A Transformation of the Growth Cycle into the Industrial Cycle in a Four-Dimensional Goodwinian Model with Leontiev Technology

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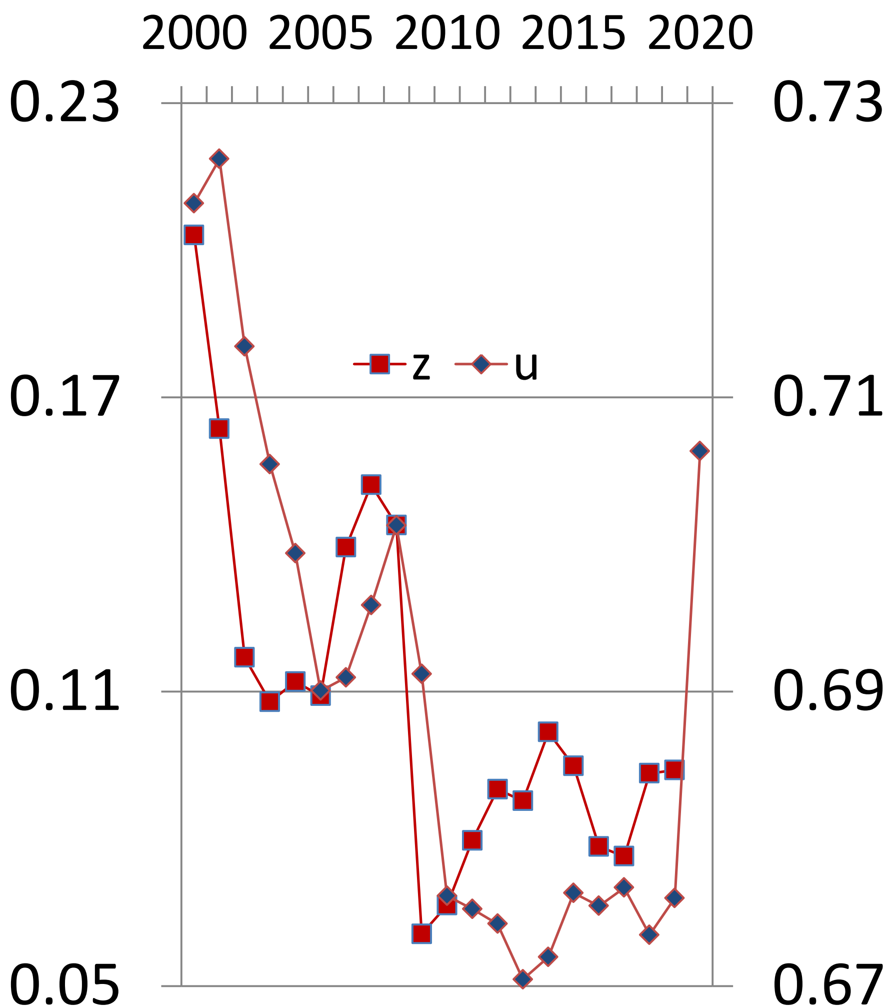
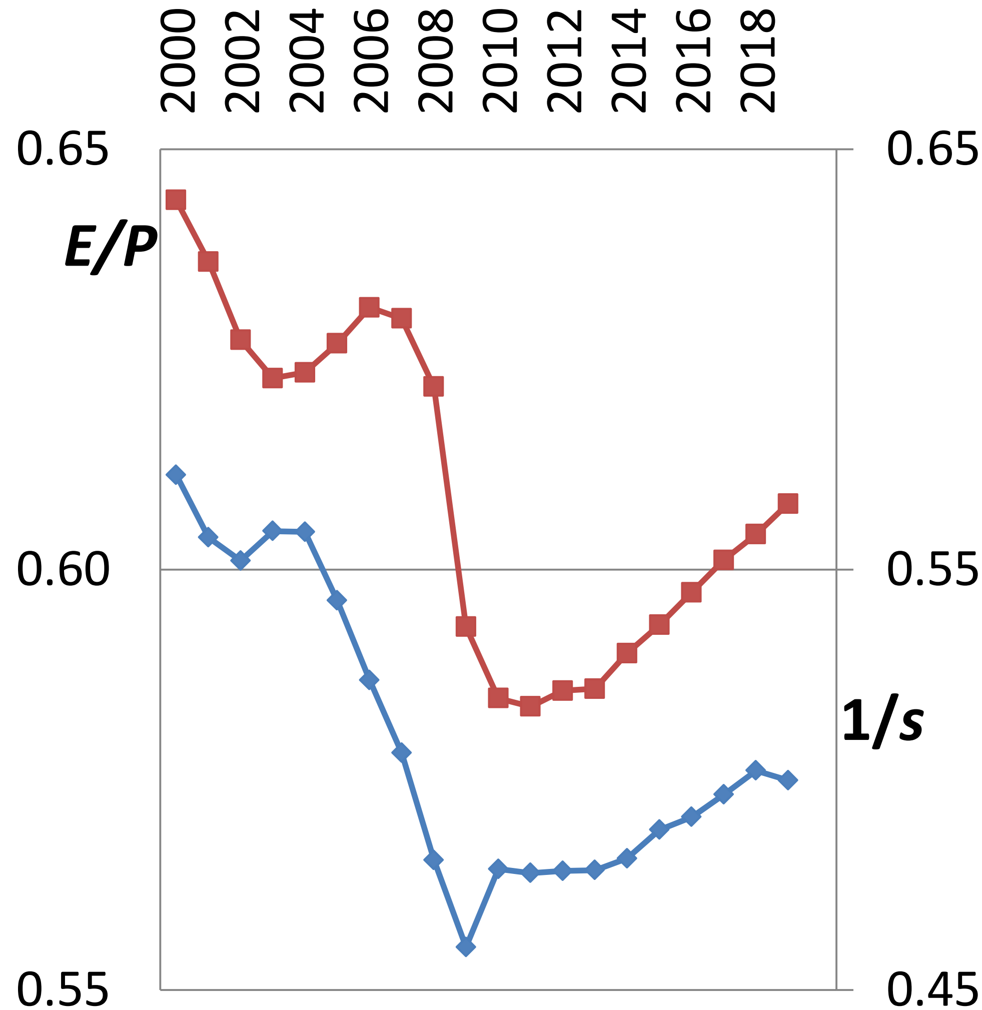
The left is winning the economic battle of ideas. This raises the issue of policy resistance

- *…working out how to meet the public's legitimate expectations of a leftward shift in economic policy without undermining opportunities for growth will be the great economic experiment of the post-pandemic world.* The Financial Times 29 4 2021
- *All too often, well-intentioned efforts to solve pressing problems lead to policy resistance, where our policies are delayed, diluted, or defeated by the unforeseen reactions of people or of nature.* John Sterman. Business dynamics. 2000, p. 3

Accumulation rate *z* 1980-2019, relative labour compensation *u* 1980-2020; employment to non-instit. population ratio *E/P*, output-capital ratio *1/s*, the USA, 1980-2019





Abstract

- A three-dimensional Goodwinian model L-1, containing the greed feedback loops, reflects destabilizing cooperation and stabilizing competition of investors. The growth rate of output per worker directly depends on growth rate of fixed capital whereas the capital-output ratio is constant.
- Oscillations imitating growth cycles are endogenous. A crisis is a manifestation of relative and absolute over-accumulation of capital.
- A knife-edge limit cycle maintains a growth cycle with the Kondratiev duration; a more structurally solid one with a period of about 7.5 years reflects business cycle except reduced net output in a recession. These limit cycles result from the subsequent supercritical Andronov – Hopf bifurcations.

Abstract (continued)

- The transformation of the growth cycle into industrial cycle gives credit to raising status of capital-output ratio from auxiliary in L-1 to the level status (phase variable) in four-dimensional L-2.
- The latter includes new 11 intensive feedback loops. The model implements proportional and derivative control over the capital-output ratio by owners of fixed capital.
- A pair of supercritical Andronov – Hopf bifurcations cause two limit cycles. The second (structurally unstable) is a remote analogue for Kuznets cycle with the period of about 18 years; the first (structurally stable) upholds the industrial cycle with period of about 7 years and declining output in the outright crisis.
- Stronger capital monopoly power unchecked by the society leads to long-term decline in relative labour compensation, rate of capital accumulation and output-capital ratio.
- This necessitates hardening labour countervailing strength for turning these tendencies around.

Demonstrating how detrimental for workers the monopolization tendency could be if left unchecked by countervailing power

- The paper emphasizes the link between ever stronger monopolies and declining labour share in net output accompanied by growing profitability and atrophying net domestic investment.
- This paper calls for setting national goals for the rate of capital accumulation and output-capital ratio besides targeting employment ratio emphasised in Ryzhenkov (2020, 2021).
- “The question today is whether we too can think big and act big.” Janet L. Yellen on A Better Deal for Americans May 18, 2021

Ancestors of L-1 and L-2

NBER working papers shed light on monopolization.

R.M. Goodwin ‘s (1972) structurally unstable predator – prey model M-1. The main variables are relative wage and employment ratio, a rate of capital accumulation (ratio of investment to profit) and capital-output ratio are constant.
F. Lordon (1995) addition of productivity scale effects and implicit rate of capital accumulation to M-1. A growth rate of output per worker positively depends on growth rate of fixed capital. When a growing relative wage exceeds a threshold both fixed capital and net output decline. The acuteness of over-accumulation in crises is hyperbolized in the bi-dimensional model.
Ryzhenkov (2021) refines the above models, particularly, by adding a flow and ceiling to the rate of capital accumulation. Growth cycle is endogenous and structurally stable.

Table 1. The main variables of L-1 & L-2

Real net output	*P*	mlrd. $ 2009/year
Employment	*L*	mln. workers
Labour force	*N*	mln. workers
Output per worker	$a = P/L$	mln $ 2009/(worker*year)
Employment ratio	$v = L/N$	fraction of unit
Fixed capital (net)	*K*	mlrd. $ 2009
Worker’s real wage	*w*	mln $ 2009/(worker*year)
Unit value of labour power (relative wage)	$u = w/a$	fraction of unit
Profit, surplus product	*M*	mlrd. $ 2009/year
Surplus value	*S*	mln. workers
Accumulation rate	*z*	fraction of unit
Investment	$dK/dt = zM$	mlrd. $ 2009/year

L-1

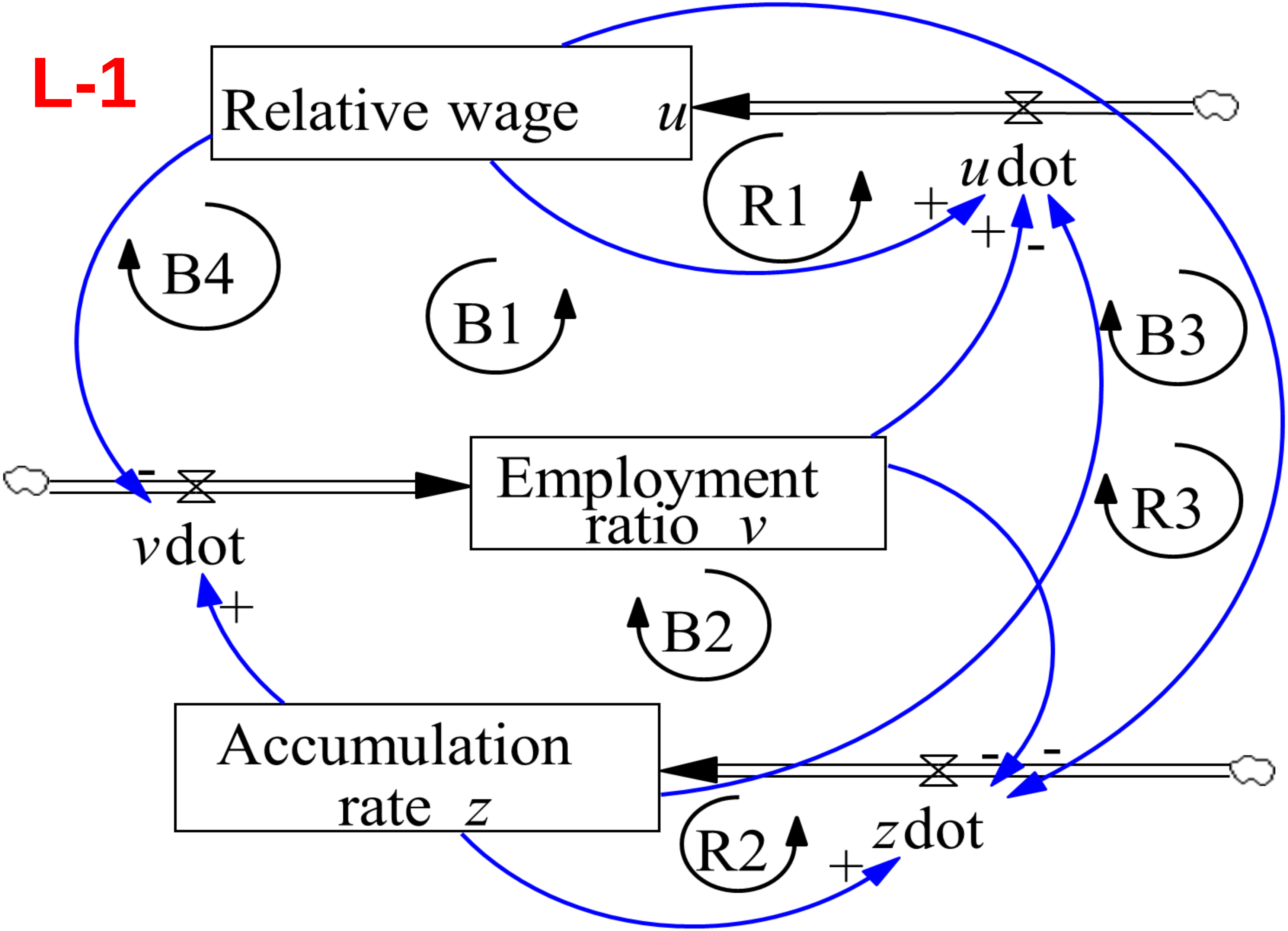
In agreement with K. Marx (1867: 634) net change of the share of investment in surplus product has an opposite sign to relative wage gains ($b > 0$, $0 < z_b < z_0 \le Z \le 1$);

logic of logistic growth and proportional control by monopoly capital applied additionally ($p > 0$):

$$\dot{z} = -b \frac{\dot{u}}{1-u} z(Z - z) + p(z_b - z)$$

L-2 generalizes this equation together with s = const.

L-1



Relative wage u
u dot
R1
B4
B1
B3
Employment ratio v
v dot
R3
B2
Accumulation rate z
z dot
R2

Conditioned structural stability of L-1

***Proposition* 1**. Stationary state $E_b = (u_b, v_b, z_b)$ is locally asymptotically stable for $b \le b_0$;

E_b loses its stability and two supercritical Andronov – Hopf bifurcations take place:

the 1st (structurally stable) at $b_{critical} > b_0 > b_{3,}$

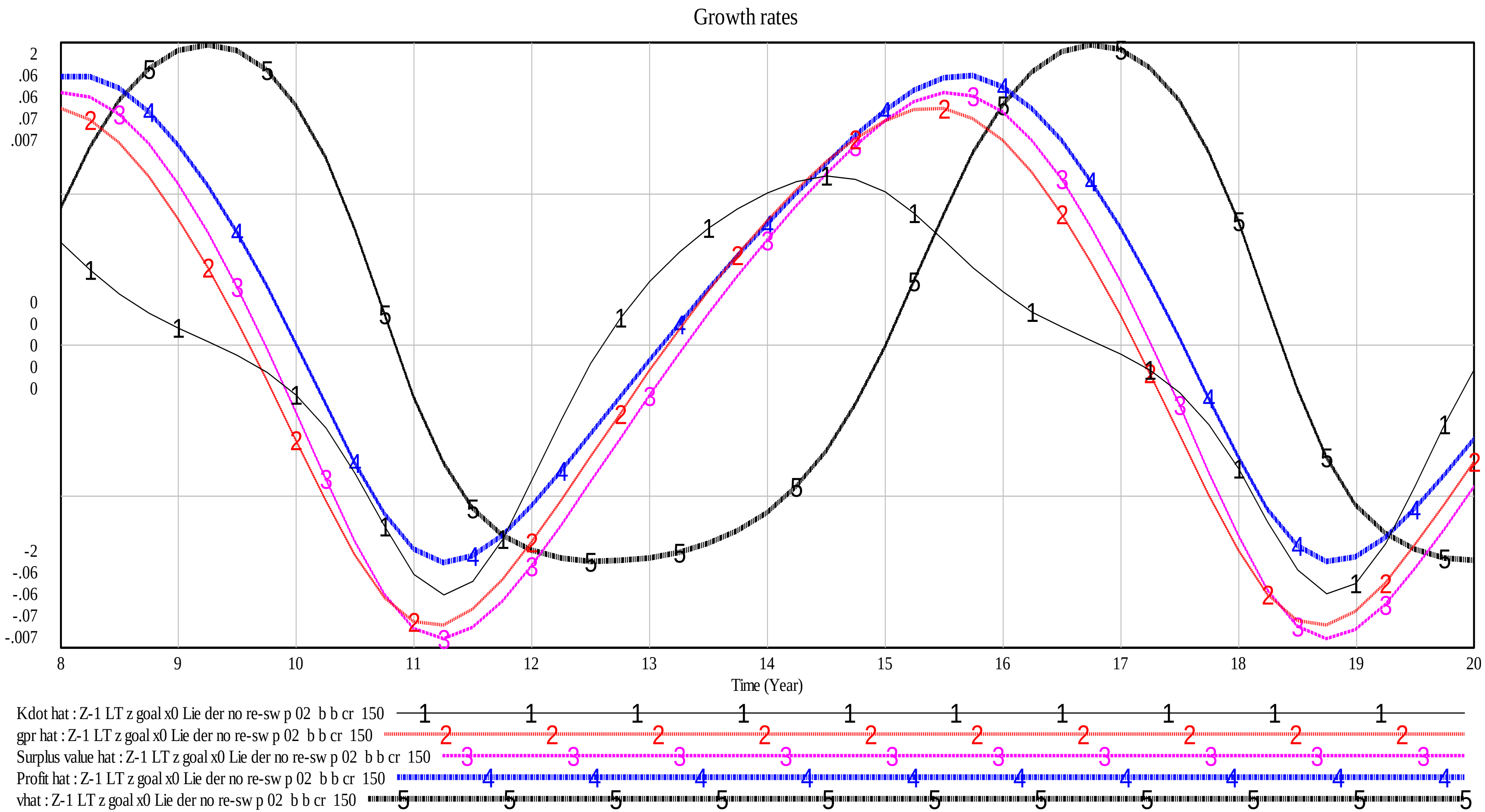
the 2nd (structurally unstable) at $0 < b_{critical} < b_3$.

The period of oscillations on limit cycles is about ≈ 8 and 41 (years), respectively.

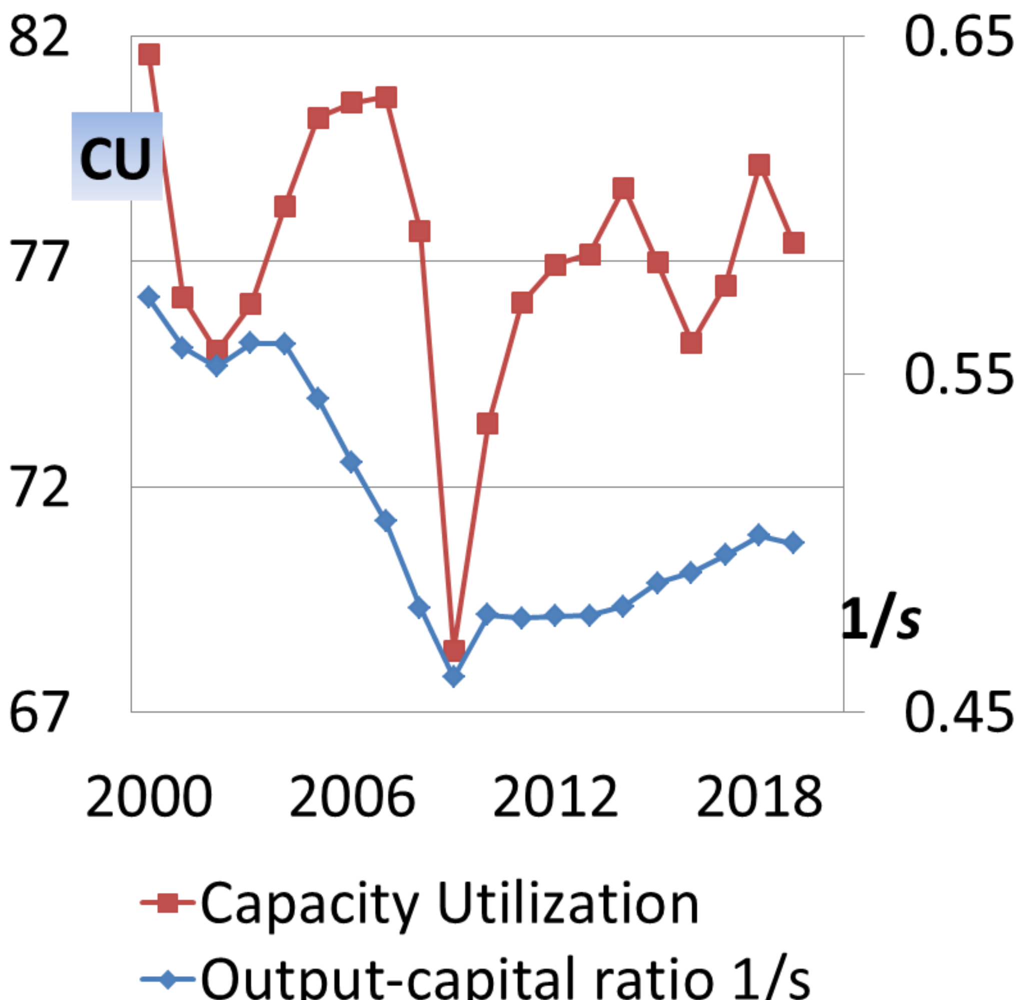
***Proposition* 2**. For sufficiently high $0 < z_b < Z \le 1$ only a limit cycle of the 1st type remains at $b_{critical} > b_0 = b_3$.

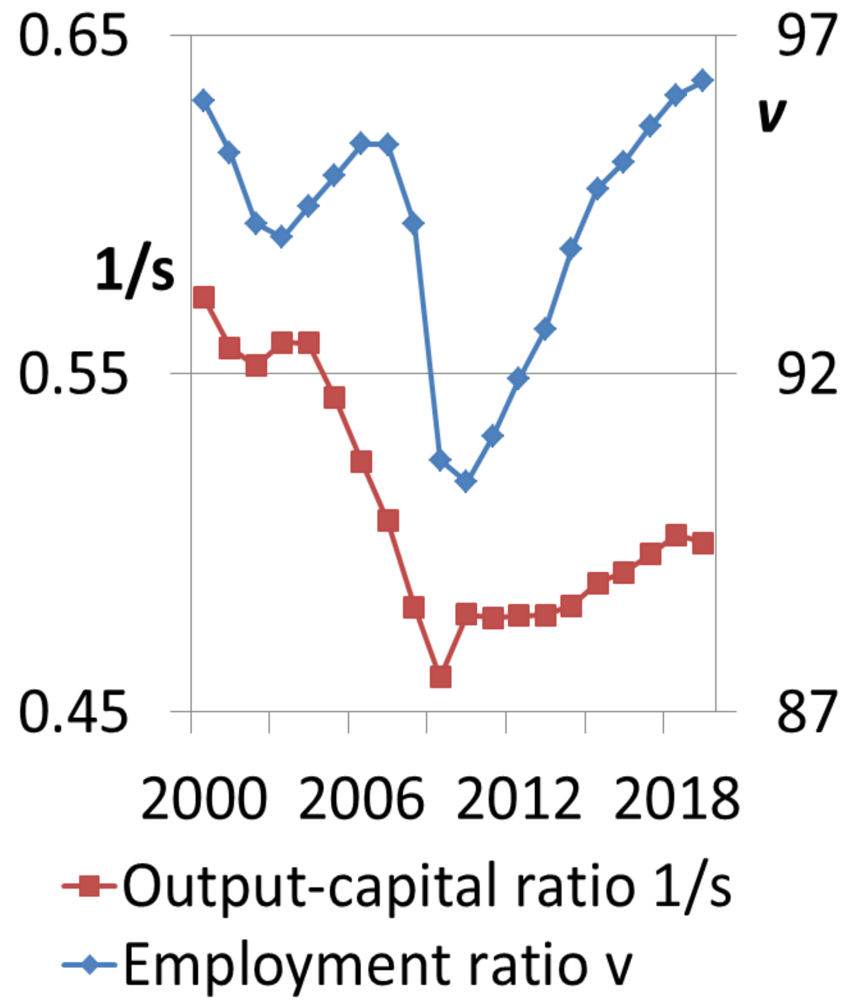
During growth cycles net output P does not decline. Indicators of reproduction on the increasing scale (profit, surplus value, their rates, accumulation rate & other indicators) fluctuate coherently.

Growth rates of investment (1), profit rate (2), surplus value (3), profit (4), and employment ratio (5) over the growth cycles along the limit cycle (related to the 1st AHB) in L-1, years 8–20



Output-capital ratio 1/*s* as indicator of total capacity utilization CU positively correlated with employment ratio *v*, the USA, 2000-2019





Derivative and proportional control by monopoly capital over capital-output ratio *s* in L-2

$$\dot{u} = \left[f(v) - \alpha - \gamma \frac{z(1-u)}{s} \right] u,$$

$$\dot{s} = [j_1 \hat{v} + j_2 (s_b - s)] s,$$

$$\dot{v} = \left[\frac{1-\gamma}{1+j_1} \frac{z(1-u)}{s} - \frac{j_2}{1+j_1} (s_b - s) - \frac{\alpha + \beta}{1+j_1} \right] v,$$

$$\dot{z} = b \left(-\frac{\dot{u}}{1-u} - \hat{s} \right) z (Z - z) + p(z_b - z) =$$

$$= b\hat{R} z (Z - z) + p(z_b - z),$$

where the growth rate of profit rate is

$$\hat{R} = -\frac{\dot{u}}{1-u} - \hat{s}; \; -1 < j_1 < 0, \; j_2 > 0.$$

New 1st, 2nd & 3rd order feedback loops in L-2

Order	Positive	Negative
1	$s \longrightarrow \dot{s}$	-
2	$s \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{s}$	-
	$s \rightarrow \dot{u} \rightarrow u \rightarrow \dot{s}$	-
3	$s \xrightarrow{-} \dot{v} \rightarrow v \xrightarrow{-}$ $\xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{s}$	$s \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-}$ $\xrightarrow{-} \dot{u} \rightarrow u \rightarrow \dot{s}$
		$s \xrightarrow{-} \dot{v} \rightarrow v \rightarrow$ $\rightarrow \dot{u} \rightarrow u \rightarrow \dot{s}$
		$s \rightarrow \dot{u} \rightarrow u \xrightarrow{-}$ $\xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{s}$

New 4th order negative feedback loops in L-2

$s \xrightarrow{-} \dot{v} \rightarrow v \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{u} \rightarrow u \rightarrow \dot{s}$
$s \rightarrow \dot{u} \rightarrow u \xrightarrow{-} \dot{v} \rightarrow v \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{s}$
$s \xrightarrow{-} \dot{z} \rightarrow z \rightarrow \dot{v} \rightarrow v \rightarrow \dot{u} \rightarrow u \rightarrow \dot{s}$
$s \xrightarrow{-} \dot{v} \rightarrow v \rightarrow \dot{u} \rightarrow u \xrightarrow{-} \dot{z} \rightarrow z \xrightarrow{-} \dot{s}$

Conditioned structural stability of L-2

***Proposition* 3**. Stationary state $E_b = (u_b, v_b, s_b, z_b)$ is locally asymptotically stable for $0 < b_3 \le b \le b_0$.

E_b loses its stability and two supercritical Andronov – Hopf bifurcations take place:

the 1st (structurally stable) at $b_{critical} > b_0 > b_3$,

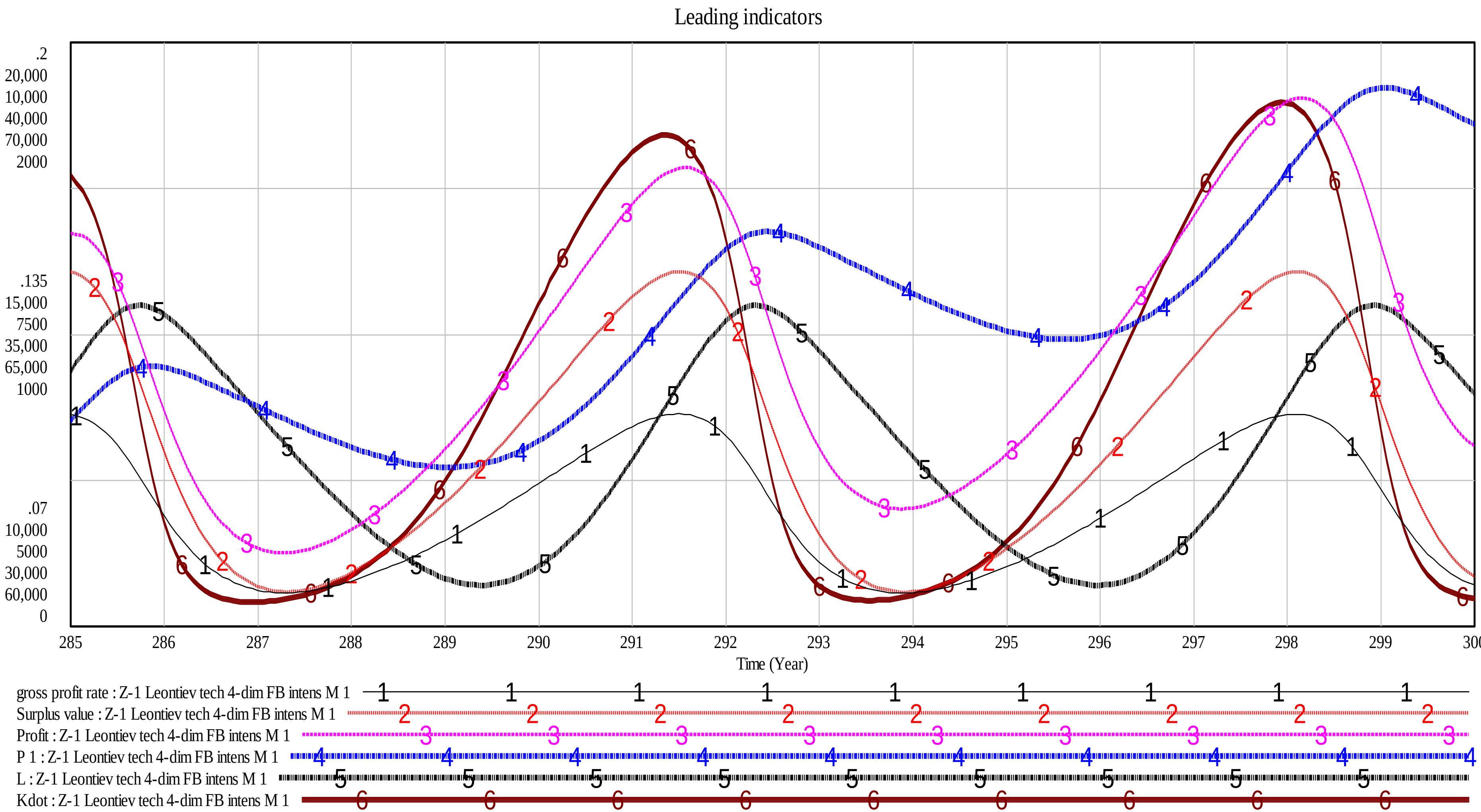
the 2nd (structurally unstable) at $0 < b_{critical} < b_3$.

The period of oscillations on limit cycles is about ≈ 7 and 18 (years), respectively.

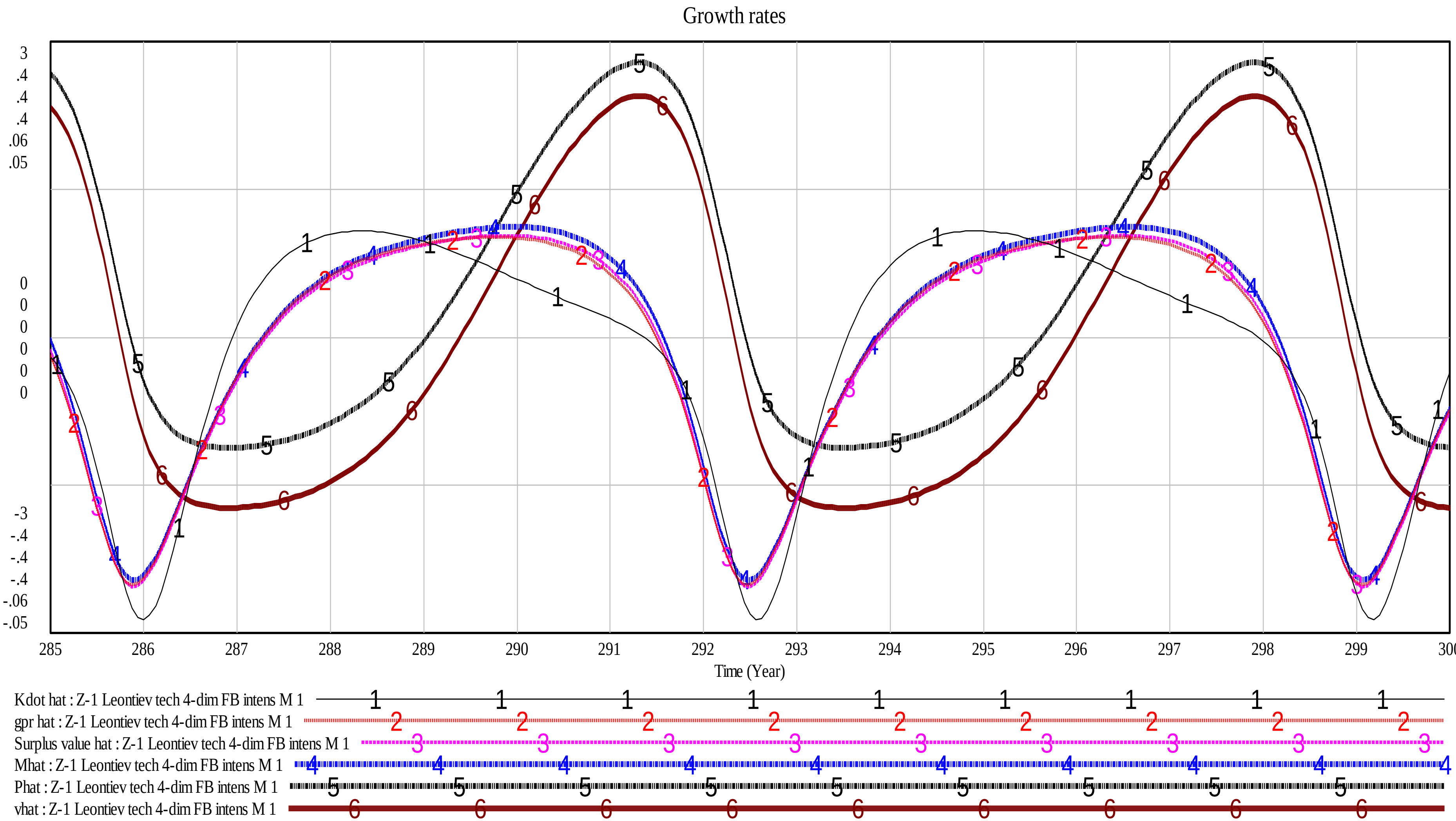
***Proposition* 4**. For sufficiently high $0 < z_b < Z \le 1$ only a limit cycle of the 1st type remains at $b_{\mathrm{neg}} << 0 < b_0 = b_3 < b_{critical}$.

During industrial cycles net output P declines. Indicators of reproduction on the increasing scale (profit, surplus value, their rates, accumulation rate & other indicators) oscillate consistently and comprehensibly.

Leading, coinciding & lagging indicators of industrial cycles due to 1st AHB in L-2: profit rate (1), investment (6), surplus value (2), profit (3), net output (4), and employment (5) in L-2, 285–300 y.

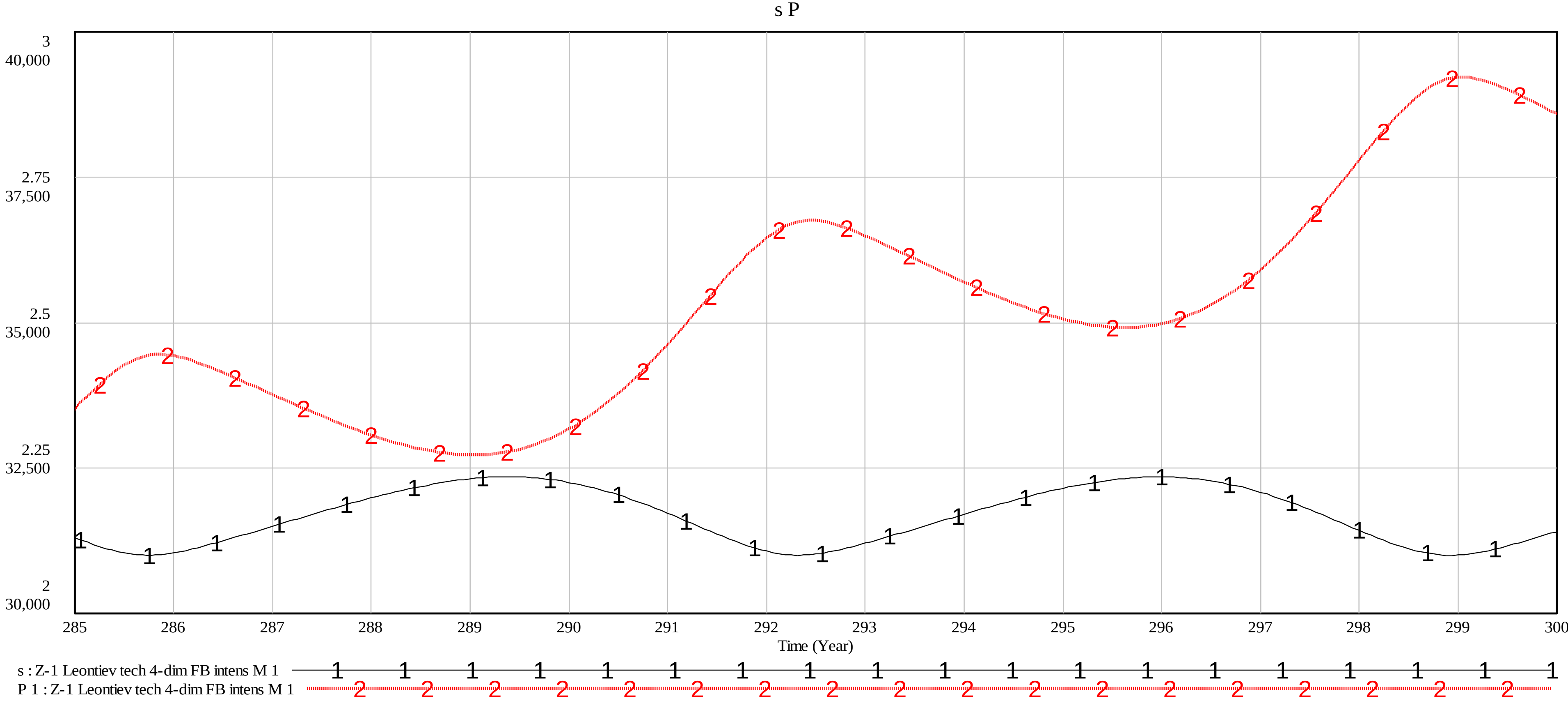


Growth rates of economic indicators in industrial cycles due to 1st AHB in L-2: investment (1), profit rate (2), surplus value (3), profit (4), net output (5), employment ratio (6), 285–300 y.



Phases of two adjacent industrial cycles (years). Counter-phases of net output *P* (2) and capital-output ratio *s* (1) in industrial cycles due to 1st AHB in L-2, years 285–300

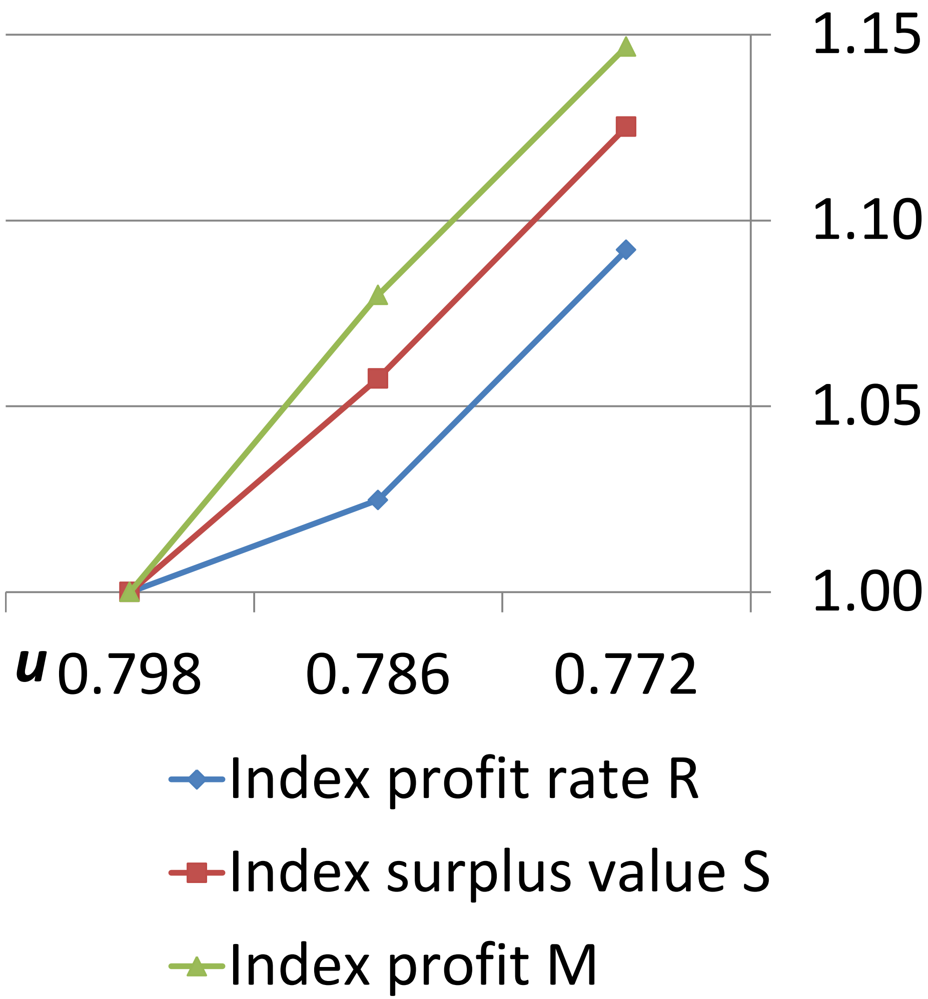
Crisis	Depression	Recovery	Boom
292.5–295.5	295.75	295.75–297.5	297.5–299



Industrial cycles in L-2: indicators’ extreme growth rates

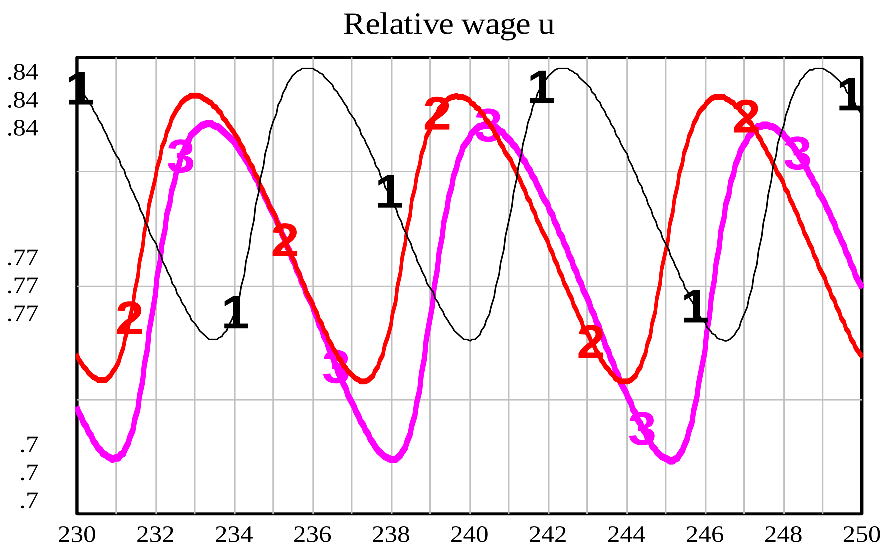
	Boom started 291					**Crisis** 292.5–295.5		**Depression** 295.75	**Recovery** 295.75–297.5			**Boom** 297.5–299	
	289	291.25	291.5	292.25	292.5	293.5	294	295.75	23.25	296.5	23.75	298	298.25
P	0	max	↓	↓	0	min	↑	0	↑	↑	↑	max	0
(1–*u*)/*s*	↑	max	0	↓	min	↑	0	↑	↑	max	↓	↓	0
(1–*u*)*L*	↑	max	0	↓	min	min	0	↑	↑	max	↓	↓	0
(1–*u*)*P*	↑	max	0	↓	min	min	0	↑	↑	max	↓	↓	0
v	↑	max	↓	0	↓	min	↑	↑	↑	0	↑	max	↓

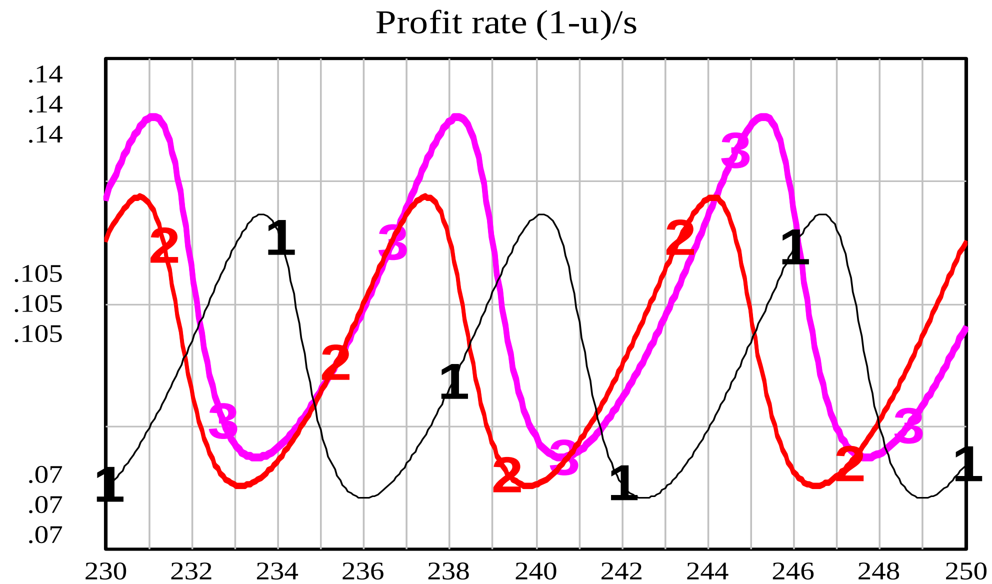
Effects of step-wise changes in targeted capital-output ratio s_b & capital accumulation rate z_b on relative wage u, profit rate $(1-u)/s$, surplus value S, and profit M averaged over 2030-2050 against basal run with index = 1 in L-2

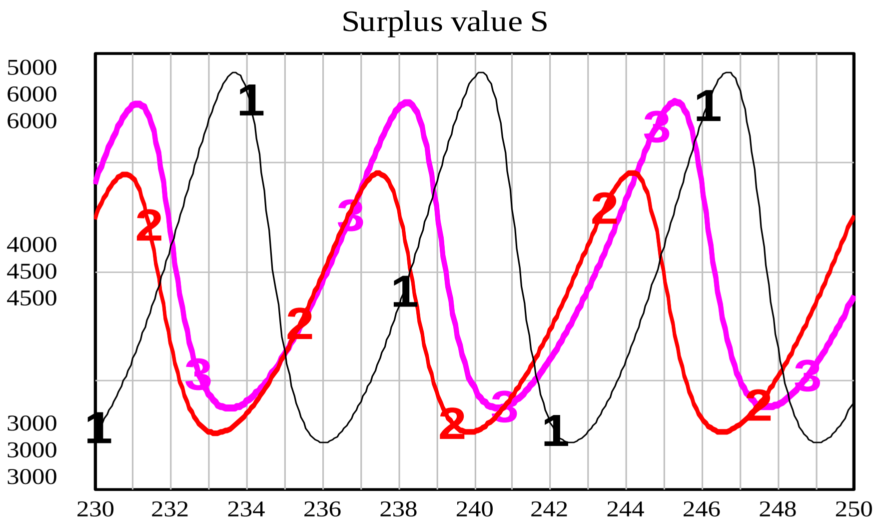


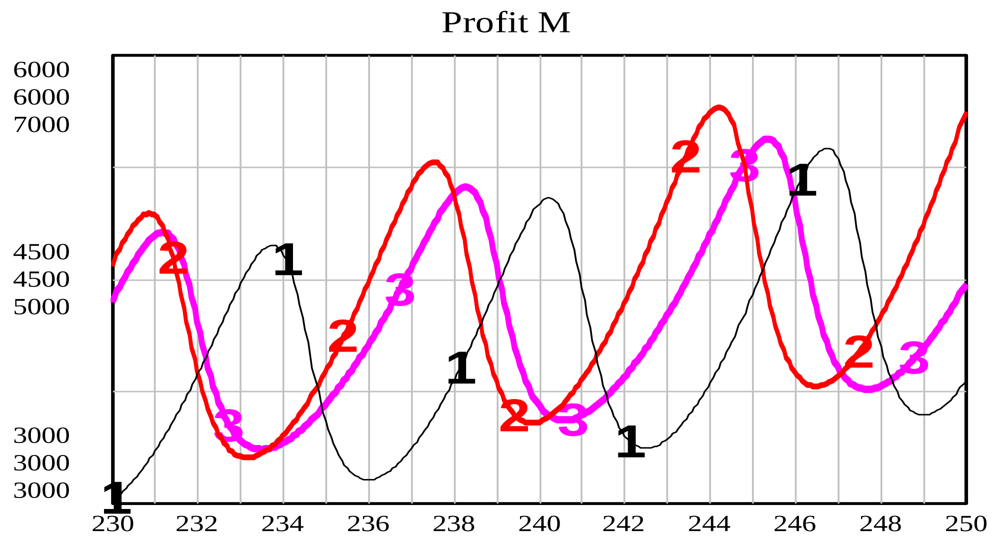
Statio-nary	Basal run 1	Run 2	Run 3
s_b	2.17	2.34	2.34
z_b	0.068	0.068	0.05
u_b	0.675	0.649	0.539

Effects of step-wise changes in targeted capital-output ratio s_b & accumulation rate z_b on relative wage u, profit rate $(1-u)/s$, surplus value S, profit M in runs 1, 2 and 3 in L-2, y. 230-250









Conclusion

The developing Marxist theory of capital accumulation has to be maintained by vanguard system dynamics methodology and by mathematical bifurcation theory.

The research carried out on these foundations ascends from growth cycle in L-1 to more relevant industrial cycle in L-2.

The proposed models explain where their substantial differences come from by demonstrating specific stock-and-flow structures, revealing particular feedback loops and by going through the elaborated simulation experiments for the theoretical models.

All these outcomes have prepared new, more empirically oriented, strides forward.

The outlined crude reality checks of the practical relevance of L-1 and L-2 are to be developed into more elaborated statistical tests in the enduring research.

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