PID Control in a Toy Bio-economic Model

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Abstract

The mathematic control theory is applied to the development of modifications of the Schaefer fishery model. The key variables are the stock of the bio-resource as well as the Man harvesting activity. The global and local analysis reveals quantitative and qualitative characteristics of closed loop control under a heuristic harvesting control rule in a two-dimensional model. Deeper analyses refine and generalize this rule in a three-dimensional model. The synthetic proportional, integral and derivative (PID) control over fisheries together with parametric policy optimization maintains a robust harvesting control rule. The latter generalizes the heuristic one.

Key words: renewable resource, maximum sustainable yield, harvesting effort, PID control, optimization, Andronov – Hopf bifurcation

Figure 1 – The Vensim diagram of Verhulst logistic model M-1

$\dot{x} = \beta x(1 - \alpha x)$. Let $\alpha = 1$ and $\beta = 1$

$x_2 = 1$ is globally stable

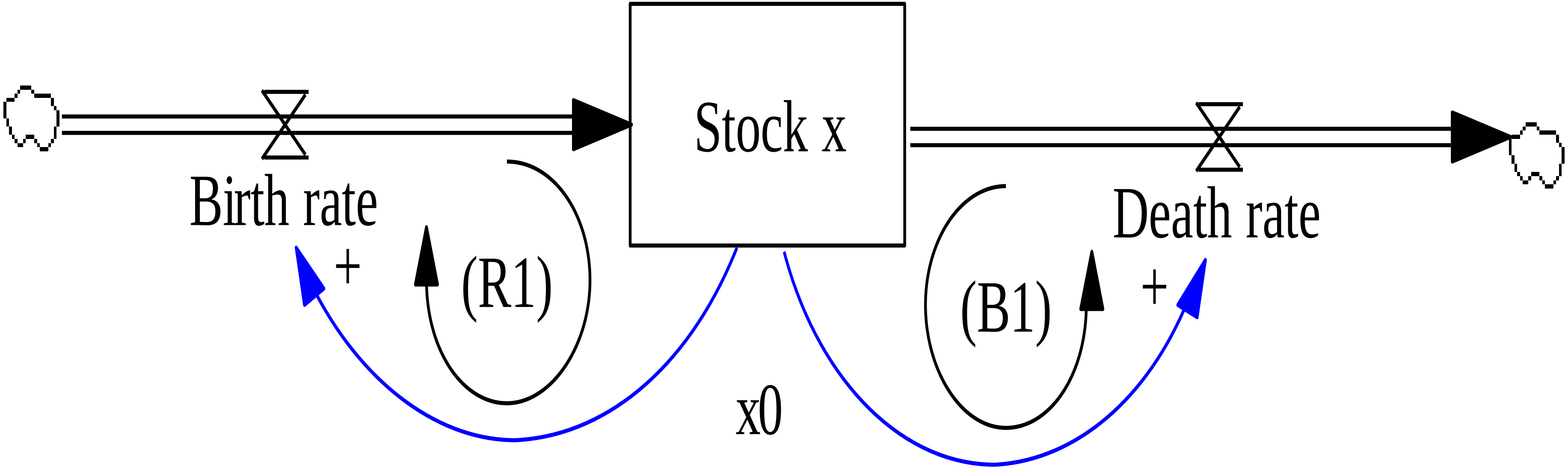


Fig. 2 – The SFD of M-2 with heuristic HCR adapted from Moxnes (2004)

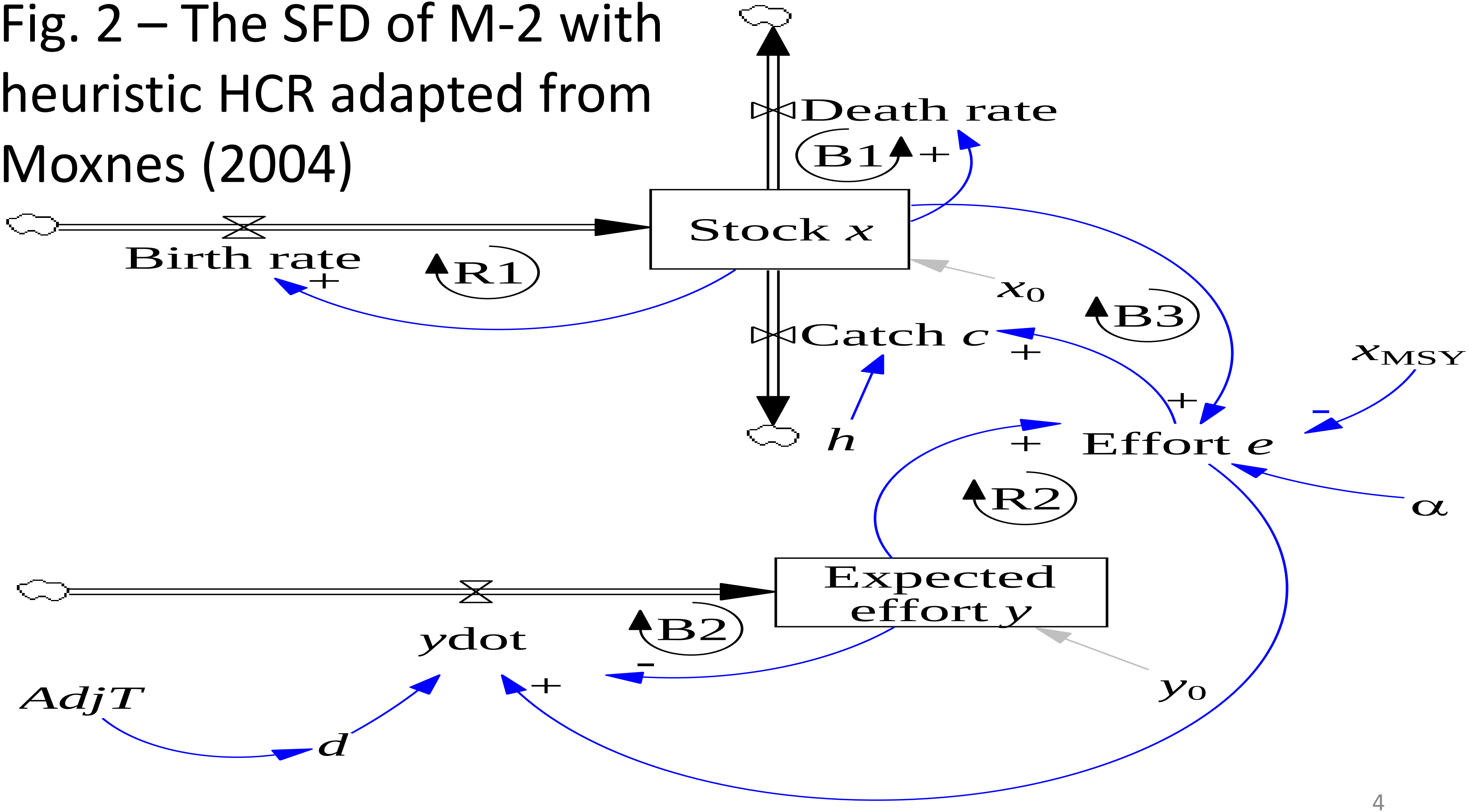


Table 2. 5 feedback loops in the extensive form of M-2 with heuristic HCR

No	Order, sign	Loop
B1	1, -	Stock $x \xrightarrow{+}$ Death rate $\xrightarrow{-}$
R1	1, +	Stock $x \xrightarrow{+}$ Birth rate $\xrightarrow{+}$
B3	1, -	Stock $x \xrightarrow{+}$ Effort $e \xrightarrow{+}$ Catch $c \xrightarrow{-}$
B2	1, -	Expected effort $y \xrightarrow{-}$ Net change of y ($y$dot)
R2	1, +	Expected effort $y \xrightarrow{+}$ Effort $e \xrightarrow{+}$ Net change of y ($y$dot)

The predator (y) – prey (x) system of two ODEs (3) and (6) in M-2

Stock x derivative w.r. to t $\dot{x} = \text{Stock } x - \text{Stock } x^2 - \text{Catch } c,$ (3)

$\text{Catch } c = (1/4)\text{Effort } e;$ given historically $x_0,\ e_0$ *and* c_0

$$\text{Effort } e = \text{Expected effort } y\left[(1-\alpha) + \alpha\frac{\text{Stock } x}{x_{MSY}}\right], \tag{4}$$

1^{st} order information delay:

$\text{Expected effort derivative w.r. to } t\ \dot{y} = d(\text{Effort } e - \text{Expected effort } y) =$

$$= \text{Expected effort } y\,\frac{\text{Stock } x - x_{MSY}}{x_{MSY}}\,d, \tag{6}$$

$$d = \frac{1}{AdjT} > 0,\ 0 < \alpha \le 1$$

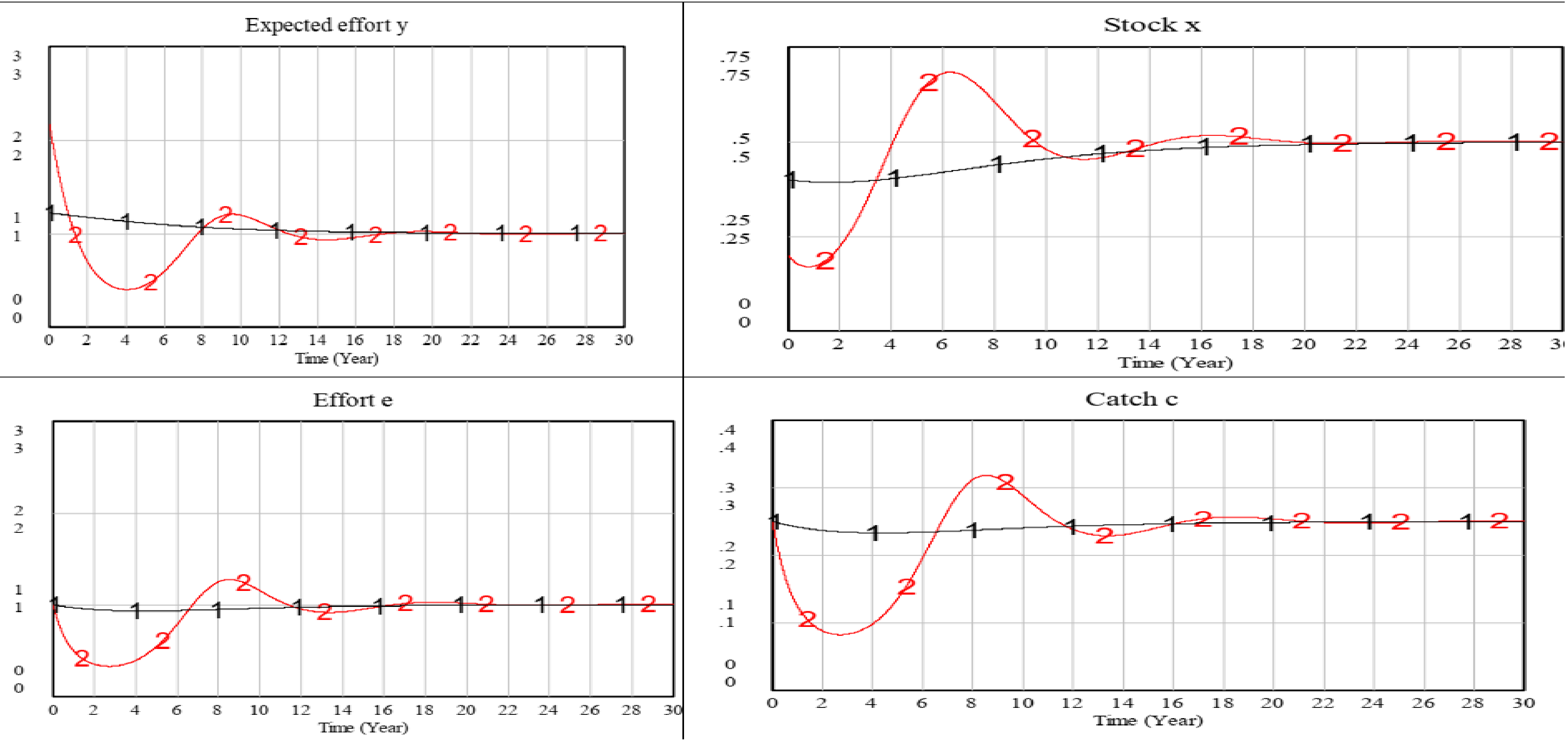
M-2 has non-trivial stationary state $E_s = (x_s, y_s)$, where Stock $x_s = x_{MSY} = 0.5$, Expected effort y_s = Effort $e_s = 1$

Proposition 1 The dynamics of the system (3) and (6) linearized in the neighbourhood of its hyperbolic stationary state E_s (8) are locally asymptotically stable (LAS). Then stationary state E_s is also LAS in the non-linear system (3) and (6).

Corollary If E_s is LAS focus, a period of fluctuations

$$T_c \approx \frac{8\pi}{\sqrt{-\alpha(\alpha - 8d)}}. \qquad (16)$$

Fig. 3 - LAS node E_s (8), d = 0.1, x_0 = 0.4; LAS focus E_s (8), d = 1, x_0 = 0.2



PID control in a three-dimensional S-2

- A proportional–integral–derivative controller (PID controller) continuously calculates an error value as the difference between a target and a measured process variable and applies a correction based on proportional, integral, and derivative terms (denoted P, I, and D respectively), hence the name.
- Consider net change of Effort *e*

Net change edot P stands for the element of proportional control,

Net change edot I expresses the element of integral control;

Net change edot D relates to the element of derivative control.

The sum of these three elements equals the derivative of effort *e* with respect to time. Discrepancy *D* (as stock) integrates the instant difference between current fish stock x and the target stock x_{MSY} that enables maximal sustainable yield.

Figure 4 – The extensive SFD of S-2 containing PID control

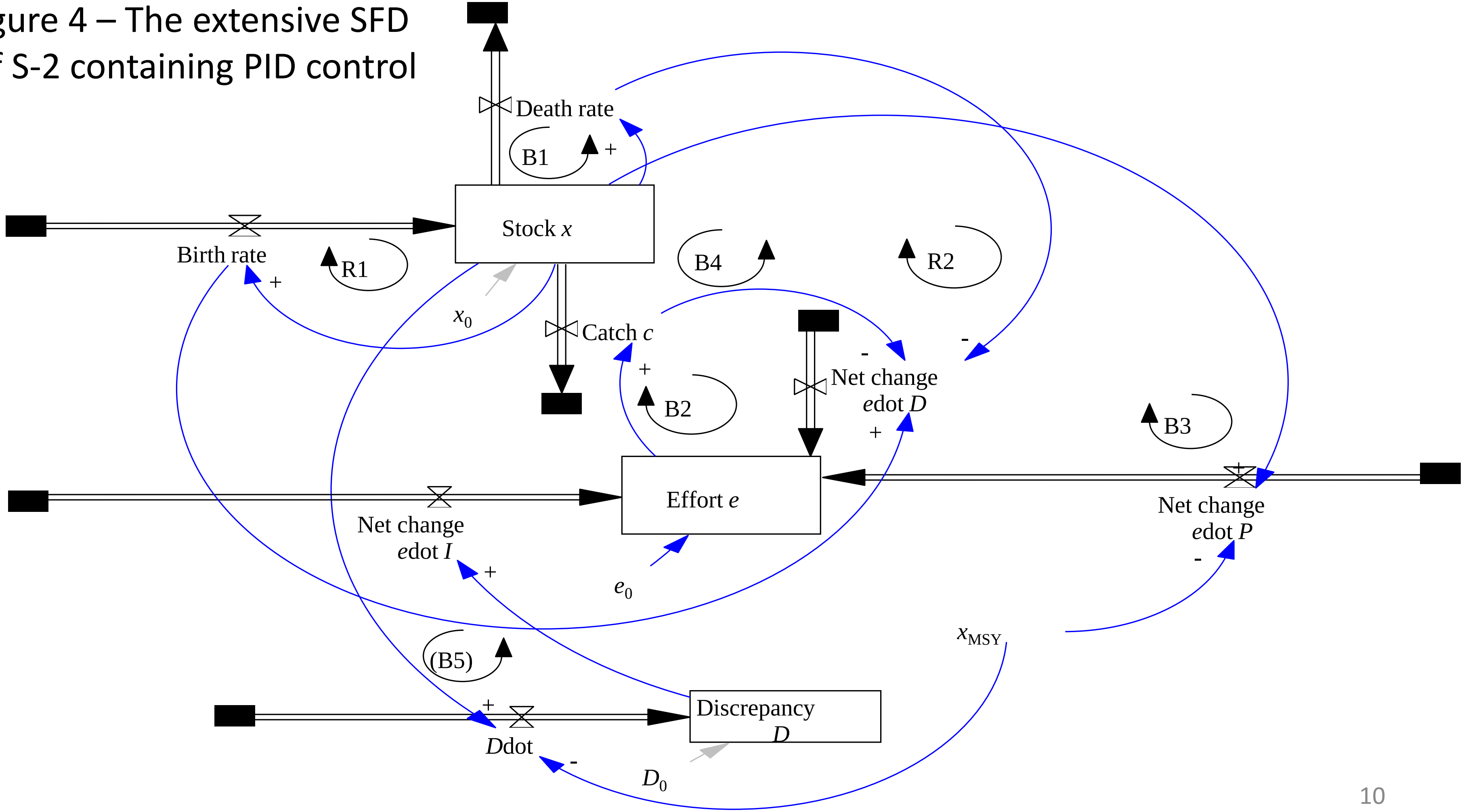


Table 6. Feedback loops in the extensive form of S-2

No.	Order, sign	Loop
R1	1, +	Stock $x \xrightarrow{+}$ Birth rate $\xrightarrow{+}$
B1	1, -	Stock $x \xrightarrow{+}$ Death rate $\xrightarrow{-}$
B2	1, -	Effort $e \xrightarrow{+}$ Catch $c \xrightarrow{-}$ Net change edot D
B3	2, -	Stock $x \xrightarrow{+}$ Net change edot $P \longrightarrow$ Effort $e \xrightarrow{+}$ Catch $c \xrightarrow{-}$
R2	2, +	Stock $x \xrightarrow{+}$ Death rate $\xrightarrow{-}$ Net change edot $D \longrightarrow$ Effort $e \xrightarrow{+}$ $\xrightarrow{+}$ Catch $c \xrightarrow{-}$
B4	2, -	Stock $x \xrightarrow{+}$ Birth rate $\xrightarrow{+}$ $\xrightarrow{+}$ Net change edot $D \longrightarrow$ Effort $e \xrightarrow{+}$ Catch $c \xrightarrow{-}$
B5	3, -	Stock $x \xrightarrow{+}$ Ddot $\longrightarrow$ Discrepancy $D \xrightarrow{+}$ $\xrightarrow{+}$ Net chande edot $I \longrightarrow$ Effort $e \xrightarrow{+}$ Catch $c \xrightarrow{-}$

The predator (y) – prey (x) system of three ODEs (3), (17) and (18) in S-2

[ODE for derivative of Stock x w.r. to t as in M-2 remains] (3)

Derivative of Effort e w.r. to t

$$\dot{e} = \frac{k_1\left(x - x_s\right) + k_2 D + k_3(x - x^2 - eh)}{h}, \tag{17}$$

Derivative of Cumulative Discrepancy between Stock x and x_{MSY}

D w.r. to t

$$\dot{D} = x - x_s, \tag{18}$$

S-2 has stationary state $F_s = (x_s, e_s, D_s)$, (20)

Stock $x_s = 0.5$, Expected effort y_s = Effort $e_s = 1$, Discrepancy $D_s = 0$,

Catch c = $c_s = 0.25$

Proposition 2 The dynamics of the system $(3),(17)$ and (18) in the neighbourhood of its hyperbolic stationary state $F_s\,(20)$ are LAS provided that $0 < k_2 < k_1 k_3$.

Proposition 4 For $k_2^{AHB} \approx k_2^{critical}$ and $k_2^{AHB} < k_2^{critical} = k_1 k_3$ the Andronov – Hopf bifurcation takes place in the system $(3),(17)$ and (18) in vicinity of $F_s\,(20)$.

There exists some periodic solution bifurcating from F_s and the period of fluctuations is about $T_c \approx \dfrac{2\pi}{\sqrt{a_1}} = \dfrac{2\pi}{\sqrt{k_1}}$.

Parametric policy optimization in S-2

Optimization criterion is cumulative catch c under penalty δ < for $c < 0$

$$\mathrm{Max}\left(0.1\int_0^T ehdt + 0.9\int_0^T \delta dt\right), \qquad (31)$$

$$\text{for}\,(x_0, e_0, D_0) = (0.4,\ 1,\ 0),$$

$$0.5 \le k_1 = 0.5 \le 5,$$

$$0 \le k_2 = 0.102827 \le 0.5,$$

$$0 \le k_3 = 0.5 \le 15$$

Solution

$$k_1 = 5, k_2 \approx 0, k_3 = 3.6256$$

A solution for LAS node or focus F_s depends on x_0

Parametric policy optimization in M-2

Optimization criterion is cumulative catch c under penalty $\delta < 0$ for $c < 0$

$$\mathrm{Max}\left(0.1\int_0^T ehdt + 0.9\int_0^T \delta dt\right), \tag{33}$$

subject to (3) and (6)
with $(x_0, y_0) = (0.4,\ 1.25)$,

$e_0 = 1$,
$0 \leq \alpha = 0.9 \leq 1$ and
$1 \leq AdjT = 10 \leq 25$.

A quasi-optimal solution for T =100: α = 1 and $AdjT$ = 1.21635.

A solution for LAS node or focus E_s depends on x_0

Revealing correspondence of harvesting control rules in M-2 and S-2

Almost perfect matching of linear proportional and derivative (PD) control over Stock x and Effort e without active integral element in S-2 and nonlinear proportional and derivative (PD) control in M-2 is achieved for the same initial conditions x_0 and e_0 with over-exploited fish stock when the congruity conditions are satisfied:

$k_1 = 0.5d\alpha$, $k_2 = 0$, $k_3 = 0.5\alpha$.

Proposition 5 It is possible for PID control in S-2 to match the heuristic HCR in M-2 even with inactive integral element when $k_2 = 0$.

Proposition 6 It is not always possible to match PID control in S-2 through the heuristic HCR in M-2.

Proposition 7 PID control in S-2 is a generalization of the heuristic HCR in M-2.

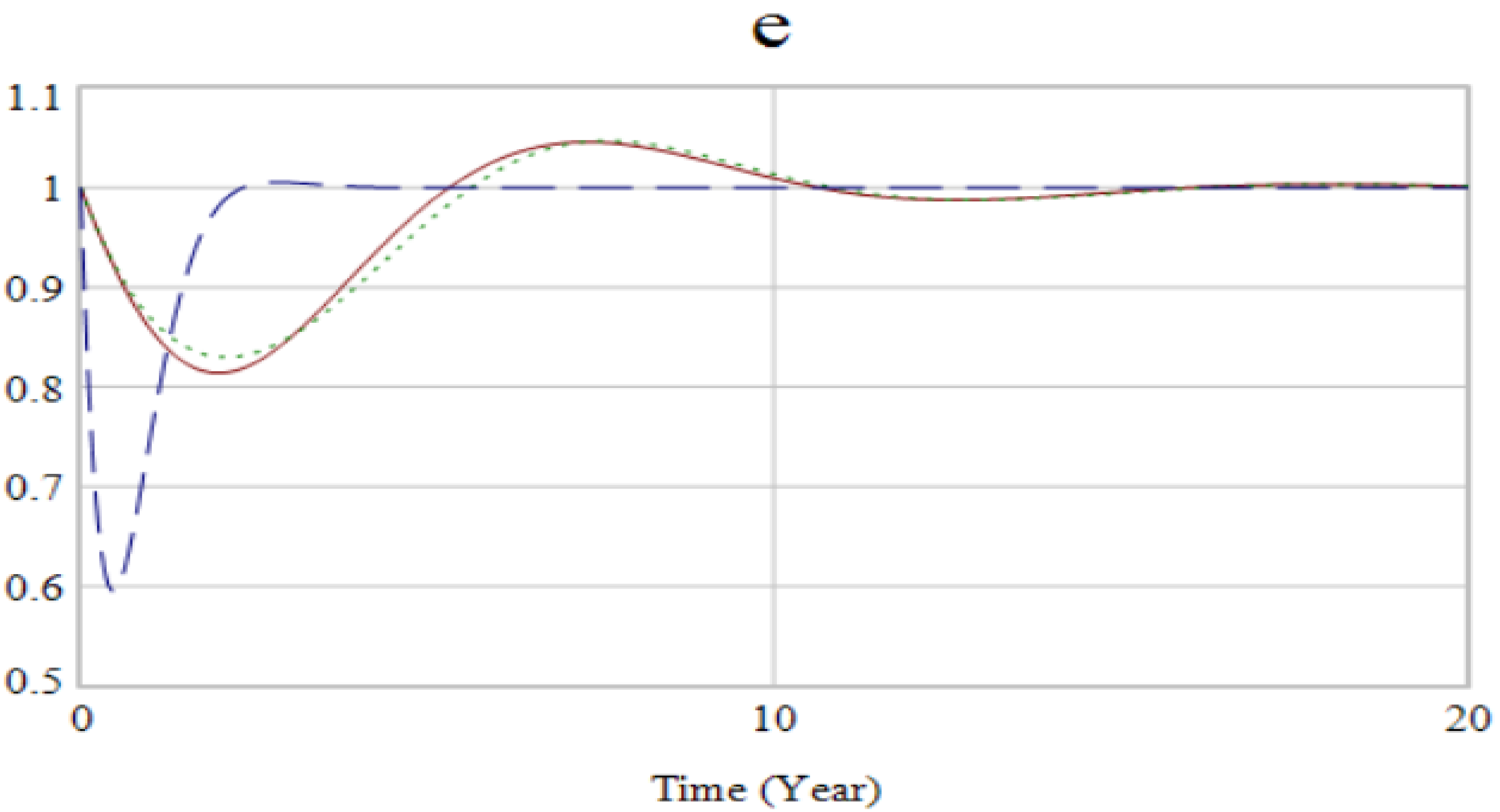


Figure 12 – Transition of effort e to fitting sustainable effort $e_s = 1$; solid curve – congruent run in S-2, dotted curve – quasi-optimal run in M-2, piece-wise curve – quasi-optimal run in S-2, 0–20 years

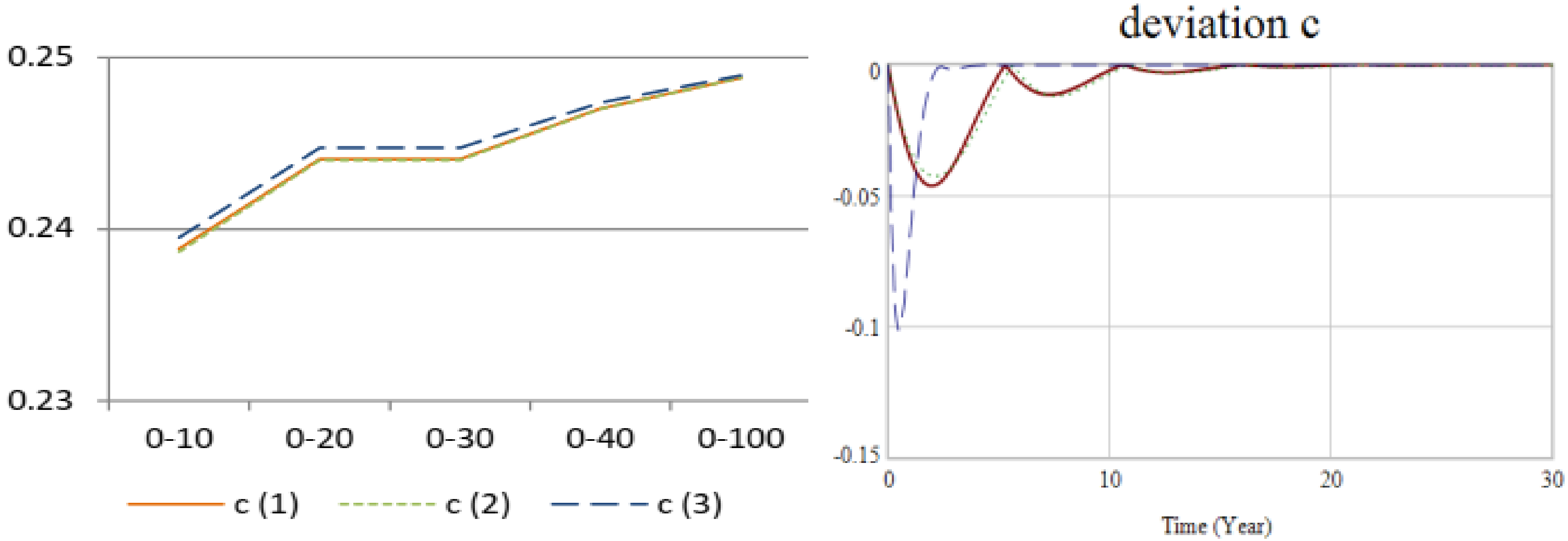


Figure 15 – Evolution of average magnitudes over decades; on the left – Catch c, 0–100 years and on the right– real-time deviation of Catch c from MSY c_s, 0–30 years; (1) solid curve – congruent run in S-2, (2) dotted curve – quasi-optimal run in M-2, (3) piece-wise curve – quasi-optimal run in S-2

Conclusion

- This paper has provided new experimental and analytical material to substantiate the strength of the system dynamics method (still under only deterministic conditions so far) with account for the impacts of alternative hypotheses on the behaviour of the complete system using systems of equations.
- The policies of improving biomass catch and renewal are elaborated in M-2 and S-2.
- Long-term policy effectiveness is improved in S-2 in relation to M-2.
- The analytical results for the proposed predator-prey models are mostly local, they are extended to broader areas thanks to Vensim simulations.
- The author intends to raise the dimension of the two main models through information delays in the measurement of the fish stock in decision-making. Obtaining knowledge of a critical delay length could facilitate the PID control further.
- Besides disaggregation of bio-mass into specific components in different geographical regions, the prospective research should also enhance the probabilistic approach to bio-economic modelling.

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