Production and Inventory Control System Dynamics under Emission Feedback

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Abstract

Green Supply Chain Management focuses on SCM's environmental (emission) aspect. Efforts by introducing technological innovation, capital investment, and capacity expansion to reduce emission impact for the supply chain are existing options in the prevailing literature. However, operational inefficiencies cause short-term environmental havoc to which corrective measures like adjustments can reduce impact. This paper proposes extension of the production inventory control system (APIOBPCS) where production emissions are explicitly modeled. The order releases are determined using emission information from the system via two pathways, either based on perceived emissions or the actual stock of net emissions. Simulation results under pulse/step change in demand and emission are performed to understand the dynamic behavior of such a system. The study's key insights include a clear tradeoff between emission reduction and system service level. A system with Perceived Emission feedback provides a higher fulfilled orders rate than a system with Net Emission feedback, but no significant reduction in net emission is observed. On the contrary, a system with Net Emission feedback as a zero-level target causes significant emission reduction to the system at a cost of unfilled demand. An environmental module with both Perceived Emission Feedback and Net Emission Feedback can contribute towards determining a balance between emission reduction and order fulfill rate tradeoff.

Keywords: sustainability, emission feedback, APIOBPCS, perceived emission feedback, net emission, stock management,

1. Introduction

Achieving a sustainable supply chain requires balancing economic, environmental, and social aspects through effective management. Extensive literature is available on the economic and the environmental aspects of supply chain management [Seuring (2008)]. A supply chain
system that gives emphasis to environmental considerations is referred to as a clean and/or green supply chain. Clean supply chain management uses technology and methods to produce and sell without causing excessive pollution and/or toxic waste. Green supply chain management, on the other hand, focuses specifically on reducing the green-house gas (GHG) emissions. Bowen et al (2001) further distinguish between “greening the supply chain process” and “product-based green supply”. A green supply chain process deals with adaptation by a firm’s suppliers for environmental consideration. Product-based green supply chain brings changes to product supplied or alterations in purchasing decisions for environmental concerns. In this paper, we consider a generic notion of ‘emissions’ to refer to GHG emissions.

Governmental regulations and public sensitivity toward carbon-related products have been important factors for organizations and firms to move towards green productions [Du et al. (2015)]. Carbon caps and governmental/societal pressures to promote green production forces firms to incorporate environmental factors into their decision-making. From a supply chain management perspective this can translate into longer-term efforts by firms to reduce their emissions through technological innovations, product redesign, improvements in the production process, etc., often involving capital expenses. However, in the short and medium-term, the supply chain may be expected to reduce/control their emissions through effective operational decisions in inventory management, especially to maintain emissions within permissible limits.

Considering emissions in inventory decisions creates a challenge of balancing the tradeoff between system service-levels and emission-levels. Lead time uncertainty creates an challenge for the supply chain regarding cost and service perspective due to dynamics arising from varying lead-time. Existing studies from the literature suggest that lead time is considered as a critical factor that impacts supply chain performance under emission consideration [Li et al. (2019)] and contributes towards the bullwhip effect [Chen et al. (2000)]. This paper reports our preliminary work on considering emissions in a firm’s inventory ordering policies via a modified Production and Inventory Control System.

The paper is organized as follows. Section 2 presents a literature review of production and inventory control systems and the emission aspect for sustainability. Section 3 presents the proposed extensions to the classical APIOBPCS, to consider emission feedback in ordering decisions Section 4 discusses the preliminary simulation results under different demand and permissible emissions scenarios, along with initial sensitivity analysis. Section 5 extends the model to a three-player decentralized supply chain. Finally, section 6 discusses the key insights, conclusions, and future work.

2. Literature Review

Production and Inventory Control Systems (PICS), in general, will affect the environmental (emission) footprint of a firm. Typically existing literature, weightage is given to policies for transport selection, capital investment, and technological innovation for emission reduction. Operational adjustments provide opportunities for emission reduction when
operational drivers of emission are different from the operational drivers of costs [Chen et al. (2011)]. Operational flexibility and collaborations provide leverage among firms of the vertical supply chain to reduce emissions. Optimization models consider the carbon emission level as a soft constraint and use optimization heuristics for optimal ordering quantities depending upon the system setup level of player collaboration [Benjaafar et al. (2012)]. Such an optimization approach based on nonlinear thinking can miss out on achieving the green target of the supply chain system due to the lack of use of environmental impact as feedback for future decision-making. consumer preferences also play an important role in creating the demand for the product. When a customer is environmentally sensitive, their perceived utility will increase as the products' carbon footprint reduces the specific consumption and vice-versa [Chitra (2007)]. Focusing on low-carbon consumption helps greater market share [Amacher et al. (2005), Conrad (2005)], higher brand value[Conrad (2005)], and more brand competitiveness [Lu et al. (2007)]. The game-theoretic model also gives consideration to emission-sensitive demand using a negative exponential form [Du et al. (2015)]. Consumers sensitive to emissions may be willing to pay a higher price for the product. [Upham et al. (2009)]. However, all such assumptions are exogenous variables and beyond the system’s control in reality. Thus, the focus towards emission reduction should be at the grassroots level of manufacturing, transportation, and operational aspects. Recent literature sets the trend towards using Economic Order Quantity (EOQ) techniques to order optimal quantities with emission considerations. Hua et al. (2011) proposed an EOQ model with a carbon-cap trade mechanism for emission reduction and total cost trade-off against no emission considerations. Further, Chen et al.(2011) derived analytical support for a significant reduction in emission without significant cost increment.

A feedback-based approach toward viewing inventory ordering decisions was the original motivation for a more systems view of such decisions, as laid out in the seminal work by Forrester’s Industrial Dynamics (1961) [Forrester (1997)]. Since then, the stock management structure has been quite useful to understand various supply chain dynamics such as the bullwhip effect, the impact of delays, ordering stability, etc. The popular (well studied) version of the same is the Inventory and Order-based production control systems (IOBPCS) [Towill (1980)] family of models. These models include the supply line, the level of available inventory, and the expected incoming demand [Disney et al. (2005)]. Over the years, the IOBPCS family has significantly improved on its variants. Feedback in IOBPCS from desired inventory creates an automated pipeline inventory and order-based production control system (APIOBPCS) [Simon et al. (1994)] by considering discrepancies in work-in-process or supply line providing stability to the system. APIOBPCS considers a fixed desired inventory target. To capture varying demand, Automatic Pipeline Variable Inventory and Order Based Production Control System (APVIOBPCS) is developed which takes in feedback from the current forecast of demand. A substantial literature on the IOBPCS family of systems varying from system stability and controllability [Disney et al. (2004, 2006), Dejonkkeere et al. (2003), Oregta and Lin (2004), Venkateswaran and Son (2007), Disney and Towill (2002)], the effect of information update [Venkateswaran and Son (2007)].
Efficient inventory policies require mitigating frequent stock-outs under emission consideration via periodic or continuous review policies to achieve a target of a green supply chain. From the literature review, introducing feedback from system emission to ordering policies is still unexplored, which we consider a contribution of this paper. PICS with carbon cost consideration increases inventory and carbon cost [Li et al., 2019]. In this study we model and analyse PICS dynamics under a mechanism for ordering policies based on emission feedback, using a modified APIOBPCS model.

3. Proposed Model

The classical stock-flow structure of the stock management system of APIOBPCS is shown in Figure 1, say for the manufacturer in a supply chain. APIOBPCS system prepares the production or replenishment orders based on the difference between the Order-up to level and inventory position for the manufacturer. The inventory position includes the physical stock (INV) and the quantity in orders outstanding, called the supply line or work-in-process (WIP). The model has two negative or balancing feedback loops, one for adjusting the WIP discrepancy and the other for adjusting the inventory discrepancy. The control parameters (or gains of the feedback loops) of this model are \( \alpha \), the fractional rate of adjustment of WIP discrepancy (or the time delay to adjust WIP discrepancy) and \( \beta \), the fractional rate of adjustments of end inventory discrepancy (or the time delay to adjust end inventory discrepancy). (Note: fractional rate = 1/time delay.) The desired work-in-process, DWIP, and desired inventory, DINV, keep changing based on the forecasted demand. The control policy is known as ‘pure OUT’ when \( \alpha = \beta = 1 \); ‘generalized OUT’ when \( \alpha \leq 1 \) and \( \beta \leq 1 \) and ‘smoothing replenishment rule’ when \( \alpha < 1 \) and \( \beta < 1 \) [Dejonckheere et al. (2003), Disney et al. (2008)].
Figure 1: Classical Stock Flow Structure for Production and Inventory Control System

The model above represents APIOBPCS derived from [Sterman (2000)] and is widely studied with a caveat considering inventory to be non-negative, i.e., unmet demand treated as lost sales. Eq (1) considers forecasting as first-order exponential smoothing, Eq(2) & Eq(3) change in WIP and Inventory level. Eq(4) typically represents a fixed pipeline delay for the production process. From Little’s Law, we know that desired WIP is equivalent to the desired throughput for a given lead time where the desired throughput is forecast [Venkateswaran and Hasti (2007)]. Based on the system feedback, orders placed are as per Eq(5). Further anchor and adjustment heuristic as per Eq(6) and Eq(7) for adjustment in WIP and INV level adjustment based on the desired level.

\[
\text{Demand Forecast: } FD_n = FD_{n-1} + \rho(CD_{n-1} - FD_{n-1}) \tag{1}
\]

\[
\text{Inventory: } INV_n = INV_{n-1} + PCR_{n-1} - CD_{n-1} \tag{2}
\]

\[
\text{Work-in-Process: } WIP_n = WIP_{n-1} + PREL_{n-1} - PCR_{n-1} \tag{3}
\]

\[
\text{Production Completion: } PCR_n = PREL_{n-L} \tag{4}
\]

\[
\text{WIP Adjustment: } WIPAdj_n = \alpha(DWIP_n - WIP_n) = \alpha(L \cdot \omega \cdot FD_n - WIP_n) \tag{5}
\]

\[
\text{INV Adjustment: } INVAdj_n = \beta(DINV_n - INV_n) = \beta(\omega \cdot FD_n - INV_n) \tag{6}
\]

\[
\text{Production Release: } PREL_n = FD_n + WIPAdj_n + INVAdj_n \tag{7}
\]

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>Fractional rate of WIP adjustment</th>
<th>(\beta)</th>
<th>Fractional rate of INV adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>Smoothing factor for the forecast</td>
<td>(\omega)</td>
<td>Scaling factor for DINV</td>
</tr>
<tr>
<td>(L)</td>
<td>Lead time</td>
<td>(\text{maxSR})</td>
<td>Unit adjustment for maximum Sales Rate</td>
</tr>
<tr>
<td>(FD_n)</td>
<td>Demand forecast for period (n)</td>
<td>(CD_n)</td>
<td>Customer demand for period (n)</td>
</tr>
<tr>
<td>(INV_n)</td>
<td>Inventory at beginning of period (n)</td>
<td>(WIP_n)</td>
<td>Work-in-Process at beginning of period (n)</td>
</tr>
<tr>
<td>(DINV_n)</td>
<td>Desired Inventory at period (n)</td>
<td>(DWIP_n)</td>
<td>Desired WIP at period (n)</td>
</tr>
<tr>
<td>(INVAdj_n)</td>
<td>Adjusted Inventory at period (n)</td>
<td>(WIPAdj_n)</td>
<td>Adjusted WIP at period (n)</td>
</tr>
<tr>
<td>(PREL_n)</td>
<td>Production order release at period (n)</td>
<td>(PCR_n)</td>
<td>Production Completion Rate at period (n)</td>
</tr>
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</table>

We propose an extension of the above classical APIOBPCS with emission feedback on the inventory ordering decisions. Based on the level of information for system emission, future orders are adjusted via—ordering policies. Higher system emission requires reduction to future order to compensate for increment past emission level. The emissions of the firm are modeled as a function of its Production completion rate (PCR) and the emissions per unit. Next, we propose two possible extensions/pathways in the above stock management structures to include emission-related feedback in ordering decisions. Here we assume another exogenous variable:
permissible emissions. The gap between emissions and permissible emissions is then adjusted in the ordering decision. These adjustments can be based on perceived emissions (discussed in Section 3.1), or actual net emissions (discussed in Section 3.2).

### 3.1 Perceived Emission based Feedback on ordering

The perceived emissions are assumed to be a first-order exponential smoothing (information delay) of the actual emissions, as shown in Figure 2. The gap between the perceived emissions (PE) and permissible emission (EQ) is then adjusted and reduced from the PREL. The equations underlying the model shown in Figure 2 are as follows. Eq (8.a) captures the rate of change in Perceived Emission under smoothing constant τ with an accumulation of Perceived Emission given in Eq (8.b). Based on the discrepancy between the two levels, an adjustment in terms of orders as per Eq (8.c) and Eq (8.d) is required and accounted into the system via., Eq (8.e) and Eq(8.f). The complete model of Perceived Emission module, includes Eq (1) - (6) as per APIOBPCS model (given earlier) along with Eq (8.a) - (8.f) (given below).

\[
\text{Change in Perceived Emission: } CPE_n = \tau \cdot (e \cdot PCR_n - PE_n) \quad (8. a)
\]

\[
\text{Perceived Emission: } PE_n = PE_{n-1} + CPE_{n-1} \quad (8. b)
\]

\[
\text{Adjustment Required: } adjReq_n = \frac{PE_n - EQ_n}{e} \quad (8. c)
\]

\[
\text{Emission Adjusted Orders: } EAO_n = adjReq_n \quad (8. d)
\]

\[
\text{Production Release: } PREL_n = DOR_n - EAO_n \quad (8. e)
\]

\[
\text{Desired Order Rate: } DOR_n = FD_n + adjWIP_n + adjINV_n \quad (8. f)
\]

<table>
<thead>
<tr>
<th>PE_n</th>
<th>Perceived Emission</th>
<th>CPE_n</th>
<th>Change in Perceived Emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQ_n</td>
<td>Permissible Emission Quota</td>
<td>( \tau = \frac{1}{T_e} )</td>
<td>Perceived Emission smoothing rate</td>
</tr>
<tr>
<td>e</td>
<td>Unit emission per unit</td>
<td>adjReq_n</td>
<td>Adjustment required for additional emission</td>
</tr>
<tr>
<td>( T_e )</td>
<td>Adjustment time to adjust emission</td>
<td>EAO_n</td>
<td>Emission Adjusted Orders</td>
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The above model with Perceived Emission uses a structure similar to the response delay system from system modeling. A delay of \( L+1 \) time units is apparent while making adjustments for emission. While lead-time \( L \) is required to complete the production process or deliver the quantity ordered. An additional one-time unit is necessary to place an order based on an estimated forecast. Eliminating lead time is not an option as well ordering still takes one-time units. The lag of these \( L + 1 \)-time units from placing order/production release to delivery/production completion cause varying dynamics into the system. Based on estimates ordering decisions are made, and stock errors are corrected. Similarly, we can perceive emission using first-order exponential smoothing.
3.2 Net Emission-based Feedback on ordering

The net Emission Feedback model assumes a simple anchor and adjustment heuristic for order adjustment. A balancing loop with some desired state (usually, zero-target level or some positive-level in emission context) to achieve by adjusting discrepancy (net emission, surplus from actual system emission rate compared to quota). Notably, a delay involved through Production Release Rate(PREL) to Production Completion Rate (PCR) creates a structure for oscillatory feedback, but non-linearity in sales rate refrains systems from oscillations.

System with Net Emission Feedback can be modeled as Eq (1) - (6) of APIOBPCS model together with Eq(9.a)- (9.f) (in below equations). Similar to the above structure in Perceived Emission Feedback, Eq (9.a) give a proportional rate of emission, Eq (9.b) represents the rate at which additional emission in the system appears and Eq (9.c) accounts for the accumulation of surplus emission. With a stock of accumulation and adjustment time $T_e$, EAO can be derived as per Eq (9.d) and (9.e). Finally, to reduce emission levels in the system via., net emission feedback ordering policies modified in Eq (9.f).
\[ \text{Emission Rate: } ER_n = e \cdot PCR_n \]  
(9. a)

\[ \text{Net Emission Rate: } NER_n = ER_n - EQ_n \]  
(9. b)

\[ \text{Net Emission: } NE_n = NE_{n-1} + NER_{n-1} \]  
(9. c)

\[ \text{Adjustment Required: } adj\text{Req}_n = \frac{NE_n}{e} \]  
(9. d)

\[ \text{Emission Adjusted Orders: } EAO_n = \frac{adj\text{Req}_n}{T_e} \]  
(9. e)

\[ \text{Production Release: } PREL_n = FD_n + adjWIP_n + adjINV_n - EAO_n \]  
(9. f)

<table>
<thead>
<tr>
<th>( ER_n )</th>
<th>Emission Rate</th>
<th>( NER_n )</th>
<th>Net Emission Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( EQ_n )</td>
<td>Permissible Emission Quota</td>
<td>( NE_n )</td>
<td>Net Emission</td>
</tr>
<tr>
<td>( e )</td>
<td>Unit emission per unit</td>
<td>( adj\text{Req}_n )</td>
<td>Adjustment required for additional emission</td>
</tr>
<tr>
<td>( T_e )</td>
<td>Adjustment time to adjust the emission</td>
<td>( EAO_n )</td>
<td>Emission Adjusted Orders</td>
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</table>

PCR as a source for system emission and an acceptable permissible emission quota creates an emission-level discrepancy (gap). Any surplus emission from the system increases Net Emissions, which is used to reduce or increase the desired order rate. Under the proposed system, we can expect oscillatory dynamics to the presence of delay within the multiple negative feedback loops.

### 4. Simulation Experiments and Results

The system is set to start in dynamic equilibrium conditions by the careful setting of the simulation parameters. We focus on dynamics arising from POUT, i.e., \((\alpha, \beta) = (1, 1)\) policies with system lead time, \(L\) as 3-time units. The exponential smoothing forecast parameter \(\rho = 0.2\) throughout the simulation study. Initial demand (demand under dynamic equilibrium is assumed to be 5000 units/month). Base Case refers to the dynamics of change in demand and/or permissible emissions without any feedback of emissions on the ordering decision.

We expect that as (exogenous) demand changes (pulse/step), the production release rate will fluctuate but the system will eventually reach equilibrium. A system with PE based feedback (see Section 3.1) can be expected to saturate with a higher service rate (or low unfilled demand) as compared to NE based feedback (see Section 3.2) which continues to reduce PREL until zero net-emission targets are achieved by the system. In contrast, when the permissible emission changes (pulse or step decrease), we can expect the system with Perceived Emissions Feedback to perform better as such a system responds rapidly against the system than with Net Emission Feedback.
4.1 Base Case and Basic Scenarios

The following scenarios are considered for simulation:

- Base Case: No Feedback of emissions to PREL (either Pulse & Step Change in Demand)
- Scenario 1: Pulse Increase in Demand, with emission feedback on PREL
- Scenario 2: Step Increase in Demand, with emission feedback on PREL
- Scenario 3: Pulse Decrease in Emission Permit, with emission feedback on PREL
- Scenario 4: Step Decrease in Emission Permit, with emission feedback on PREL

The above scenarios are modeled in Vensim, and the simulation results are as shown in Figure 4 to 8. In Figure 4, the response of the system to pulse increase (shown in black line) in demand and step increase (shown in green lines) in demand is shown. Once customer demand increases (pulse/step), the production rate also increases as per the classical mechanism of PICS to maintain sufficient supply-line and inventory, and the system moves from a state of dynamic equilibrium to a transition state. Note that for the base case, emission feedback is not incorporated into PREL. The bottom two plots in Figure 4 show the dynamics of stock of Perceived Emission and Net Emission. As expected, without emission feedback, Net Emission increases linearly for a step increase, and a minute change in emission level can be validated for pulse increase in demand. Whereas, the Perceived Emission saturates at a value where the emission rate is equivalent to emission for the desired order rate. The right top graph in Figure 4 shows the unfilled demand. A step-change in demand had an unmet demand for a larger number of periods. However, this was eventually corrected since the PREL stabilized at the required level to meet the increased demand.

![Figure 4: Pulse Increase(black) and Step Increase(green) in demand without Emission Adjusted Orders](image)
Scenario 1 considers Pulse change(increase) in demand but with feedback from Emission Adjusted Orders (EAO) (see Figure 5). Blue curves represent the dynamics obtained when Perceived Emission Feedback is used in computing PREL, and orange curves represent the dynamics obtained when Net Emission Feedback is used in computing PREL. As the system moves away from dynamic equilibrium, higher orders immediately change the desired INV and WIP. Due to the delay from PREL to Product Completion Rate (PCR), the system response will lag in the emission context and require emission adjustment PREL reduction. Instant change in demand will only result in lost sales. Under the pulse demand increase scenario, no stockout will be observed in both Perceived Emission feedback and Net Emission feedback. Net Emission feedback will refrain from negative Emission Adjusted Orders (overshooting PREL) due to tight budget constraints on permissible emission (as Net Emission will never go below zero). Eventually, both Perceived Emission and Net Emission overlap once the system reaches a dynamic equilibrium state.

![Figure 5: Pulse Change(Increase) in demand with EAO dynamics from Perceived Emission(blue plot) and Net Emission(orange plot)](image)

On the contrary, for Step change(increase) in demand, i.e., scenario 2 (see Figure 6), some interesting dynamics include PREL saturates below Customer Demand (CD) due to Emission Adjusted Orders. PE feedback to PREL is represented by the blue curve and the orange curve represents NE feedback to PREL (refer to top-left plot). The saturation level for INV (refer to middle-left plot) and other stock in Perceived Emission (refer to top-right plot) is higher than Net Emission (refer to middle-right plot). Both mechanisms to reduce emission succeed in
achieving the emission target, but the tradeoff is visible. As emission reduction is higher for the Net Emission feedback model but creates a compromise in service-level context with respect to the Perceived Emission feedback (refer to bottom-left plot). A significant difference in emission can be observed in Figure 6 of the Net Emission plot, where Net Emission Feedback has a logarithmic growth, moving towards saturation than linear emission growth in emission for Perceived Emission Feedback (refer to bottom-right plot).

**Figure 6:** Step Change(Increase) in demand with EAO dynamics from Perceived Emission(blue plot) and Net Emission(orange plot)

Under scenario 3 (see Figure 7), we try to understand the resulting dynamics with PE feedback (blue curve) and NE feedback (orange curve) when the permissible emission limit reduces (maybe due to governmental regulations or any other external factor beyond system control). Initial investigation about pulse change(decrease) in permissible emission level suggests that EAO (refer bottom-right plot) will directly come into effect as emission permit has pulse decline. Perceived Emission (refer top-right plot) is first-order exponential smoothing with smoothing constant, $\tau$ to which system depends on the saturation effect. Perceived Emission will have a sudden shortfall as PE estimation takes emission permit into effect to which system responds suddenly by reducing EAO under Perceived Emission feedback without delay (system lag) as compared to Net Emission (refer middle-right plot) which reduces Emission Adjusted Orders slightly. Thus, variability in PREL(refer top-left) will be on the higher side for systems with PE feedback as compared to a system NE feedback system. Thus, the system re-enters
dynamic equilibrium almost at the same time units once perturbation in pulse emission is introduced. Just only for the initial decrease in emission permit demand will not be fulfilled but the early saturation effect of PE will result in the following time period to fulfill all demands (refer to bottom left plot of unfilled demand). But under NE feedback, PREL adjusted on a level of NE with adjustment time $T_e$ which creates a response delay and impact will be visible in terms of the occasions for demand being unfilled will be on the higher side as compared to PE dependent system for EAO.

![Graph showing Pulse Change (Decrease) in permissible emission with EAO dynamics from Perceived Emission (blue plot) and Net Emission (orange plot).](image)

**Figure 7:** Pulse Change (Decrease) in permissible emission with EAO dynamics from Perceived Emission (blue plot) and Net Emission (orange plot)

Figure 8 (below) gives insights to the dynamics of step-change in emission permit can provide some interesting insights. A step-change in permit has an immediate impact on PREL for both PE feedback (in a blue curve) and NE feedback (in an orange curve) which can be visualized from the top-left plot. PE feedback cause oscillations to PREL for a while and eventually stabilizes whereas, NE feedback causes goal-seeking behavior to PREL due to a zero-emission target. PE plot (refer top-right) and NE plot (refer middle-right) represent perception about perceived emission and net emission respectively based on step emission decrease as a perturbation. Net Emission will continue to increase linearly under PE feedback against very steady growth in NE feedback to the system. Emission Adjustment Orders (refer bottom right) will be higher for NE-Feedback as compared to PE-Feedback. The discrepancy
between both Perceived Emission/ Net Emission and emission permits will refrain from ordering Desired Order Rate (DOR) as some adjustment due to EAO is required. Order unfulfilled (refer bottom-left plot) will be higher for NE feedback system as compared PE feedback system due discrepancy continues to increase in the earlier case and later case PE saturates to emission permit.

![Figure 8: Step Change(Decrease) in permissible emission with EAO dynamics from Perceived Emission(blue plot) and Net Emission(orange plot)](image)

In scenarios 4 and 5, the dynamics of step-change in emission permit can provide some interesting insights (refer to Figures 7 and 8). Dynamics suggests and based on an intuitive understanding that with a step/pulse-change in emission, PE change immediately without any delay depending upon smoothing rate $\tau$. From the Net Emission point-of-view, PE already ignores history about emission. Instead, it uses exponential smoothing at a prefixed rate. Stock for Net Emissions will continue to increase in PE feedback until the system reaches a state of dynamic equilibrium. On the contrary, NE feedback will continue to reduce order until Net Emission Stock reaches zero-level.

Finally, similar to scenario 4, the system will respond to PE feedback by stabilizing or saturating Perceived Emission to step-changed emission in scenario 5. Net Emission will continue to increase linearly under PE feedback against very steady growth in NE feedback to the system. Emission Adjustment Orders will be higher for NE-Feedback as compared to PE-Feedback.
4.2 Sensitivity Analysis of $T_e$ and $(\alpha, \beta)$

The sensitivity of the parameters $T_e$ and $(\alpha, \beta)$ on the inventory dynamics under emission feedback is analyzed. The following scenarios are used to indicate the same.

- Scenario 5: Step Change in Demand and varying time to adjust emission, $T_e = 7,5,3$
- Scenario 6: Sensitivity of $(\alpha, \beta)$ on PREL under Perceived Emission Feedback and Net Emission Feedback under Pulse change in Demand
- Scenario 7: Sensitivity of $(\alpha, \beta)$ on PREL under Perceived Emission Feedback and Net Emission Feedback under Pulse change in Emission Permit

$T_e$ is time to adjust emission feedback into the system and rate of emission adjustment, 
$\tau = \frac{1}{T_e}$. In the base setup above for scenarios 1, 2, 3, and 4, we considered $T_e = 5$ and $\tau = 0.2$. In Figure 9-11 the dynamics of PREL (or ordering rate) for the system under Perceived Emission Feedback (left) and Net Emission Feedback (right) are shown. Under step increase in demand (scenario 2 above), Perceived Emission Feedback treats emission permits as a soft constraint, on the contrary, Net Emission Feedback creates hard constraint into the system. Therefore, Figure 6 gives a basic idea about expected dynamics when $T_e$ varies over $(7, 5, 3)$. A system with emission feedback from Perceived emission will cause the system to saturate PREL marginally below Desired Order Rate. Whereas, a system with Net Emission feedback will saturate PREL back to the level where the system kicked off from dynamic equilibrium. In both feedbacks system reaches dynamic equilibrium eventually, but the latter system has higher unfilled additional demands as compared to the earlier system with Perceived Emission feedback which has a relatively very high service level.

As $T_e$ changes from 7 to 3 (see Figure 9), PREL with Net Emission feedback will reach an equilibrium state as less time is required to adjust net emission. Lower $T_e$ implies higher $\tau$ giving higher weightage to recent information about emission as compared to past information. Thus such a system will have relatively some fractional reduced emission as compared to the low value of $\tau$ (or higher value $T_e$).

Figure 10 represents dynamics of sensitivity analysis for Scenarios 6 and 7 discuss the sensitivity of WIP adjustment rate and Inventory adjustment rate for PE feedback (shown by blue curve) and NE feedback (shown by orange curve) to PREL. We assumed $\alpha$, work-in-process adjustment rate and $\beta$, inventory adjustment rate to vary from $[(0.25, 0.25), (0.5, 0.5), (0.75, 0.75), (1,1)]$ Under $(\alpha, \beta) = (1, 1)$, ordering policies are known as Pure Out Policies (POUT), any error between the desired value and actual value of the stock (WIP and Inventory) is adjusted fully as compared to $(\alpha, \beta) \neq (1,1)$ known as generalized ordering policies where partial adjustment is considered. The magnitude of dynamics for any perturbation from dynamic equilibrium state depends on $(\alpha, \beta)$ (thus, they are
considered to be system control parameters). In scenario 6, a pulse change in demand (in Figure 10 above) reaches near (1,1) on DE line PREL with Perceived Emission feedback and Net Emission feedback overlapping each other. A lower value for \((\alpha, \beta)\) in DE line implies the system is relatively slow in response to error state correction. Thus, highly responsive control parameters will have overlapping dynamics for PREL under the feedback.

**Figure 9:** PREL dynamics under Perceived Emission (left) and Net Emission (right) for \(T_e \in \{7, 5, 3\}\)

**Figure 10:** Pulse change in Demand for PREL dynamics under varying \((\alpha, \beta)\) with Perceived Emission and Net Emission Feedback
From figure 11 PE feedback (shown by blue curve) and NE feedback (shown by orange curve) to PREL under pulse change in emission permit considered for varying $(\alpha, \beta)$. A visualization for the change in adjustment rate on pulse change in emission permit has an almost insignificant impact on PREL under both PE and NE feedback. For EAO with NE feedback, the system will reach equilibrium after perturbation with low variability in PREL compared with PE feedback.

![Figure 11: Pulse change in Emission Permit for PREL dynamics under varying $(\alpha, \beta)$ with Perceived Emission and Net Emission Feedback](image)

### 6. Conclusion and Future Work

Sustainability requires considering aspects of economic, environmental, and social dimensions. Existing supply chain literature weighs environmental and economic aspects, but social aspects are ignored. We consider a green supply chain where we look forward to minimizing environmental, specifically emission impact from operational activities. For improving the green supply chain, capital investment, technological innovation, and capacity expansion seem to result in reduced emission. The non-existence of literature on emission feedback to ordering policies motivated us to explore and design for a system with emission feedback.
The proposed model uses system emission feedback to the APIOBPCS model to understand the impact of emissions and ordering dynamics. Feedback from emissions is considered in the order based on two pathways: (i) Perceived Emissions (PE) and (ii) Net Emissions (NE). The models were built and simulated using Vensim. Preliminary experiments were conducted to understand the dynamics arising from pulse and step change in demand and permissible emission. Initial results confirm that PE feedback fails to reduce system emission, as satisfying demand seems to be the primary objective of achieving the reduced emission target under step change(increase) in demand. An argument about such dynamics can be due to first-order exponential smoothing information making emission objective a soft constraint. Another setup discusses using net emission as feedback to adjust future order rates, where a change in emission permit causes PE to respond immediately without any delay involved. A system under PE feedback for pulse/step change in emission permit performs better for emission reduction. In contrast, under pulse/step change in demand, NE feedback performs better from the operational aspect to reduce system emission.

A trade-off between both service-level and emission reduction is quite evident. Since this proposed study is part of ongoing research, we look at trade-offs as a part of future work. Further, we try to integrate both models into a single environmental module for feedback using different weights for the level of emission information and a different look at the trade-off between system performance in economic and environmental aspects.

We know from the literature that the short-term objective of using an operational adjustment to have a sustainable supply chain can create havoc on the system. Ongoing work is to expand the scenario to a multi-player supply chain setting to understand the dynamics of different players adjusting for emission independently. Further, we plan to extend the above model to account for capacity expansion, technological innovation, and capital investment which reduces the per-unit emissions on the longer terms. The eventual goal is to develop key insights on managing supply chain emissions towards a net-zero target.

References


