Supplementary material
Modelling the long-term evolution and collapse of societies

Based on prior work on the mathematical modelling of the societal development, and ultimately the collapse, on Easter Island [1], of the Classic Maya [2] and of the Western Roman Empire [3], we propose a framework for understanding long-term societal evolution. Analyzing social systems at a large scale, in a aggregate way over a long time span, can provide models that with significant predictive power, as has been the case with Limits to Growth [4]. The success of the method lies in identifying feedback mechanisms that operate over centuries and that survive the numerous changes in rulers, social norms and external threats. After studying several ancient societies, and developing mathematical models that reproduce their archaeological record, we have identified an archetypal scheme of feedback mechanisms that is common to these historical cases and proves informative for modern society as well. The key variables in the feedback mechanisms involve the complexity of a society, its resources and a measure of returns from investments, and the framework allows for a direct interpretation of these abstract notions in concrete cases, see Fig. 1.

In Fig. 1 the feedback mechanism is illustrated in a causal loop diagram (CLD). In the growth phase of a society, the reinforcing (outer) loop is dominant and leads to the system's growth, while during its decline the counter-acting loop is most significant, leading to a continuous decrease in the system's complexity due to very low returns. This latter phenomenon constitutes the collapse of a society. The CLD captures the key relationships found in several mathematical models of ancient societies, validated by comparison with archaeological data [1, 2, 3]. For different societies, various metrics of complexity, resources and returns apply, depending on the features of the societies in question. In the case of agricultural societies, such as Easter Island or the Classic Maya, the population engaged in crop production is an adequate measure of complexity in light of our framework. This section of the society is the most representative for the activity of extracting energy from the environment, a key feature of societal complexity [5]. For other societies, such as the Roman Empire, the army is an appropriate measure of complexity, as it is the main method of guaranteeing returns (precious metals, taxes) from resources (territory), see Fig. 1. Overall, our work extends the theory of [6] and provides a useful tool to move from conceptual models of societal dynamics to more quantitative ones.

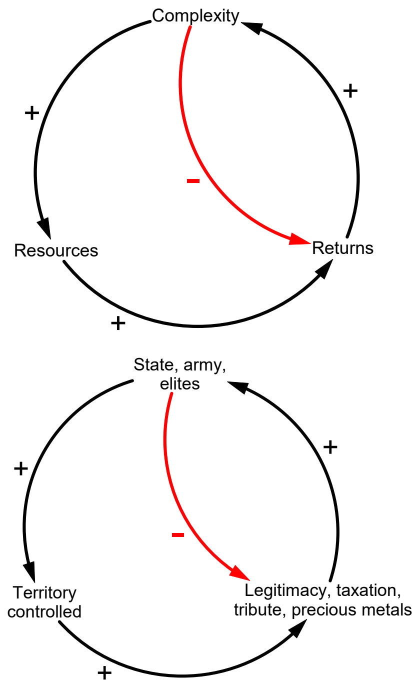

Fig. 1. The causal loop diagram illustrates how complexity, resources and returns relate. With increasing complexity, resources are more rapidly depleted; the decline in resources translates to an increase in returns, which in turn lead to an increase in complexity. This represents the reinforcing feedback (outer, black) loop of the diagram. But, complexity also has costs that draw on returns (red arrow). Higher complexity implies higher costs and a decline in returns, which constitutes a counter-acting loop in the diagram. In the case of the Roman Empire, the complexity is best encapsulated by the army, the resources by the territory and returns by precious metals and taxation [3].

[1] Roman, Sabin, Seth Bullock, and Markus Brede. *Coupled societies are more robust against collapse: A hypothetical look at Easter Island.* Ecological Economics 132 (2017): 264-278.

[2] Roman, Sabin, Erika Palmer, and Markus Brede. *The dynamics of human–environment interactions in the collapse of the Classic Maya.* Ecological economics 146 (2018): 312-324.

[3] Roman, Sabin, and Erika Palmer. *The Growth and Decline of the Western Roman Empire* Cliodynamics 10, no. 2 (2019).

[4] Turner, Graham M. *A comparison of The Limits to Growth with 30 years of reality.* Global environmental change 18, no. 3 (2008): 397-411.

[5] Tainter, Joseph A., and Tadeusz W. Patzek. *Drilling down: The Gulf oil debacle and our energy dilemma.* Springer Science & Business Media, 2011.

[6] Tainter, Joseph. *The collapse of complex societies.* Cambridge University Press, 1988.