Socially Efficient Stabilization of Industrial Cycles via Organic Profit-sharing and its Crucial Tension

©Alexander V. Ryzhenkov
Novosibirsk State University & IEIE SB RAS
17 Acad. Lavrentiev Avenue Novosibirsk 630090 Russia
ryzhenko@ieie.nsc.ru

Keywords: capital accumulation, relative wage, surplus value, industrial cycle, stabilization policy

Abstract. Goodwinian model Z-1, containing the greed feedback loops, reflects destabilizing cooperation and stabilizing competition of investors. In a system of three ODEs, the rate of capital accumulation becomes the new main variable in addition to relative wage and employment ratio. Oscillations imitating industrial cycles are endogenous. A crisis is a manifestation of relative and absolute over-accumulation of capital. Limit cycle with a period of about 6 years results from supercritical Andronov – Hopf bifurcation.

The reshaped non-linear three-dimensional model is named S-1. It implements proportional and derivative control over the growth rate of surplus value maintaining organic profit-sharing. The growth rate of surplus value depends on a gap between the target and current employment ratios and growth rate of employment ratio. Parametric policy optimization shortens a transient to a deliberately high target employment ratio without lowering stationary relative wage against Z-1.

The proposed stabilization policy enhances the stability and efficiency of capital accumulation; it also provides stronger gains for workers' well-being. A deep-rooted tendency to producing relative surplus value challenges workers' adherence to organic profit-sharing.

1. Model Z-1 of industrial cycles as capital accumulation cycles

A variable's time derivative is denoted by a dot, its growth rate – by a hat. An ordinary differential equation is abbreviated as ODE. Table 1 lists the variables of Z-1 and S-1.

Table 1. Main variables in Z-1 and S-1

Variable	Expression
Real net output	q
Implicit produced commodity price	1
Fixed production assets	k
Capital-output ratio	$s = k/q$
Employment	l
Employment in efficiency units	l_e
Rate of capital accumulation	z
Output per worker	$a = q/l$
Labour force	$n = n_0 e^{\beta t}$, $\beta \geq 0$
Wage	w
Employment ratio	$v = l/n$
Consumption per head	wv
Total wage	wl
Unit labour value of produced commodity	$1/a$
Relative wage (unit value of labour power)	$u = w/a = wl/q$
Profit (surplus product)	$M = q–wl = q(1–u)$
Surplus labour (value of surplus product)	$M/a = q(1– u)/a = l(1–u)$
Profit rate	$R = (1–u)/s$
Investment (net)	$\dot{k} = zq(1-u)$

A CES production function is applied for determining the net output

$$q = F(k, l_e) = c\left[\mu(m_\kappa k)^{-\delta} + (1-\mu)(m_e l_e)^{-\delta}\right]^{-1/\delta}, \quad (1)$$

where μ is the distribution parameter, $0<\mu<1$, c is the efficiency parameter, and δ is the substitution parameter. Both van der Ploeg (1985, p. 223) and Aguiar-Conraria (2008, p. 522) insist that $\delta>0$. Ryzhenkov (2016) offers a critical assessment of this "neoclassical" interpretation.

A simplified non-linear Phillips equation defines the growth rate of wage as a function of employment ratio v

$$\hat{w} = f(v), \quad (2)$$

where $f'(v) > 0$, for $v \to 1$ $f(v) \to \infty$. The function has a negative lower bound for low magnitudes of v. At a stationary employment ratio, the growth rate of wage has to be positive.

Following Blatt (1983, p. 213), a specification satisfying these requirements is applied in Z-1

$$f(v) = -g + r/(1-v)^2, \quad (3)$$

where g and r are positive coefficients with plausible magnitudes (see below in this section).

The definition of employment in efficiency units l_e in Table 1, taken from (Aguiar-Conraria, 2008) in a slightly refined form (normalised fixed assets k/k_0 instead of k) that allows the efficiency of labour to be influenced by the size of the fixed assets embodying advancing technology:

$$l_e = l e^{\alpha t}(k/k_0)^{\gamma}, \quad (4)$$

where $\alpha > 0$, $0 \le \gamma < 1$. Consequently, a direct scale effect manifests itself as a linear positive dependence of a growth rate of output per worker on a growth rate of fixed production assets.

Goodwin (1972), van der Ploeg (1985), and Aguiar-Conraria (2008) reflected the rate of accumulation z as a constant. Ryzhenkov (2016) has turned it into a new phase variable.

An intensive form of Z-1 is a system of three ODEs:

$$\dot{u} = \left[f(v) - \alpha - \gamma\frac{z(1-u)}{s(u)}\right]\frac{\delta u}{1+\delta}, \quad (5)$$

$$\dot{v} = \left[(1-\gamma)\frac{z(1-u)}{s(u)} - \frac{1}{\delta}\frac{\hat{u}}{1-u} - (\alpha+\beta)\right]v, \quad (6)$$

$$\dot{z} = -b\frac{\dot{u}}{1-u}z\left(Z - z\right) + p(z_b - z), \quad (7)$$

where $b \ge 0$, $p > 0$, $0 < z_{\text{infimum}} < z_b < Z \le 1$.

The equation (7), first, takes into account, in agreement with the views of K. Marx, that net change of the share of investment in the surplus product has an opposite sign in response to relative wage gains (Marx, 1867, Ch. 25, section 1). This equation, second, reflects capitalists' targeting of the rate of capital accumulation at $z_b = z_{\text{goal}}$; restrictions $p > 0$ is a prerequisite for proportional control over capital accumulation, the requirement $0 < z_{\text{infimum}} < z_b$ is necessary for a positive stationary relative wage. Third, the product of multiplication $z(Z - z)$ reflects the logistical dependence of $\dot{z}$ on z that restricts trajectories in the phase space. Fourth, the upper bound Z is set here lower than in the version of (7) in Ryzhenkov (2016, 2019) where $Z \ge 1$. This modification permits better accounting for the real long-term tendency of capital accumulation rate z to decline, observed in the US economy. Besides that, greater structural stability of closed orbits in Z-1 is achieved.

Relative surplus value is created in the capitalist production whenever for given employment l = const, $-\dot{u}l = (\hat{a} - \hat{w})ul > 0$. Absolute surplus value is defined for the given unit value of labour power u = const as $(1-u)\dot{l} > 0$. Still, the net change of surplus value

$\dot{S} = \dfrac{\partial(1-u)l}{\partial t} = -\dot{u}l + (1-u)\dot{l}$ is not automatically positive in each distinct moment.

Declining profit rate ($\hat{R} < 0$) is the indicator for a relative over-accumulation (excess) of capital. The latter can be secular and/or cyclical.

Marx's distinguishes in the third volume of "Capital" two forms of absolute over-accumulation of capital: of type 1 when the increased capital produced just as much, or even less, surplus value than it did before its increase; of type 2 when the increased capital produced just as much, or even less, profit than it did before its increase.

Forms of relative and absolute over-accumulation of capital constitute the immediate endogenous factors of a crisis in an industrial cycle. The conception of real business-cycle (associated with the Chicago school of economics in the "neoclassical" tradition) substitutes these crucial factors by surmised mostly exogenous causes for the recessions in US history. This conception even led to the unwarranted conclusion (Goldman Sachs' economics research, 2019, p. 1): "While some new risks have emerged, on net we see the US economy as structurally less recession-prone than in the past."

Proposition 1. In Z-1, stationary state E_b = (u_b, v_b, z_b) exists that is locally asymptotically stable for the parameter of (7) $b \le b_0$; E_b loses its stability, and Andronov – Hopf bifurcation (Gandolfo, 2010) does take place at $b_{critical} > b_0$. A proof is omitted.

An increase in the stationary rate of economic growth $d = (\alpha+\beta)/(1-\gamma)$ affects relative wage u_b negatively; $u_b < 1$ is true only if $d > 0$. *Ceteris paribus*, the higher is the rate of capital accumulation z_b, the higher are stationary relative wage u_b and capital-output ratio s_b, and the lower is the stationary profit rate R_b.

In the simulation runs, the economy in the starting year – numbered for certainty 2009 – was at a brink to a crisis. Declining net output, elevating relative wage, surging unemployment, and the falling rate of capital accumulation characterise this phase within the industrial cycle.

The plausible magnitudes served in simulation runs: $\alpha = 0.005$, $\beta = 0$, $\gamma = 0.75$, $\delta = 1$, $\varepsilon = 0.5$, $\mu = 0.3$, $b = 1800$, $p = 0.2$, $c = 0.4444$, $g = 0.04$, $r = 0.001$, $d = 0.02$, $u_0 = 0.6859 > u_b = 0.6646$, $v_0 = 0.9073 > v_b = 0.8709$, $z_0 = 0.0507 < z_b = 0.12 < Z = 0.25$, $s_0 = 2.1490 > s_b = 2.0125$.

A supercritical Andronov – Hopf bifurcation (Gandolfo, 2010) occurs in simulations. The period of oscillations is about 6 years. Amplitude of cycles is slightly higher and period is shorter when $b = 3000$ than when $b = 1800 >> b_0 = 711.44$. If $b = 1000$ in scenario (exploratory) 1, industrial cycles become growth cycles without declines of net output q except for the initial cycle.

2. Model S-1 with organic profit sharing under state-monopoly capitalism

Following Ryzhenkov (2015, 2020), for overpowering greed and abating consequent relative and absolute capital over-accumulation under state-monopoly capitalism, the State and owners of capital, urged by the organised working class, set a target growth rate of surplus value depending on the difference between the indicated X_1 and current v employment ratios and on the growth rate of employment ratio:

$$\hat{S} = c_2(X_1 - v) + (1 - c_1)\hat{v}. \qquad (9)$$

where $c_1 > 1$, $c_2 > 0$, target employment ratio $X = X_1 - \beta / c_2 > v_b$, β is the stationary growth rate of labour force (see Table 1). The non-linear ODE for relative wage u follows:

$$\dot{u} = \left\{ c_2(v - X) + c_1(1-\gamma)\left[\frac{z(1-u)}{s(u)} - d\right]\right\}\frac{\delta u(1-u)}{c_1 + \delta u}. \qquad (10)$$

The ODEs (10), (6), and (7) comprise the intensive form of S-1. The target employment ratio X is an element of the stationary state of S-1defined as

$$S_X = (u_b, X, z_b), \qquad (11)$$

where u_b and z_b are taken from stationary state E_b in Z-1.

Proposition 2. Stationary state S_X (11) in S-1 is (a) locally asymptotically stable and (b) hyperbolic. The proof is skipped. In simulations, S_X is broadly asymptotically stable.

The growth rate of wage is a sum of bargained $\hat{w}^m$ and stimulating $\hat{w}^b$ terms:

$$\hat{w}^m = d_1 + \frac{(1+\delta)(1-u)}{c_1+\delta u} c_2 (v - X), \tag{12}$$

$$\hat{w}^b = d_1 + \left[\frac{(1+\delta)(1-u)}{c_1+\delta u} c_1 (1-\gamma) + \gamma \right] \left[\frac{z(1-u)}{s(u)} - d \right], \tag{13}$$

then stationary growth rates $\hat{w}_b^m = \hat{w}_b^b = \hat{w}_b / 2 > 0$ if $d_1 = const = \hat{w}_b / 2 > 0$.

Figure 1 reflects the stock-and-flow structure near stationary state S_X (11) in S-1, whereby abbreviations udot, vdot, and zdot stand for the derivatives of u, v, and z with respect to time. Compared to Z-1, feedback loops, marked by red turns from negative into positive, marked by blue – from positive into negative.

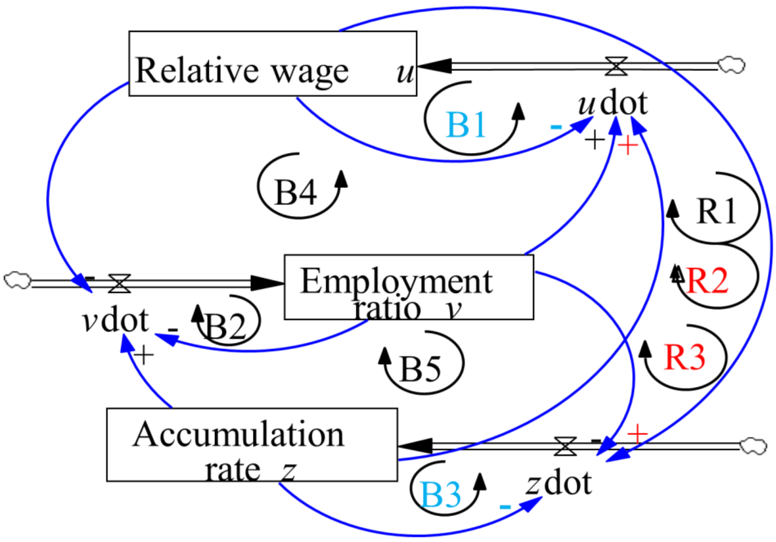

Figure 1 – A condensed stock-and-flow structure of S-1 near stationary state S_X (11); the total number of feedback loops – 8, among them: 1st order – 3 negative ones, 2nd order – 3 (2 negative ones, 1 – *positive* one), 3rd order – 2 *positive* ones

The stationary growth rate of surplus value is zero for β=0. Therefore increases in employment ratio v vanishing with time are to be accompanied by slower and slower decreases in relative wage u. Thus for the steady labour supply, the tendency to greater social inequality in primary income distribution cannot be overcome for this setting – it could be only mitigated.

3. Parametric optimization of stabilization policy in S-1

The initial 2009 year is the appropriate moment for starting the stabilization policy in the crisis-ploughed economy. A policy optimization is carried out in Vensim founded on a restricted dynamic optimization problem for S-1 embracing ODEs (10), (6), and (7):

$$\text{Minimize} \left[\int_{T_0}^{T} |v - X| dt + 10^5 \int_{T_0}^{T} \operatorname{sgn}(v - X) dt \right], \tag{14}$$

$$\text{subject to } \dot{y} = f(y, b, c_1, c_2),\ y = (u, v, z),$$

where T_0 = 2009, T = 2072, $y_0 = (u_0, v_0, z_0)$, $100 \le b \le 1000$, $0.5 \le c_1 \le 10$, $0.01 \le c_2 \le 1.5$.

The sub-optimal solution for the stable node-focus S_X implies b = 300 in (7), c_1 = 2.5 and c_2 = 1.4953 in (10), (12), and (13). These magnitudes are used in scenario (stabilization) 2 in S-1 that smooths out business volatility of scenarios (exploratory) 1 and 1 b typical for Z-1.

Simulations exhibit that employment ratio v moves to target X = 0.965 without over-shooting whereby $v_{\max}$ = 0.9649. Table 2 displays results that are generally superior in S-1 to those in Z-1 with b = 1000 or even more so for b = 1800, respectively, in scenarios (exploratory) 1 and 1 b.

The magnitudes of the phase (stock) variables in 2009 and magnitudes of common parameters are the same in scenarios 1, 1 b, and 2. The latter in S-1 outperforms the former two in Z-1 in the long run judged by all indicators except relative wage u for the identical stationary magnitude u_b.

Table 2. Shortened summary statistics for three scenarios, 2009–2025

Indicator	Min	Max	Mean	Norm. St.Dev.	Range	Scenario
Net output	1.000	1.481	1.248	0.113	0.481	2
	0.968	1.270	1.098	0.090	0.302	1
	0.967	1.248	1.093	0.081	0.281	1 b
Profit	0.314	0.482	0.405	0.118	0.168	2
	0.294	0.390	0.338	0.093	0.096	1
	0.293	0.387	0.336	0.084	0.093	1 b
Employment ratio	0.907	0.965	0.957	0.015	0.058	2
	0.853	0.907	0.869	0.012	0.054	1
	0.853	0.907	0.866	0.014	0.055	1 b
Wage	0.385	0.528	0.449	0.101	0.144	2
	0.385	0.520	0.447	0.086	0.134	1
	0.385	0.505	0.446	0.082	0.120	1 b
Relative wage	0.674	0.686	0.676	0.004	0.012	2
	0.686	0.697	0.693	0.003	0.011	1
	0.686	0.697	0.693	0.004	0.011	1 b

Conclusion

Contrary to the conception of real business-cycle, the model Z-1 reflects industrial cycles as capital accumulation cycles. Uncovered is the dual nature of capital as the endogenous driver and barrier of capitalist production that subordinates production of use-values to creation and capitalists' appropriation of surplus value in agreement with Marx (1867).

The designed stabilization policy via organic profit-sharing not only smooths out endogenous industrial (or growth) cycles typical for Z-1 in the reshaped model S-1; it is also advantageous for such characteristics of capitalist reproduction on the increasing scale as domestic investment, net output, profit, surplus value, wage, total wage and consumption per head. This stabilization policy could be applied for the revival of the economy plunged under the crisis related to Covid-19 without procrastination just after the social isolation safely finished as suggested by Shierholz (2020) and Ryzhenkov (2020).

The considered structural changes remain in the domain of the capitalist mode of production with its tendency of producing relative surplus value that expectedly manifests itself in declining relative wage on the transient to the stationary state (particularly under a constant labour supply assumed). This tendency challenges the workers' adherence to organic profit-sharing and creates a crucial tension under state-monopoly capitalism.

Organic profit-sharing is to be maintained by the stronger workers' involvement in business and governmental management, by more progressive income and property taxation, by large-scale direct and indirect financial subsidies. State-monopoly capitalism has to accept these changes and overcome neo-liberalism for the benefit of society as a whole. Otherwise more radical (up to revolutionary) measures will enter into the political agenda – even stronger than nowadays. Additional efforts are needed for modelling contrasting modes of socio-economic development.

References

Aguiar-Conraria L. 2008. A note on the stability properties of Goodwin's predator-prey model. *Review of Radical Political Economics* **40**(4): 518–523.

Blatt JM. 1983. *Dynamic Economic Systems*. Wheatsheaf Books, Brighton.

Gandolfo G. 2010. *Economic Dynamics*. Springer, Heidelberg.

Goldman Sachs' economics research. 2019 (20 January). Learning from a century of US recessions. Retrieved September 14, 2020, from https://www.goldmansachs.com/insights/pages/learning-from-a-century-us-recessions/report.pdf

Goodwin RM. 1972. A growth cycle. In C.H. Feinstein (ed.). *Socialism*, *Capitalism and Economic Growth*. Cambridge University Press, Cambridge MA, 54–58.

Marx K. 1867. *Capital*. *A Critique of Political Economy*. Vol. I. The Internet English edition. Retrieved September 14, 2020, from https://www.marxists.org/archive/marx/works/1867-c1/

Ploeg F. van der 1985. Classical growth cycles. *Metroeconomica* **37**(2): 221–230.

Ryzhenkov A. 2015. Socially efficient stabilization policies for growth cycles. *Advances in Economics and Business* **3**(11): 502–527.

Idem. 2016. Endogenous industrial cycles in a reshaped "neoclassical" model. In *Proceedings of the 2016 International System Dynamics Conference*. Delft, System Dynamics Society.

Idem. 2019. Stabilization of industrial cycles by profit-sharing policies localized near stationary states. In *Proceedings of the 2019 International System Dynamics Conference*. Albuquerque, NM, System Dynamics Society.

Idem. 2020 (26 May). A prospective study of the absolute and relative surplus value in the United States based on macroeconomic models: consequences for Russia. The Research Gate preprint. Doi: 10.13140/RG.2.2.33416.85763/1.

Shierholz H. 2020 (20 May). The coronavirus recession will become a long depression unless federal policymakers act now. Retrieved May 22, 2020, from https://www.epi.org/blog/the-coronavirus-recession-will-become-a-long-depression-unless-federal-policymakers-act-now/