time, called the loop power. The power of *R1* peaks before the transition, whereas *B1*'s power peaks after the transition. At the transition, they are numerically equal. From the energy viewpoint, the loop transition point is where the two loops are transferring energy at the same rate but with opposite directions.

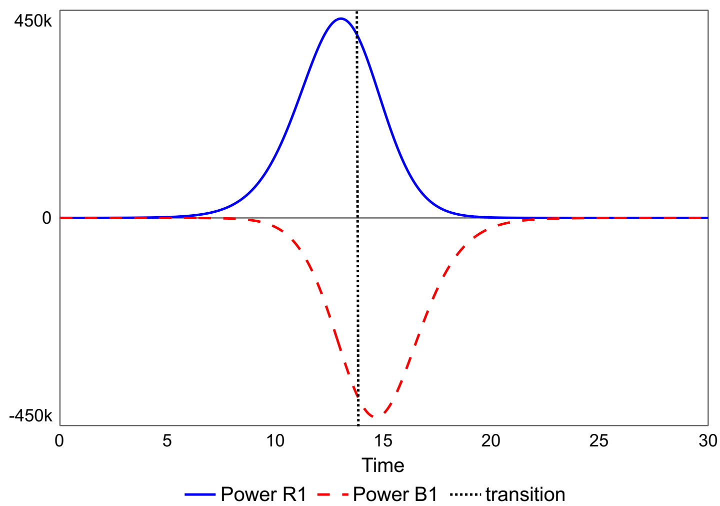

Figure 25: Power of loops (energy transferred per unit time) in the SI model.

SIR Model

Next, we examine the loop dominance methods for the SIR model, Figure 26. In this model, the infected recover from the disease and are removed. There is now an extra loop, B2, on an outflow from *Infected*.

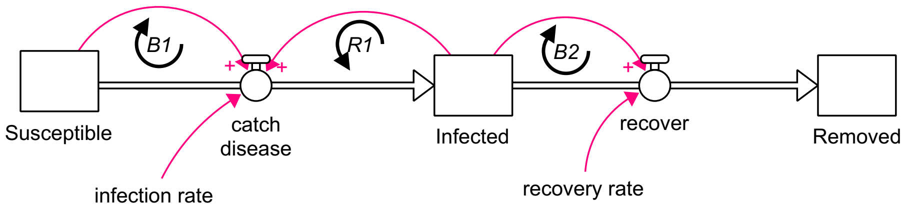

Figure 26: SIR model.

Loop impact explains the rise and fall of the infected using three loops. *R1* dominates the accelerating growth, B1 dominates the turnaround and B2 the decline, Figure 27. Note that *B1* acts on *Infected* exogenously. The infected stock is not part of the loop, but its flow is affected by the pathway from *Susceptible* via the flow *catch disease*, Figure 26.

Loop impact explains the susceptible behaviour using *B1* in the middle phase, with *R1* on either side, Figure 27b. *R1* affects the Susceptibles exogenously, so it can both accelerate and slow down its behaviour depending on the net flow of infected.

LTM effectively combines the two loop impact analyses to explain the system with a similar pattern to loop impact on infected but with different transition points, compare Figure 27c with Figure 27a.

EEA determines the initial phase as two growing exponentials, an oscillation in the middle phase and two decaying exponentials in the decline, Figure 27d. From the eigenvalue loop influences, *R1* dominates the first phase, and half the second phase, explaining the turnaround as a change of dominance from reinforcing to balancing loops, Figure 27e. The second half of the simulation differs from the pathway methods, with B2 taking a greater role. In the final phase, *B1* dominates one of the exponential modes, with the other exponential dominated by *B2*. To map this pattern onto stock behaviour would require further analysis using dynamic weights (Naumov and Oliva, 2018).

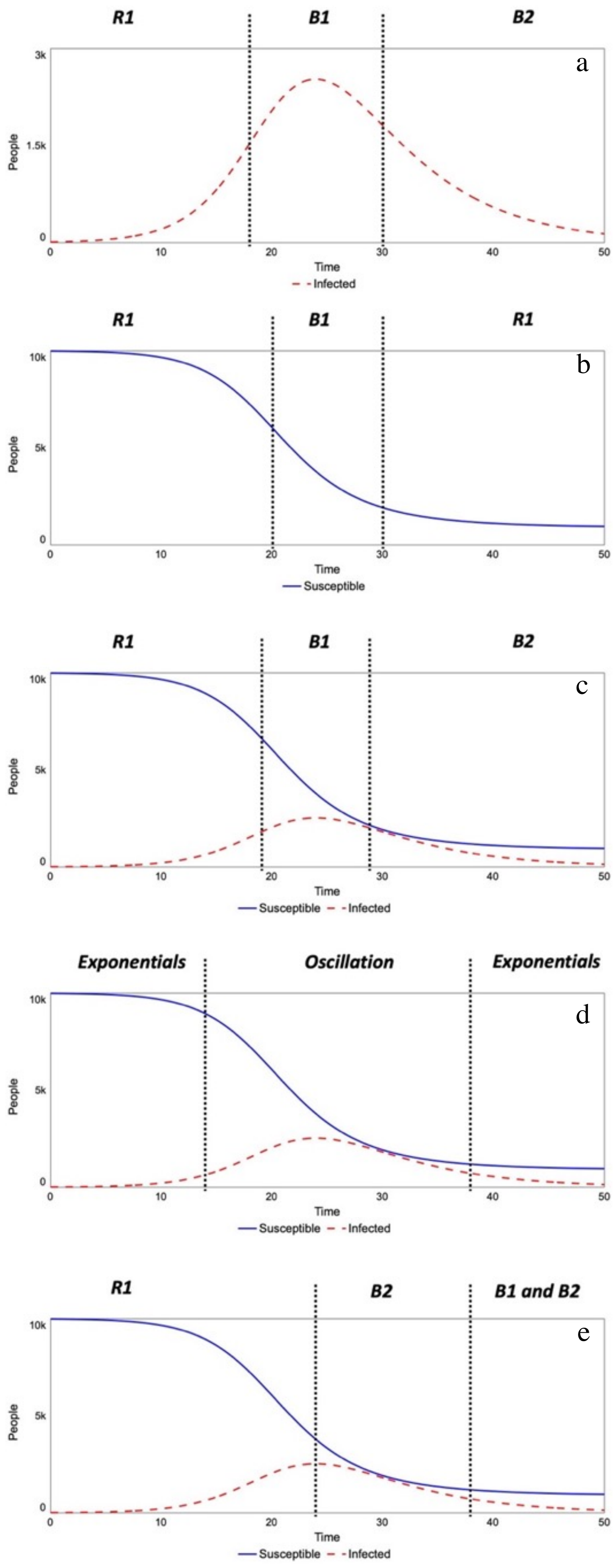

Figure 27: Loop dominance analysis of SIR model using: a) Loop impact on infected; b) loop impact on susceptible; c) LTM on system; d) EEA behaviour modes; e) eigenvalue loop influence (EEA).

From the Energy viewpoint, there is now an additional drain of energy from the system through *B2*, Figure 28. This drain is through the infected stock. Thus, not all the input energy is drained through the susceptibles. As such, the infection ends before all susceptibles catch the disease, a well-known result in epidemic models.

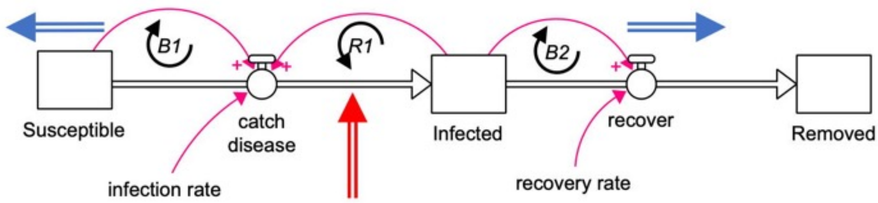

Figure 28: Energy flows on the SIR model.

Conclusion

We have compared the loop dominance measures of Loop Impact, the Pathway Participation metric, Loops That Matter, Eigenvalue Elasticity Analysis and Energy. The measures differ in their meaning; the number associated with each loop; their scope with regard to exogeneity and use of time. The differences are summarised in Table 1.

Measures	Meaning	No. Measures per Loop	Scope	Time
Loop Impact	Acceleration Curvature	One per stock	Endogenous Exogenous	Instantaneous
PPM	Percentage Influence	One per stock	Endogenous Exogenous	Instantaneous
LTM	Percentage Influence	One	Endogenous	Instantaneous
EEA	Sensitivity of modes	One per behaviour mode	Endogenous	Instantaneous
Energy	Stock Change	One or One per stock	Endogenous Exogenous	Interval

Table 1: Comparison of Loop Dominance Methods.

The differences in the measures can produce different dominance patterns, as noted with the three models examined in this paper. However, none of the methods are wrong, and they can all be related to each other. We encourage system dynamicists to try these methods for themselves. Toolsets are available for Loops That Matter in the software Stella (Schoenberg et al., 2020), and for EEA using the Structural Dominance Analysis Toolset (Oliva, 2015; Naumov and Oliva, 2018). PPM may be automated using the Digest software (Mojtahedzadeh et al., 2004). Loop Impact and the Energy method may be used in any software simulator, see Hayward and Boswell (2014) and Hayward and Roach (2017; 2022). The results in this paper were computed with these methods and, for the Workforce-Inventory model, confirmed analytically[4].

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[4] For further details, see the Sociomechanics website https://sociomechanics.com/.

Future Work

The scope of our paper was deliberately limited to the meaning and quantification of the measures used in each dominance method. We did not compare the benefits of one method over another or test their limits. Such comparisons have been published previously (Kampmann and Oliva, 2008; Mojtahedzadeh, 2008; Hayward and Boswell, 2014; Sato, 2016; Hayward and Roach, 2017; Kampmann and Oliva, 2020; Schoenberg et al., 2020). However, none of these studies compares all the methods, nor do they relate their results to the meanings and measures described in this paper. Potential future work could test the usefulness of the methods to understand system behaviour and the effect of interventions, basing the tests on the characteristics of the methods described in this paper.

At the beginning of this paper, we stated that the structure of a system dynamics model is expressed in its stocks, flows and feedback loops. We are aware that there is a fourth structure that is essential to a model – the formulae used in each model element. Future work could investigate the connection between different types of formulae and equations and system behaviour for given stock and feedback structures. Although it is believed that dominance patterns are largely determined by loop structure, we are not aware of work that examines the effects of variations in formulae on system behaviour.

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