Improving Loops that Matter

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Abstract

The Loops that Matter approach to understanding behavior has proven easy to use and broadly applicable, but it has a shortcoming in its original formulation in that it does not give the same results when flows equations are combined or separated. This is because the original formulation treats the impact of a flow on a stock relative to the net flow, so that all scores tend to get very large in magnitude as a stock approaches equilibrium, but how big depends strongly on how the flows are specified. By reformulating the link scores from a flow to stock as the score from the flow to the net flow for the stock, this topological dependency is removed. Using this approach makes it easier to see how two loops, especially balancing and reinforcing loops, can work together to achieve an equilibrium or steady state. This makes the analysis of models showing a transition to a steady state both easier and more insightful. In addition, the mathematics behind this approach lines up more closely with the Pathway Participation and Loop Impact analysis methods making the relationship among these different approaches clear. The result of this, when applying the analysis to a variety of models, is that the determination of the structure responsible for behavior is clearer, and more clearly tied to work already done using other techniques.

Introduction

Understanding models, and therefore reality, from a structure-based feedback perspective is of tantamount importance to the System Dynamics method. For over 50 years the field has worked to develop tools and methods to perform automated, objective, loop dominance analysis (Graham, 1977; Forrester 1982; Eberlein, 1984; Davidsen, 1991; Mojtahedzadeh, 1996; Ford, 1999; Saleh, 2002; Mojtahedzadeh et al., 2004; Goncalves, 2009; Saleh et al., 2010; Kampmann, 2012; Hayward and Boswell, 2014; Moxnes and Davidsen, 2016; Oliva, 2016; Sato, 2016; Hayward and Roach, 2017; Naumov and Oliva, 2018; Oliva, 2020; and Schoenberg et. al, 2020). A recent development in that long-standing stream of research is the invention of the Loops That Matter (LTM) method (Schoenberg et. al, 2020) and for the first time, the inclusion of an automated loop dominance analysis method (LTM) in commercially available software (Stella Architect & Professional 2.0) (Schoenberg & Eberlein, 2020). LTM advanced the state of the art, not only in its ability to analyze a wide range of models including those with discrete and discontinuous elements, but also by enabling powerful visualizations including animated stock and flow diagrams, as well as algorithmically generated, machine simplified, animated causal loop diagrams (Schoenberg, 2019, Schoenberg & Eberlein, 2020).

The LTM method uses link scores (a measure of a links contribution to behavior) across not only individual links between auxiliaries, but also across links between flows and their stocks while still allowing those scores to be chained together via multiplication. To support this, LTM has two methods for measuring the link score, one for instantaneous connections (stock/auxiliary/flow to auxiliary /flow) and one for integration-based connections (flows to stocks). While these two forms for the link score were designed to measure the same concept, the specific mathematical steps taken to compute them, is different to account for the impact of the integration.

In the course of continued experimentation, we discovered that the original LTM analysis is sensitive to the structure of the flows into a stock. This means that choices made about the aggregation of flow components (for example combing two flows into a net flow) can make a significant difference to the results of the analysis.

We solve this shortcoming by updating the method used to measure the link score between a flow and a stock and demonstrate the efficacy of the improved formulation. We will show that the cause of the identified shortcoming in LTM is a product of the original formulation of the flow to stock link score being based on the value of the flows rather than the change in value. Furthermore, this paper will present an updated analysis of LTM relation to existing automated loop dominance analysis techniques such as the Pathway Participation Metric (Mojtahedzadeh et. al, 2004) (PPM), and the Loop Impact method (Hayward & Boswell, 2014).

Problem Demonstration

The link score is a measure which approximates the link gain and measures the *"... contribution of a value change in an independent variable to a value change in a dependent variable and also the associated polarity* [of that relationship]" (Schoenberg et. al, 2020). Link scores are measured for all links in the network of model equations, including those which exist between flows and stocks (and therefore represent the integration process). The original method for measuring the link score of a flow (i for inflow, o for outflow) to stock (S) relationship is reproduced below as Equation 1:

$$Inflow: LS(i \rightarrow S) = \left(\left|\frac{i}{i-o}\right| * 1\right) \quad Outflow: LS(o \rightarrow S) = \left(\left|\frac{o}{i-o}\right| * -1\right) \tag{1}$$

In Equation 1 the contribution of a flow to the change in behavior of a stock is the portion of the net change in the stock resulting from the flow under analysis.

This is in contrast to Equation 2 which defined the link score x → z where z is a flow or auxiliary defined by the equation $z = f(x, y)$

$$LS(x \rightarrow z) = \begin{cases} \left(\left|\frac{\Delta_x z}{\Delta \mathrm{z}}\right| \cdot sign\left(\frac{\Delta_x z}{\Delta \mathrm{x}}\right)\right), \\ 0, \qquad \Delta \mathrm{z} = 0 \text{ or } \Delta \mathrm{x} = 0 \end{cases} \tag{2}$$

The first term in Equation 2 measures the contribution of x to z by reporting the proportion of the change in z which originated from x, where the partial change in z, $\Delta_x z$, is the change in z due to x alone with y held constant. The second term measures the polarity of the link using Richardson's (1995) method. For an in-depth discussion see *Defining link scores for links without integration* in Schoenberg et. al, 2020.

Let's now demonstrate the problem with the original flow to stock form of the link score shown in Equation 1 using a simple model with one stock (S), one inflow (in) and one outflow (out) with the values shown in Table 1:

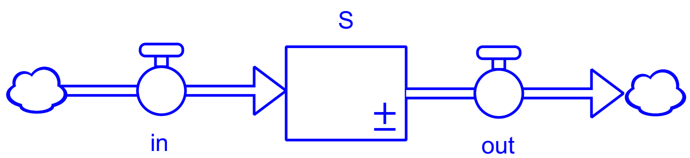

Figure 1: Diagrammatic depiction of system demonstrated in Table 1

Table 1: Link score between out and S with a disaggregated flow structure using Equation 1

Variable	Time 1	Time 2	Link score magnitude Time 2
in	5	10	$\left\|\frac{in}{in - out}\right\| = \frac{10}{5}$
out	4	5	$\left\|\frac{in}{in - out}\right\| = \frac{5}{5}$
$\boldsymbol{S = \int (in - out)}$ $initial = 100$	101	106	-

Now let's change the network structure of the model but keep it mathematically identical by aggregating the flows into a net flow (net) and therefore turning the variables in and out into auxiliaries. To get a value which is comparable to the link score between out and s compute the link score from out to net which is shown in Table 2 (the link between net and S is 1 so does not change the result when going to the stock).

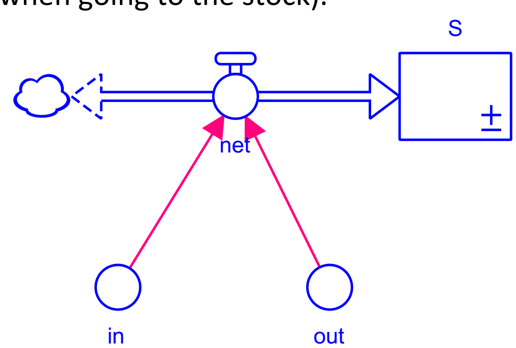

Figure 2: Diagrammatic depiction of system demonstrated in Table 2.

Table 2: Link score between out and net

Variable	Time 1	Time 2	Variable Change	Partial Change in net	Link score magnitude Time 2
in	5	10	$\Delta \text{in} = 5$	$\Delta_{in} net = (10 - 4) - 1 = 5$	$\left\lvert \frac{\Delta_{in} net}{\Delta \text{net}} \right\rvert = \frac{5}{4}$
out	4	5	$\Delta \text{out} = 1$	$\Delta_{out} net = (5 - 5) - 1 = -1$	$\left\lvert \frac{\Delta_{out} net}{\Delta \text{net}} \right\rvert = \frac{1}{4}$
$net = in$ - out	1	5	$\Delta \text{net} = 4$	-	-

Comparing the analyses of the two mathematically equivalent models represented in Table 1 and Table 2 we can clearly see that the link score magnitude from out or in to S is not equivalent (1 does not equal 0.25). The discrepancy is the result of the difference in the way that the link score is calculated for the flow to the stock. Focusing on just the outflow, the original flow to stock link score formulation (Equation 1) overweighs the impact of the relatively small change in out on S. Focus for the moment on the outflow link score calculation demonstrated in Table 1. Using the original flow to stock link score formulation (Equation 1), we are dividing out (5) by the change in S (5) to get a link score magnitude of 1. Now focusing on the instantaneous form of the outflow to net flow relationship demonstrated in Table 2, using the instantaneous link score equation (Equation 2), we are dividing the change in net caused by out (-1) by the change in net (4) yielding 0.25. These are two very different calculations which obviously give different results, demonstrating the flaw in having such different calculation methods for determining the link score.

Problem Solution: An improvement to calculating the link score from flows to stocks
Ultimately the solution to this problem is simple, convert all disaggregated flows into a single aggregated net flow, then use a link score of 1 for all net flow to stock links. At that point by definition the analysis is completely insensitive to the aggregation level of the flows because the flows are always aggregated before performing the analysis. While tidy, this solution deserves an explanation demonstrating that it is theoretically sound and not arbitrary.

To demonstrate that a special form for measuring the link score between flows and stocks isn't necessary, we restate the instantaneous form of the link score (Equation 2) into Equation 3, the updated flow to stock link score, accounting for the integration process, but ultimately producing the same set of mathematical operations as if all flows were aggregated into a single net flow. Equation 3 uses i for an inflow, o for an outflow, (S) for the stock, and (t) represents time.

$$Inflow: LS(i \rightarrow S) = \left(\left\lvert \frac{\Delta i}{\Delta S_t - \Delta S_{t-dt}} \right\rvert * 1 \right) \quad Outflow: LS(o \rightarrow S) = \left(\left\lvert \frac{\Delta o}{\Delta S_t - \Delta S_{t-dt}} \right\rvert * -1 \right)$$

(3)

Comparing the updated flow to stock link score equation (Equation 3) to the instantaneous link score equation (Equation 2), Δi and Δo represent the first order partial change in the stock S with respect to the flow. $\Delta S_t - \Delta S_{t-dt}$ is the change in the net flow which is the second order change in the stock S. The flow to stock link score magnitude in Equation 3 now measures the first order partial change in the stock due to the flow relative to the second order change of the stock. This is a clear conceptual difference from the instantaneous form of the link score which is necessary to account for the integration process, but from an operational perspective the instantaneous and updated flow to stock link score equations now produce the same set of calculations which is demonstrated in Table 3 using the same example values from above.

Table 3: Link score for in and out to S using Equation 3

Variable	Time 1	Time 2	Variable Change	Link score magnitude Time 2
in	5	10	$\Delta \mathrm{in} = 5$	$\left\lvert\frac{\Delta in}{\Delta S_t - \Delta S_{t-dt}}\right\rvert = \frac{5}{4}$
out	4	5	$\Delta \mathrm{out} = 1$	$\left\lvert\frac{\Delta out}{\Delta S_t - \Delta S_{t-dt}}\right\rvert = \frac{1}{4}$
$\boldsymbol{S} = \int(\boldsymbol{in} - \boldsymbol{out})$ $initial = 100$	101	106	$\Delta S_t - \Delta S_{t-dt} = 5 - 1$	-

Looking at Table 3 and Table 2, the result, and all of the steps along the way are now the same. Reinterpreting the instantaneous form of the link score shown in Equation 2, into the updated flow to stock link score equation shown in Equation 3, demonstrates that there is no need for a separate calculation method for measuring the link score between flows and stocks as long as all flows are aggregated during analysis. Ultimately it is now up to the implementor of the LTM method to determine whether or not automated flow aggregation is advantageous to their implementation because the updated flow to stock link score equation (Equation 3) demonstrates that the level of flow aggregation is now irrelevant to the analysis.

Placing LTM into the literature

With the removal of the special case of the link score between flows and stocks the relationship of the link score to the Pathway Participation Metric (PPM) (Mojtahedzadeh et. al, 2004) and Loop Impact (Hayward & Boswell, 2014) becomes clearer and easier to understand. The link score is closely related to the PPM with a single key difference in the interpretation of the sign of a link or path score. In PPM the sign measures the effect that the causal pathway has on the behavior of the stock, a positive value means the behavior of the stock is exponential (increasing or decreasing), whereas a negative value means the stock's behavior is logarithmic (increasing or decreasing). In LTM, the sign measures the structural polarity of the causal pathway.

To demonstrate this relationship lets again look at the link x → z where z is characterized by the equation $z = f(x, y)$. Assume z is not a stock. This link score for this link is shown in Equation 2, which can be restated as

$$LS(x \to z) = \begin{cases} & \left(\frac{\Delta_x z}{\Delta \mathrm{x}} \cdot \left|\frac{\Delta \mathrm{x}}{\Delta \mathrm{z}}\right|\right), \\ 0, & \Delta z = 0 \text{ or } \Delta x = 0 \end{cases} \tag{4}$$

Or if we let all our deltas approach 0 (fundamentally dt approaches 0):

$$LS(x \to z) = \begin{cases} & \left(\frac{\partial z}{\partial \mathrm{x}} \cdot \left|\frac{\dot{x}}{\dot{z}}\right|, \right), \\ 0, & \dot{z} = 0 \text{ or } \dot{x} = 0 \end{cases} \tag{5}$$

This expression of the link score (Equation 5) contains the gain between adjacent auxiliary variables $\frac{\partial z}{\partial x}$ (Kampman, 2012, p. 373; Richardson, 1995, p. 75). These link gains are used in PPM (Mojtahedzadeh et. al, 2004, equation 3) and definition of impact (Hayward & Boswell, 2014, appendix 2). The link gains obey the chain rule of partial differentiation so that $\frac{\partial z}{\partial x}$ remains the gain regardless of the number of auxiliary variables between x and z. Although loop score weights these gains by the value of time derivatives of the variables, $\left|\frac{\dot{x}}{\dot{z}}\right|$, these weights cancel each other when applying the chain rule so the path score is always the gain multiplied by the relative time derivative of the two variables.

For the link between a flow and stock, the denominator of Equation 3 is the change in ΔS over time, or said another way, the second order change in the stock. Letting dt approach zero allow us to restate Equation 3 as,

$$Inflow\colon LS(i \to S) = \left|\frac{\frac{di}{dt}}{\frac{d^2S}{dt^2}}\right| \tag{6}$$

where we have assumed i is an inflow (there would be a corresponding formula for outflows.)

We next consider the link score between adjacent stocks in a causal chain in order to compare the LTM metric with PPM and Loop Impact.

Let S_1 be a source stock and S_2 be the target stock with flow f. Stock S_1 is connected to S_2 through f. The link score between S_1 and S_2 is the link score between S_1 and f using equation (5) multiplied by the link score between f and S_2 using equation (6), our revised formula. Ignoring the cases where the derivatives are zero,

$$LS(S_1 \to S_2) = LS(S_1 \to f) \times LS(f \to S_2) = \frac{\partial f}{\partial S_1}\left|\frac{\dot{S_1}}{\dot{f}}\right|\left|\frac{\dot{f}}{\ddot{S_2}}\right| = \frac{\partial f}{\partial S_1}\left|\frac{\dot{S_1}}{\ddot{S_2}}\right| \tag{7}$$

We note that the PPM and Loop Impact methods are related. Whereas *impact* measures the *absolute* value of the curvature in stock behavior, due to a source stock, PPM, from which loop impact is derived, measures the *relative* change in curvature compared with other influences on the stock. As the link score in equation (7) is an absolute measure, we first compare it with the impact between the two stocks. The relationship between stocks S_2 and S_1 can be written as

$$\frac{dS_2}{dt} = f(S_1, \dots) + \cdots \tag{8}$$

where the ellipses indicate the possible presence of other variables. The impact between the stocks is obtained by differentiating equation (8), (Hayward & Roach, 2017, appendix C; c.f. Mojtahedzadeh et. al, 2004, appendix 1)

$$\frac{d^2S_2}{dt^2} = \frac{\partial f}{\partial S_1}\frac{dS_1}{dt} + \cdots = \left(\frac{\partial f}{\partial S}\frac{\dot{S_1}}{\dot{S_2}}\right) \times \frac{dS_2}{dt} + \cdots \tag{9}$$

Impact measures the contribution of the stock S_1 to the acceleration of S_2 relative to its rate change $\frac{dS_2}{dt}$. Thus, the impact of S_1 on S_2 is given by the bracketed expression in equation (9)

$$\mathrm{Impact}(S_1 \to S_2) = \frac{\partial f}{\partial S_1}\frac{\dot{S_1}}{\dot{S_2}} \tag{10}$$

Comparing equations (7) and (10) gives:

$$LS(S_1 \to S_2) = \mathrm{Impact}(S_1 \to S_2) \times \left|\frac{\dot{S_2}}{\ddot{S_2}}\right| \mathrm{Sign}(\dot{S_1})\mathrm{Sign}(\dot{S_2}) \tag{11}$$

Link score and impact differ in two aspects, the weighting by acceleration of the target stock, $\left|\frac{\dot{S_2}}{\ddot{S_2}}\right|$, and the polarity of the link, noted by the presence of the Sign functions in equation (11). Link score measures the impact between the stocks relative to the acceleration of the stock due to all influences. If the influence from stock S_1 were the only influence, link score would be unity (Schoenberg et. al, 2020). Additionally, the polarity of the link score reflects the *structure* of the model, whereas impact (and thus PPM) measures the polarity (curvature) of the link's *behavior*. Ultimately this difference is due to a difference in goals and design. LTM is designed to report the polarity of the causal pathways it measures. Whereas PPM and loop impact are designed to measure if a causal pathway is acting with or against a chosen stock's behavior.

For single stock models, the source and target stocks are the same, $S_2 = S_1$ and form a first-order loop. Thus, from equation (10), impact is the loop gain G_1 as defined by Kampman, 2012:

$$\text{Impact}(S_1 \to S_1) = \frac{\partial f}{\partial S_1} \triangleq G_1 \tag{12}$$

From equation (11), link score is a weighted loop gain:

$$LS(S_1 \to S_1) = \frac{G_1}{\left|\frac{\ddot{S_1}}{\dot{S_1}}\right|} \tag{13}$$

referred to as *loop score*. In models with many loops, both PPM and LTM present normalized measures of loop influence. PPM is the percentage loop impact, equation (12), compared with all loops on a given stock (Mojtahedzadeh et. al, 2004; Hayward & Boswell, 2014). In LTM, relative loop score is the percentage form of the loop score, equation (13), (Schoenberg et. al, 2020). Because the loop scores of all loops in a single stock will be weighted by $\left|\frac{\ddot{S_1}}{\dot{S_1}}\right|$, then the relative loop score will be identical to the PPM in single stock models. Thus, we expect LTM to produce the same analysis as PPM and the Loop Impact method for first-order systems.

For models with more than one stock, LTM differs from PPM and Loop impact by providing a single measure for the whole loop. By contrast, the other two methods have a measure for each stock in the loop. For example, consider a two-stock loop where S_2 in equations (10–11) is connected back to S_1. Using the loop impact theorem (Hayward & Boswell, 2014, appendix 3), the product of loop impacts in the loop equals the loop gain G_2. Thus $\text{Impact}(S_1 \to S_2) \times \text{Impact}(S_2 \to S_1) = G_2$. The loop score of the loop is the product of the link scores, which becomes:

$$\begin{aligned} LS(S_1 \to S_1) &= LS(S_1 \to S_2) \times LS(S_2 \to S_1) \\ &= \frac{\text{Impact}(S_1 \to S_2) \times \text{Impact}(S_2 \to S_1)}{\left|\frac{\ddot{S_1}}{\dot{S_1}}\right|\left|\frac{\ddot{S_2}}{\dot{S_2}}\right|} \times \left(\text{Sign}(\dot{S_1})\text{Sign}(\dot{S_2})\right)^2 \\ &= \frac{G_2}{\left|\frac{\ddot{S_1}}{\dot{S_1}}\right|\left|\frac{\ddot{S_2}}{\dot{S_2}}\right|} \end{aligned} \tag{14}$$

using equation (11) and the loop impact theorem. Again, the loop score is directly related to the loop gain. In models with more than one stock, a loop dominance analysis using LTM will give different results to that of PPM and Loop Impact. However, if the link scores on a single stock were compared, the results would be the same as that of the other two methods, except for the link polarities following the model structure rather than behavior.

Thus, we have shown that LTM is derived from PPM/Loop Impact apart from its treatment of link polarity. But LTM is distinct in its application, measuring dominance across the entire model (or connected components for models which are not fully connected via feedback). It is this very difference between the link score and PPM, the sign of the link score measuring structural polarity, which yields the ability to chain through multiple stocks (e.g. equation 14) making for the largest difference between the other PPM based methods and LTM. This is what allows for dominance to be measured model-wide as described by Schoenberg, et. al., 2020:

> *[In LTM] We define loop dominance as a concept which relates to the entirety of a model, as opposed to loop dominance being something that affects a single stock. For loop dominance to apply to the entire model, we require that all stocks are connected to each other by the network of feedback loops in the model. For models where there are stocks that do not share feedback loops, we consider each subcomponent of interrelated feedback loops individually, and we refer to each model substructure as having a separate loop dominance profile. [In LTM,] our measurement of loop dominance is specific to the particular time period selected for analysis. We say that a loop (or set of loops) is dominant if the loop(s) describe at least 50% of the observed change in behavior across all stocks in the model over the selected time period.*

The implications of Equation 14, and therefore the meaning of the loop score are threefold. First, loop scores always measure the structural polarity of loops because of the absolute values of the loop impacts in the denominator. Second if a stock is not changing (reaching a maximum, minimum, or equilibrium value), e.g., as $\dot{S_1}$ or $\dot{S_2}$ approaches 0, then the loop score approaches 0. As a corollary, when loop gains are 0, the loop score is 0, which means inactive loops are never explanatory. Third, as the acceleration in a stock ceases, for instance at inflection points, when stocks are changing the most, e.g., as $\ddot{S_1}$ or $\ddot{S_2}$ approaches 0 then the loop score approaches infinity. All of this aligns well with the goal of the loop score, to measure the change in the behavior of the stocks in the model. This leads to the understanding that LTM favors loops with large gains, which pass through stocks changing the most. It also demonstrates the necessity of the relative loop score (where the loop scores are normalized across all feedback loops which interact) in order to make the infinities which happen at inflection points more easily interpreted. Ultimately, dominance is a measure of relative importance, and therefore the values of the loop scores are only meaningful in relation to each other as relative numbers. The only exception to that rule may be that the sum of the absolute values of the loops scores may have some use to express the magnitude of the change in the model at an instant in time.

Understanding carrying capacity models using LTM

With the identified shortcoming of the LTM method addressed, now it can be used to better analyze models with disaggregated flows. A well-known, simple model we can start with is a carrying capacity model which is looking at a population of critters, pictured below in Figure 3 in its aggregated and disaggregated form.

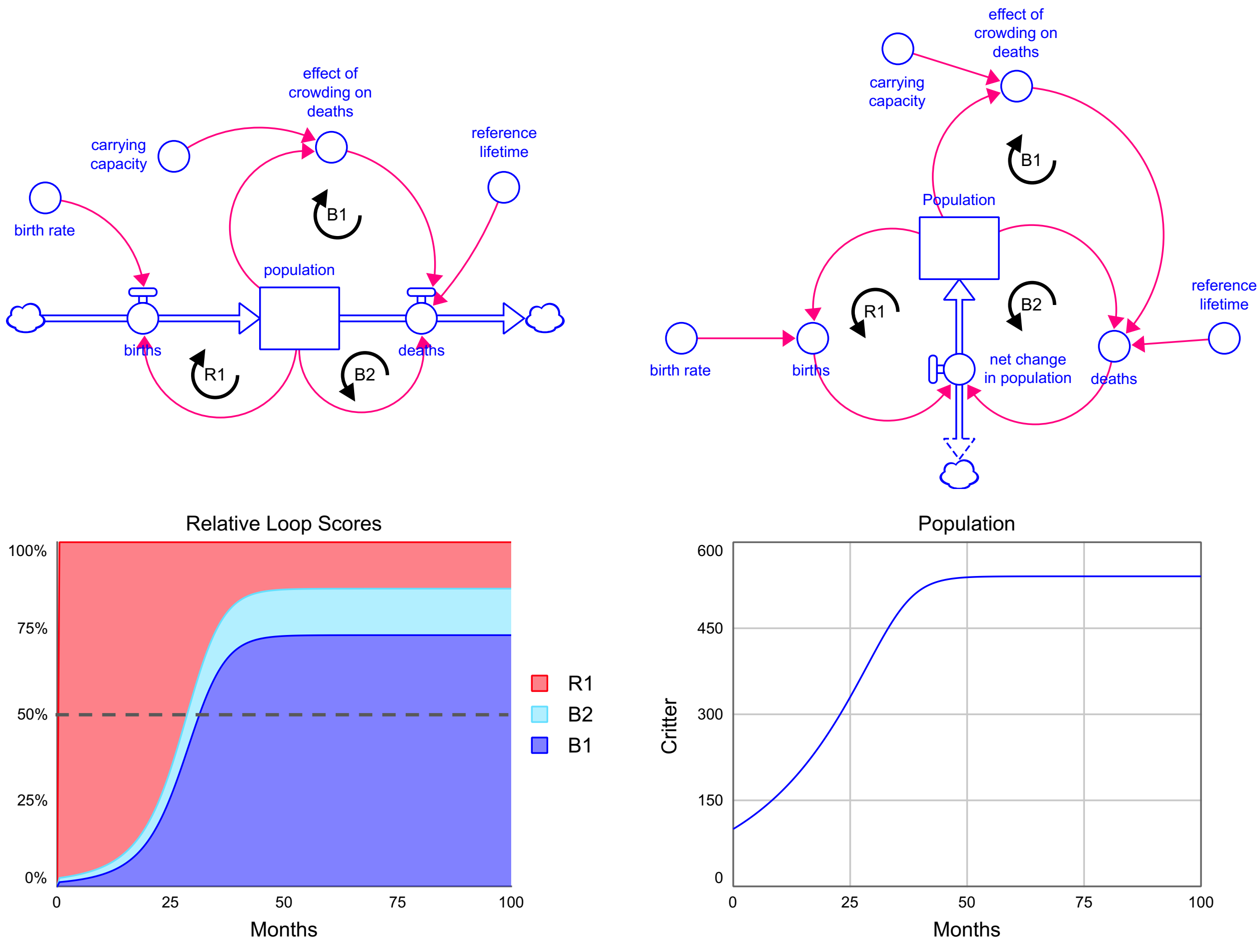

Figure 3: A simple bathtub style carrying capacity model with its behavior and LTM analysis. Birth Rate = 0.05, Reference Lifetime = 50, Carrying Capacity = 500, Population=100, effect of crowding on deaths = EXP(5(Population/carrying_capacity-1))*

The analysis of this model in the lower left of Figure 3 shows us a straightforward explanation for the behavior. There are two perspectives that can be used to explain the relative loop score plot. From a mechanistic perspective, which is more useful for those already skilled in System Dynamics and understand the impacts of loop dominance analysis; the model starts with the reinforcing births loop (R1) as dominant up until the inflection point in Population at Time 29. Then, the model enters a short phase from Time 29 to 31.5 where it is dominated by both B1 (the crowding loop) and B2 (the natural deaths loop). After 31.5 the model behavior is dominated by B2, the crowding loop.

The second perspective on the LTM analysis is more intuitive, especially for those not so well versed in System Dynamics and the mechanistic concepts of dominance. Initially the births process (R1) is most important to the governing of the behavior of the critter population by a very large margin, nearly completely overwhelming the natural deaths process (B2) and the deaths process from crowding (B1). Because the births process (R1) is reinforcing, critters are being added to the population at an exponential rate. This exponential increase in critters causes the magnitude of the relative loop score, or more colloquially, the relative importance of the natural deaths process (B2) and the deaths by crowding process (B1) to grow at the expense of the births process (R1). It also important to note that the large majority of the relative importance lost by the births process (R1) is gained by the crowding process (B1), not the natural deaths process (B2). As the reinforcing birth process (R1) is slowing, eventually the two deaths processes (B1 and B2) take over the governing of the behavior of the critter population. This first happens at the inflection point (Time 29) and from this point forward the critter population is no longer growing exponentially, but rather logistically towards an equilibrium value because both of the deaths processes (B1 and B2) are balancing. Because of this change in governance, the reinforcing births process (R1) is no longer giving up its relative importance exponentially, but rather logistically. By Time 31.5 because the reinforcing births process (R1) is still adding more critters faster than they can be killed, the relative importance of the crowding process (B1) no longer relies on the natural deaths process (B2) to maintain control over the critter population. Finally, because the population of the critters is still governed a balancing process (the crowding loop B1) by Time 60 the critter population reaches equilibrium. At this point the births process (R1) is no longer shedding any relative importance, and its remaining relative importance is exactly the same as that of the natural deaths process (B2).

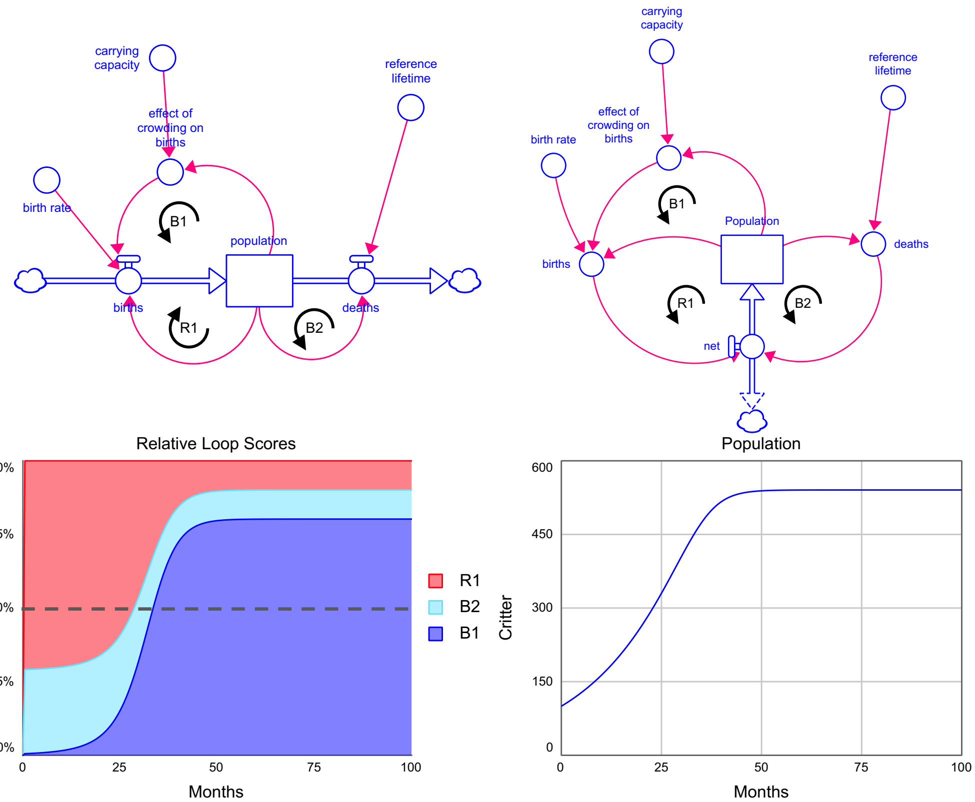

*Figure 4: Carrying capacity model in non-standard formulation with with an effect of crowding on births. Its stock behavior is exactly the same as the model in Figure 3. Also plotted is the system behavior with its LTM analysis. Birth Rate = 0.05, Reference Lifetime = 50, Carrying Capacity = 500, Population = 100, effect of crowding on births = 1 + ((1/birth_rate)/reference_lifetime) - (((1/birth_rate)/reference_lifetime) * (EXP(5*(Population/carrying_capacity-1))))*

To demonstrate the robust nature of the updated LTM method we've taken the standard carrying capacity model from Figure 3 and created its exact behavioral analogue using an effect of crowding on births. While the model in Figure 4 has the exact same stock values as the one in Figure 3, the effect of the change to the feedback structure is captured and reflected in the analysis. While the same general mechanistic (shifting dominance pattern) and intuitive understanding of the model still applies from the first analysis there is one key structural difference which is reflected clearly by LTM. Because the crowding process (B1) now retards the births process (R1) directly (rather than indirectly through creating more deaths), the births process is always relatively less important than it otherwise was in the standard formulation. Early on this allows the natural births process (B2) to be relatively more important (because crowding isn't important in the beginning), but during the equilibrium phase this reduction in the relative importance of the births process (R1) is linked to the reduction in the relative importance of the natural deaths process (B2) as the two must be of equal relative importance during this period which transitively means the crowding process is relatively more important in this second formulation. Finally, the inflection point is still Time 29 (the stock behavior is identical after all) and Time 29 is still when the two balancing processes (B1 and B2) take over from the reinforcing births process (R1).

The analysis of these carrying capacity models demonstrates that the updated LTM method is insensitive to the aggregation level of the flows, and properly captures the effects of changes to the feedback structure on the subtleties of loop dominance progression.

LTM and oscillation

One of the key advantages to LTM over the other PPM based methods is its clear interpretation of oscillatory models (Schoenberg, et. al., 2020). To demonstrate that the change to the flow to stock link score hasn't materially affected that advantage we analyze the two-stock oscillator pictured below in Figure 5. The analyzed instance of the model has been parameterized to exhibit a dampened oscillation. The analysis was completed over the full-time range of [0, 100], but for the purposes of ease of understanding just a single cycle of the dominance pattern was chosen for a focused analysis [10, 30]. This time period corresponds with exactly one half period of the oscillation in either stock, x or y. The reason that the second cycle of the oscillation was chosen is because it is good practice to avoid any potential effects from the initialization of the system, although in this specific case that precautionary measure was not necessary because the pattern is unchanged throughout the entire simulation period.

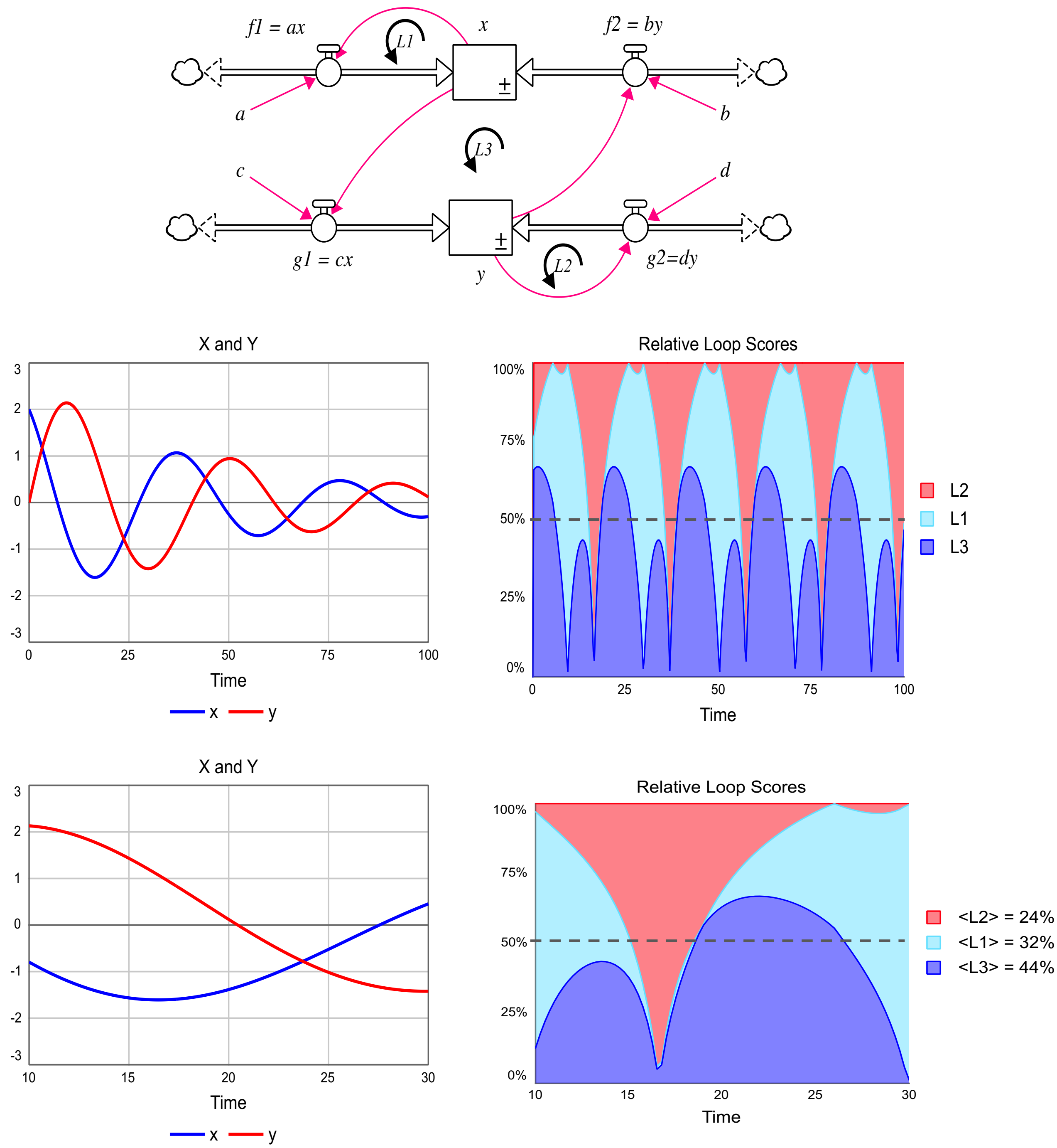

Figure 5: Oscillatory system showing analysis over the full simulation time period and one period of the dominance analysis (half a period of the oscillation). a=-0.1, b=-0.15, c=0.2, d=0.6, X=2, Y=0. <L1> <L2> and <L3> are the average relative loop scores of the loops, shown to the right of the lower right graph.

The pattern of the shifting dominance does not change in this model which is logical because the causes of behavior are constant, and the behavior itself is fractal. Also remember the relative loop score is a relative measure, therefore it does not distinguish between the larger earlier oscillations and the later smaller oscillations. Studying the relative loop score plot in the lower right of Figure 5 yields the following understanding. The strongest loop over this range (and the full simulation period) is the major loop L3, the two-stock oscillatory loop. We know this is the oscillatory loop because it connects two stocks, but if we ignore that information, LTM shows us that it is the most explanatory loop over the period, explaining 44% of the change in both stocks because the average magnitude of its relative loop score over that time range is 44% (Schoenberg, 2019). Continuing our analysis, we notice that the minor loop L2 describes 100% of the behavior of the system at time 16.5 and the minor loop L1 does the same at time 29.8. In fact, the relative loop scores of both minor loops L1 and L2 are following the same pattern just shifted in time from each other. Each minor loop grows from being completely unexplanatory to very nearly fully explanatory in an exponential fashion, and then loses its relative importance quickly in the same exponential fashion. So, the explanation now becomes further clearer, L3 is the oscillatory loop whose relative importance is being changed by L1 and L2 as they undergo swings in their relative importance depending upon the amount of change currently being exhibited in the stock they are directly attached to. L1 is dampening the oscillation because it's a balancing loop, and L2 is attempting to explode the oscillation because it's a reinforcing loop. The reason this model produces a dampening oscillation is because L1 is relatively more important than L2 over the time period of the oscillation. L1's average relative loop score magnitude of 32% means its more explanatory then L2 whose average relative loop score magnitude is 24%. This then begs the question of what is happening when L1 and L2 are fully explanatory? At time 16.5 when L2 explains nearly all of the behavior of the system, the stock x is at its minimum value, it's not changing, which means L1 and L3's loop scores approach 0 because both include links to and from x. At 29.8 when L1 is fully explanatory the opposite is true, the stock y is at its maximum, not changing which means L2 and L3's loop scores approach 0 because both include links to and from y.

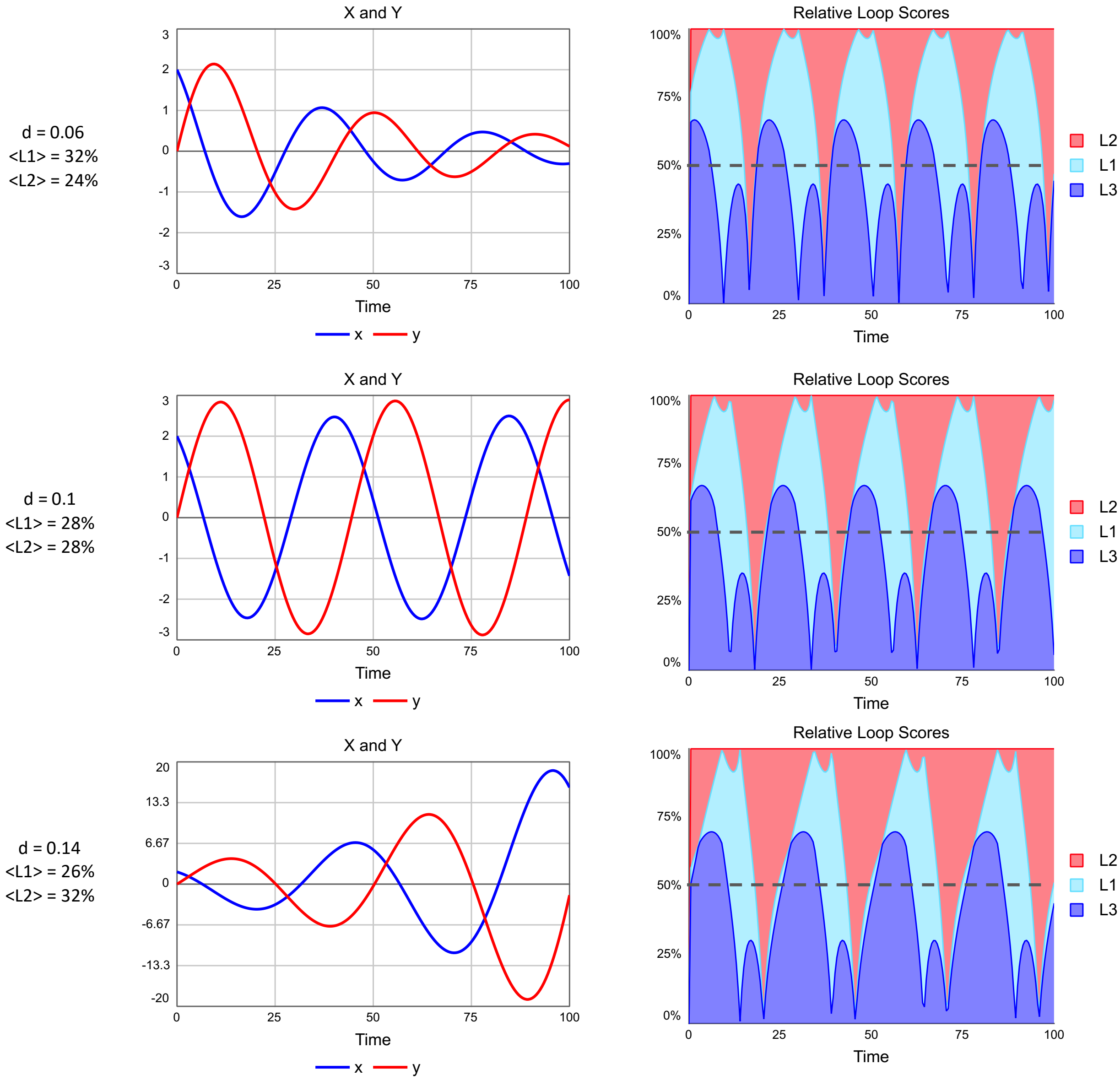

Figure 6: Analysis of the two stock oscillator from above with different values for d. This adjusts the relative importance of L1 and L2 relative to each other to create dampened, standing, and exploding oscillation. <L1> and <L2> are the average relative loop scores of the loops, shown to the left of the graphs.

The first analysis shown in Figure 4 repeats the analysis from Figure 5 above and again shows us that L3 is the oscillatory loop, L1 dampens the oscillation, L2 tries to explode the oscillation but L2 is relatively less important then L1 therefore the oscillation dampens. When the relative importance of L2 is increased above that of L1 by increasing the value of d above 0.06 the behavior of the model moves to a slower dampening, until the point where the relative importance of L1 and L2 overtime are equal at d=0.1 when a standing oscillation is achieved. By further increasing the relative importance of L2 by increasing the value of d above 0.1 L2 then becomes relatively more important then L1 which causes the oscillation to explode. The analysis of this two-stock oscillator shows that LTM is still able to clearly and concisely explain the origins of oscillation and identify the purpose of loops in oscillatory systems.

Conclusions

The updates to the LTM method have resolved the identified shortcoming that the aggregation level of flows changes the analysis. This change makes analysis more straightforward and has helped to better situate the LTM method within the pre-existing literature, bringing it closer to other PPM based methods in its theoretical underpinnings. This paper has demonstrated that the update to LTM improves model understanding while maintaining LTMs ability to clearly and correctly analyze a variety of models, including oscillatory models.

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