Title: Ecological Economic System Modelling With a Focus on Endogenous Innovation and Resilience: Preliminary Results

by

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Abstract

This paper capitalises on recent developments in the modelling of production functions in dynamic systems and presents an analysis of the effect of innovation on an Ecological Economic Ssystem (EES). Using a dynamic two-sector (called 'resource' sector and 'manufacturing' sector), three-input (called 'labour', 'resource' and 'man-made capital') model of an EES, this paper contributes to the existing literature in four ways. First, the model employs a normalised CES (NCES) production function that helps us to study the effect of relative scarcity between the natural versus man-made capital--the primary issue concerning the strong sustainability criterion, in a time-consistent manner. Second, the production technology, represented by the input substitutability parameter between the natural and man-made capital, is endogenously determined, driven by the relative scarcity of the two inputs. Third, the model extends the literature that support the analytical focus on a decentralised system outcome with myopic agents, rather than a centralised system based on a benevolent social planner's infinite-time optimisation. And last but not the least, we employ system dynamics (SD) modelling that enables us to build and run an analytically unsolvable complex model.

Preliminary simulation results confirm that letting innovation on input substitutability respond to the relative price of inputs alters the dynamics of the system outcome. By enodogenising innovation on input substitutability in response to changes in the relative scarcity of inputs, innovation relieves pressure on the demand for the natural resource as an input and makes the dynamics of the system less volatile.

1. Introduction

This paper extends the branch of literature that addresses the dynamics of an Ecological Economic System (EES) that embodies exhaustible supply of productive resources and innovation as possible, but not necessarily viable, means to escape from the Malthusian kind of limits (Pearce, 2002).

There are known approaches to the study of properties of growth models. Traditional studies of growth dynamics of economic systems adopt infinite-time optimisation approaches and employ the Cobb–Douglas or CES production function and conclude that steady growth is possible when the elasticity of substitution (σ) is greater than or equal to one (*e.g.*, Growiec and Schmacher, 2008). Models in more recent studies depart from this traditional approach in two ways such that they can be integrated into the study of the sustainability of an ecological-economic system (EES). The first group of studies introduce endogenous innovation to their models. Through the applications of the New Growth Theory in the field of resource economics, studies such as Di Maria and Valente (2008) and Breschger & Smulders (2012) incorporate factor-specific endogenous innovation and find that poor substitutability ($\sigma < 1$) is not an issue for sustainability with sufficiently high innovation rates and low discount rates.[1] The second group of studies employ Variable Elasticity of Substitution (VES) production functions to embody the evolution of σ as the capital-labour ratio changes over time (Karagiannis *et al*., 2005; Antony, 2010). While the results of these studies still maintain the standard relation between $\sigma (> 1)$ and sustainable growth, the introduction of a VES production function opens the door to the modelling of endogenous evolution of input substitutability driven by relative input scarcity.[2]

This paper capitalises on these recent developments in the modelling of production functions in dynamic systems and presents an analysis of the effect of innovation on an EES. Using a dynamic two-sector (called 'resource' sector and 'manufacturing' sector), three-input (called 'labour', 'resource' and 'man-made capital') model of an EES, methodologically this paper contributes to the existing literature in four ways. First, the model employs a normalised CES (NCES) production function that helps us to study the effect of relative scarcity between the natural versus man-made capital--the primary issue concerning the strong sustainability criterion, in a time-consistent manner. An NCES is a suitable choice for the purpose of our analysis due to its theoretical advantages. The main benefit of using the normalised weights as factor shares is that, *ceteris paribus*, an increase in σ results in an increase in output. Normalisation indicates that there is a reference value for the resource-capital share at a given point (*cf*., Krump *et al*., 2011). This is critical in studying dynamic responses of the EES to varying σ in a consistent and meaningful manner. Graphically speaking, this means that as σ changes isoquants at the reference (or baseline) point that corresponds to the initial output level are all tangents. The isoquants will not cross at the baseline point as σ changes; instead, a larger σ will result in a higher isoquant (except at the tangency), representing higher productivity.[3]

Second, the production technology, represented by the σ between the natural and man-made capital, is endogenously determined, driven by the relative scarcity of the two inputs. In

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[1] In a typical neo-classical style, these studies use an infinite-time optimisation and focus on the steady state, which results in their findings. Their models adopt a set of common features--competitive goods market, "intermediary" goods that are produced by monopolists and depend on endogenous innovation, non-renewable resource as an essential input, and the Hotelling Rule.

[2] It may also be good to note that existing empirical studies suggest that using aggregate data tend to yield high estimate of σ (Koeste *et al*., 2008).

[3] See de la Grandville (2009) for a detailed description of this type of function.

other words, σ is no longer an invariant, exogenous parameter as it is in earlier studies. The structure of the model will drive the variation of the endogenous technology variable σ, to be recalibrated for each time period (*cf.* Temple, 2012). Namely, the increase in σ in our EES will be driven by the relative scarcity (the relative price) between the two capital inputs. This assumption of σ responding to (or dependent on) changes in the relative factor price is consistent with the empirical studies (*cf.* Koetsea *et al*., 2008; Bretscher, 2005).

Third, the model extends the literature (*e.g*., Nagase and Uehara, 2011) that support the analytical focus on a decentralised system outcome with myopic agents, rather than a centralised system based on a benevolent social planner's infinite-time optimisation (*e.g*., André and Cetrá, 2005). In reality, economic agents seldom implement infinite-time optimisation of their activities. Therefore, although theoretically beautiful, the assumption of perfect foresight into the very distant future is a rather challenging one in a real-world context. An alternative approach is that of bounded rationality, *i.e*., agents can take account of, and hence are concerned about, some foreseeable near future. This approach follows the position by Sterman's (2000) advice that "[in order t]o mimic the behaviour of real systems models must capture decision making as it is, not as it should be, nor how it would be if people were perfectly rational" (p. 597).[4]

And last but not the least, we employ system dynamics (SD) modelling that enables us to build and run an analytically unsolvable complex model. Several features of SD modelling are essential for the model building and simulation. For example, the hill-climbing optimization technique (Sterman, 2000) is essential to solve and run the complex mathematical model presented below.

Using a system dynamics (SD) approach, in analysing the simulation results we focus on the transitional dynamics of the model, rather than its steady state. The evolving literature on thresholds of EESs warns that the viability of an EES depends critically on the transitional paths (*e.g*., Baumgärtner and Quaas, 2009; Uehara, 2013). Even if an EES has a long-run equilibrium steady state that is sustainable, it may take a very long time for the system to reach its equilibrium (Dasgupta and Heal, 1974, as stated by Bretschger and Pittel, 2008). Depending on the viability of the EES, the system may cross the threshold in the transitional stage and collapse. Solow also asserts that the out-of-equilibrium state has a non-negligible impact on the resource allocation (Solow, 1974). Our study extends a SD model developed by Uehara *et al*. (2016), which is a multi-sector EES model of population-resource dynamics. Contrary to standard economics models, our study addresses out-of-equilibrium states and adaptations striving to find new equilibria (period-by-period equilibrium prices and output and input levels). The SD approach has been employed to investigate the viability of an EES, by (i) identifying thresholds that lead to the collapse of the system , (ii) relating the collapse processes that lead to crossing thresholds to the underlying system structure, and (iii) allowing alternative hypotheses on key parameter values related to innovation, degree of government intervention, etc., and comparing the resulting system outcomes (*cf.* Cumming and Peterson, 2017).

Our use of an SD model allow us to use an NCES function without depending on analytic solutions and introduce an endogenous innovation process for input substitutability. Our simulation results confirm that letting σ respond to the relative price of inputs alters the dynamics of the system outcome. By enodogenising σ in response to changes in the relative scarcity of inputs, innovation relieves pressure on the demand for the natural resource as an input and makes the dynamics of the system less volatile.

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[4] Conlisk (1996) provides both empirical evidence and theoretical discussions about the importance of bounded rationality in the context of economics.

The rest of the paper is organised as follows. Section 2 presents out model and explains the simulation method. Section 3 reports our preliminary results from the simulation exercise of the baseline model and the model with endogenous innovation. Section 4 provides a discussion on the issues to be explored further, based on the preliminary result and concluding remarks.

2. Methodology

2-1. The base model:

We adopt a two-sector (called "resource" sector and the "manufacturing" sector) capital-resource model. In contrast to a macro-based model that would obscure both the varying capital-resource substitution possibility at the sector level and the varying relative growth of the sectors as resource availability changes, a sector-level model of microeconomic foundation has the advantage of capturing such structural changes and providing "a robust fundament for theoretic modelling" (Bretchger, 2005).[5]

In our model, the availability of the flow of natural resources as inputs, represented by *R*, depends on the overall natural resource stock size, represented by *S*. The transitional dynamics of *S* is governed by the difference between the growth of the resource *G*(*S*) and the impact on the use of *S* by the resource sector (subscript *t* denotes time period):

$$\frac{dS_t}{dt} = G(S_t) - g_R R_t = \mu S_t \left(1 - \frac{S_t}{S_{max}}\right)(S_t - S_{min}) - g_R R_t \tag{1}$$

G(*S*) takes the form of a critical depensation growth function with the intrinsic growth rate μ, the carrying capacity $S_{\max}$, and the tipping point $S_{\min}$ (*cf.*, Uehara, 2013). g_R is a parameter representing the impact of *R* on the natural resource stock.

Our base model embodies the issue of capital-resource substitution through the production function for the manufacturing sector (*M*-sector), using a normalised CES (NCES) function:

$$M_t(L_{Mt}, K_t, R_{Mt}) = AM_0 \left[\pi_K \left(\frac{K_t}{K_0}\right)^{\rho_t} + \pi_R \left(\frac{R_{Mt}}{R_{M0}}\right)^{\rho_t}\right]^{\frac{1}{\rho_t}\gamma} {L_{Mt}}^{1-\gamma};$$

$$\pi_K = \frac{r_t K_0}{p_{Rt} R_{M0} + r_t K_0}, \qquad \pi_R = \frac{p_{Rt} R_{M0}}{p_{Rt} R_{M0} + r_t K_0}. \tag{2}$$

K_t and R_{Mt} denote man-made capital input and natural resource input for *M*-sector in period *t*, respectively. L_{Mt} denotes the labour input for this sector in period *t*. r_t and p_{Rt} denote the prices of *K* and R_M in period *t*, respectively. These five variables are endogenously determined in each time period. The elasticity of substitution between *K* and R_M is given by

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[5] The appreciated value of NCES functions among macroeconomists seems to be its "empirical fitness" (but a simple one-sector structure that requires an aggregate production function must meet highly restrictive conditions such as equal input intensities across all sectors--see Bretschger and Pittel (2008) and Felipe and McCombie (2013). NCES functions have been used in empirical estimations of σ (*e.g.*, León-Ledesma *et al*., 2010b for *L* vs. *K* for US, with $\sigma \approx$ 0.5-0.6). Meanwhile, for our research purpose, the importance of using a properly weighted CES such as norm CES is that it is purely based on the fact that it is a power mean function and hence behaves *as expected*.

$\sigma_{KR} = 1/(1-\rho)$. $A > 0$ is a (Hicksian-neutral) productivity parameter, and by assumption the output elasticity parameter γ is between 0 and 1.

Besides meeting the standard properties of production functions, an NCES is a suitable choice for the purpose of our analysis due to its theoretical advantages.[6] First, the main benefit of using the "normalised" weights as factor shares is that, *ceteris paribus* an increase in σ results in an increase in output. Normalisation indicates that there is a reference value for the K-R_M share at a given point (*cf.*, Krump *et al.*, 2011). This is critical in studying dynamic responses of the EES to varying σ in a consistent, and meaningful, manner. Graphically speaking, this means that as σ changes isoquants at the reference (or baseline) point (K_0, R_{M0}) that corresponds to the initial output level M_0 are all tangents. The isoquants will not cross at the baseline point as σ changes, instead, and a larger σ will result in a higher isoquant (except at the tangency) representing higher productivity. [7]

Second, a known property of a CES function in general that suits our specific focus on natural resource scarcity is that, when $\sigma_{KR} < 1$, $\lim_{R \to 0} M = 0$, making R_M a *growth essential* input (*cf.* Groth, 2007). A useful definition associated with the case of $\sigma < 1$ is that an input is *more important* for production if it is used in a *smaller amount* (Growiec and Schumacher, 2008).

Finally, because the factor weights are such that $\pi_R = 1 - \pi_K$, the effect of a change in π_K on output depends on the relative scarcity of K and R.[8] Namely,

$$\frac{\partial M}{\partial \pi_K} > 0 \text{ if } \frac{K}{K_0} > \frac{R_M}{R_{M0}}$$

and

$$< 0 \text{ if } \frac{K}{K_0} < \frac{R_M}{R_{M0}} .$$

Output of the resource sector (R-sector) is given by the following production function:

$$R_t(L_{Rt}) = \alpha S_t L_{Rt} \tag{3}$$

where $\alpha > 0$ is a productivity parameter and L_{Rt} denotes the labour input for this sector; the latter is endogenously determined in each period. This sector produces both the final good Q_R

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[6] An NCES function is homogeneous hence maintains the known properties in neoclassical theory, that are (i) we can always define a total scale elasticity (ε) which is equal to the degree of homogeneity of the function; (ii) Euler's theorem applies; and (iii) σ can be defined using the natural log of the input ratio and input price ratio in equilibrium (Bairam, 1994). For a three-input model, two-level nesting of (KR)L is the popular one among the three alternatives in CGE model, the GTAP-E model being a well-known example. But it is also known that the outcome of CGE models depend critically on the choice of nesting (Dissou *et al*., 2015).

[7] See de la Grandville (2009) for a detailed description of this type of function.

[8] (*Cf.* Leon-Ledesma *et al*., 2010a) If we add factor-specific innovation parameters (or variable) Γ^K and Γ^R to the function so that

$$Q(L, K, R; \rho) = AQ_0 \left[\frac{r_K K_0}{r_R R_0 + r_K K_0} \left(\Gamma^K \frac{K}{K_0} \right)^{\rho} + \frac{r_R R_0}{r_R R_0 + r_K R_0} \left(\Gamma^R \frac{R}{R_0} \right)^{\rho} \right]^{(1/\rho)\beta} L^{1-\beta}$$

Then, in equilibrium, both $\partial\left(\frac{Q_K}{Q_R}\right) / \partial\left(\frac{\Gamma^K}{\Gamma^R}\right)$ and $\partial\left(\frac{r_K}{r_R}\right) / \partial\left(\frac{\Gamma^K}{\Gamma^R}\right) > 0$ if $\sigma > 1$ and < 0 if $\sigma < 1$. Therefore, capital-augmenting technical progress results in capital-*biased* technical progress only when $\sigma > 1$.

for consumption and the intermediary good R_M to be employed in the manufacturing sector. Therefore, $R_t \equiv Q_{Rt} + R_{Mt}$.

We assume that, in equilibrium, profit maximisation in these two sectors requires that the contribution of an additional unit of an input in monetary value (*i.e.*, the marginal revenue product) must equal its price. This is given by the following equations (time subscript suppressed):

$$p_R \alpha S = w \tag{4}$$

$$\frac{P_M(1-\gamma)M}{L_M} = w \tag{5}$$

$$p_M A M_0 (L - L_H)^{1-\gamma} \gamma \left[\pi_K \left(\frac{K}{K_0}\right)^{\rho} + \pi_R \left(\frac{R_M}{R_{M0}}\right)^{\rho}\right]^{\frac{\gamma}{\rho}-1} \pi_R \frac{{R_M}^{\rho-1}}{{R_{M0}}^{\rho}} = P_R \tag{6}$$

$$p_M A M_0 (L - L_H)^{1-\gamma} \gamma \left[\pi_K \left(\frac{K}{K_0}\right)^{\rho} + \pi_R \left(\frac{R_M}{R_{M0}}\right)^{\rho}\right]^{\frac{\gamma}{\rho}-1} \pi_K \frac{K^{\rho-1}}{{K_0}^{\rho}} = r \tag{7}$$

where the population L must meet the labour demand from the two sectors, *i.e.*, $L \equiv L_{Rt} + L_{Mt}$, and w denote the unit price of labour, endogenously determined in each time period.

Population L is given for each time period, and its transitional dynamics is governed by the following birth and death functions:

$$\frac{dL}{dt} = [b(q_R, q_M) - d(q_R, q_M)]L;$$

$$b(q_R, q_M) \equiv b_0\left(1 - \frac{1}{e^{b_1 q_R}}\right)\frac{1}{e^{b_2 q_M}} \text{ and } d(q_R, q_M) \equiv d_0 \frac{1}{e^{q_R(d_1 + d_2 q_M)}} \tag{8}$$

where $b_0, b_1, b_2, d_0, d_1, d_2 > 0$. b_i and d_i, $i = 0, 1, 2$ denote the birth and death rates, respectively. Fertility is positively correlated with per-capita consumption of the resource good Q_R (q_R) and is negatively correlated with per-capita consumption levels of the manufactured good Q_M (q_M). Mortality is negatively correlated with both q_R and q_M. The term $b_0\left(1 - \frac{1}{e^{b_1 q_R}}\right)$ depicts that, as the consumption of the resource good increases, so does the birth rate. The term $\frac{1}{e^{b_2 q_M}}$represents a decrease in the birth rate due to increases in the consumption of the manufactured good. The term $d_0 \frac{1}{e^{q_R(d_1+d_2 q_M)}}$ tells that a higher consumption level of the resource good reduces the death rate, but that a higher consumption level of manufactured good reduces the death rate via the term. Equation (8) represents a Malthusian population dynamics in the sense that the higher per-capita consumption of a harvested good leads to higher population growth. However, parameters b_2 and d_2 make this model non-Malthusian (*cf.*, Nagase and Uehara, 2011).

In each time period, a representative consumer maximises utility subject to the budget constraint (time subscript suppressed):

$$\max_{\{q_R, q_M\}} \quad u = {q_R}^{\beta} {q_M}^{1-\beta}$$

$$\text{s.t.} \quad p_R q_R + p_M q_M = (1-\eta)\left(w + \frac{rK}{L}\right).$$

q_R and q_M denote per-capita consumption levels of the resource good Q_R and manufactured good Q_M, respectively.

The budget constraint contains two basic assumptions. First, for simplicity each agent has one unit of labour to be allocated across the two sectors, and the rental price of capital is evenly distributed back to all agents. Second, a fraction η of each individual's income is invested in the formation of the capital input *K*, which yields the following transitional dynamics for *K*:

$$\frac{dK}{dt} = \theta\frac{\eta(wL+rK)}{p_M} - \delta K \tag{9}$$

The first term on the right-hand side specifies that output from *M*-sector is used to form *K* as an input, and θ is the unit-productivity parameter. Parameter δ represents the capital depreciation rate. Note that man-made capital accumulation depends indirectly on natural resources through the production of manufactured good.

This optimisation problem yields the consumption demand functions for the outputs from the two sectors:

$$Q_R = L\cdot q_R = \frac{(1-\eta)\beta}{p_R}(wL+rK)$$
$$Q_M = L\cdot q_M = \frac{(1-\eta)(1-\beta)}{p_M}(wL+rK)$$

The static equilibrium $\{L_R^*, Q_M^*, w^*, r^*, p_R^*, p_M^*\}$ is given by equations (4), (5), (6), (7), and the following two equilibrium conditions (10) and (11) for the good markets:

$$\frac{(1-\eta)\beta}{p_R}(wL+rK) + R_M = \alpha S L_R \tag{10}$$

$$\frac{(1-\eta)(1-\beta)}{p_M}(wL+rK) + \frac{\eta(wL+rK)}{p_M} = AM_0\left[\pi_K\left(\frac{K}{K_0}\right)^{\rho} + \pi_R\left(\frac{R_M}{R_{M0}}\right)^{\rho}\right]^{\frac{1}{\rho}\gamma} {L_M}^{1-\gamma} \tag{11}$$

The transitional dynamics of the EES from one period to the next is governed by equations (1), (8) and (9).

2-2. System Dynamics methodological tools employed for the simulation of the model

We translate the baseline model presented in equations (1) through (11) into a system dynamics model. System dynamics is a computer-aided approach to a system of coupled, nonlinear, first-order differential (or integral) equations (Richardson, 2013). There are two justifications for this choice. First, due to the complexity of the model, it is not possible to derive analytically the static equilibrium $\{L_R^*, Q_M^*, w^*, r^*, p_R^*, p_M^*\}$ as is typically done for mathematical models in economics. Second, and more importantly, the instantaneous equilibrium states are highly criticized (*cf.* Dasgupta, 2000), and the importance of disequilibrium states has been noted (*e.g.* Costanza *et al.*, 1993). A system dynamics model can capture disequilibrium states which could move toward equilibrium states. While equations (1) through (11) assume an instantaneous equilibrium state all the time, our system dynamics model allows such disequilibrium states in which there are mechanisms moving the systems state towards equilibrium.

The hill-climbing optimization technique is used to capture the dynamics of moving towards equilibrium states from disequilibrium ones (Sterman, 2000). Uehara (2013) and

Uehara *et al*. (2016) applied the technique to EES. Hill-climbing is a technique to adjust a state of system until it reaches the desired state of the system. The structure of hill-climbing using a stock-and-flow diagram is presented below.

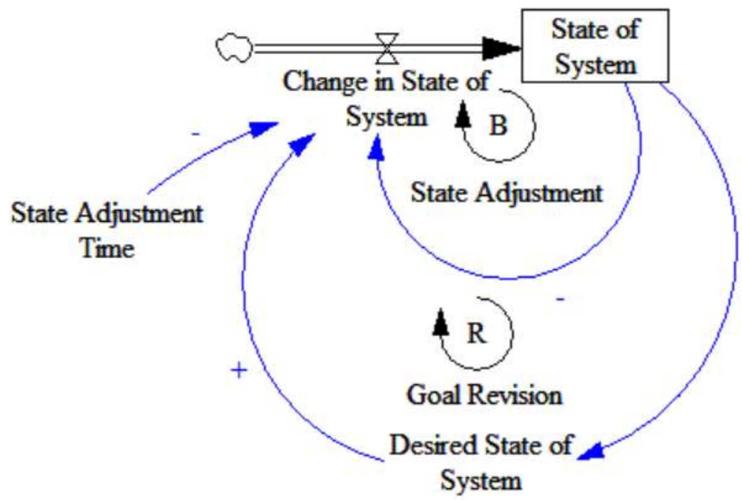

Figure 1. A general structure of hill-climbing search
Adopted and simplified based on Figure 13-6 in Sterman (2000, p.539). A box is stock; blue arrows are information arrows; an arrow enters into the box is a flow into the stock; B surrounded by an arrow indicates a balancing feedback; R surrounded by an arrow indicates a reinforcing feedback.

In mathematical form,

$$\text{State of System}_t = \int_{t_0}^{t} \left(\text{Change in State of System}\right) ds + \text{State of System}_{t_0},$$

$$\text{Change in State of System} = \frac{\left(\text{Desired State of System} - \text{State of System}\right)}{\text{State Adjustment Time}}.$$

As the model shows, the "State of System *S*" changes until it in identical to "Desired State of System." It is, for example, an equilibrium price of manufactured goods, P_M^*. Since it could change over time in our model, it could happen that the system does not reach the equilibrium state $\{L_R^*, Q_M^*, w^*, r^*, p_R^*, p_M^*\}$. The full system map is presented in the Appendix (see Figure A1).

To find parameter values that would yield a particular scenario of interest, we used the optimization method available in the Vensim Pro software, as follows. We started with a trajectory for population that reflected an "overshoot and collapse" scenario. We then wrote an objective function to be maximized that simultaneously attempted to accomplish the desired population trajectory and to maximize the difference in the *M* good production rate over time between a fixed exogenous (fixed) ρ of -10 and an endogenous (variable) formulation in which ρ is a function of τ and an innovation variable *x*.
To be able to run both model formulations at the same time and calculate the difference in production rate trajectories, we created a copy of the entire model in another "view" in the Vensim software in which each variable had a prefix of "BBB" added to its name (representing the "B" version of the model) in contrast to the "A" version without the prefix.

Both models were identical, and since we wanted the optimization to vary thirteen parameters (in both models), the names of the 13 parameters being optimized did not have the prefix in their name.
The thirteen parameter were: the regeneration rate for the natural resource (μ), six parameter governing the birth and death rates of the population (b_0, b_1, b_2, d_0, d_1, d_2), R-sector productivity parameter (α), the M-sector output elasticity parameter (γ), and four "adjustment time" parameters for factor demand R_M, prices of the resource good and manufactured good (p_R and p_M), and the rental price for capital (r).

We experimented with the weights used to identify the comparative scenario with a large difference in M-sector good production rate between the exogenous and endogenous ρ formulations. With a large weight on population scenario, it was not possible to achieve a large difference in the production rates of M, so that weight was reduced. We eventually obtained the scenario in which the population L achieves an equivalent increasing trajectory over time, and at the same time endogenous variables yield different dynamics between the exogenous and endogenous ρ formations. The set of parameter values (and initial values of the endogenous variables) are reported in Table 1A in the Appendix. The difference in the dynamics of endogenous variables are reported at length in the next section.

3. Simulation Results

3-1. The baseline model

The baseline model presented above yields a set of system outcomes with a fixed value of ρ (and σ) and provides a system in which factor (input) prices and factor weights are interdependent. In our simulation, factor prices r and p_R reflect the relative input scarcity in each period, and the factor weights, also known as "share parameters" (though they are not parameters but variables in our model) will evolve over time.

We identified a set of parameter values (see Table A1 in the Appendix) that yield a system outcome that we regard as the reference behavioural pattern, with the population and resource stock levels oscillating across time without a collapse, with the assumption that the natural resource input R is growth essential, which is consistent with the strong sustainability criterion (*i.e.*, $R < 1$).[9] The time horizon of 200 years has been chosen for our analysis, so that the time horizon is long enough for the system to exhibit the dynamic transition of the system towards a steady state, with the chosen set of parameter values. The reference behavioural pattern consisting of the dynamics of a set of key variables is reported along with the dynamics of the model with endogenous ρ (and σ) in Section 3-2, to facilitate the comparison between the two sets of results.

3-2. The effect of endogenous technological changes in input substitutability

To be compared with the reference behavioural pattern is the system outcome, with a mechanism of endogenous innovation. Following the traditional *ceteris paribus* approach in economics, we hold all other aspects of the system unchanged between the two versions of the models. We modify the basic model so that whenever either of the two types of input became "scarce" (represented by the normalised relative price), input substitutability (σ) increases.

[9] Although our study is not an empirical one, there are many empirical supports for complementarity ($\sigma < 1$) between natural resource and man-made capital input, and also between labour input and man-made capital input. See, *e.g.*, Albarez-Cuadrado *et al*. (2018) and Nagase and Uehara (2011).

This mechanism of σ responding to (or dependent on) changes in the relative factor price is consistent with the empirical studies (*cf.* Koeste *et al*., 2008; Bretscher, 2005). This approach also responds to the criticism that σ should depend on other endogenous variables (Temple, 2012).

The introduction of endogenous innovation as described above to the system is done, by adding the following set of equations. In each period, the value of ρ is given by

$$\rho = \tau\left(\frac{1}{1+e^{-x}} - 1\right);\ \tau > 0 \tag{12}$$

where variable x is governed by the following transitional dynamics: [10]

$$\frac{dx}{dt} = \varsigma\left[\left|\frac{p_{Rt}/p_{R0}}{r_t/r_0} - 1\right| + \left|\frac{r_t/r_0}{p_{Ht}/p_{H0}} - 1\right|\right];\ x_0 = 0, \varsigma > 0 \tag{13}$$

With equations (12) and (13) added to the system, Figure 2 demonstrate the changes in the dynamics of ρ changes (the value of ρ for the baseline model is set at -10):

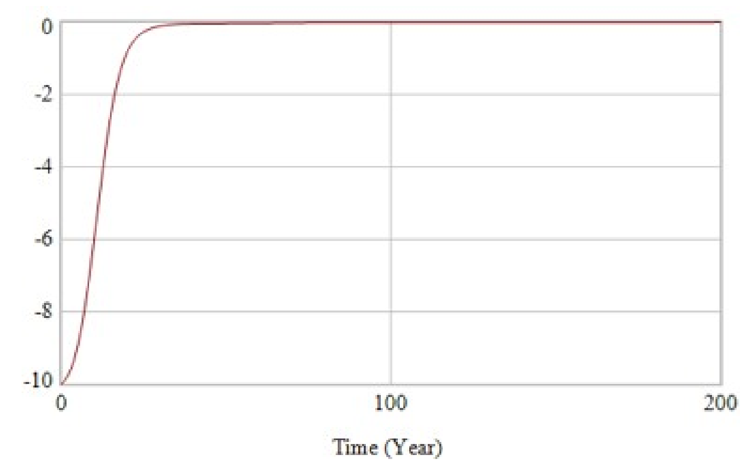

Figure 2: dynamics of innovation parameter ρ

In the baseline case, the value of ρ is fixed at -10. Compared with the baseline case, changes in (normalised) relative price between the two inputs increases the value of input substitutability parameter σ. As the value of ρ increases towards zero, σ approaches to unity from below, improving the substitutability between natural resource R and man-made capital K, while leaving the natural resource input growth essential.[11]

The population dynamics is the same across the two sets of the simulation results.

[10] Our modelling approach lets economic agents respond to prices that reflect relative resource scarcity, without introducing a separate innovation sector to the system. An approach commonly employed by endogenous growth models is to introduce a monopoly (or competitive) sector dedicated to innovation (*e.g*., Bretschger, 2005; Di Maria and Valente, 2008; Bretschger and Smolder, 2012). This approach will increase the number of variables, as the degree of innovation is represented by the increase in the number of intermediary goods. Fenichel and Zhao (2014) impose the "conservation law" (which is an identity between production and consumption) that determines the amount of "output" used for knowledge capital accumulation. This approach is simple and works for central planner's optimisation, but our model is decentralised. See Nagase and Uehara (2011) for more information on approaches concerning innovation taken by earlier studies using similar model structures.

[11] By construction of equation (12) which keeps σ below unity, we are imposing the assumption of complementarity between the two types of input (R and K). This assumption follows the general assumption on the natural resources as essential in production of goods and services. See Uehara *et al*. (2016).

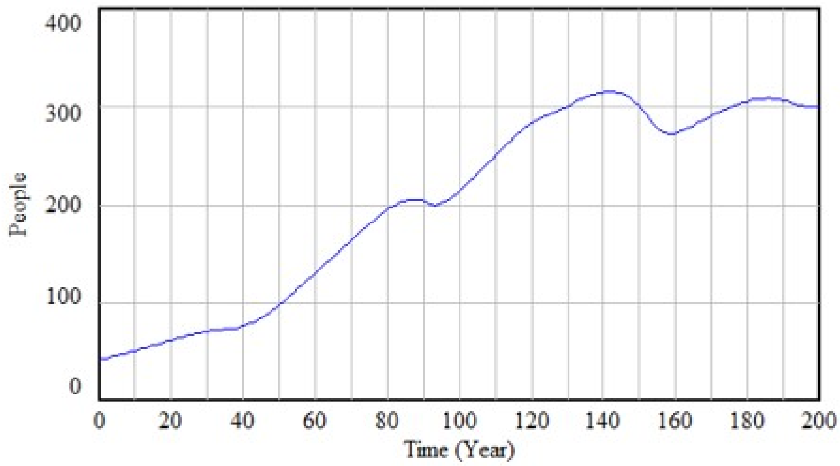

Figure 3A. Population (*L*) dynamics

The chosen population dynamics it that of an increasing trajectory, as is the case of many economies, historically. We have chosen to compare the two system outcomes with the same underlying population dynamics, to highlight how different the experience for the individuals who live in the otherwise likewise EES might be, due to the difference in the innovation process.

How does the system respond to endogenised input substitutability? Let's start with the stock variable, natural resource stock *S*.

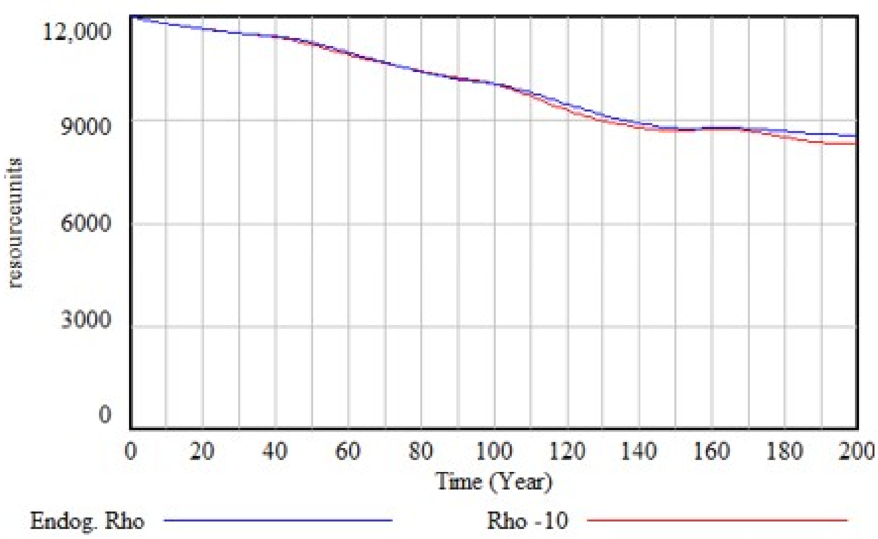

Figure 3B. Resource (*S*) dynamics

Figure 3B shows the 'smoothing out' effect of endogenous innovation on resource stock *S*, and also shows that, over time, the size of the resource stock will be higher with endogenous innovation. Given the same size of the population at any point in time for both scenarios, the dynamics of the two stock variables as displayed by Figures 3A and 3B depict a picture that the individuals in the EES with endogenous innovation will experience a higher sense of abundance in their surrounding ecosystem.

Figures 4A and 4B provide additional dimensions to convey the sense of individuals' well-being, under the two scenarios.

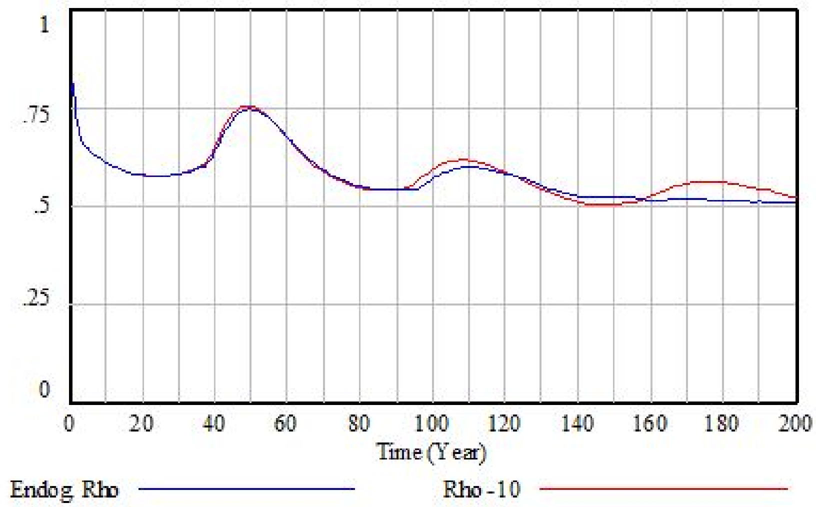

Figure 4A. Dynamics of per-capita consumption of the resource good (q_R)

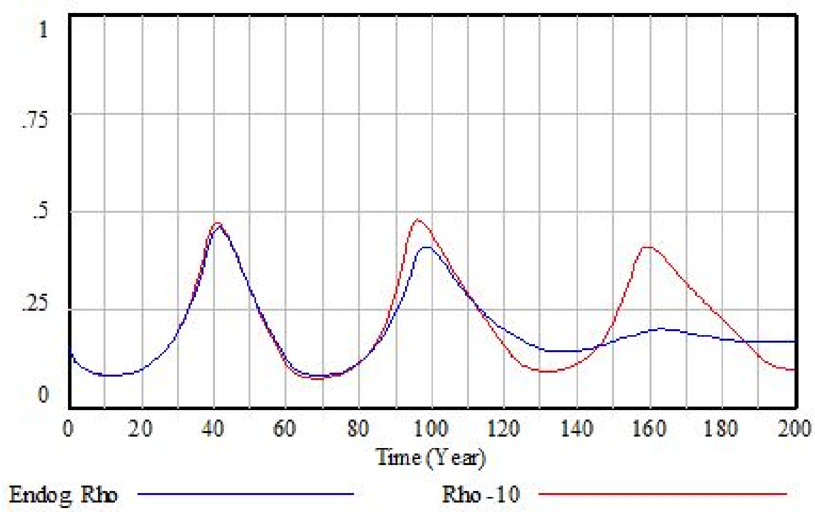

Figure 4B. Dynamics of per-capita consumption of the manufactured good (q_M)

Figures 4A and 4B show that, with endogenous innovation, consumption levels of the resource good per capita q_R and q_M become smoothed out, compared with those of the baseline case. The directions of smoothing out are consistent between the two variables (the blue curve above/below the red, during the matching time periods). This is consistent with the equivalent population dynamics for both scenarios, based on equation (8). The smoothing effect is more pronounced for q_M. The dynamics of the production levels of M (Figure 4C) provides the explanation.

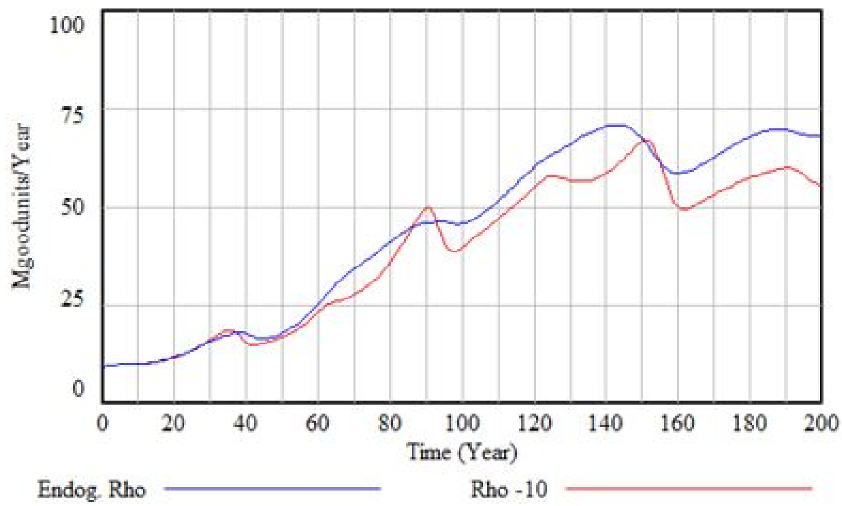

Figure 4C. Dynamics of manufactured good (M)

As a result of endogenous innovation, the overall output (and consumption) level of the manufactured good M is higher for the latter half of the simulation period (Figure 4C). This is facilitated by improvements in the substitutability between inputs R and K.

Next question: how does endogenous innovation affect the employment of resource R and man-made capital input K? Figures 5A and 5B provide a clear answer.

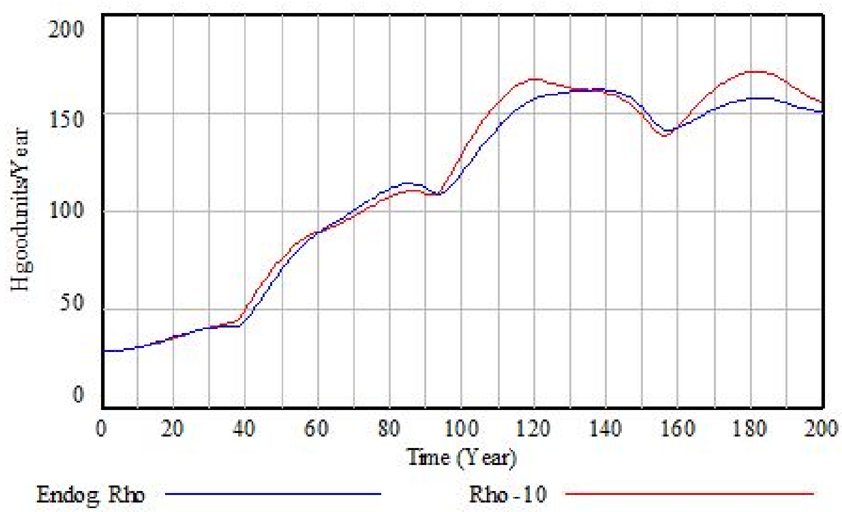

Figure 5A. Dynamics of natural resource input (R)

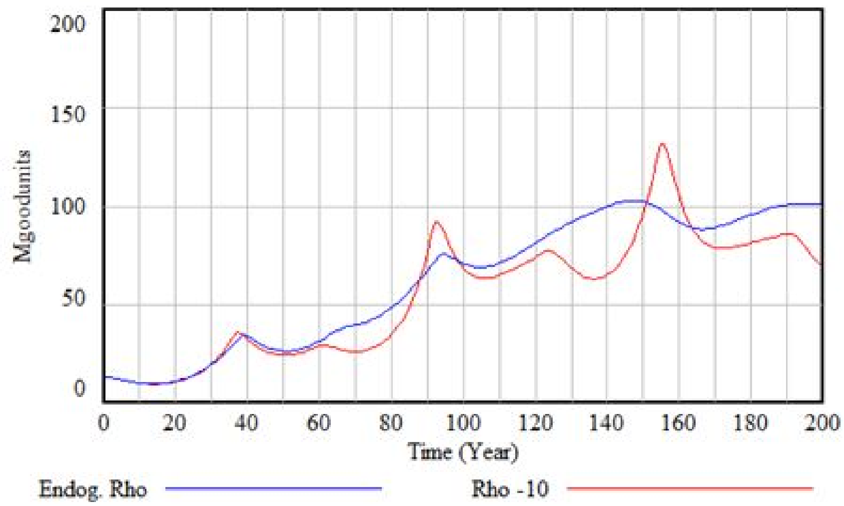

Figure 5B. Dynamics of man-made capital input (K)

Figures 5A and 5B show that the natural resource input R, often labelled in the sustainability literature the 'natural capital' input, is replaced by the man-made capital input K.

The Dynamics of the relative price between the two goods can help explaining decisions made by the economic agents in the system.

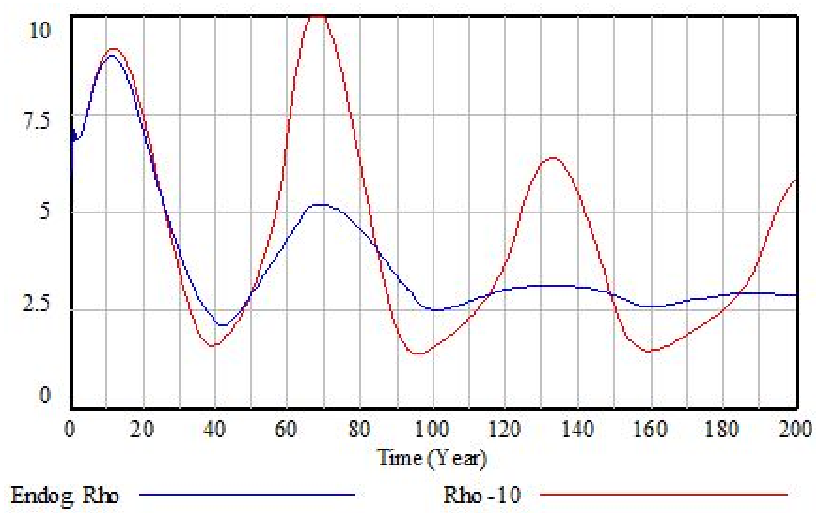

Figure 6. Dynamics of the relative price (p_M/p_R)

Once again, we are observing the 'smoothing effect' of endogenous innovation on the oscillation of the relative price. By design, innovation is driven by the relative scarcity between 'natural capital' and 'man-made capital', as represented by the (normalised) relative price of these inputs (equation (13)). Input substitutability in the 'M'-sector improves as a result, making the relative price of the two goods less volatile across time. The direction of the changes in per-capita consumption of the two goods as a result of endogenous innovation, shown by Figures 4A and 4B, are consistent with the changes in the relative price shown in Figure 6, in light of the Law of Demand; as the law of demand predicts, a relatively more expensive good (M) become consumed less.

To conclude, our modelling exercise successfully addressed the issue of endogenous innovation driven by the price signal, and the simulation results are consistent with the basic principle of a market system, that is, price signals should convey the relative resource scarcity so that economic agents will change their behaviour accordingly. As a result, the dynamics of key variables show that the endogenous innovation can alleviate the volatility of the EES.

4. Discussion and Conclusion

This study aims to address the criticisms raised against the branch of literature that addresses the dynamics of an EES that embodies exhaustible supply of productive resources and innovation. By introducing into an EES an NCES production function, endogenous innovation driven by price signals, and a decentralised economic structure without agents' perfect foresight into the future, our simulation exercise using SD yield logically sound preliminary results, providing a basis for further exploration of the sensitivity, boundaries, and variations of the basic model.

Our results from the baseline model and the model with endogenised innovation deliver an anticipated outcome in view of economics; prices serve the signalling role for relative scarcity of productive inputs, and innovation driven by such price signals contributes to smoothing out the dynamic oscillation of the key economic variables in the system.

Based on the results obtained, there are issues that warrant further exploration. First, it is worthwhile to investigate the sensitivity of the system to certain parameters. The model is sensitive to changes in some parameters. There are three types of sensitivity caused by a change in assumptions: numerical, behaviour mode, and policy sensitivity (Sterman, 2000). Numerical sensitivity is about changes in numerical values of the model. Behavioural mode sensitivity is about changes in the patterns of behaviour generated by the model. Policy sensitivity is about changes in the policy recommendation. Since our model is more about thought experiments rather than about real case studies, behavioural model sensitivity is of our concern. For example, a change to the output elasticity parameter γ in equation (2) has a significant impact on the patterns of the behaviour of the production of the manufactured good M. Therefore, there exists a threshold value of γ with which the model exhibits different patterns of behaviour. Because of the model complexity, it is almost impossible to test all the possible variations. However, it is critical to conduct further sensitivity analysis by focusing on key assumptions of the model to test corroborate the robustness of our findings (Sterman, 2000). Second, another variation of the model, with the capital accumulation process driven by the relative input scarcity, will generate further useful results. This can be done by expanding the set of variables on which the price signals have direct effects, by making the accumulation of man-made capital stock K a function of the return on the investment in K. Currently, accumulation of K is accomplished by each individual's investing a fraction η of the income (see equation (8)). One way to introduce to the model the effect of the price signal, namely, changes in the rate of return r, on households' investment decisions, is by making η a function of r. An increase in r can be interpreted as the capital input becoming relatively scarce. We anticipate that making households to respond positively to a higher rate of return on investment will contribute further to the smoothing out of the dynamics of the system.

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Appendix

Table A1. Baseline parameter values and initial values

Parameters	**Notations used in Vensim***	**Values**
S_{min}		0
S_{max}	Carrying Capacity Smax	12000
g_R	NatRes per unit of Hgood	1
α	alpha	.00012
μ	Regeneration Rate r	0.2
A	a	0.33
ρ	rho	–10
γ	gamma	0.6
b_0		0.5
b_1		2.4
b_2		1
d_0		0.8
d_1		14
d_2		0.5
β	beta	.4
η	Saving Rate s	.2
θ		1
δ	Capital Depreciation Rate	0.2
τ	tau	20
ζ	sensitivity to price gap	0.05
State variables		**initial values**
S	Natural Resource S	12000
L_R	Lh	246
R_M	Current Factor Demand Hm	7.4
L_M	Lm	16
K		12.85
p_R	Ph	.37
r		.16
R	H inventory	39.4
M	M inventory	8.75
p_M	Pm	2.2

* Names provided in the second column correspond to the labels used for parameters and variables (where they differ from those used for the mathematical model in Section 2) that we used in representing the model in Vensim. These notations are shown in the system map provided in Figure A1.

Figure A1. System Map