Representing feedback in a computable stock/flow model

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Abstract

This paper addresses the disconnect between feedback structure of a problem and its computable stock/flow model, which has created a separation between systems thinking and system dynamics. This separation on one hand has led to creating complex feedback maps and inferring their behavior without knowing its contingencies. On the other, it has prompted creating complex stock/flow models whose behavior can also not be easily understood. The paper shows how the two representations are related and how they should be connected in a formal computer model.

Introduction

Mapping a stock and flow map to causal feedback diagram offers many problems as pointed out by George Richardson (Richardson 1986). As described by Forrester in his Principles of Systems, the two representations are maps of different but related characterizations of the system structure (Forrester 1971). The feedback map articulates the understanding of the behavior as it arises from structure; the stock and flow structure creates a computable model whose behavior can be obtained through computer simulation. Unfortunately, the relationship between two is often left to intuition that can lead to multiple interpretations of the behavior of a stock and flow model, which negates the requirement of uniqueness of a theory. The extended use of the two representations in isolation from one another has even led to a separation between systems thinking and system dynamics – the former limited to qualitative explanations and the later focused on computer simulation, although the two must go hand in hand (Forrester 1994).

The two representations must be seen as different views of the system and how they relate to each other. A stock/flow model is an intuitive representation of the integration process carried out by the computer, while the feedback map is a mental model of the aggregate *information* relationships explaining system behavior. Thus, in a computer model of the integration process, the feedback diagram must show relationships between clusters of the model structure rather than between each element of it. The relationship between the two must be unique and easily discernable.

Representing feedback in a stock/flow model

The relationship between a feedback map and computable stock/flow structure can be illustrated by programing a one stock model using the hierarchy in Stella software. Figure 1 shows the stock/flow structure of a simple population growth model consisting of the stock of population, births computed as a compounding process and deaths as a draining process.

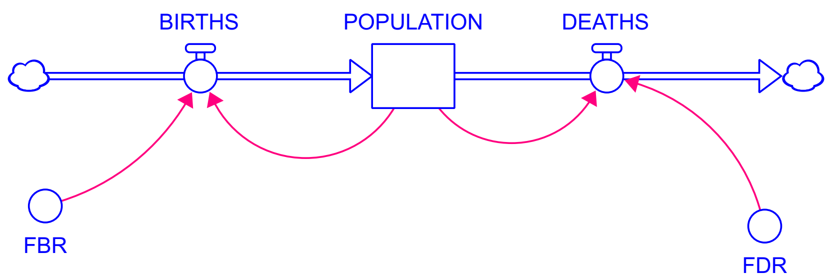


Figure 1 Stock/flow structure of the simple population growth model.

A feature in Stella allows you to place signs on the information links in the stock/flow model, which I think is anomalous since this diagram does not define feedback loops as *information relationships* that Forrester surmised. The inflow in Figure 1 has an information link going from population to births, but it designates a policy that computes a flow into the stock. It creates a feedback effect, but not an information path a feedback loop must represent. The feedback effect associated with the outflow is even more confusing. An information link connects the stock to the policy, but the outflow it creates does not even give the appearance of feedback in view of the direction of the outflow. Forrester talks briefly about feedback loops in Chapter 4 of Principles of Systems (Forrester 1971). The two relevant principle he states are summarized below:

> **Principle 4.2-1**. <u>Decisions always occur within feedback loops.</u>
>
> Every decision is made within a feedback loop. The decision controls action which alters the system levels which influence the decision. A decision process can be part of more than one feedback loop.
>
> **Principle 4.2-2.** Feedback loop — the structural element of systems
>
> The feedback loop is the basic structural element in systems. Dynamic behavior is generated by feedback. The more complex systems are assemblies of interacting feedback loops.

Principle 4.2-1 describes, how stocks affect flows creating iterative adjustments, while principle 4.2-2 ties behavior to feedback structure embodied in the information network connecting clusters of stocks and flows. Both must be discerned for explaining behavior and its contingencies. Forrester never intended the two to be separated, but the separation arose possibly out of the software limitation to connect the two.

In my interpretation, Forrester construed feedback loop as an information relationship that is the basic building block of the system, not as a part of the computable stock and flow structure but as a relationship between clusters of structure. Let me illustrate this point by reconstructing the simple population growth model of Figure 1as a hierarchy between information relationships and computable stock/flow structure.

Let us construct population, births and deaths as three separate computations represented in the stock/flow diagrams if Figure 2 parts a, b and c.

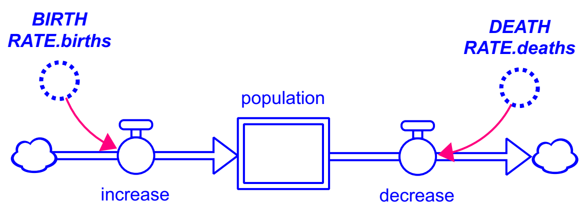
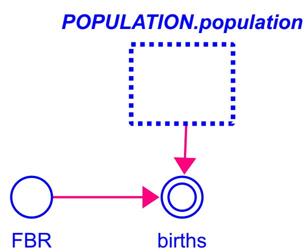


Figure 2(a): Module representing population stock and its associated flows



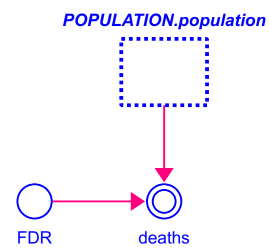


Figure 2(b): Module computing births **Figure 2(c): Module computing deaths**

Figure 2 Population model reprogrammed using module/stock flow hierarchy

Now, if the structure of the diagrams in Figure 2(a), 2(b) and 2(c) is placed in different modules, the top layer showing information relationships between modules will generate a feedback map making no distinction between computable elements of the model and displaying the feedback loops entirely created by information links as shown in Figure 3. The information feedback exists between clusters of the computable stock/flow sub-models and is shown as the relationship between the modules containing those clusters. The modules of course can be represented as names only thus reducing Figure 3 to a conventional hand drawn feedback map describing intuitive relationships that lead to the understanding of the behavior (or solution) the system.

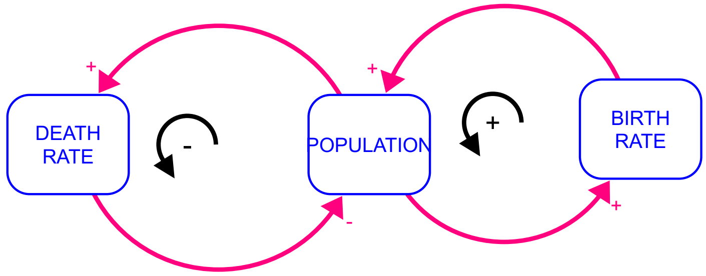


Figure 3 **Information relationships in the simple population model generated in the top layer of Stella**

Another simple example is a simple CO2 cycle model shown in Figure 4. It contains a draining process driven by a goal seeking negative feedback loop and a tipping point structure driven by a positive feedback loop containing a nonlinear relationship. The nonlinearity creates two equilibria, one of them unstable, which creates a tipping point making the efficacy of policy interventions path dependent. Yet, these feedback loops are not evident in this map and must be left to imagination. However, when the flows, the stock and the overload condition are represented by separate modules containing computable clusters of icons for each as shown in Figure 5, the information relationships between them clearly generate the coupled feedback loops shown as the module map of Figure 6 that explain the behavior of this system.

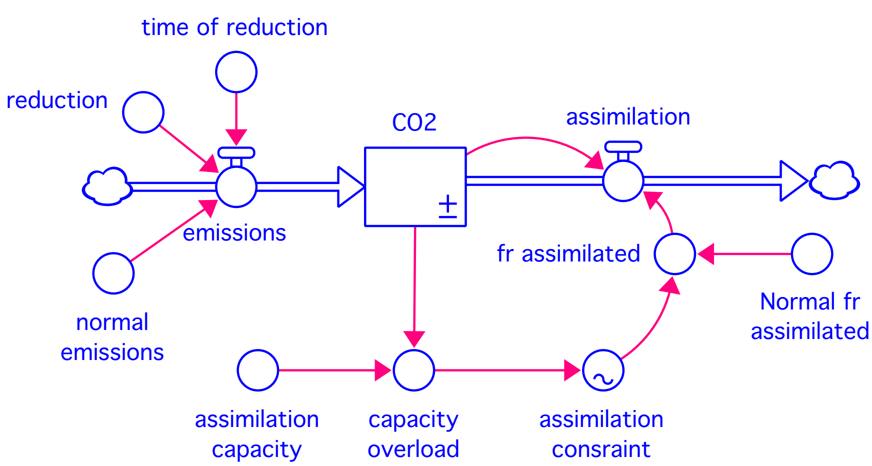


Figure 4 **Simple CO2 cycle model**

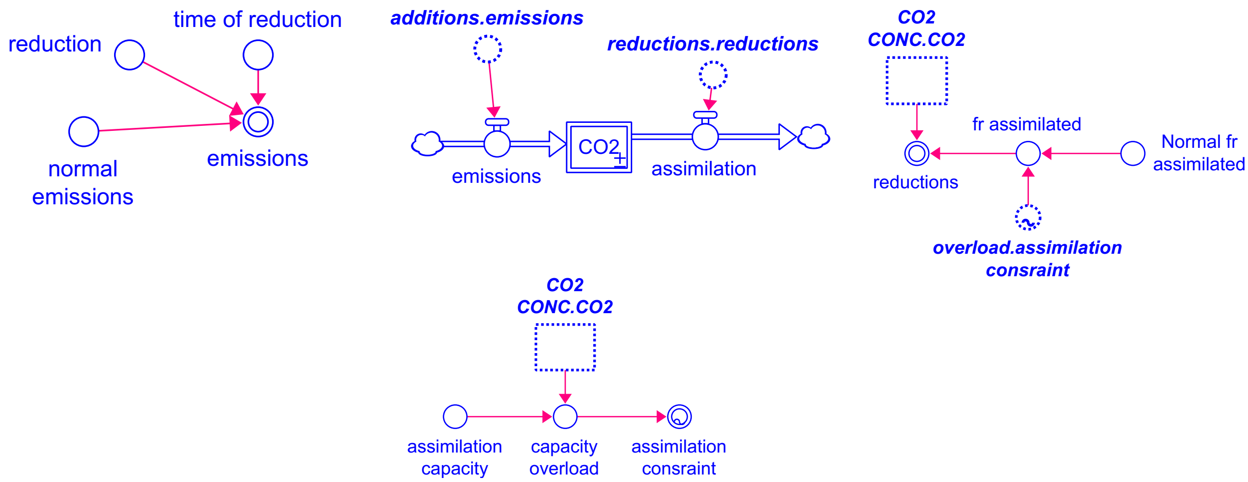


Figure 5 **Modules for inflow, stock, outflow and overload in the simple carbon cycle model**

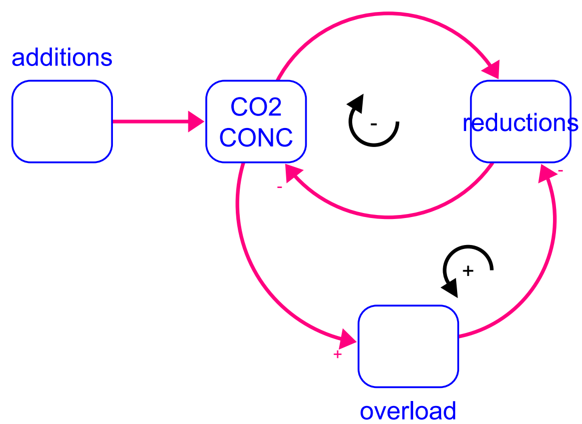


Figure 6 **Module map giving feedback relationships between computable clusters of Figure 5**

Forrester's pioneering use of feedback/stock-flow hierarchy

Forrester recognized the hierarchy between feedback loops and computable stock/flow structure from the inception of his modeling practice and it appeared both in his hand-drawn diagrams and the equation clusters of his models. Figure 7a shows an original diagram Forrester created to articulate the feedback structure of his market growth model (Forrester 1968). His feedback map represented relationship between subsystems of his model, each consisting of computable stock/flow structure. Figure 7b shows the top layer of Forrester's market growth model programmed by the author using the module hierarchy in Stella software. Each variable in this top layer is an aggregate representation of its underlying computable subsystem. Through this hierarchical relationship, it becomes possible to unambiguously tie the feedback map with the stock/flow structure.

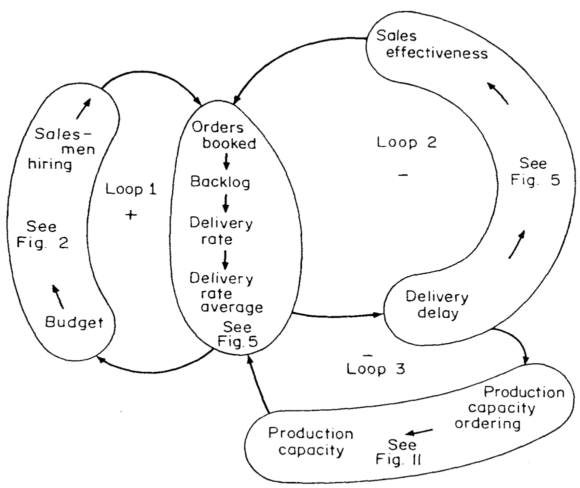


Figure 7a: Feedback loops connecting Subsystems in Forrester's market growth model.

Source: (Forrester 1968)

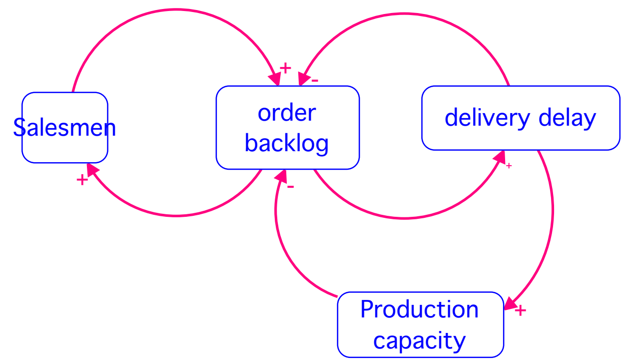


Figure 7b: Sector map created in the top layer of Forrester's model programmed in Stella software. Each module in this layer is linked with its respective stock/flow structure.

(Forrester's model reprogrammed by the author)

Figure 7 Feedback map representation between model sectors in Forrester's Market Growth model.

Articulating the relationship between the feedback map and the computer model was a challenge when our software presented models in flatland, but the possibility to create hierarchical views has made it possible to relate the aggregate feedback map of model sectors to detailed stock and flow structure, which is of great value for organizing the model as well as explaining its behavior. It also provides a direct link between the dynamic hypothesis and the computable model.

Figure 8 shows the stocks, flows and other icons, and the information links connecting them within each aggregate variable in Forrester's feedback map shown in Figure 7. Forrester is quite clear about what each representation means and how are the two intertwined. Stocks and flows are parts of the integration process subsumed in the model, while feedback loops are formed by information relationships in the system, and they drive behavior of the system. He surmised the more complex systems to be assemblies of interacting feedback loops (Forrester 1971). Without the benefit of having an explicit way to represent the relationship between feedback loops and the stock and flow structure, he organized his model equations into clusters that were connected by information feedback.

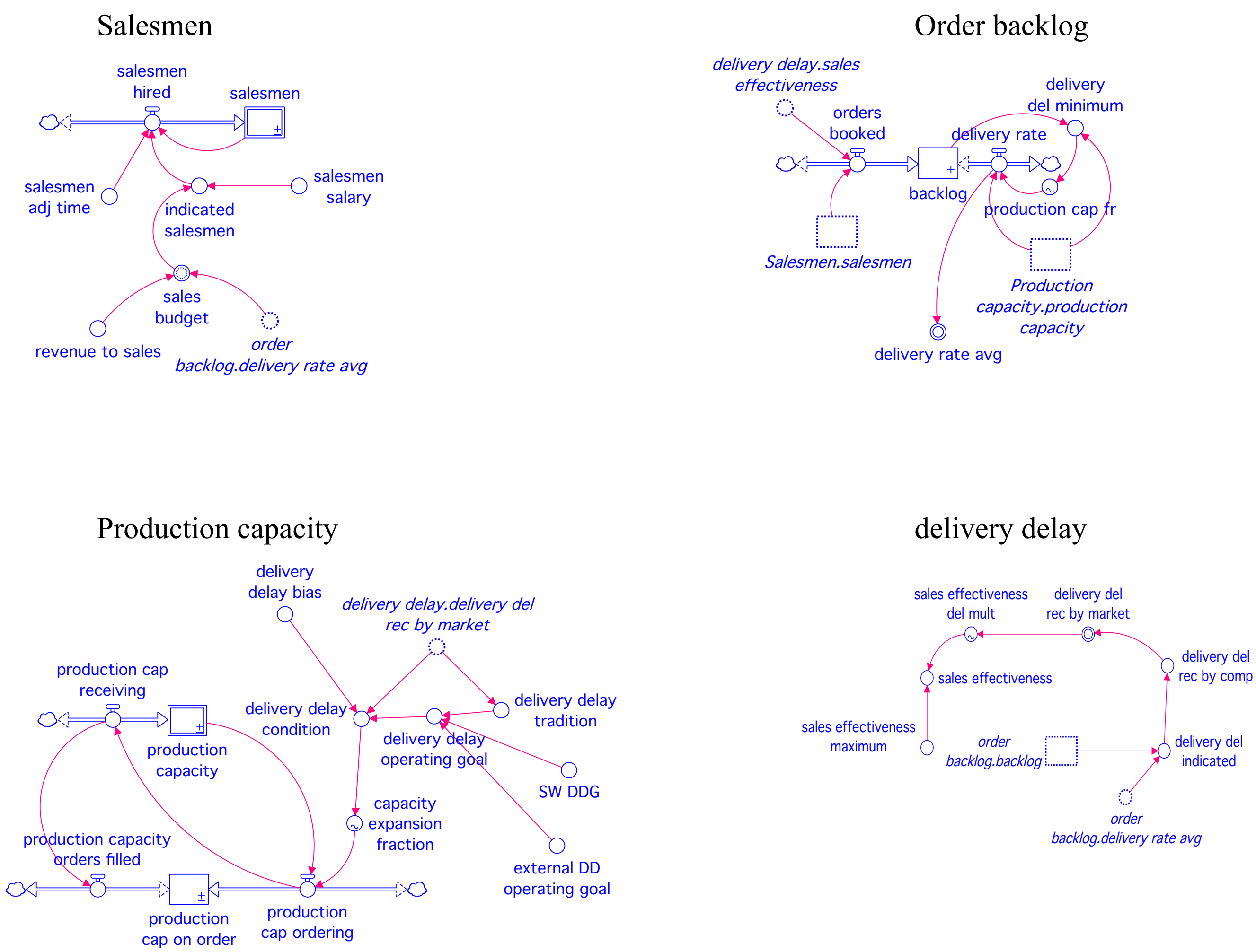


Figure 8 **Stock and flow structure underlying the feedback maps of Figure 7**

As Forrester's models grew bigger, the feedback maps representation became less explicit, perhaps due to software limitations. For example, Forrester explains the behavior of his model in his World Dynamics (Forrester 1973) book using narrative alluding to the feedback relationships shown in Figure 9, which is not coincidental. This map appeared when I programmed his world3 DYNAMO equation clusters into modules in Stella software.

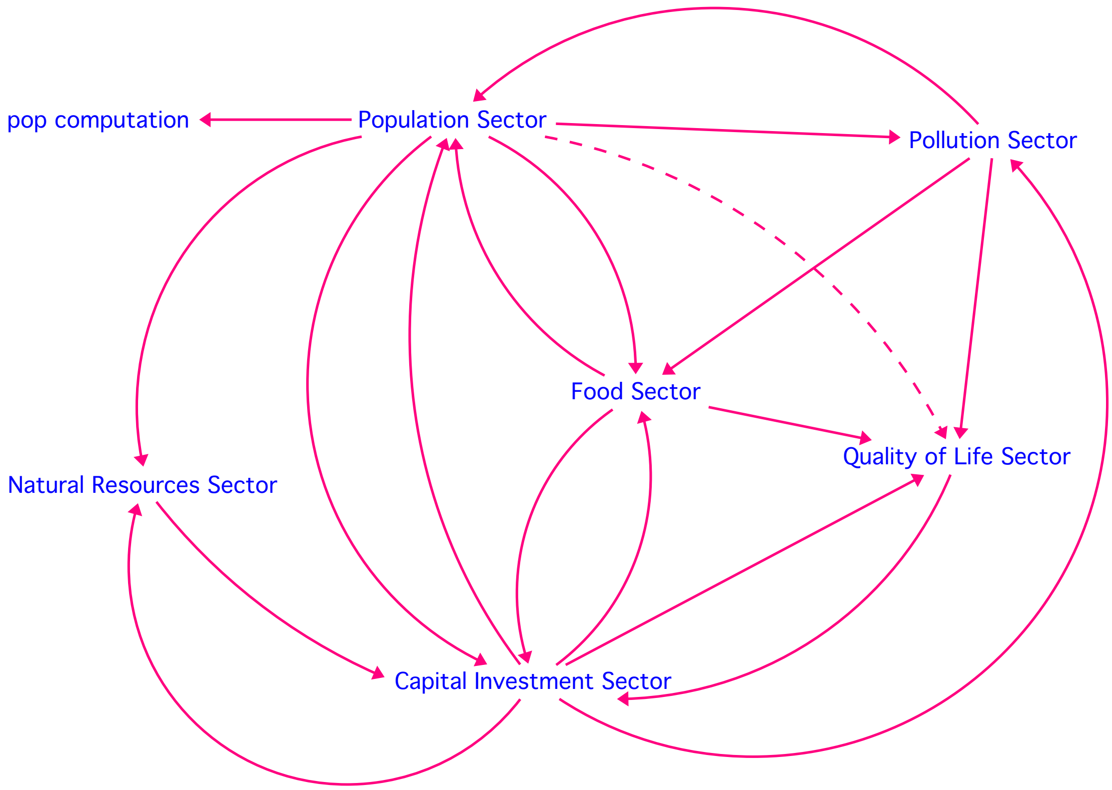


Figure 9 **Top layer feedback map of Forrester's World3 model sectors programmed into modules**

The aggregate feedback organization, although not as explicitly stated as in his market growth model, was now implicit in the way he organized the Worls3 model equations into sectors. This organization came from his dynamic hypothesis whose variables were aggregates of his model sectors each containing computable stock/flow structure.

The stock and flow structure residing in each module constructed from Forrester's equation clusters is shown in Figure 10.

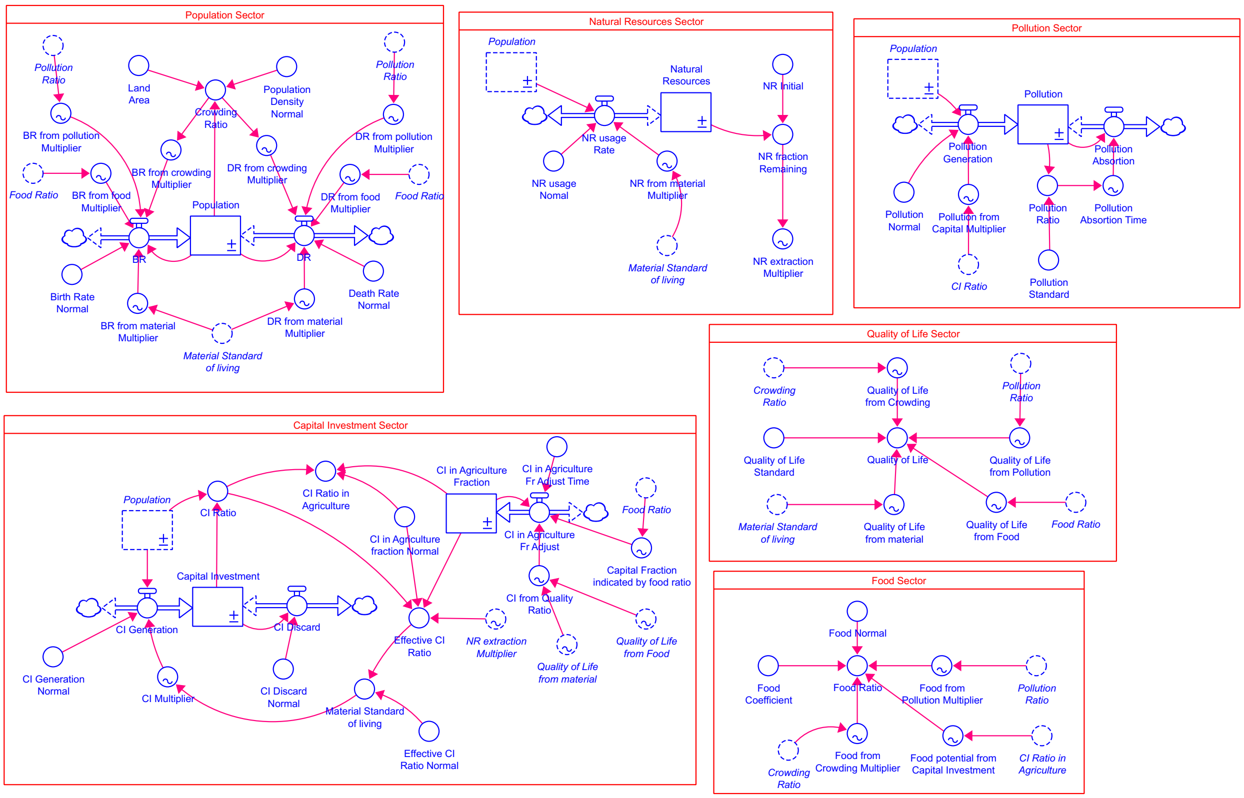


Figure 10 **Forrester's DYNAMO equation clusters of World3 representing its subsystems**

Perhaps Urban Dynamics (Forrester 1969) was the largest published model Forrester constructed. I am excluding the System Dynamics National Model (SDNM) as it remained unpublished. My friend Karim Chichakly and I tried to reconstruct its equation clusters into modules and, in our first attempt, arrived at the module map shown in Figure 11, which is obviously useless as an explanatory instrument. Karim then attempted to organize the equation clusters into a two-level hierarchy of modules as Stella allows multi-level hierarchy. This reorganization created the module map shown in Figure 12 in the top layer.

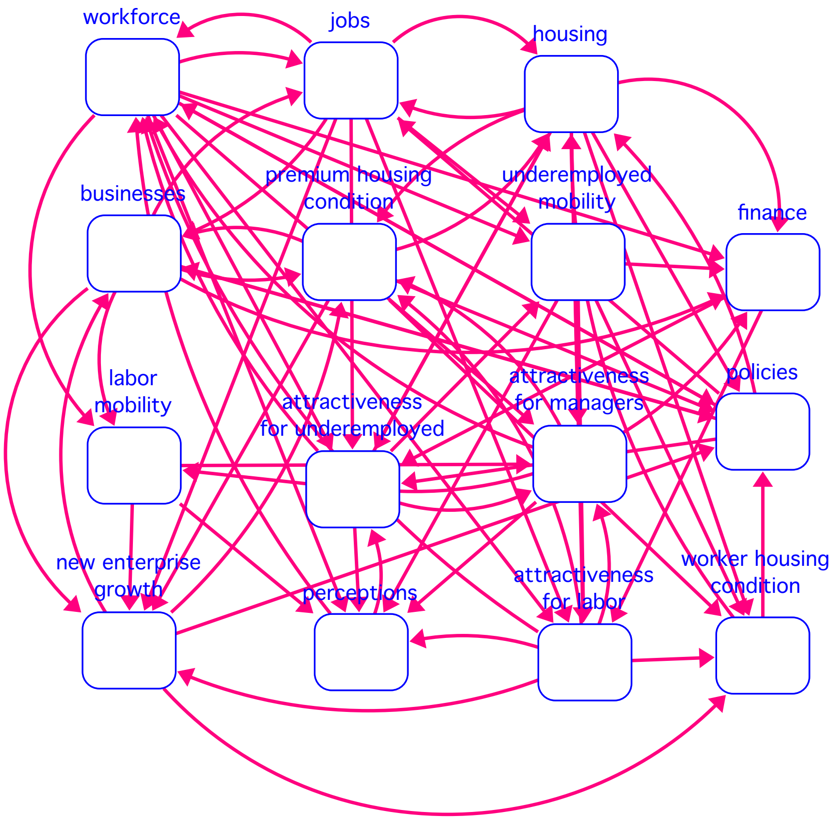


Figure 11 **Module map of Urban Dynamics equation clusters.**

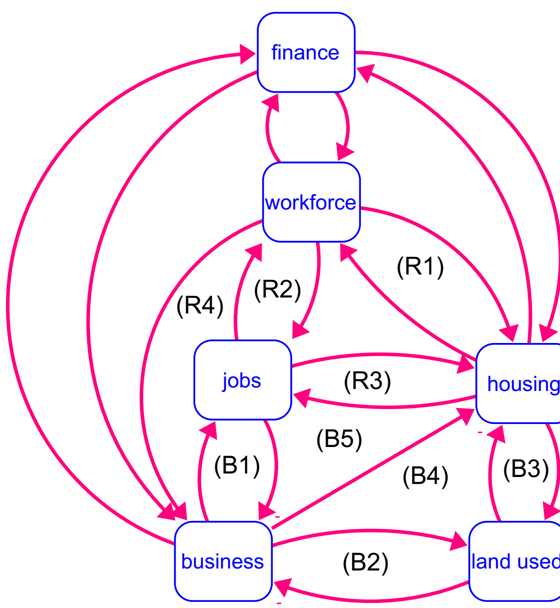


Figure 12 **Urban Dynamics equation clusters programmed into two level module hierarchy**

Using a multi-level hierarchy will call for creating dynamic hypothesis at the multiple levels of feedback pyramid so created, which further helps explain behavior at various levels of aggregation in a system. For example, in Urban Dynamics, Workforce comprises of the subsystems shown in Figure 12(a); Businesses in 12(b) and Housing in 12(c)

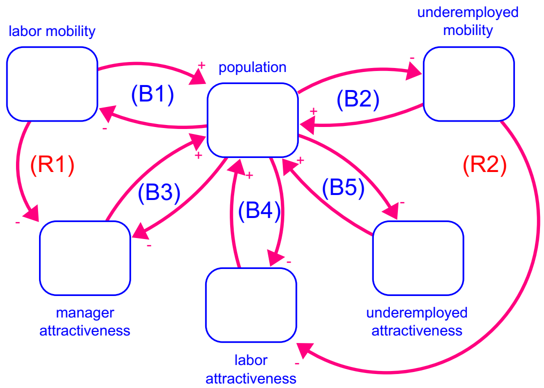


Figure 13(a): workforce subsystems

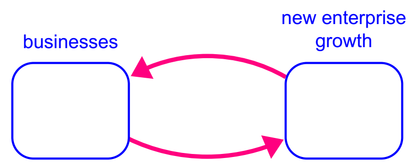


Figure 13(b): Businesses subsystems

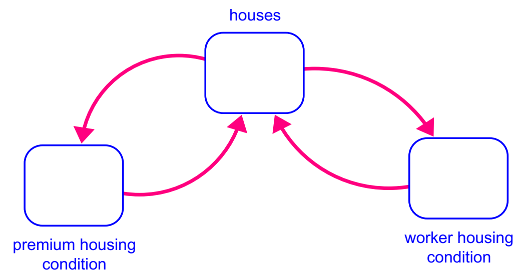


Figure 13(c): Housing subsystems

Figure 13 Module maps within modules in Forrester's Urban Dynamics model of Figure 12

Since the feedback map explains the behavior generated by the stock and flow structure in terms of the information relationships between subsystems of the model rather than between each variable, the problems of mapping between stock/flow diagrams and feedback loops are moot. We are better off understanding the hierarchical relationship between the two instead of being caught up in the problem of trying to exactly map one representation to the other.

Degree of aggregation in feedback maps

The examples of relationship between the feedback map and stock flow structure in the preceding sections are vastly different with respect to the level of aggregation of the stock-flow structure represented in the feedback maps. The variety of the levels of aggregation represented in these maps implies that there is no set rule defining the level of aggregation between feedback loops consisting only of qualitative information relationships and computable stock/flow structure, which is an important finding. It is sometimes recommended that the feedback map might be conceived as a relationship between stocks, but this can truncate causal information that might make a feedback map ambiguous. I am in favor of keeping this level of aggregation flexible, so a clear dynamic hypothesis can be formulated and tied to the computable model. Stella allows for a multilevel hierarchy and I encourage using it when representing complex models if they cannot be avoided even after having sliced the problem to identify a pattern of interest subsumed in the historical behavior (Saeed 1992).

Connecting systems thinking and system dynamics

There is a disconnect between system dynamics modeling and systems thinking. The former entails building a computable stock and flow model and experimenting with it to draw inferences about system behavior. The latter is often limited to drawing inferences from feedback maps. Divorced from each other, both practices have become runaway trains. System dynamics modeling has ended up creating large computational instruments, euphemistically called high fidelity models, whose simulated behavior is difficult to explain. Yet they draw their eminence from their complexity and rather judgmental calibration practices that validate the non-verifiable forecasts they are often used to create (Saeed 2017). System thinking on the other hand has become limited to creating complex feedback maps and drawing inferences from them whose contingencies cannot be evaluated. Unfortunately, both practices cannot qualify as science. The disconnect between the feedback maps and stock/flow structure was never intended as Forrester emphasized in his Principles of Systems (Forrester 1971).

This paper has demonstrated how a feedback diagram representing a dynamic hypothesis explaining system behavior can be wired to the detailed stock and flow structure of a system dynamics model that can be experimented with to understand the contingencies driving that behavior. This removes the separation between the two types of representation enhancing the explanatory power of the model and possibly reconnecting systems thinking and system dynamics (Forrester 1994).

Conclusion

Articulating the relationship between feedback map and computable stock/flow structure was a challenge when our software presented models in flatland, but the possibility to create hierarchical views has made it possible to relate the aggregate feedback map of model sub-systems to their respective stock/flow structure, which is of great value for organizing the model as well as explaining its behavior. It also provides a direct link between the dynamic hypothesis and the computable model.

The aggregate feedback maps sitting on top of the stock and flow structure of the integration processes in a model creates a penetrating representation of a verifiable theory as well as the explanatory process connecting structure and behavior that resides in explicit feedback loops existing in the system. The stock and flow structure can replicate the managerial decision process manifest in the bounded rational roles in the system being modeled thus creating an opportunity to verify the model. The feedback loops replace the foggy explanations, especially in economics, the supply and demand schedules and the invisible hand. Together, they offer a powerful process to operationalize theory, whose actors are managers making the best of available information to make sensible decisions. The outcome of these decisions calls for another round of sensible decision creating an iterative process driven by feedback loops that can converge into rational and not so rational outcomes. It is thus a gateway to operational policy that can rectify problematic behavior in firms, markets, regions and national economies (Saeed 2014).

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