



THE 38TH INTERNATIONAL CONFERENCE OF THE SYSTEM DYNAMICS SOCIETY
Virtual (was to have been in Bergen, Norway)

Addressing Parameter Uncertainty in SD Models with Fit-to-History and Monte-Carlo Sensitivity Methods

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[Link to video of the talk made beforehand](#)

Background: assessing parameter sensitivity

- Conventional MC-based sensitivity testing of uncertain parameters
 - Has been done for decades^{1,2}
 - The key idea is to vary parameters systematically over properly established ranges
- More recently Markov Chain Monte Carlo (MCMC) has been preferred^{3,4,5}
 - An optimal strategy for sampling from very large parameter spaces
 - And, more efficient than “brute force” MC
- But we had not seen a practical procedure for what we typically need: (1) to narrow MC or MCMC parameter spaces based on *fit-to-history* (data) constraints; and (2) to systematically test the narrowed space for *policy sensitivity*.

¹ Ford 1990, ²Sterman 2000, ³Osgood 2015, ⁴Fiddaman & Yeager 2015,

⁵Sterman et al. 2018

Aim of the paper

- How should useful parameter sets be identified?
 - Why not let model parameters run free over their plausible ranges and report all the results?
 - Because randomly selected sets of parameter values are very unlikely to produce results that resemble the historical data
- But isn't the answer already known? Isn't MCMC the method of choice? Why not just use it?
- Indeed. This is where we started
 - But the samples identified were puzzling, not even close to Gaussian; rather they looked rather like spruce trees.
 - We experimented with the settings that control the MCMC algorithm; we consulted with MCMC/Vensim experts; but to no avail.
 - Either MCMC itself is not optimal for our application (12+ data variables, not just 1 or 2); or, perhaps Vensim/MCMC is somehow sensitive to user settings.
- Our paper offers an overall approach designed to work with MCMC or MC, and demonstrates the latter

The approach

1. Specify parameter uncertainty ranges, define error metrics (MAEMs) for data variables, and find optimal parameter set (OPS)

2. Monte Carlo Testing:
a. Markov Chain (MCMC), or
b. Very large (e.g. $1e7$ runs) standard MC

3. Qualifying Parameter Sets (QPS):
a. MCMC gives “statistically valid sample”, or
b. From MC, select QPS based on MAEM criteria

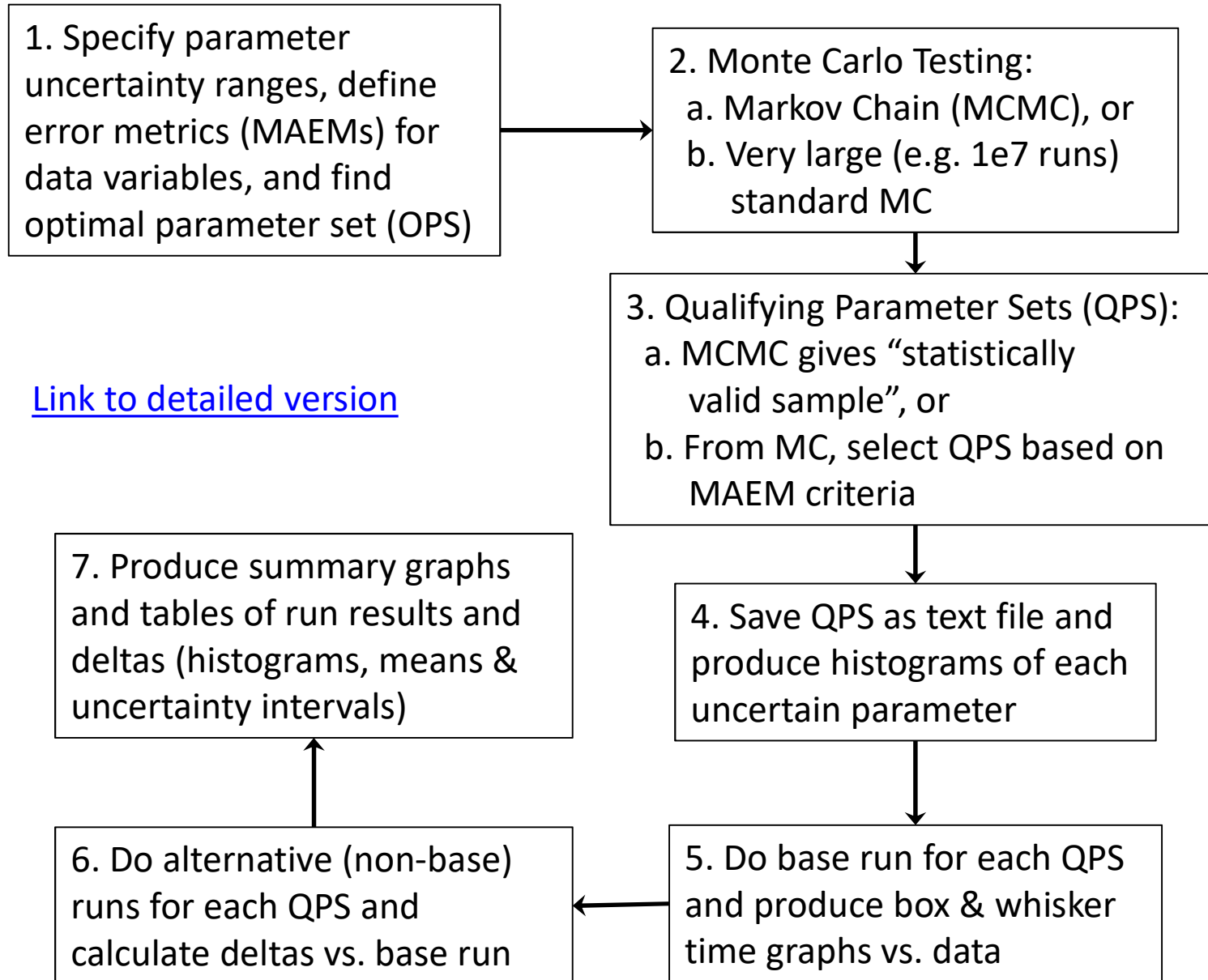
4. Save QPS as text file and produce histograms of each uncertain parameter

5. Do base run for each QPS and produce box & whisker time graphs vs. data

7. Produce summary graphs and tables of run results and deltas (histograms, means & uncertainty intervals)

6. Do alternative (non-base) runs for each QPS and calculate deltas vs. base run

[Link to detailed version](#)

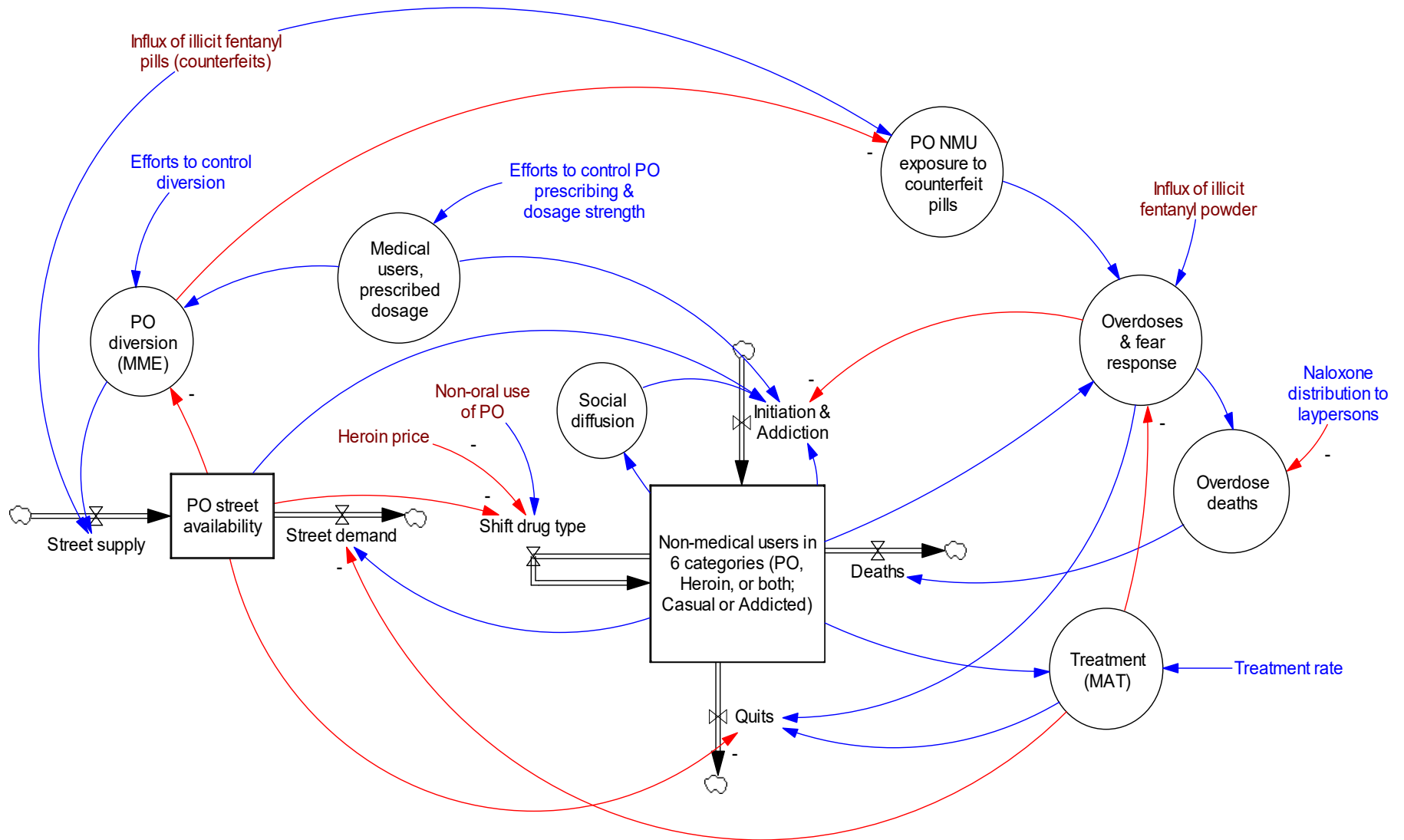


Illustrative results

- We applied the MC-version of the method to an SD model of significant complexity¹
 - An SD model of the opioid epidemic in the United States from 1990 to 2030
 - Designed to replicate the history and current status of the opioid “crisis”
 - And then to evaluate policy options for abatement

1. Homer J, Wakeland W. A Dynamic Model of the Opioid Drug Epidemic with Implications for Policy. *Am J Drug Alcohol Abuse* June 2020.

High level model diagram (for context)



1. Add model fit statistics to the model (using the SSTATS¹ macro)

| | | | |
|-------------------------------|-----------|-----------------------------|-----------|
| Time (Year) | 2020 | Time (Year) | 2020 |
| MAEM PO Abusers | 0.0635467 | R2 PO abusers | 0.920513 |
| MAEM Addicted PO Abusers | 0.0904467 | R2 Addicted PO abusers | 0.792607 |
| MAEM Addicted frac PO Abusers | 0.0565265 | R2 Addicted frac PO abusers | 0.797944 |
| MAEM PO abuse initiates | 0.108133 | R2 PO abuse initiates | 0.822349 |
| MAEM H users | 0.121204 | R2 H users | 0.865612 |
| MAEM Addicted H users | 0.0900014 | R2 Addicted H users | 0.896018 |
| MAEM Addicted frac H users | 0.0569164 | R2 Addicted frac H users | 0.293248 |
| MAEM H initiates | 0.18316 | R2 H initiates | 0.292869 |
| MAEM frac H users also PO | 0.138318 | R2 frac H users also PO | 0.691761 |
| MAEM frac H initiates also PO | 0.1005 | R2 frac H initiates also PO | 0.0539205 |
| MAEM street price PO | 0.179008 | R2 street price PO | 0.69684 |
| MAEM OD deaths from PO | 0.0509725 | R2 OD deaths from PO | 0.956034 |
| MAEM OD deaths from illicit | 0.0392809 | R2 OD deaths from illicit | 0.998476 |
| MAEM OD deaths total | 0.0363724 | R2 OD deaths total | 0.997179 |
| Simple avg of all MAEM | 0.0938847 | | |
| Weighted avg of all MAEM | 0.0892688 | | |

¹ Sterman 2000, pg. 875; the code, written by Tom Fiddaman, is available in the appendix

2. Use optimization to help find parameter values that yield best fit

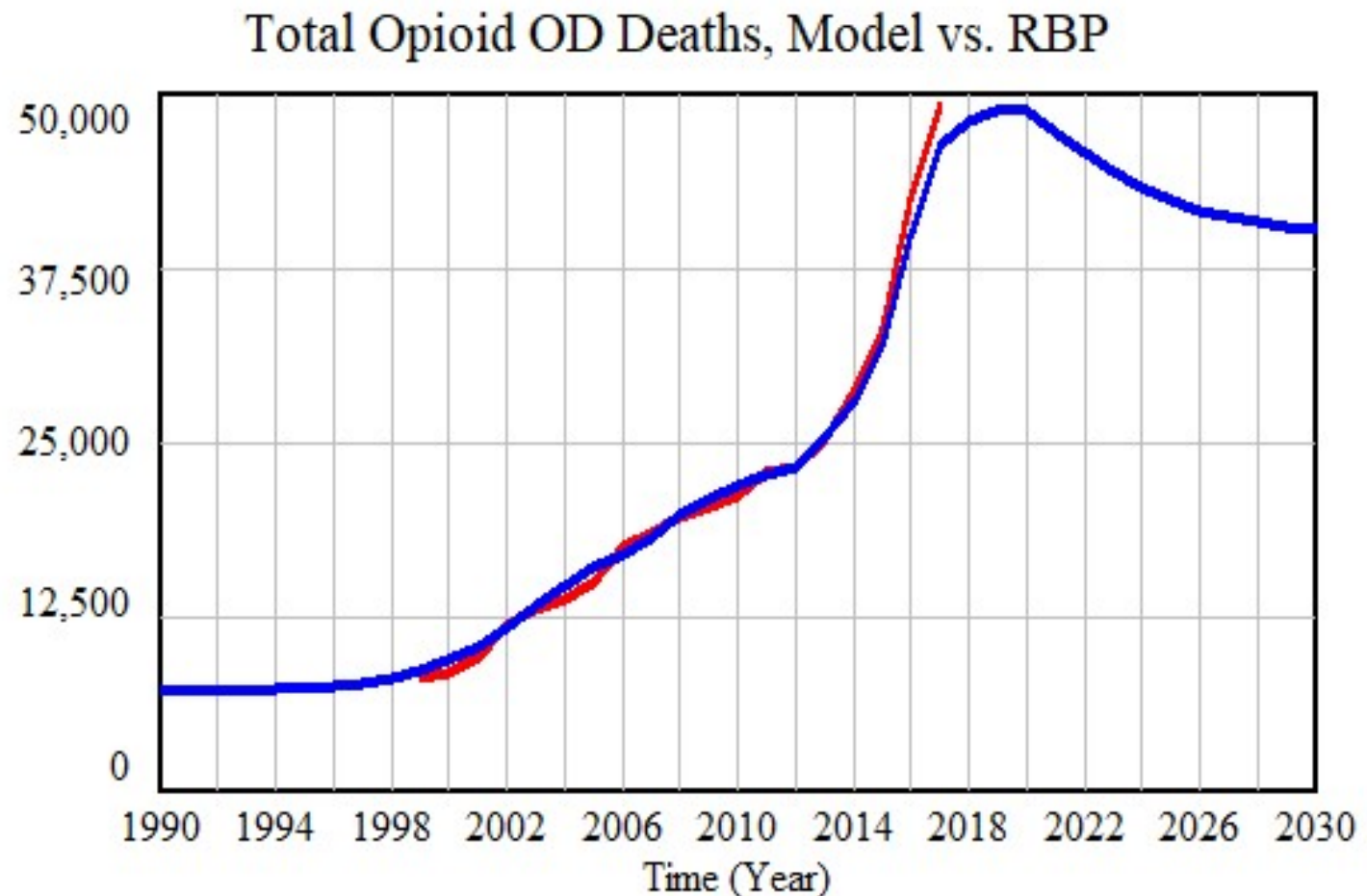
- Balance of modeler judgment & optimization
 - Powell optimization provided with Vensim Pro
- Must select which input parameters to allow to vary, the min/max values & distribution for each
- And specify the objective function
 - E.g. to minimize the mean error (MAEMs) in the model-calculated outcome trajectories vs. data
 - And weights for each term of the objective function
 - Each term is the fitness error for an outcome trajectory
 - Each weight is related to the std. deviation in that trajectory, adjusted possibly for missing data

Parameter documentation, incl. min/max

| Parameter | units | Value | Sources | min value | max value |
|--|--------|--------|--|-----------|-----------|
| Addicted frac of H users initial | Fract | 0.65 | Optimized; our NSDUH analysis shows 60.8% 2000, 61.1% 2005. | 0.6 | 0.7 |
| Addicted frac of PONHA initial | Fract | 0.123 | Optimized; our NSDUH analysis shows 11.4% 2000, 14.2% 2005. | 0.1 | 0.15 |
| Addicted H user OD death rate initial | 1/year | 0.010 | Optimized | 0.005 | 0.015 |
| Addicted H user quit rate initial | 1/year | 0.138 | Optimized | 0.07 | 0.21 |
| Addicted opioid abuser misc death rate | 1/year | 0.0045 | Ray et al 2016 gives mortality hazard ratio of 1.94 vs general popn for "high dose users" (>60 mg ME). Multiply by general popn: average of NVSR death rates for [age 25-34, 35-44, 45-54] = .0023 for 2000-2010 x 1.94 = .0045. | | |
| Addicted PONHA move to heroin rate initial | 1/year | 0.021 | Optimized | 0.01 | 0.03 |
| Addicted PONHA OD death rate initial | 1/year | 0.0059 | Optimized | 0.004 | 0.007 |
| Addicted PONHA quit rate initial | 1/year | 0.149 | Optimized | 0.08 | 0.22 |

3. Model Fitness (w/o uncertainty)

- To establish how well model with the “optimal” parameter values capture the dynamics of the target system



4. Make **large** Monte Carlo run (millions)

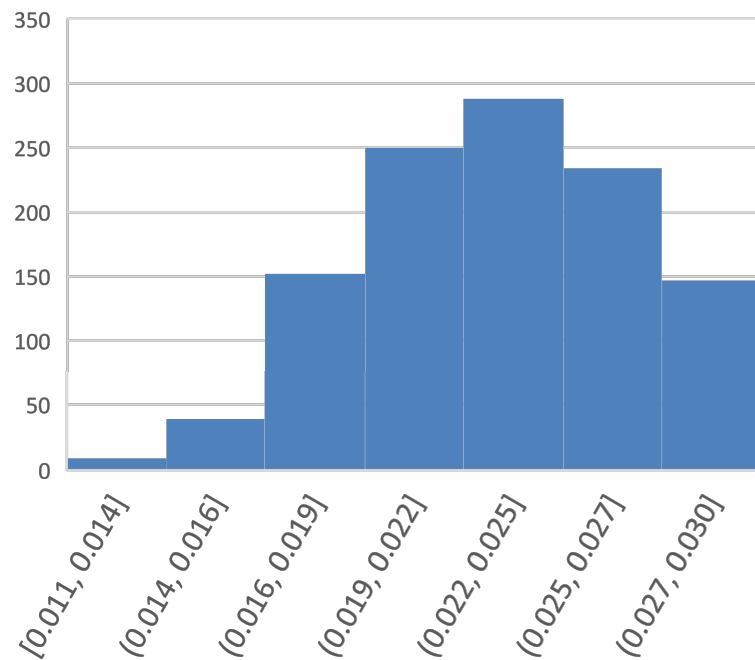
- Used Vensim sensitivity analysis feature in our case
- Could be Unif distributions between approp. min and max
 - Could be the same min/max used during calibration (or smaller)
 - Could instead use a Triangular dist. w/optimum value as mode
 - Would increase samples near the optima
 - This option was chosen for our illustration
- Must specify a list (.lst file) of variables to be saved for ea. run
 - Sensitivity runs automatically saves the varied parameters
 - It is also useful to know the maxMAEM and avgMAEM
 - Used to select “qualified” runs
 - Since we don’t need to know the time trajectories for this step, we set the model SAVEPER to 40 to keep the output file modest in size
- The weighted average MAEM was below .11 for 300-600 runs in each batch of 1M runs
 - These rows were kept. The file was then sorted by max MAEM
 - Yielding 100-130 runs with max MAEM < .20; these were kept
 - Made additional runs of 1M until ~1000 qualifying runs were found

Excel file illustrating MC results

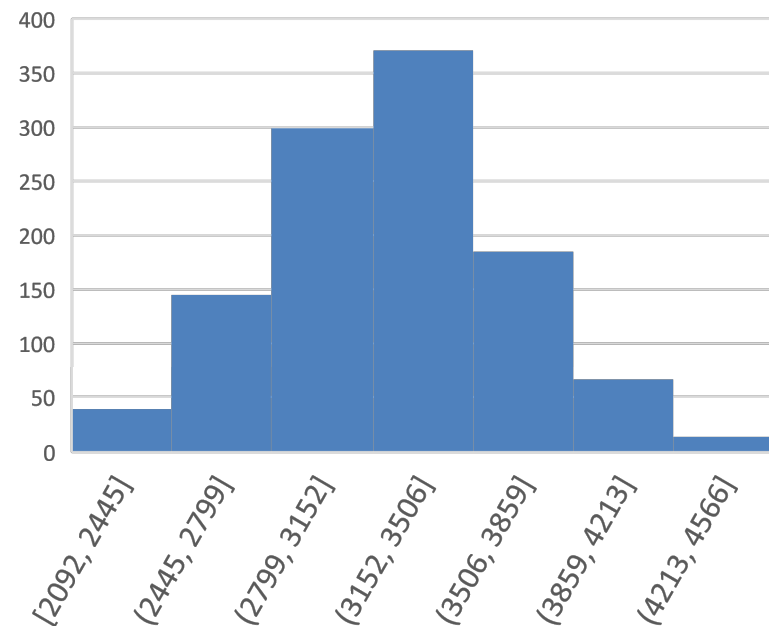
| Simulation # | Addicted frac of H users initial | Addicted frac of PONHA initial | Addicted H user OD death rate initial | ... | maxofMA EMs | Simple avg of all MAEM | Weighted avg of all MAEM |
|------------------|--|--------------------------------------|---|-----|----------------|---------------------------|--------------------------------|
| 681526 | 0.630252 | 0.126897 | 0.012121 | | 0.199446 | 0.100208 | 0.095847 |
| 376905 | 0.691254 | 0.118593 | 0.012554 | | 0.197496 | 0.101902 | 0.09691 |
| 131761 | 0.645974 | 0.118036 | 0.009821 | | 0.19669 | 0.105488 | 0.097978 |
| 67350 | 0.684089 | 0.117152 | 0.007821 | | 0.171332 | 0.101261 | 0.098184 |
| 726864 | 0.650072 | 0.124617 | 0.010844 | | 0.183783 | 0.101797 | 0.098296 |
| 736791 | 0.653768 | 0.123594 | 0.010931 | | 0.190377 | 0.115002 | 0.109998 |
| 358518 | 0.688724 | 0.122379 | 0.009984 | | 0.184925 | 0.114706 | 0.109999 |
| | | | | | | | |
| MIN of sims | 0.6012 | 0.1003 | 0.005936 | | 0.1612 | 0.1002 | 0.0958 |
| MAX of sims | 0.6998 | 0.1488 | 0.014533 | | 0.2000 | 0.1191 | 0.1100 |
| MIN allowed | 0.6 | 0.1 | 0.005 | | | | |
| MAX allowed | 0.7 | 0.15 | 0.015 | | | | |
| Optimized value | 0.65 | 0.123 | 0.010 | | 0.1795 | 0.0994 | 0.0935 |
| | | | | | | | |
| Sample Mean | 0.6487 | 0.1247 | 0.010454 | | | | |
| Sample std. dev. | 0.0204 | 0.0100 | 0.001473 | | | | |

Check distributions in the qualified parameter set for each input parameter (QPS)

Addicted PONHA move to heroin
rate initial



Consumption mgs ME per addicted
PO abuser per month initial

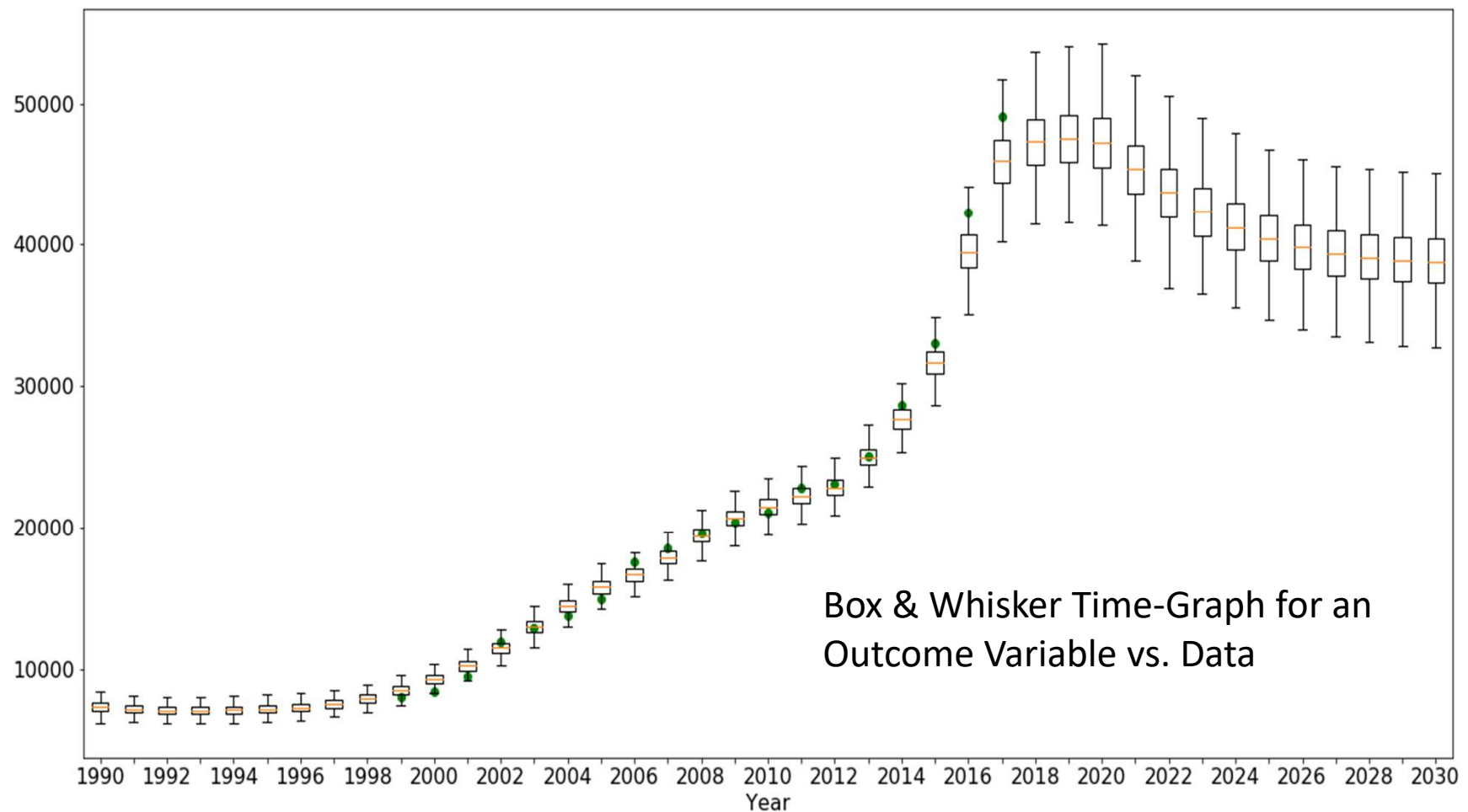


5. Use QPS to run a file driven sensitivity run

- This time, saving all of the outcome trajectories for each qualified parameter set
- Excel can be used to create a visualization, but Python provides more flexibility
- To display the uncertainty interval at time at each time point along the trajectory in a box & whisker format, along with the actual data

Example outcome trajectory uncertainty intervals

Total opioid OD deaths



Make policy analysis runs and compute uncertainty intervals

- Use the QPS to run each policy alternative for each qualified set of parameters
- The key here is that the differences at each time point for each outcome, are computed exactly, run by run
- Yielding a meaningful distribution of the effects of parameter uncertainty on the predicted impact of the policy change
 - Perhaps better on average, but could be worse

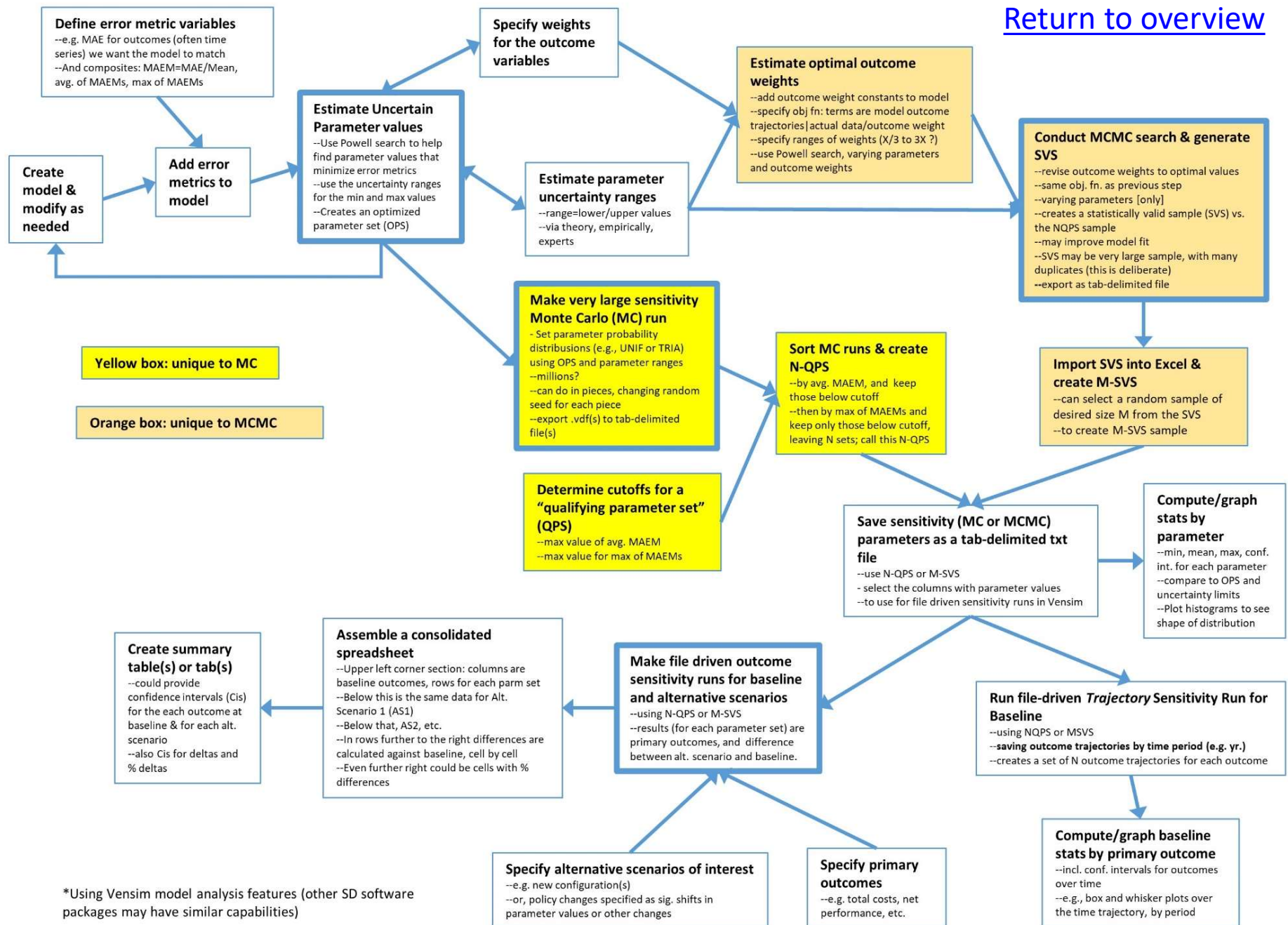


Example Policy Analysis Results

| | | Optimized parameter set | | QPS 1119 MC Result | | | QPS 1119 MC, % change vs baseline | | |
|-------------------------|--------------------------------|-------------------------|-------------------|--------------------|------------------|---------|-----------------------------------|------------------|-------|
| OUTCOME MEASURE | TEST CONDITION | Result | % chg vs Baseline | Mean | Range (min, max) | | Mean %Δ | Range (min, max) | |
| Persons with OUD (thou) | Baseline | 1,694 | | 1,593 | 1,111 | 2,084 | | | |
| | Avg MME dose down 20% | 1,510 | -10.9% | 1,416 | 1,035 | 1,823 | -11.1% | -25.7% | -3.4% |
| | Diversion Control 30% | 1,428 | -15.7% | 1,339 | 1,007 | 1,716 | -15.9% | -37.4% | -4.6% |
| | Treatment rate 65% (from 45%) | 1,713 | 1.1% | 1,585 | 1,054 | 2,130 | -0.5% | -9.0% | 5.0% |
| | Naloxone lay use 20% (from 4%) | 1,728 | 2.0% | 1,624 | 1,150 | 2,111 | 1.9% | 1.3% | 2.3% |
| | All 4 policies combined | 1,285 | -24.1% | 1,189 | 905 | 1,560 | -25.4% | -60.2% | -6.5% |
| Overdoses seen at ED | Baseline | 154,710 | | 149,450 | 124,745 | 179,297 | | | |
| | Avg MME dose down 20% | 152,686 | -1.3% | 145,473 | 118,491 | 176,363 | -2.7% | -8.2% | 3.8% |

Process for Addressing SD Model Uncertainty*

[Return to overview](#)



*Using Vensim model analysis features (other SD software packages may have similar capabilities)