Inventory Management in Response to an Unfolding Epidemic

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Abstract

A generic production-inventory (PI) management framework is developed for a hospital to respond to an unfolding epidemic. The framework is modelled as a closed feedback loop, where the future epidemic behaviour is governed by the medicine supply of current period which further influences the demand for medicine in the future. A Susceptible-Infected-Recovered (SIR) disease diffusion model is coupled with the PI model as a forecasting tool to anticipate the medicine demands of a novel epidemic during production lead time, where the forecasting model parameters in every decision cycle are estimated by calibrating forecasting disease model with past epidemic data. With an illustrative example of a hypothetical outbreak, the performance of SIR model is compared against a naïve forecasting method (defined as next period's forecast is current period's demand) and found that SIR model outperforms naïve method in terms of reducing epidemic impact and inventory leftover.

Keywords: production-inventory, integrated framework, SIR, naïve model

1 Introduction

An epidemic could cause extreme damages by claiming many human lives and incurring huge economic losses. The emergence of an epidemic requires immediate attention to prevent it from going beyond controllable proportions. Thus, for any treatable disease epidemic, an efficient management of medicine supply chain in conjunction with epidemic outbreak information is required to alleviate the suffering of infected people and reduce further spread of the disease (Dasaklis et al., 2012). The medicines distributed in earlier periods' will take effect in future periods and reduce demands for medicine (Paul & Venkateswaran, 2015). This shows the importance of combined study of an epidemic and its corresponding medicine supply chain.

There are not much literature until recent (Chick et al., 2008; Duintjer Tebbens et al., 2010; Liu et al., 2015; Paul & Venkateswaran, 2015; Liu & Zhang, 2016) to study epidemics in light of medicine supply chain. Past literature in planning and control of epidemics (Lee & Chen, 2007; Arinaminpathy & McLean, 2008; Ren et al., 2013; Yarmand et al., 2014) have primarily focused on the allocation of critical resources such as medicine but overlooked the supply chain aspect of these resources. But without ensuring the supply of resources, resource allocation models are pointless (Dasaklis et al., 2012). The stream of literature, combining epidemic diffusion and supply chain, are discussed below. Chick et al. (2008) have proposed a variant of cost sharing contract between government, healthcare sector, and medicine manufacturer to improve the overall performance of vaccine supply chain using a game theoretic approach. Duintjer Tebbens et al. (2010) have integrated a polio epidemic model with its vaccine production model to minimize total public health cost and vaccine production cost by solving an optimization problem to select vaccine filling flows and production flows at each time period. Liu et al. (2015) have proposed a time-space network model to study the effect of early periods' medicine supply on the demands of later periods by defining a so-called linear growth factor. This factor along with past supply information is used to forecast demand for future periods. Later on, in another publication (Liu & Zhang, 2016), the authors have pointed out following limitations of their earlier work (Liu et al., 2015): the naïve forecasting method served as a quick corrector but not cross-validated against actual demand data, production lead time and order sizes were not considered. Paul & Venkateswaran (2015) have shown that the effect of inventory management on epidemic dynamics is significant by developing an integrated system dynamics (SD) model. They further concluded that a communicable disease epidemic could occur due to either severity of the disease or insufficient supply of medicine. Although they have captured the feedback from medicine supply chain to disease diffusion process and vice versa, they used exponential smoothing demand forecasting technique which is inadequate for estimating demand in disease diffusion process. Liu & Zhang (2016) have addressed the limitations of their previous work and solved a more realistic epidemic logistics planning problem with a large-scale mixed integer programming formulation. In this model, the authors used a disease diffusion model to forecast demands, leveraging past demand information. However, they did not consider the effect of unmet demands (i.e., infected patients) on the spread of the epidemic in subsequent periods.

On the other hand, in supply chain modelling literature, Forrester (1961) and Sterman (2000) have provided a generic production-distribution system which serves as a basic framework for many decision-making analyses like, stock management control, raw material ordering etc. Most of the analyses in this domain (Disney & Towill, 2003; Venkateswaran & Son, 2007; Bijulal et al., 2011) were conducted for stationary demand setting while epidemic demand pattern follows a bell-shaped curve over time.

In this paper, we attempt to fulfil the research gaps as pointed out in literature survey. An integrated framework, similar to the one discussed in (Paul & Venkateswaran, 2015), is proposed for managing inventory and ordering decisions in healthcare institutions in response to an unfolding epidemic. We describe the problem under study as follows: although in post-epidemic response planning process, the disease true behaviour of a novel epidemic remains unknown, but it is known that the disease can be alleviated by providing medicines to infected patients on time otherwise the infected people would keep spreading the disease into other susceptible population. This situation demands a good inventory management which depends on the ordering policy and hence the forecasting method. Due to lack of information about the disease true behaviour, we try to anticipate its dynamics through other disease diffusion models. The novelty of this work lies in combining both epidemic diffusion and inventory management models with a dynamic disease forecasting model. The proposed framework is simulated for three different settings of actual disease diffusion model and the overall performance is compared against a naïve forecasting method.

The rest of the paper is organised as follows: Section 2 describes the conceptual framework of the proposed model, then a detailed disease outbreak model and an inventory management model are outlined. In section 3, simulation results of the proposed framework with an illustrative example are demonstrated and performance is compared with the naïve method. Section 4 discusses observations and future work.

2 Model Description

In this section, we will first describe the conceptual framework of our proposed integrated model and then provide a detailed description of each module in the subsequent sections. The symbols, notations and then abbreviations as used in this section are listed in Table 1.

2.1 Conceptual Model

The basic framework of *integrated model*, presented by Paul & Venkateswaran (2015), is adopted here to capture the interaction of disease outbreak and its corresponding medicine supply chain.

In this paper, we have modified the major components viz. *inventory management module*, *disease diffusion module* (which depicts patients flow in the hospital), and demand *forecasting module* of (Paul & Venkateswaran, 2015) in order to capture the reality better and build a decision support system in response to an unfolding epidemic.

The mental model of our proposed system is displayed in Fig. 1. In this model, with the inception of a novel epidemic, infected patients start coming to the hospital, denoted as compartment "I", and generate demands for

Table 1: Notations Used

Symbol	Description	Units
γ	Infection probability (p)× Contact rate (λ)	(rate) 1/day
N	Total population	person
P	Incubation period	day
TRM	Time to recovery with medicine	day
TR	Time to recovery without medicine	day
TW	Waning time	day
S_d	Susceptible population at time d	person
E_d	Exposed population at time d	person
I_d	Infected patients seeking treatment at time d	person
T_d	Infected patients under treatment at time d	person
R_d	Recovered population at time d	person
IR_d	Infection rate at time d	person/day
ER_d	Exposure rate at time d	person/day
AR_d	Hospital admission rate at time d	person/day
RRM_d	Recovery rate with medicine at time d	person/day
RR_d	Recovery rate without medicine at time d	person/day
WR_d	Waning rate at time d	person/day
α	Fractional rate of adjustment of medicine on order discrepancy	(rate) 1/week
β	Fractional rate of adjustment of medicine in stock discrepancy	(rate) 1/week
ζ	Fractional rate of adjustment of medicine backlog discrepancy	(rate) 1/week
n	Medicine unit require per person	units/person
TL	Production lead time	week
TS	Time to ship medicine	week
TO	Time to fill order	week
DC	Desired stock coverage	week
SS	Safety stock coverage	week
DI_w	Desired inventory of medicine at time w	units
DO_w	Desired order of medicine at time w	units
FD_w	Demand forecast for time period w	units/week
MB_w	Medicine order backlog at time w	units
MI_w	Available inventory or stock at time w	units
MO_w	Medicine quantity on order at time w	units
AI_w	Adjustments for inventory discrepancy at time w	units/week
AO_w	Adjustments for medicine on order discrepancy at time w	units/week
AB_w	Adjustments for medicine backlog discrepancy at time w	units/week
DR_d	Medicine demand rate at time d	units/day
OR_w	Medicine purchase order rate at time w	units/week
PR_w	Order receiving rate at time w	units/week
SR_d	Shipment rate of medicine at time d	units/day
MS_d	Maximum Shipment rate of medicine at time d	units/day

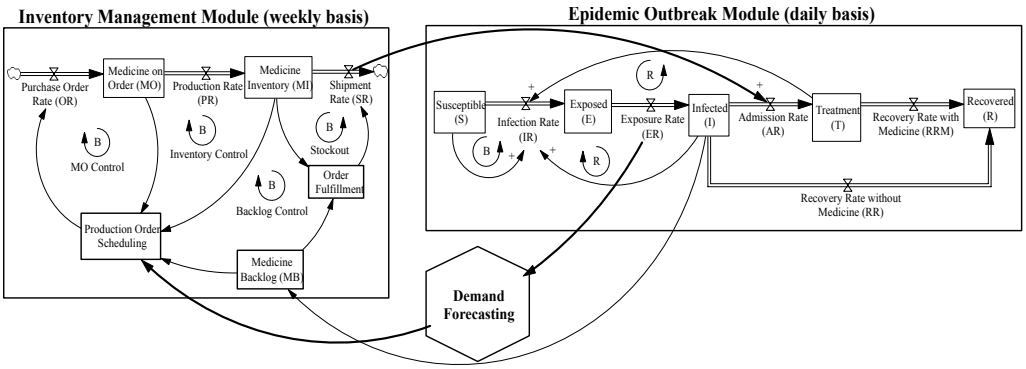


Figure 1: Schematic of proposed framework.

medicine which in turn are fulfilled from the medicine stock of hospital, "MI", subject to its availability. The hospital records its daily demand of medicine from incoming epidemic patients and leverages it for predicting future demand for medicine. It is assumed that the hospital sends medicine order to the manufacturer after every "h" days, defined as the length of planning horizons or decision cycles ($= T_f/h$, where T_f is the final simulation runtime) i.e., the ordering takes place only at the beginning of each planning horizon. This assumption is justified by frequent changes in order rate incurs a high cost and causes an increase in dynamics as the order flows to upstream of the supply chain (Lee et al., 1997). The hospital sends the order to the manufacturer based upon the forecasted demand for the next planning horizon and inventory profile (measured as units/h days) and receives shipment after an average production lead time. Thus, it is noted that the actual epidemic outbreak and the forecasting process run on a daily basis, but the production-inventory model runs on the time-length of planning horizon basis (i.e. variables are updated after every "h" days). The simulation process of this model is shown in Fig. 2.

2.2 Integrated Inventory Management and Disease Diffusion Framework

In this section, each component of integrated framework is demonstrated.

2.2.1 Hospital Disease Diffusion Component

To depict the real unknown epidemic at hospital, a variant of popular compartmental treatment model (Brauer, 2008) viz. Susceptible-Exposed-Infected-Treatment-Recovered-Susceptible (SEITRS) is taken here (see Fig. 3). In a manner similar to the conventional SD epidemic models (Sterman, 2000; Brauer, 2008), this model also divides the total population into five compartments namely, Susceptible (S), Exposed (E), Infected patients seeking treatment (I), infected patients under Treatment (T), and Recovered (R).

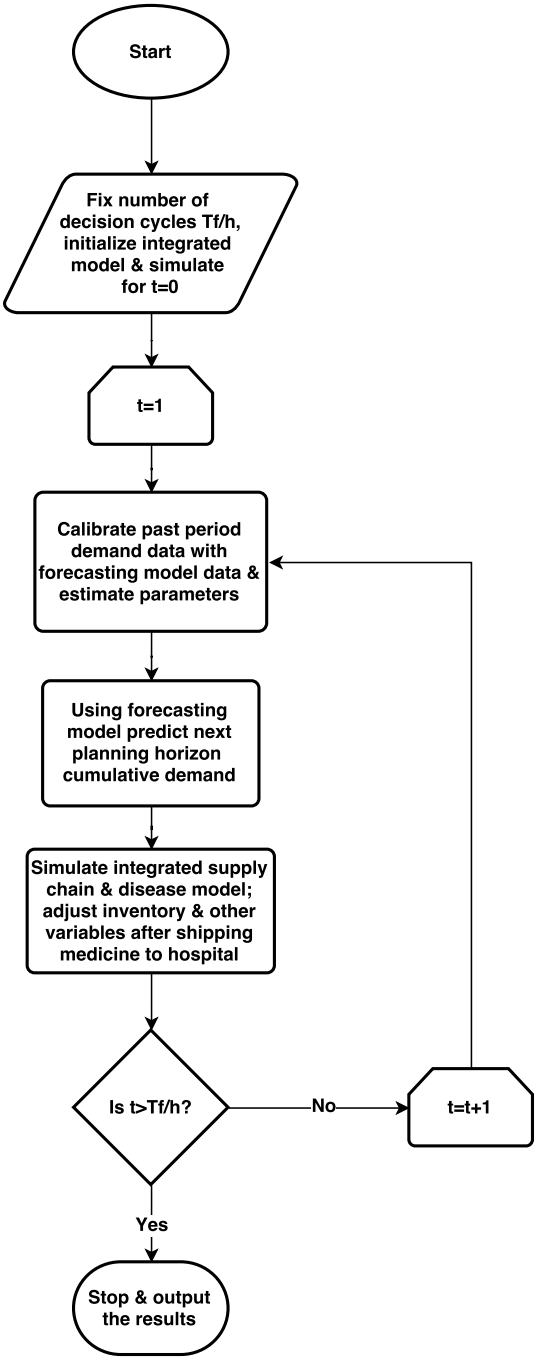


Figure 2: Flow chart diagram of proposed framework.

This model is an extension of the model proposed by Paul & Venkateswaran (2015), where the earlier Infected (I) compartment is split into two compartments viz. Infected (I) and Treatment (T) to capture the medicine supply delay explicitly. The assumptions of our model are as follows:

1. The hospital was in stable condition before the introduction of the epidemic.
2. Each patient requires "n" units of drugs to recover.
3. The total population remains fixed during the course of the epidemic.
4. The mixture of people within each compartment is homogeneous.

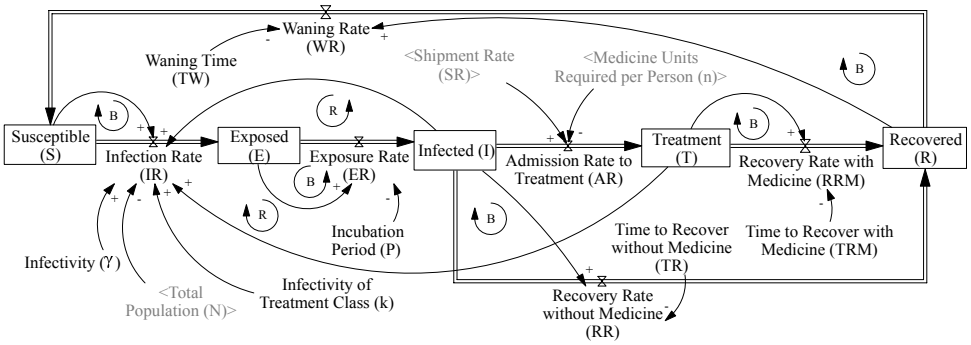


Figure 3: Stock flow diagram of disease diffusion component.

A susceptible (S) individual may get exposed to the disease if he comes in contact with an infected (I) or treatment (T) class individual (see Eq. (1)). It is assumed that for the under treatment class (T), the *infectivity* of an individual is $k\%$. The infection rate of S is governed by the *infectivity* parameter γ, which is defined as the product of contact rate (λ, the number of people who interact per time period) and infection probability (p, the chance of getting the infection from contacting an infected person). Thus $\gamma = \lambda \times p$. The exposed individuals remain asymptomatic for an average incubation time period P (see Eq. (2)). As soon as the patient develops a symptom or falls sick, he is shifted from the "E" to the "I" compartment (see Eq. (3)). The admission rate (AR) to treatment class ("T") is governed by the shipment rate of medicine from the inventory management model and number of patients waiting for treatment, i.e., $AR_d = \min(SR_d, ER_d + I_d - RR_d)/n$. It is assumed that when medicines are not available, then the patients recover from the disease after an average time of natural recovery (TR) (i.e., natural burnout). Patients under treatment (T) are assumed to recover after an average recovery time (TRM) (see Eq. (4)). To make the treatment beneficial, it is assumed that $TR > TRM$. After recovering from the disease, an individual loses immunity (this loss could be due to the mutation of the disease virus) and again becomes susceptible to the disease after an average time period

of "TW" (see Eq. (5)). It is noted that hospital component variables are updated every day.

$$S_{d+1} = S_d + \frac{R_d}{TW} - \frac{\gamma \times (I_d + k \times T_d) \times S_d}{N} \quad (1)$$

$$E_{d+1} = E_d + \frac{\gamma \times (I_d + k \times T_d) \times S_d}{N} - \underbrace{E_d/P}_{ER_d} \quad (2)$$

$$I_{d+1} = I_d + \frac{E_d}{P} - \frac{\min(SR_d, ER_d + I_d - RR_d)}{n} - \underbrace{I_d/TR}_{RR_d} \quad (3)$$

$$T_{d+1} = T_d + \frac{\min(SR_d, ER_d + I_d - RR_d)}{n} - \frac{T_d}{TRM} \quad (4)$$

$$R_{d+1} = R_d + \frac{T_d}{TRM} + \frac{I_d}{TR} - \frac{R_d}{TW} \quad (5)$$

2.2.2 Inventory Management Component

A generic production-distribution system, similar to those discussed in (Forrester, 1961; Sterman, 2000; Venkateswaran & Son, 2007) is taken here for depicting the inventory management of hospital (see Fig. 4). Although the inventory management component, in contrary to SEITRS disease diffusion component, runs with a time step of "h" days (i.e., inventory management model variables are updated after every planning horizon) but there are few variables such as, medicine inventory (MI), backlog (MB) etc. which interacts with actual disease model, are updated on both daily and "h" day (planning horizon) basis.

Ordering policy and demand forecasting

The ordering strategy is the key decision variable to control on hand inventory. In the case of demand uncertainty, ordering strategy becomes highly dependent on the accuracy of forecasting method. Production-inventory models studied in past literature (Sterman, 2000; Venkateswaran & Son, 2007), have considered conventional exponential smoothing method for forecasting demand which is designed for not to respond immediately to a sudden change of demands. However, in the case of an epidemic, the inventory management model is expected to react proactively to a small change in demands to reduce the further mismatch between demand and supply. Fig. 5, demonstrates the effect of demand fulfillment of only one planning horizon (in e.g. 2^{nd} planning horizon is taken considering h=7 days) on the dynamics of the entire epidemic span.

Since the actual disease outbreak information is unknown, we chose the popular Susceptible-Infected-Recovered (SIR) disease diffusion model (Sterman, 2000) arbitrarily from many other compartmental models as the demand forecasting tool and compared its performances with naïve forecasting model,

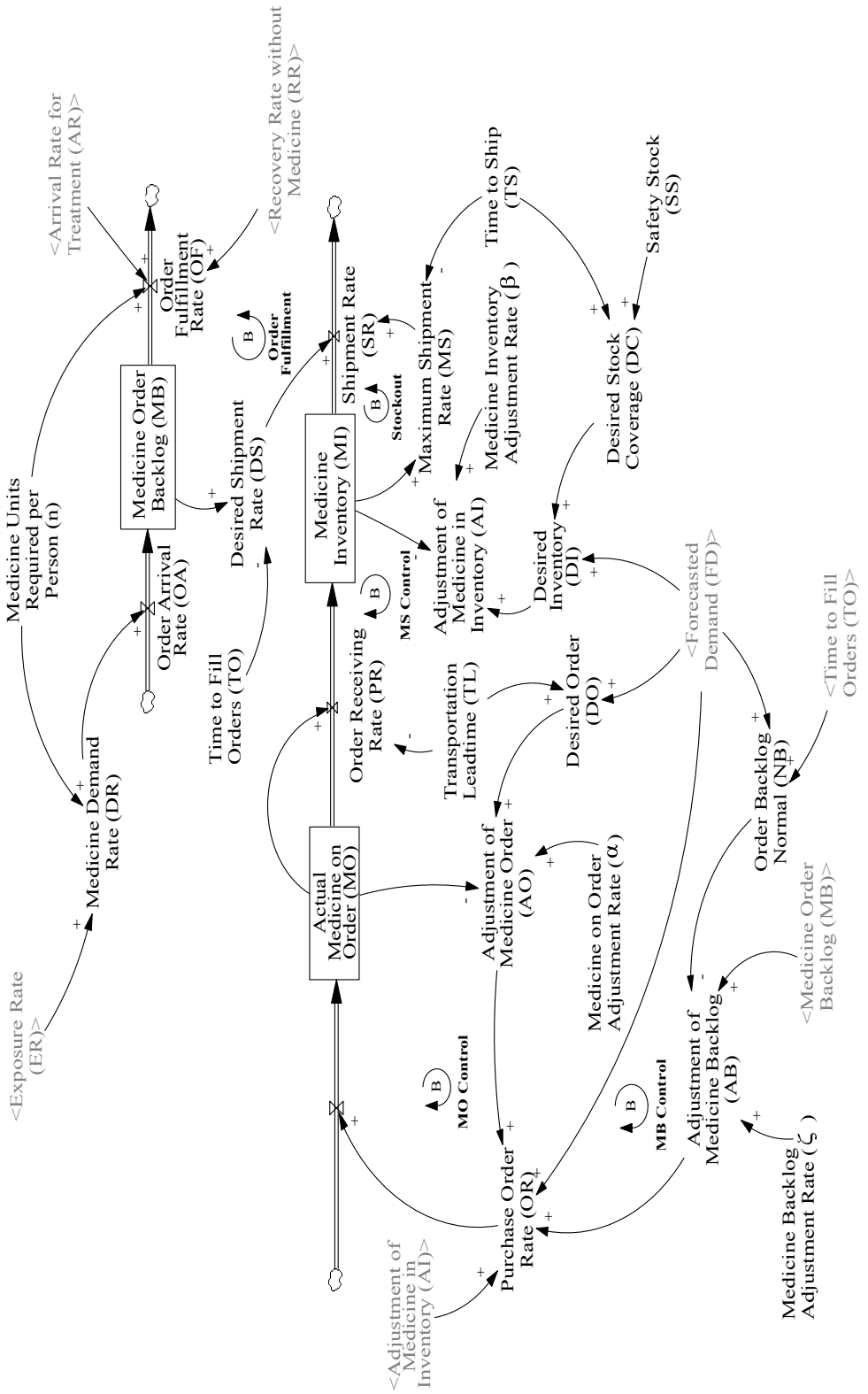


Figure 4: Stock flow diagram of supply chain component.

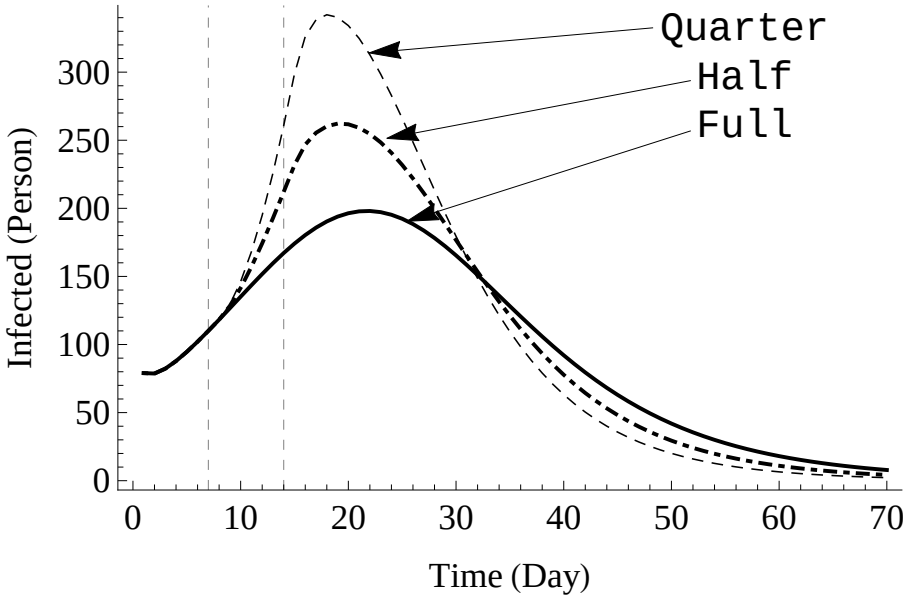


Figure 5: Epidemic dynamics when medicine supply between 7^{th}-14^{th} day is made full, half and quarter of the actual demand respectively.

defined as the next period's forecasted demand is current period's demand, for an unfolding epidemic. We have used superscript "F" to distinguish the disease forecasting model variables from the actual outbreak model (see Section 2.2.1) variables. We elucidate the SIR forecasting process below.

The governing equations of the SIR model in discrete time are

$$S^F_{d+1} = S^F_d - \underbrace{\gamma^F \times S^F_d \times I^F_d / N^F}_{IR^F_d} \tag{6}$$

$$I^F_{d+1} = I^F_d + \frac{\gamma^F \times S^F_d \times I^F_d}{N^F} - \frac{I^F_d}{TRM^F} \tag{7}$$

$$R^F_{d+1} = R^F_d + \frac{I^F_d}{TRM^F} \tag{8}$$

The demand of medicine depends on the patient's inflow rate to infected compartment (i.e., ER_d in hospital model; see Eq. (2)) and medicine vials required per person (n), as shown in Eq. (9). It is noted that this equation serves as a linking constraint from hospital disease model to inventory management/supply chain model.

$$DR_d = ER_d \times n \tag{9}$$

In order to estimate the disease forecasting (SIR) model parameters, the demand rate, DR^F_d ($= IR^F_d \times n$, where IR^F_d is given by Eq. (6)), is calibrated

with previous planning horizon actual demand data (DR_d, i.e. the demand signal sent from the hospital; see Eq. (9)) using the least square regression method. That is, in i^{th} decision cycle $\forall i = 1, 2, \ldots, T_f/h$, the objective is to minimize the root mean squared error (RMSE) between the forecasted model demand data and the original demand signal data of $(i-1)^{th}$ planning horizon, sent from the hospital as shown in Eq. (10), where, Ω, and Γ are the set of all parameters to be optimized, and input parameter ranges respectively.

$$RMSE_i = \min_{\Omega \in \Gamma} \sqrt{\frac{\sum_{d=(i-1)h}^{ih-1} \left(DR_d^F - DR_d\right)^2}{h}} \tag{10}$$

After estimating the parameters from Eq. (10) in i^{th} planning horizon, forecasting model (i.e., SIR model, see Eq. (6) & (7)) is run with these new set of parameters to forecast the demand for i^{th} planning horizon, denoted by FD_i, as shown in Eq. (11). In Eq. (11), we accumulate forecasted demand rate because our inventory management model runs on planning horizon basis while forecasting model runs on a daily basis.

$$FD_i = \sum_{d=ih}^{(i+1)h-1} DR_d^F \tag{11}$$

Model description

The functionality of inventory management model is described below. In the following discussion, it should be noted that the variables which updated every day, are denoted with subscript "d" and those updated every "h" days, are denoted with subscript "w".

In inventory management model, cumulative forecasted demand is calculated for next planning horizon as per Eq. (11). The hospital maintains a daily order backlog for medicine (MB_d), as shown in Eq. (12), where the inflow and outflow are modelled as a coflow of infected class (I). Here, the coflows keep track of the amount of medicine in backlog based upon the number of people waiting in compartment I. Since, the inventory model runs with a time step of "h" days, so the hospital also keeps a record of the medicine backlog of every "h" days, denoted as MB_w, calculated as the sum of MB_d's of last planning horizon (see Eq. (13)). The order quantity, modelled as a general replenishment-rule depends on the forecasted demand FD_w, the discrepancy between desired and actual orders in pipeline (AO_w), the discrepancy between desired and actual stock or inventory on hand (AI_w) and the discrepancy between actual and normal medicine order backlog (AB_w) (see Eq. (14)). The discrepancy in orders in the pipeline, in stock, and in backlog are adjusted using the tuning parameters α ($= 1/T_{MO}$, the reciprocal of time to adjust MO_w), β ($= 1/T_{MI}$, the reciprocal of time to adjust MI_w), and ζ ($= 1/T_{MB}$, the reciprocal of time to adjust MB_w), respectively. The desired orders in the pipeline, as per Little's Law (Little, 1961), is computed as a product of FD_w

with the production lead time TL (see Eq. (15)). The desired stock is also governed by the FD_w and the desired coverage level DC and safety stock SS (see Eq. (16)). Equations (18), (20), and (21)) are the balance equations for orders in the pipeline (MO_w), daily medicine inventory (MI_d) and "h" days medicine inventory (MI_w) respectively. The production order receiving rate (PR_w) is modelled as a first order material delay to capture the mixing of products and variabilities in production times. The delivery or sales rate SR_d of medicines, Equations (22-24), govern the actual supply of medicines from the stock to the hospital, subject to actual availability.

$$MB_{d+1} = MB_d + \underbrace{DR_d}_{OA_d} - \underbrace{(AR_d + RR_d) \times n}_{OF_d} \tag{12}$$

$$MB_w = \sum_{d=(w-1)h}^{wh-1} MB_d \tag{13}$$

$$OR_w = \max\left\{0, FD_w + \underbrace{\alpha(DO_w - MO_w)}_{AO_w} + \underbrace{\beta(DI_w - MI_w)}_{AI_w} + \underbrace{\zeta(MB_w - NB_w)}_{AB_w}\right\} \tag{14}$$

$$DO_w = TL \times FD_w \tag{15}$$

$$DI_w = (SS + DC) \times FD_w \tag{16}$$

$$NB_w = FD_w \times TO \tag{17}$$

$$MO_{w+1} = MO_w + OR_w - PR_w \tag{18}$$

$$PR_w = \frac{MO_w}{TL} \tag{19}$$

$$MI_{d+1} = MI_d - SR_d \tag{20}$$

$$MI_w = MI_d + PR_w \tag{21}$$

$$SR_d = \min\{MS_d, DS_d\} \tag{22}$$

$$DS_d = \frac{MB_d}{TO} \tag{23}$$

$$MS_d = \frac{MI_d}{TS} \tag{24}$$

2.3 Model Verification

Before going into the simulation experimentation, verification of our proposed *integrated model* is examined in two ways. Firstly, the dimensional consistency is verified and secondly, model robustness at extreme conditions are tested (Barlas, 1996). For verification of robustness, initially exposed and all infected population (i.e. I & T) of disease model was set to zero and con-

firmed that no epidemic occurs and all stock values remain unchanged while rate variables become zero. Further, the stability of inventory management module is confirmed against constant and step input of demand.

3 Experiments and Analyses

In this section, we study and compare the performances of naïve and SIR forecasting models in terms of three performance measures namely, infected peak (I_{Peak}; see Eq. (25)), final epidemic size (R_∞; see Eq. (26)), and inventory leftover (MI_{left}).

$$I_{Peak} = \max_{0 \le i \le T_f} (I_i + T_i) \tag{25}$$

$$R_\infty = \sum_{i=0}^{T_f} (RRM_i + RR_i) \tag{26}$$

The proposed framework is simulated for three different settings of epidemic severity, measured in terms of infectivity parameter γ.

3.1 Simulation Settings and Optimization

Simulation experiment of our difference equation based *integrated model* is carried out on Microsoft Excel (2010). Euler integration method with a time step of 1 was used for discrete time simulation and approximating the model variable trajectories. The total simulation runtime for the integrated model was set to 70 days (T_f). For stock management component, we chose $L = 1$ week (i.e. $L = h = 7$ days), $TO = 1$ week, $TS = 1$ week, $\zeta = 1$/week, $\alpha = 1$/week, $\beta = 1$/week, $SS = 0.1$ week (chosen arbitrarily but lesser system inventory preferred). For the SEITRS (epidemic) component of *integrated model*, we have taken $k = 0.5$ (chosen arbitrarily), $n = 1$ unit/person, $TW = 365$ days (Paul & Venkateswaran, 2015), $P = 2$ days (CDC, 2015), $TR = 10$ days, $TRM = 3.8$ days, $S_0 = 2211$ persons, $E_0 = 15$ persons, $I_0 = 1$ person, $T_0 = 78$ persons, and $R_0 = 0$ person. We chose three settings for infectivity parameter (γ) as 0.572 (low infectivity), 0.596 (base infectivity), and 0.62 (high infectivity). For MO_w, MI_w, MI_d, MB_w, MB_d, and FD_w initial values are chosen as DO_w, DI_w, DI_w, FD_w, I_0, and 100 units/week respectively. Initial FD_w is kept lesser than first planning horizon actual demand so that the system does not possess excess inventory and the problem becomes interestingly dependent upon the forecasting technique.

Calibration of the models using the least square regression (see Section 2.2.2) was carried out using the Generalized Reduced Gradient (GRG) (Fylstra et al., 1998), a gradient-based method, in *Solver*® optimisation toolbox of Microsoft Excel (2010). This simulation-based optimisation works as follows. The optimisation solver (i.e. *GRG*) chooses a set of parameter values and passes it

on to the simulation (SD) model. The model runs the simulation, evaluates the objective function (i.e. *RMSE*), and returns the same to the optimisation solver. The solver, then intelligently chooses the next set of parameters for evaluation by the simulation model. This iterative scheme continues until a predefined stopping criterion is met or the current solution meets a "slow progress" test (Fylstra et al., 1998), and the best result obtained is reported.

3.2 Comparison of Disease Forecasting Models

The input parameter ranges for calibration of SIR forecasting model with actual disease outbreak demand data are taken as $S_0^F \in [2000, 10000]$, $\gamma^F \in [0, 1]$, $TRM^F \in [2.5, 10]$ and I_0^F is always set to 79 persons (i.e. $I_0 + T_0$ of actual outbreak model).

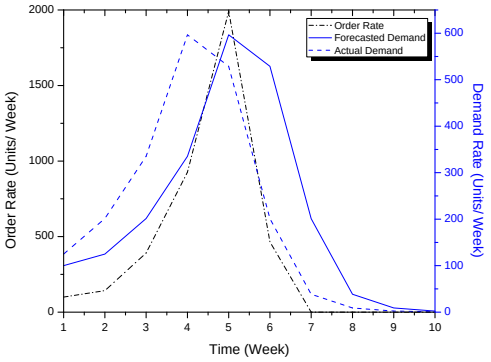
Anticipated medicine demands from the naïve and SIR forecasting models are presented in Table 2. Fig. 6 and Fig. 8 shows the comparison of actual demand data and forecasted demand data in right axis while corresponding order rate data are plotted on the left axis, for naïve and SIR forecasting models respectively. The dynamics of the corresponding supply chain of naïve and SIR models are also shown in Fig. 7 and Fig. 9 respectively, in terms of medicine backlog (MB_d), inventory (MI_d) and shipment rate (SR_d) on a daily basis. Finally, the overall comparison of demand forecasting models is summarised in Table 3.

Table 2: Predicted demands of forecasting models

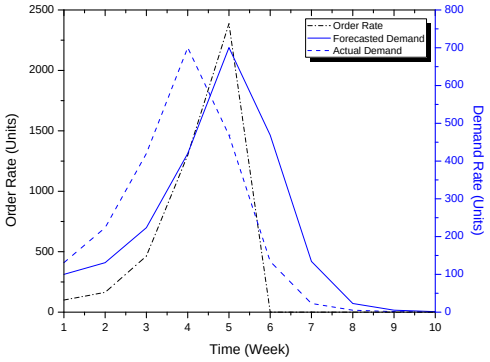
	Low Infectivity ($\gamma = 0.572$)		Base Infectivity ($\gamma = 0.596$)		High Infectivity ($\gamma = 0.62$)	
Time (Week)	Naïve	SIR	Naïve	SIR	Naïve	SIR
1	100	100	100	100	100	100
2	124.897	218.795	131.051	238.700	137.331	259.877
3	201.448	268.333	223.629	328.504	247.658	394.290
4	335.240	514.030	419.983	672.289	512.190	844.009
5	596.551	375.369	700.309	390.138	764.730	359.713
6	528.830	196.567	468.568	127.053	381.071	82.599
7	201.118	65.926	134.276	41.215	86.403	20.660
8	38.500	19.004	23.001	10.356	14.496	5.530
9	9.134	6.407	5.134	3.250	3.189	1.664
10	2.384	2.257	1.265	1.067	0.768	0.526

Fig. 6 shows that for naïve demand forecasting model, estimated epidemic peak demand rate is exactly delayed by one week (i.e. length of planning horizon) of actual epidemic peak demand rate which further worsen the epidemic. In Fig. 8, it is seen that the forecasting SIR model has overestimated the epidemic peak demand rate by 4.38%, 14.086%, and 28.12% for low, base, and high infectivity (γ) parameters respectively. Moreover, the forecasted peak demand occurred at exactly the same time of actual epidemic peak demand which is good from epidemic planners' perspective. However, the realization of the epidemic peak is just one planning horizon before actual which results

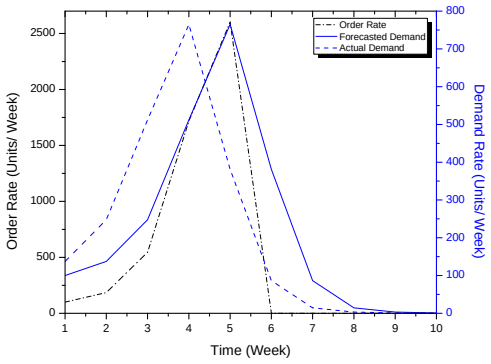
in high order rates and eventually increases the medicine inventory even after the eradication of epidemic. This situation is reflected in Fig. 9, where a sharp increase in inventory is seen after day 35 or week 5, i.e. a week after the epidemic peak demand (see Fig. 8; peak occurs at week 4) where the maximum order is placed and transformed into inventory after a week later (since the lead time TL=1 week). For the same reason, we see sharp growth in inventory of Fig. 7.



(a) Low Infectivity ($\gamma = 0.572$)



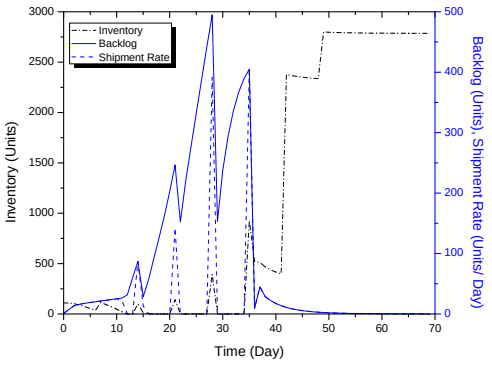
(b) Base Infectivity ($\gamma = 0.596$)



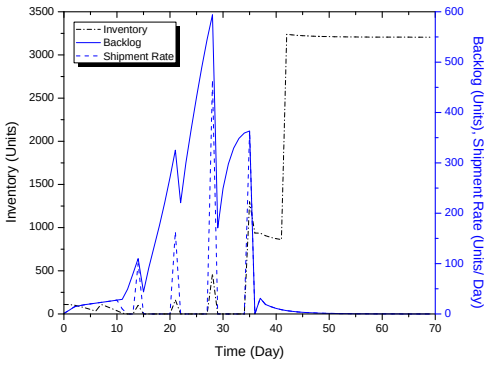
(c) High Infectivity ($\gamma = 0.62$)

Figure 6: Comparison of weekly forecasted demand rate (FD_w), actual demand rate (DR_w), and order rate (OR_w) for naïve forecasting method.

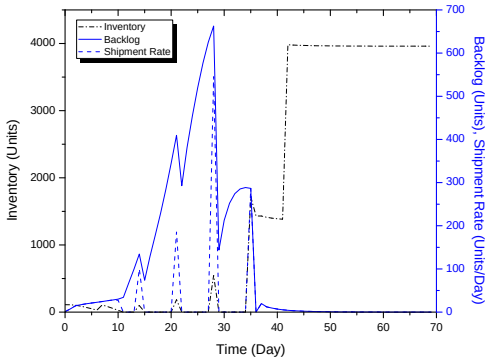
From Table 3, we see that SIR model outperforms naïve method in all three settings of γ, in terms of I_{Peak}, R_∞, and MI_{left} reduction. For e.g.,



(a) Low Infectivity ($\gamma = 0.572$)

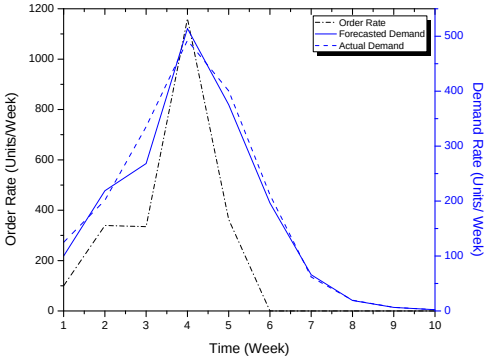


(b) Base Infectivity ($\gamma = 0.596$)

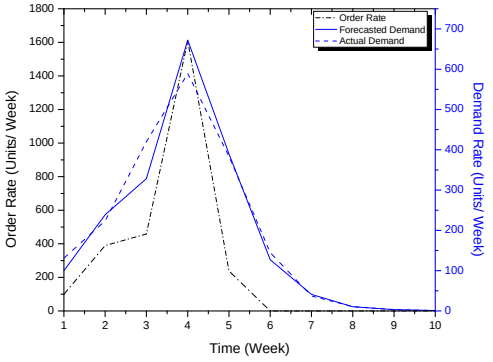


(c) High Infectivity ($\gamma = 0.62$)

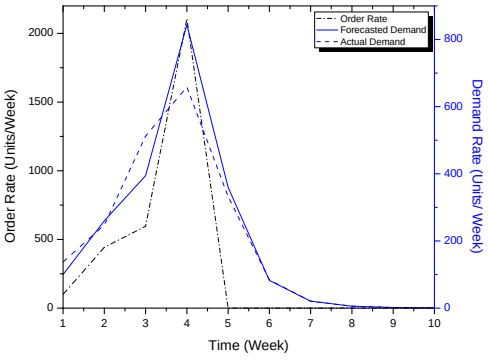
Figure 7: Comparison of daily backlog (MB_d), shipment rate (SR_d), and inventory (MI_d) dynamics of naïve forecasting method.



(a) Low Infectivity ($\gamma = 0.572$)

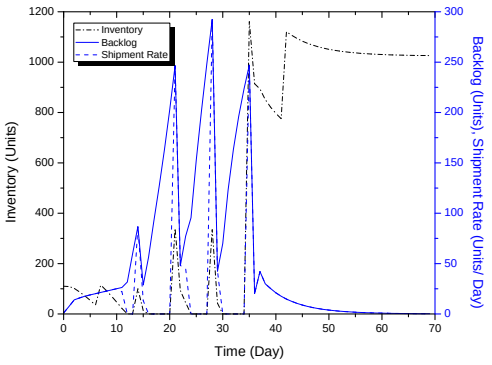


(b) Base Infectivity ($\gamma = 0.596$)

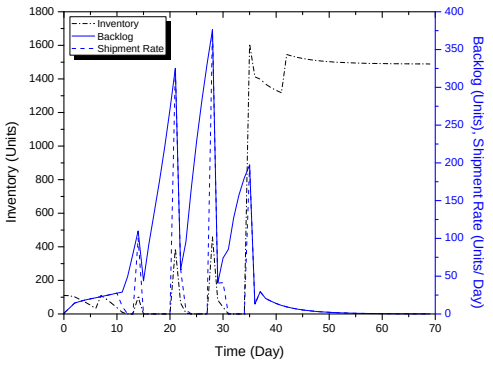


(c) High Infectivity ($\gamma = 0.62$)

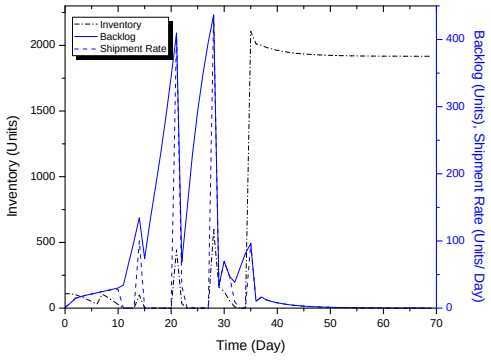
Figure 8: Comparison of weekly forecasted demand rate (FD_w), actual demand rate (DR_w), and order rate (OR_w) for SIR forecasting method.



(a) Low Infectivity ($\gamma = 0.572$)



(b) Base Infectivity ($\gamma = 0.596$)



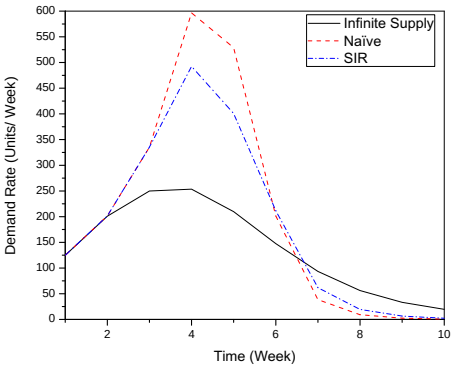
(c) High Infectivity ($\gamma = 0.62$)

Figure 9: Comparison of daily backlog (MB_d), shipment rate (SR_d), and inventory (MI_d) dynamics of SIR forecasting method.

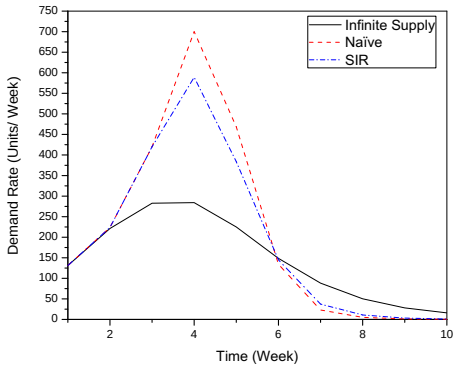
in case of low γ, 32.02%, 8.67%, and 63.18% reductions in I_{Peak}, R_∞, and MI_{left} are possible with SIR model than naïve forecasting model.

Table 3: Comparison of forecasting techniques

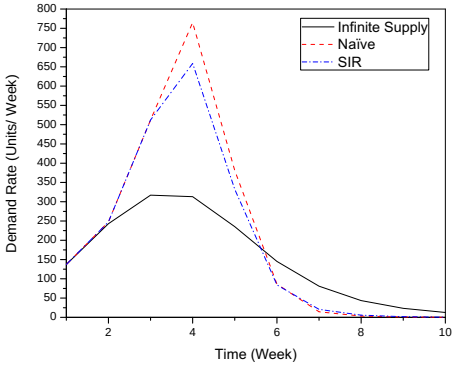
	Low Infectivity (γ)		Base Infectivity (γ)		High Infectivity (γ)	
	Naïve	SIR	Naïve	SIR	Naïve	SIR
I_{Peak}	563.593	383.139	656.316	467.870	712.648	525.034
R_∞	2117.224	1933.683	2186.260	2023.300	2226.859	2079.768
MI_{left}	2786.161	1025.977	3206.021	1489.188	3960.500	1916.775



(a) Low Infectivity ($\gamma = 0.572$)



(b) Base Infectivity ($\gamma = 0.596$)



(c) High Infectivity ($\gamma = 0.62$)

Figure 10: Comparison of weekly epidemic demand rate (DR_w) for forecasting policies under consideration and infinite supply.

In Fig. 10, actual demand rates of two forecasting models under consideration are plotted against the demand rate when infinite/unlimited medicine

inventory are available (i.e. no stock out) to reflect the scope of further improvement. In Fig. 10, the demand rate forecasted by SIR model can be reduced by 48.508 %, 51.797%, and 52.467% for low, base, and high infectivity respectively while the same can be done by 57.493%, 59.439%, and 59.054% respectively for naïve forecasting method.

4 Conclusions and Future Work

A framework for inventory management in response to the post-epidemic declaration is developed, where the demand forecasting is made using SIR disease diffusion model and updated dynamically based on past epidemic behaviour. An SEITRS epidemic model is integrated with the inventory management model for simulation purpose of a real epidemic diffusion. The performance of two different forecasting models (SIR and naïve) are compared in anticipation of the unknown epidemic. It is found that SIR model outperforms naïve method in terms of reducing epidemic impact (i.e., infected peaks and final epidemic sizes) and is also proven to be relatively economic in terms of inventory leftover. However, there is significant scope for improving the proposed model as shown in Fig. 10.

Further, analyses are currently being carried out in the following directions: comparing and finding the appropriateness of other variants of disease diffusion models as a forecasting method; quantifying the effect of planning horizon length (h), and inventory management parameters (for e.g., α, β) on disease dynamics and system cost. In our current model, we have considered one parameter (h) for both reviewing demand and the number of days to accumulate forecasted demand. Future research can explore what if there are two different parameters say, a fixed demand review period (r) and another for the number of periods, to accumulate forecasted demands (h) which will help to cope with managing the excess demand during the peak of an epidemic.

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