Exploring Harrod Domar and Solow Models of Economic Growth

Abstract

This paper models economic growth as described by the Harrod Domar and Solow models using the System Dynamics framework. The modelling exercise highlights the boundaries of the Harrod Domar model and presents alternative solutions to overcome these boundaries. As an extension to the Harrod Domar model, this paper demonstrates the endogenous effect of savings and population changes on economic growth. Solow's enhancement of the Harrod Domar model has been simulated to highlight the capital output constraint in determining income growth. The System dynamic modelling toolset in this paper provides markedly different modelling perspectives that are unavailable in static models. In addition, the models developed for this paper allow easier testing of sensitivities to a range of macroeconomic variables.

Keywords: Harrod Domar, Solow, Economic Growth, Demographic transition

Contents

Section 1

Introduction

1.1 Development Economics - Traditional modeling techniques & their limitations

Advances in economics till the 1940's centered on the mechanisms and sources of economics and prosperity (rather than those deprived by it) in general. As a consequence of this, development economics has not been a natural focus area for most economists till the high period of 1940s [1](Krugman, 1995). Given the wider range and reasons for economic outcomes, the understanding of multi-faceted subject of development is relatively recent [2](Hosseini, 2003). The multi-dimensional nature of Developmental economics makes it an ideal field for applying System Dynamics methods.

Most standard development economics text books, explain economics theory through models that tend to be static. For instance, the Harrod Domar model or the Solow model[3] (Ray, 1998), including Solow's [4] (Solow, 1956) original paper, have been explained through both text, block diagrams and mathematical equations. Some of these equations have multiple variables, all moving across time. These models help give insight and also help in offering a perspective toward development. But the static models are severely limited in bringing forth the dynamics between variables and also the endogeneity among variables. Furthermore, most of the static models covered in economics theory do not offer test cases that can help enhance the practical working of a model.

The inherent dynamic ability that the System Dynamics framework contributes is especially useful in the context of economics. Economics system spaces are complex. Most economic models deal with imperfect information about the state of the real world and have interconnected and/or ambiguous variables. These attributes of dynamic complexity make economics models suitable for System Dynamics based frameworks [5](Sterman, 2000).

Section 2

Harrod Domar Model

2.1 Background

Harrod and Domar developed their models independently but their assumptions and results were similar. The background to their work (1939 Harrod, 1947 Domar) was that industrialized economies had faced cataclysmic upheavals – the Great Depression followed by the Second World War. The Harrod-Domar model therefore perhaps, has attempted to explain as to how economies would need to grow (or would be left to stagnate) over time.

2.2 Model description

The Harrod Domar model, has two main actors. The model assumes a closed economy in which households consume & save. Firms, the other "actor" in the Harrod Domar world, produce both capital goods and consumer goods based on investments. The investment in firms, in turn, is a result of the shortfall between the consumption expenditure that households have and the income the households earn. The income that households earn, is the income generated by firms as a result of investments.

The schematic for this description has been shown in Figure 1 [6](Ray, 1998). It may be noted that the schematic has "stocks, "inflows" and "outflows".
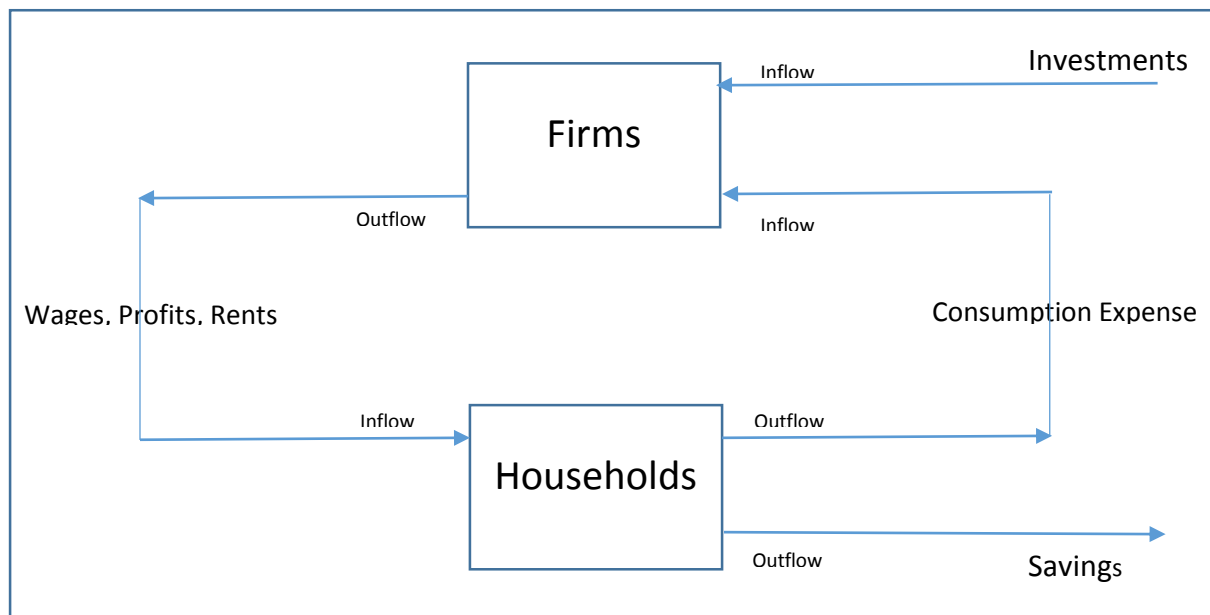
Harrod Domar model Schematic (Debraj Ray, Development Economics, Page 52, Figure 3.1)
Figure 1

2.3 Inbuilt equations in the Harrod Domar model

The key equations that govern the Harrod Domar model include (Ray, 1998):

1) $Y(t) = C(t) + S(t)$

Equation 1) states that the income denoted by ($Y(t)$) (at time t) is used by households to consume ($C(t)$) and save ($S(t)$). This has been depicted on the left side of the schematic in Figure 1.

2) $Y(t) = C(t) + I(t)$

Equation 2) above states that income ($Y(t)$) is also a derivation from consumption ($C(t)$) and investment ($I(t)$).

3) $S(t) = I(t)$

It follows from Equations 1 & 2 that the identity could be derived. In literature, this identity has also been referred to as the macro-economic balance (Ray, 1998)[7]. From this equation, it implies that all savings in a Harrod Domar world, translate to investments.

4) $K(t + 1) = (1 - \delta) * K(t) + I(t)$ (δ is the depreciation; $K(t)$ and $K(t + 1)$ denote capital at times t and $t + 1$)
5) $\frac{S(t)}{Y(t)} = s$ (s is the savings rate)
6) $\theta = \frac{K(t)}{Y(t)}$ (θ is the capital output ratio)
7) $\frac{s}{\theta} = g + \delta$ (g denotes the growth rate; Equation 7 is derived through substitutions in equations via equations 8,9 and 10 below)
8) $K(t + 1) = (1 - \delta) * K(t) + S(t)$
9) $\theta * Y(t + 1) = (1 - \delta) * \theta * Y(t) + s * Y(t)$
10) $\frac{Y(t+1) - Y(t)}{Y(t)} = \left(\frac{s}{\theta}\right) - \delta$ (the left hand side of this equation is the percent change in income denoted as g)

The subsequent sections will seek to reconcile the description of the Harrod Domar model in this section with the schematic representation in Figure 1 as well the equations 1 through 10.

2.4 System Dynamics – Modelling Harrod Domar equations

Equations 1 through 10 that govern the Harrod Domar Model are complete and consistent. But these equations do not provide insights into the casualty between the variables. For example the first equation describes income as being made up of two components – savings and consumption. Equation 1 however does not provide enough detail to provide a direction for the flow inbuilt inside the equation.

A system dynamics representation for equations 1 through 10 is provided in Figure 2 that seeks to bring casualty to these equations.
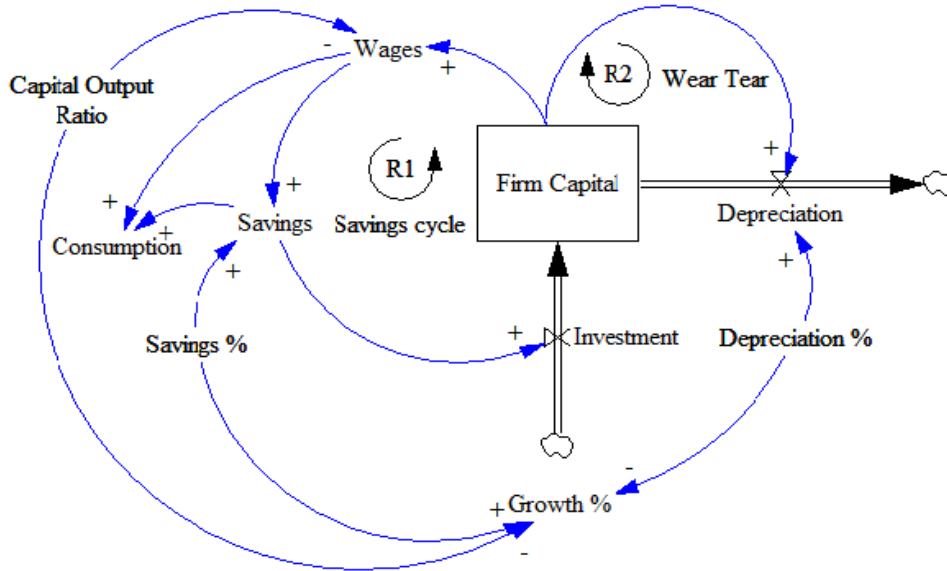
System Dynamics: Modeling Harrod Domar equations

Figure 2

Using the System Dynamics approach, each equation (i.e. Equations 1-10) in the Harrod Domar model has been used to build a stock and flow representation for the model. The SD model in Figure 2 is a complete and exact representation of equations 1 through 10.

With reference to Figure 2 and all other System Dynamics representations of Harrod Domar, Table 1, provides a list of the entities and the units that constitute the System Dynamics representation of the Harrod Domar model. It may be noted that the "units" for each of the entities are just an exact "unit" logical representation for the description of that entity.

| Entity	Description	Unit
Capital Output Ratio	Ratio of Firm Capital to Income	Year
Consumption	Household consumption per period	Money/Year
Depreciation	Markdown to account for asset's useful life	Money/Year
Depreciation %	Markdown of asset value in percentage terms	Ratio
Firm Capital	Accumulated firm capital	Money
Growth	Percent change in Wage-Profits-Rents per period	Ratio
Household Capital	Accumulated household capital	Money
Investment	Investments in firms per period	Money/Year
Savings	Savings per period	Money/Year
Savings %	Ratio of Savings to Wages	Ratio
Wages-Profits-Rents	Wages, profits, rents that households earn per period	Money/Year
Per Capita Values	Per person values for flow or stock variable	Units/Person

System Dynamics: Description of each entity with unit used

Table 1

The System Dynamics model in Figure 2 adds value to the equations 1 through 10 as it gives a better insight into the casual relationships that exist. For instance, it is clear now that Savings and Consumption exist because of Wages. Wages in turn are a result of Investment being transformed through the capital output ratio.

But, with reference to Figure 1 (Harrod Domar schematic) and Figure 2 (Harrod Domar SD representation), it is to be noted that the "Households" "stock" (from Figure 1) is completely missing from Figure 2. That there is a gap in the description of the model (as shown in Figure 1) in terms of stock and flow connectivity and the equations that represent this model is a discrepancy that System Dynamics allows the modeler to uncover.

The output graphs for two key variables – Wages-Profits-Rents (Income) and Firm Capital – using the System Dynamics model in Figure 2, have been shown in Figure 3.
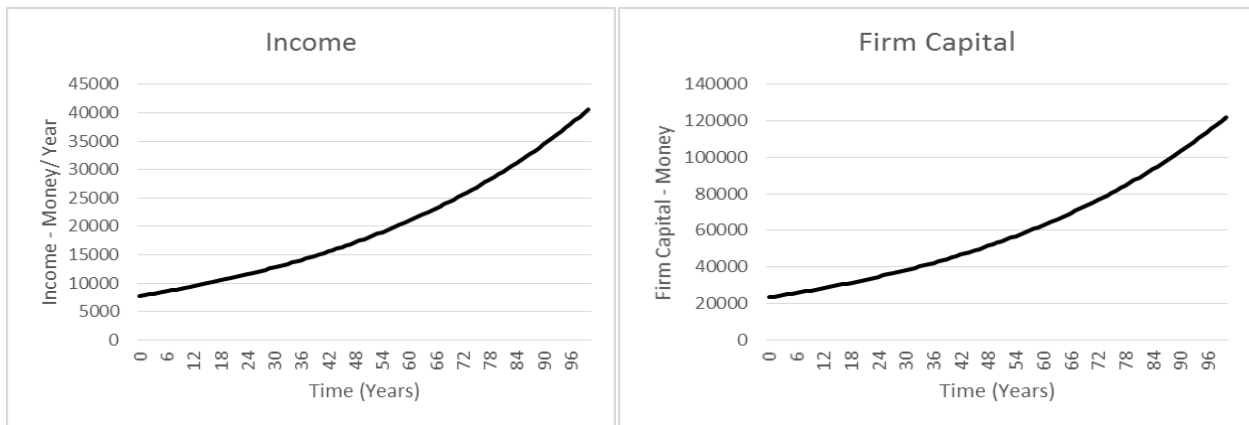
Income and Firm Capital output graphs using System Dynamics
Figure 3

For reference, control variables for the model were provided the following input:

Savings Rate ($s = 0.35$); Capital Output ration ($\theta = 3\ years$); Depreciation ($\delta = 0.10$); Starting Firm Capital (1 "Money")

Growth rate given by the formula ($\frac{s}{\theta} - \delta = g$) is 1.67%, matches with the output produced by the System Dynamics model in Figure 3.

2.5 Reconstructing Harrod Domar through the descriptive text

This section reconciles the difference between the descriptive text of the Harrod Domar model and equations 1 to 10 that represent the model mathematically.

"Households" have a prominent place within the Harrod Domar model description as well as within the schematic shown in Figure 1 (text book description of the Harrod Domar mode). Wages, Savings and Consumption, shown as auxiliary variables in Figure 2, in reality, carry the units of a flow (i.e.

"Money/Year"). In order to appreciate this in a System Dynamics framework, a new "Household capital" stock was introduced in the System Dynamics model of Figure 2. The three auxiliary variables (Wages, Savings & Consumption) were converted into flows (or rates). Without disturbing the sanctity of equations 1 through 10, and through incorporating the "Household capital" stock and three additional flows (Wages, Savings & Consumption) a new System Dynamics representation was attempted.

Building an exact replica for the schematic (in Figure 1) along with keeping the sanctity of equations 1 through 10 faced two inherent contradictions:

1. Consumption "flow" has been shown as an inflow in the schematic in Figure 1 but this representation is completely missing out from equations 1 through 10.
2. Similarly, Wages-Profits-Rents "flow" has been shown as an outflow in the schematic in Figure 1. Again, this information is not represented at all in equations 1 through 10.

Overlooking these two contradictions, a System Dynamics representation, that attempts to match the schematic (Figure 1) and equations 1 through 10 has been modelled. The block diagram for this model has been shown in Figure 4 and the output graph for Firm Capital and Wages-Profits-Rents have been shown in Figure 5. The input values for the control variables have been left unchanged (i.e. Savings Rate ($s = 0.35$); Capital Output ration ($\theta = 3\ years$); Depreciation ($\delta = 0.10$))
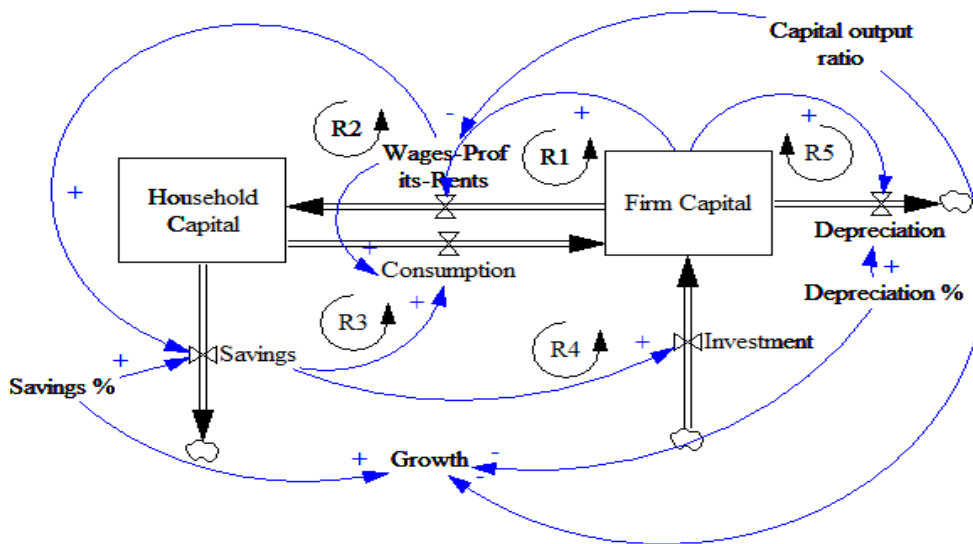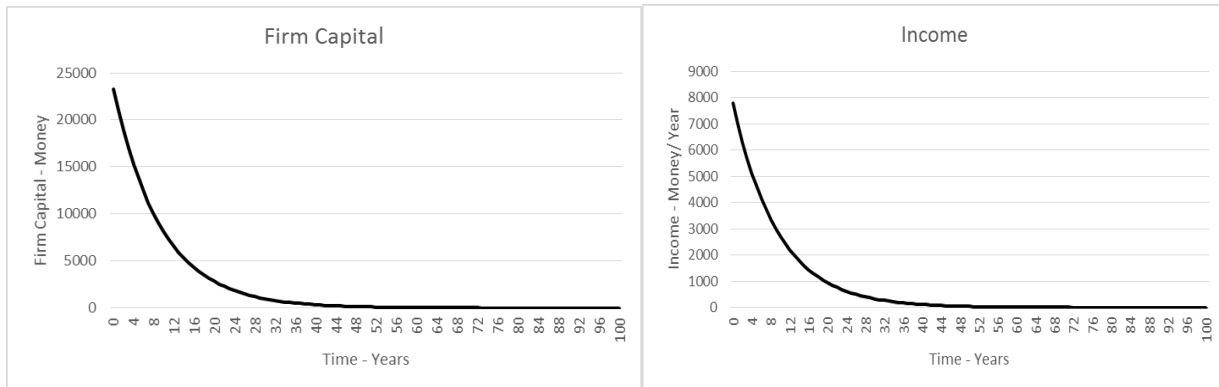

Reconciling Harrod Domar description with Harrod Domar equations
Figure 4

The graphs in Figure 5 show that both the Income (Wages-Profits-Rents) and the Firm Capital has collapsed within 50 years. Since the model in Figure 4 contravenes two rules highlighted earlier, unsurprisingly, the result in Figure 5, comes as a complete contradiction compared to the results shown in Figure 3.

It should also be noted that "Household" capital "stock" in the model described by Figure 4, finds itself isolated and unused. In fact, the software used to build the model complains and highlights this fact as a warning. A stagnant "Household" capital "stock" at a time when other entities in the model are changing

contradicts Solow's observation that all entities in a Harrod Domar world grow (or change) at the same speed

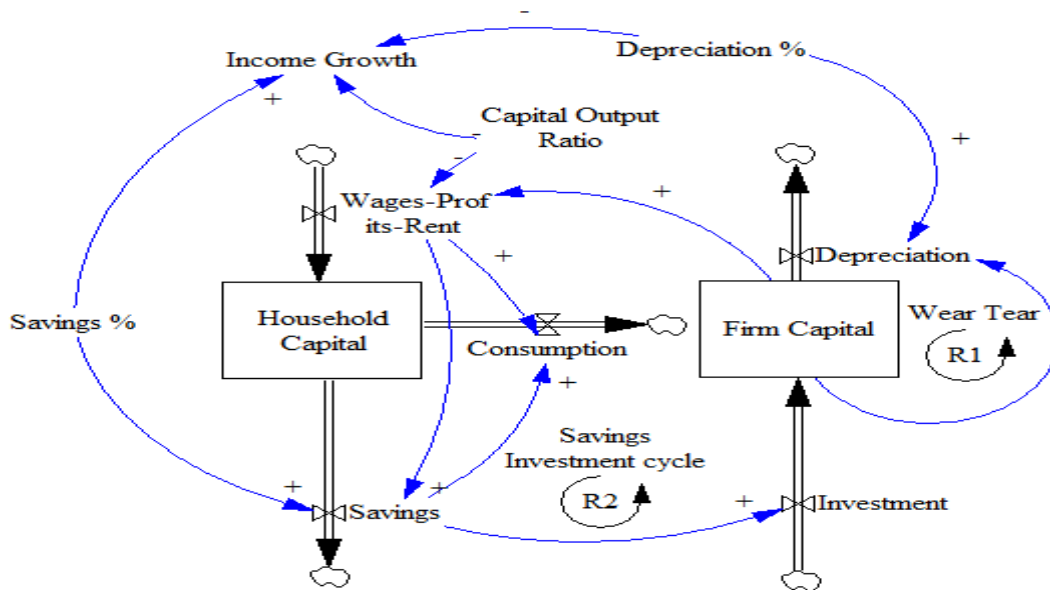
Reconciling HD description with equations
Figure 5

2.6 Reconciling Harrod Domar model description with the equations

For improvising the models described in Figure 2 and Figure 4 (Sections 2.4 and 2.5 respectively), an additional System Dynamics model was constructed to reconcile the contradictions presented in Figure 4 (section 2.5, i.e. extraneous "flow" links for wages & consumption). The improvised model has been shown in Figure 6. Figure 6 tells us that:

a. Consumption is now flowing out to a cloud (and not flowing into "Firm Capital")
a. Wages-Profits-Rents are a flow but this flow originates from a cloud with inputs from the "Firm Capital"
b. Wages-Profits-Rents, are now an inflow into a newly created "Household stock" as described in the original Harrod Domar
c. Savings and Consumption have now been shown as outflows from the "Household stock"
b. However, given the lack of mathematical equations connecting the "Household stock" to the flows, the model in Figure 6 informs that the "Household stock" would cease to play a role in determining flow of "Wages", "Consumption" and "Savings"

In reality, Consumption would need to be connected with the "Firm Capital" and Wages too would need to be derived from the production/ value addition that firms bring. But this would mean enhancing the basic Harrod Domar framework.

The improvised model shown in Figure 6 has produced exactly the same output as was shown in Section 2.4.

Reconciling Harrod Domar description with true stock and flow representations
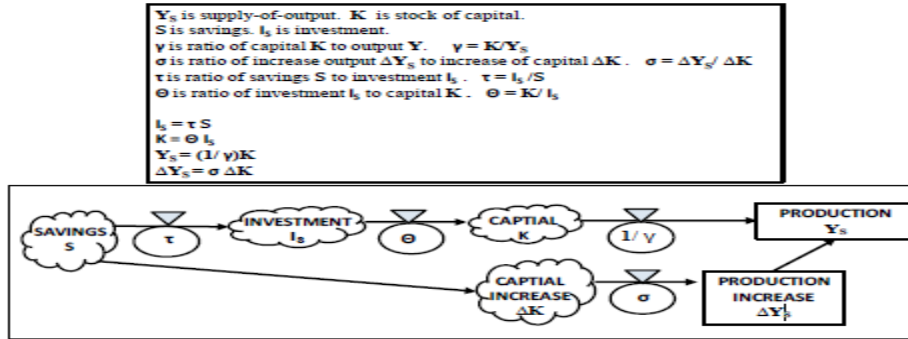
Figure 6

2.7   Literature - Harrod Domar in System Dynamics representation

Betz et al (Betz, 2015)[8] have presented used System Dynamics framework and various models in the economics literature to explain the evidence left by the 2008 financial crisis.

The schematic (Figure 7) shows the Harrod Domar modeling used by Betz et al. In the schematic below, Betz et al, give the following descriptions:

1) Clouds represent sources of flow
2) Arrow represent direction
3) Rectangles denote Stocks
4) Triangle over oval denote rates

Although not present in the original Harrod Domar model, Betz et al, in addition to (τ), the ratio for savings to investment, have introduced two other variables: (σ) a ratio for change in output to change in capital; (σ) as ratio of increased output (ΔY) to increased capital (ΔK). Betz et al have not supported the model representation with data sets/ output graphs. Their focus largely was to look at the Harrod Domar model (and other economic models) in the context of the 2008 financial crisis.

Harrod Domar used by Betz et al

Figure 7

2.8   Discussion

Harrod Domar model and equations give an elegant and aggregative view for a country's economy. The model links the growth rate in income (also referred to as the economic growth rate) to two fundamental variables – the ability of the economy to save and the influence of the capital output ratio to convert this savings into income (referred to as Wages-Rents-Profits). System Dynamics representation provides insight on the boundaries of the original model. In particular, the original Harrod Domar model equations do not account for any linkages such as savings and consumption, between household capital and firm capital.

By pushing up the savings rate it is possible to accelerate economic growth. The capital output ratio too influences the rate of growth. By requiring lesser capital to produce income (more efficiency, lesser θ), economic growth can be accelerated as well.

System Dynamics representation of the Harrod Domar models provides the following important contributions:

    a.   Identifies boundaries in the Harrod Domar model – one key finding is that the Household stock described in the model is never really used in the equations used in the model

    b.   Helps users, test and simulate the effect of changes in input variables on parts of the model including the main output variable i.e. economic growth

The following Sections build on the extension to the basic Harrod Domar model and highlight the sensitivities of input variables as well.

Section 3

Harrod Domar – Population extensions

3.1 Background

This section introduces two important extensions to the Harrod Domar model. The first extension brings population as a variable in the model. Bringing population into the Harrod Domar equations make the model more complete in describing real economies. The second extension in this Section discusses important effects of applying the Harrod Domar population extension to the real world.

## 3.2 Introducing per capita variables

Population is an integral part for describing a real world economy. Especially for developing countries, population plays an influential role in determining a country's prospect in lifting its population out of poverty.

If population (P) grows at a rate of (n), so that $P(t + 1) = P(t)(1 + n)$ for all time (t), equation 7 can be rewritten as:

11) $\frac{s}{\theta} = (1 + g) * (1 + n) - (1 - \delta)$ (g), now denotes the per capita growth rate; (n) is the population growth rate. Equation 11 is derived through substitutions via equations 7,8,9 and 10)

12) $\frac{s}{\theta} = g + n + \delta$ is an useful approximation of equation 11, given the relatively small values of (g) and (n).

Equation 12 is an important per capita extension of the Harrod Domar model. The equation informs that the per capita growth rate is now influenced by three variables. In addition to the savings rate (s), depreciation and the capital output ratio (θ), per capita growth is also influenced by the population growth rate (n). A high rate of population growth adversely affects the economic growth rate.

The per capita enhancements to the Harrod Domar model resulting from equations 11 and 12 above have been made to Figure 6. The enhanced model has been shown in Figure 8 (a).
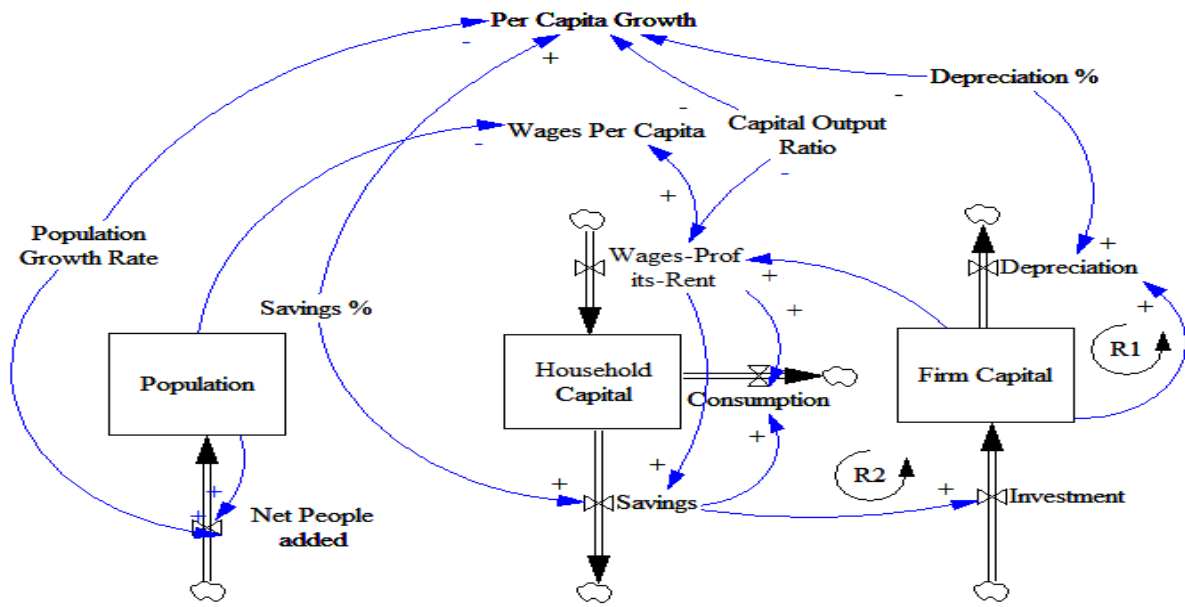
(a)
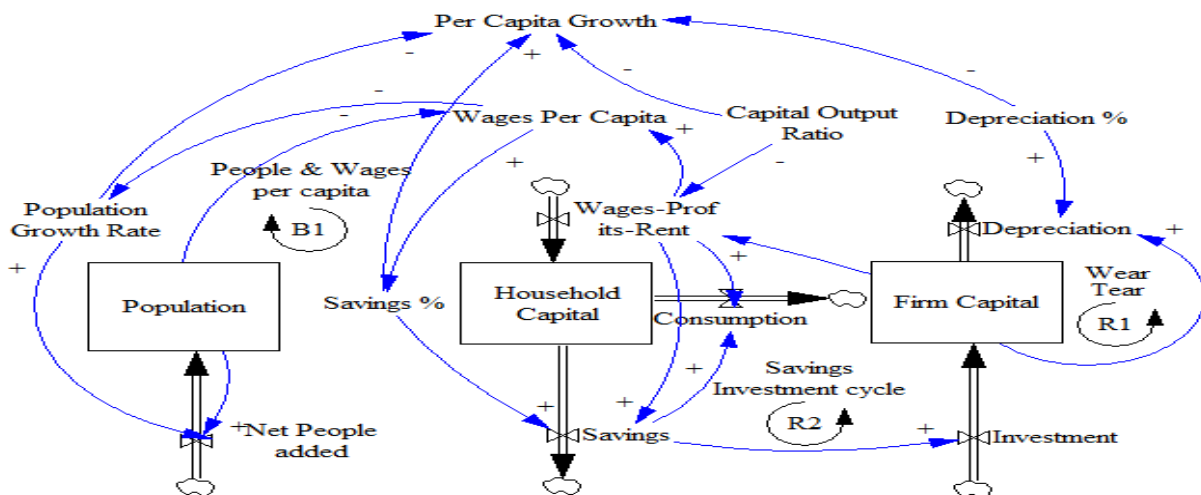
(b)

Per Capita implementation (a) savings/ population rates as exogenous (b) savings/ population rates as endogenous

Figure 8

3.3 Sensitivity of input variables on economic growth

Giving values to the input variables in the Harrod Domar equation (equation 12), provides a realistic perspective of using the Harrod Domar equation to model a real economy.

The Harrod Domar world view described in equation 12, requires inputs on four variables:

Savings rate, Capital Output ratio, Population Growth rate, Depreciation.

Table 2 shows the absolute effect on per capita growth if each of these variables is changed independently in steps of 5%.

The tables shows that:

a) On average, changes in the savings rate and capital output ratios have a bigger influence on economic growth than population growth
b) A decrease in the capital output ratio influences economic growth the most – this can only be expected as capital output is a variable that appears in the denominator
c) Population growth has the least influence compared to the other variables

Variable Name	-5%	-10%	Base Line	5%	10%
Savings	-0.42	-1.00	0.17	0.75	1.33
Capital Output	0.78	1.46	0.17	-0.39	-0.89
Population Growth	0.24	0.32	0.17	0.09	0.02

Sensitivity of growth rate to univariate changes in each of the three input variables in the Harrod Domar per capita growth equation. Values show percentage changes.
Table 2

# 3.4 Endogeneity in Harrod Domar variables

The simplicity from the growth equation (equations 7 and 12) in the Harrod Domar model hides the interrelationships amongst the input variables that determine growth and also the influence of the growth process on these input variables.

3.4.1 Savings Rate

A sensitivity analysis of the four input variables in Table 2 informs that the savings rate is among the top drivers of growth. But can the average savings rate be influenced easily by policy? It is intuitive to understand that a wealthy person (i.e. more per capita income) is in a better position to save more than a poor person (i.e. less per capita income). In fact lower levels of income may disallow a person from any savings at all. As an economy grows it increases the per capita income levels which in turn sets the pace for increased economic growth. Enhancing per capita income levels beyond the minimum threshold required for subsistence and thus allowing the savings rate to increase, becomes an important policy objective[9] (Ray,1998).

The dependence of the savings rate on per capita income levels informs that the savings rate is not an exogenous variable but it is rather an endogenous variable.

In the simulation exercise described in Figure 8 (b), the savings rate along with the population rate has been transformed to be a dependent variable. The savings rate changes to corresponding changes in per capita income have been shown in Figure 9.
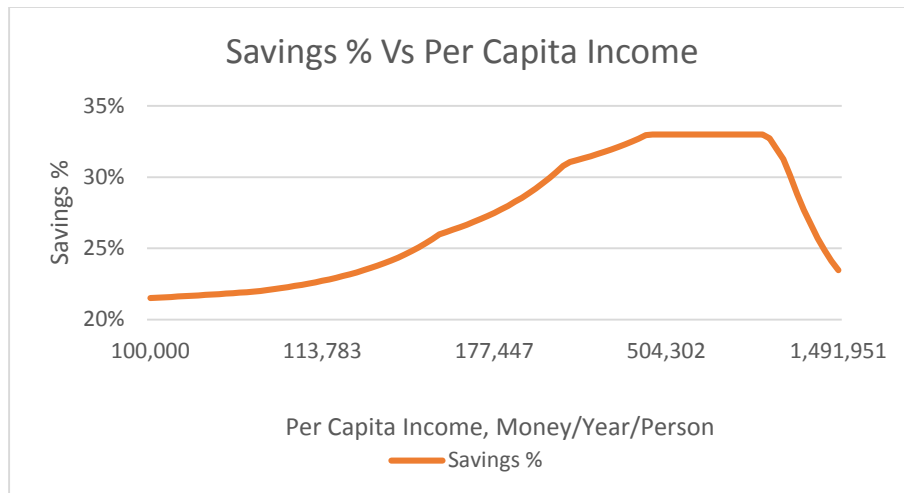
Savings as a dependent variable used in simulation – inverted U curve
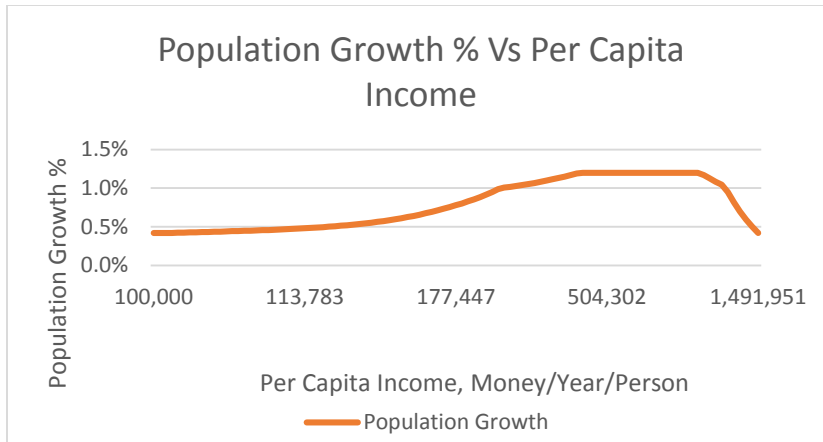
Figure 9

3.4.2   Population Growth Rate

In addition to the savings rate, population growth rates are also influenced by the per capita income levels of an economy[10] (Tsen et al, 2005). The feedback relationship between the per capita income level and the population growth rate is not as straightforward (and intuitive) as the relationship between savings rate and per capita income. Social scientists however, appreciate the variation in a country's population growth rates with level of economic development. The demographic transition – a change from higher to lower rates of mortality and fertility – with the level of development has been well documented [11](World Bank, 2000).

Death rates in poor countries (low per capita income) are higher given malnutrition and poorer health care access. To compensate, the birth rates in such countries are higher too. The combination of higher mortality and fertility rates keeps overall population growth rate in check. As income levels start to rise, the death rates fall but birth rates adjust at a slower pace. This leads to an increase in the net population growth rate – in some countries this has created "baby boom" generations [12](Bloom et al, 2011). In the longer run, fertility rates revert down, bringing down the net population growth. The inverse "U" shaped behavior of population growth has been noted in different countries [13](Valli et al, 2011).

In the simulation exercise described in Figure 8 (b), the population growth rate (in addition to the savings rate) has been transformed to be a dependent variable. The population rate changes to corresponding changes in per capita income have been shown in Figure 10.

Population growth rate as a dependent variable changes as the per capita income varies
Figure 10

3.4.3 Modelling endogeneity of savings rate and population rates in the Harrod Domar equation

The System Dynamics framework allows setting up of an enhanced version of the Harrod Domar model discussed in Figure 6 to allow for changes in per capita income growth levels to influence the savings rate and the population growth rate. Simultaneous testing of these two variables was conducted to understand and test the discussion highlighted in 3.4.1 and 3.4.2.

Of the four variables that influence economic growth from Harrod Domar's per capita extension (Equation 12: $\frac{s}{\theta} = g + n + \delta$), the depreciation rate has been set to a standard 10% per year and the capital output ration has been set to 1.6, at the lower end of the capital output scale in emerging economies [14](Taguchi et al, 2014).

Per capita income is used to determine the savings rate as well as net population growth rate. Figure 9 and Figure 10 highlight the values taken by savings rate and population growth rates. The shape of these curves resembles as closely as possible the inverted "U" curve discussion in 3.4.1 and 3.4.2.

Figure 11a shows the effect of the simultaneous changes in the savings rate and population growth rate on per capita income. Keeping the Savings Rate constant, Figure 11b provides additional insight in the role that population growth plays in deciding per capita income.

Simulating the effect of savings rate and population growth rate, provides a better understanding of the discussion in 3.4.1 and 3.4.2:

a. Per capita growth increases with savings, but it slides down at both lower as well as higher savings levels. It is to be noted that lower savings levels are a result of an inability to save whereas at higher level of savings there is a lack of inclination to save.

b. From Equation 12, it is clear that population growth reduces a nation's total growth. For per capita income to grow, total income growth has to be higher than population growth. From Figure 11b, below the "Trap" level, population growth is less, so per capita income growth is increasing. But if one starts just above the "Trap" level, population growth has the ability to outstrip total income growth and the economy can actually become poorer in per capita terms. In addition, if the economy has the growth thrust to move beyond (right of the) the "Threshold"

level, the economy would go into sustained growth. Policy to boost savings and or population control measures can guide an economy to sustained growth[15] (Ray, 1998).
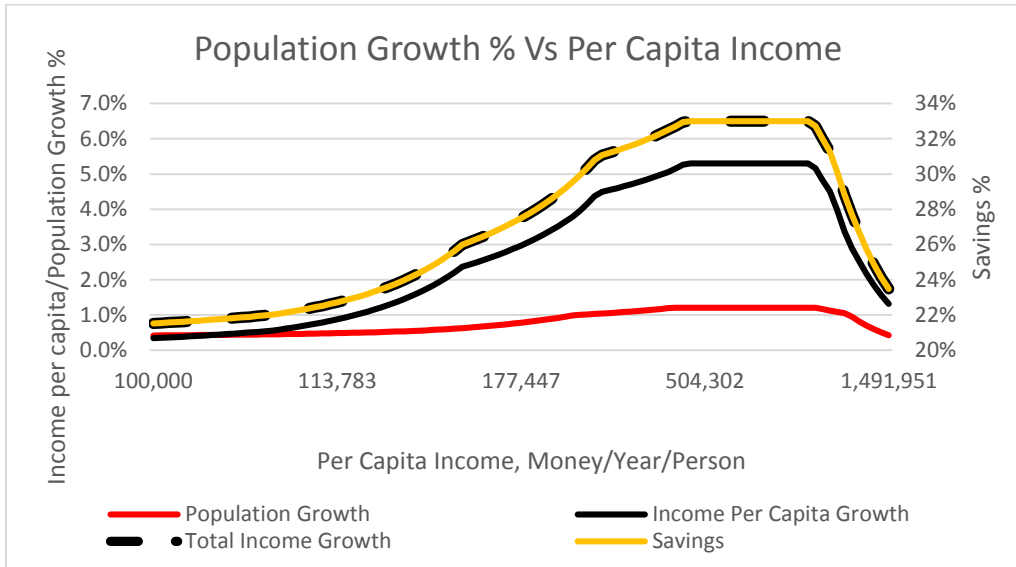
Simultaneous savings rate & population growth changes & effect on per capita growth

Figure 11a
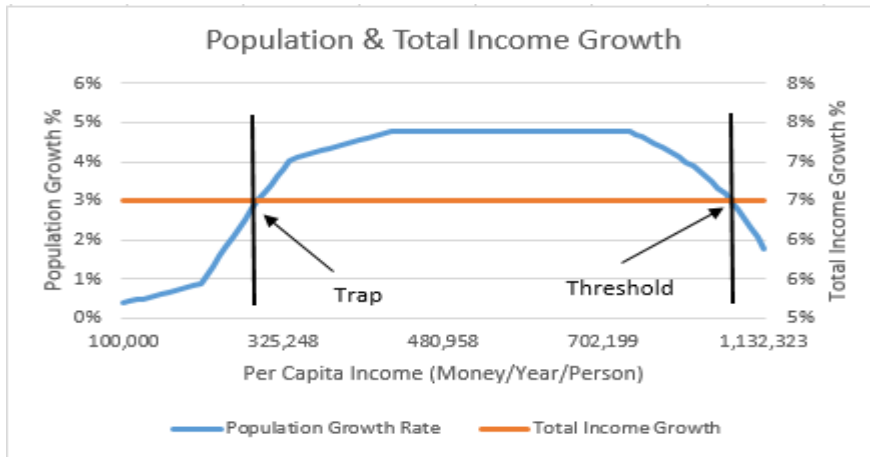
Population growth changes & effect on per capita growth

Figure 11b

3.5 Discussion

Extending the basic Harrod Domar model and using a System Dynamics framework to model per capita levels gives a practical tool for analysts and policy makers to plug and simulate real inputs and observe the effects on some of the most important macroeconomic variables. Testing the interconnectedness and endogeneity of variables on per capita income levels provides insights into the workings of an economy, beyond the understanding provided by standard static models.

Section 4

Solow's extension to the Harrod Domar model

4.1 Background

Solow provided arguments to think beyond the "long range" values of variables in the Harrod Domar model – i.e. the assumption of a stable capital output ratio [4](Solow, 1956). Solow's argument is based on the law of diminishing returns [16] (Ricardo, 1815) to individual factors of production. Capital and human effort (labor) work in tandem to produce. If there is plenty of labor relative to capital, a small amount of capital will suffice. On the other hand, if labor is in short supply capital intensive measures such as automation are required to raise output levels [17](Ray, 1998). Solow, proposed that the capital output ratio (θ) is an endogenous variable – not exogenous as was assumed in the basic Harrod Domar model.

4.2 Model Description

Solow's proposed endogeneity of the capital output ratio being influenced by the level of income and the level of capital is an additional layer of dynamism for the basic Harrod Domar model. The modular nature of System Dynamics tools provides modelers the ability to seamlessly add this feature and enhance the existing model.

Solow's extension focuses on the per capita output to the per capita capital stock. Per capita capital stock in turn is derived through a continuation of the basic Harrod Domar equations – in particular, equation 4.

Equation 4 is $K(t + 1) = (1 - \delta) * K(t) + I(t)$ δ is the depreciation; $K(t)$ and $K(t + 1)$ denote capital at times t and $(t + 1)$.

Dividing through by population (P) growing at constant rate (n), so that, $P(t + 1) = (1 + n)P(t)$, and using the relationship between savings rate, total income and investment $(I(t) = s * Y(t))$, equation 4 transforms to:

13) $(1 + n) * k(t + 1) = (1 - \delta) * k(t) + s * y(t)$ where $k(t), y(t), s$ denote per capita values of capital stock, income and savings rate

Solow's extension connects the per capita values of the capital stock and income using a Production function [18](Ray, 1998). The production function represents the technical knowledge of the economy, bringing capital and labour together to produce output. One of the popular forms [19] (Koch, 2013) of the production function – the Cobb Douglas production function – has been used in the simulation exercise in this paper.

The Cobb Douglas production function has been represented in the simulation through the following equation:

14) $Y(t) = \alpha * A * K^{\alpha} * P^{1-\alpha}$ where $Y(t)$ is the total income, α is the ratio of capital income to total income [20] (Piketty,2015), K is the firm capital, A is the technology multiplier and P is the total labor.

The Cobb Douglas function is notable for its constant returns to scale function (output doubles if the input variables – capital and labor double) of labor and capital.

Another property of the Production function is the diminishing returns to per capita capital increases.

Figure 12, shows the Production function run for a simulation of the Solow extension to the Harrod Domar model. This function plots the changes in wages per capita in response to changing per capita capital.
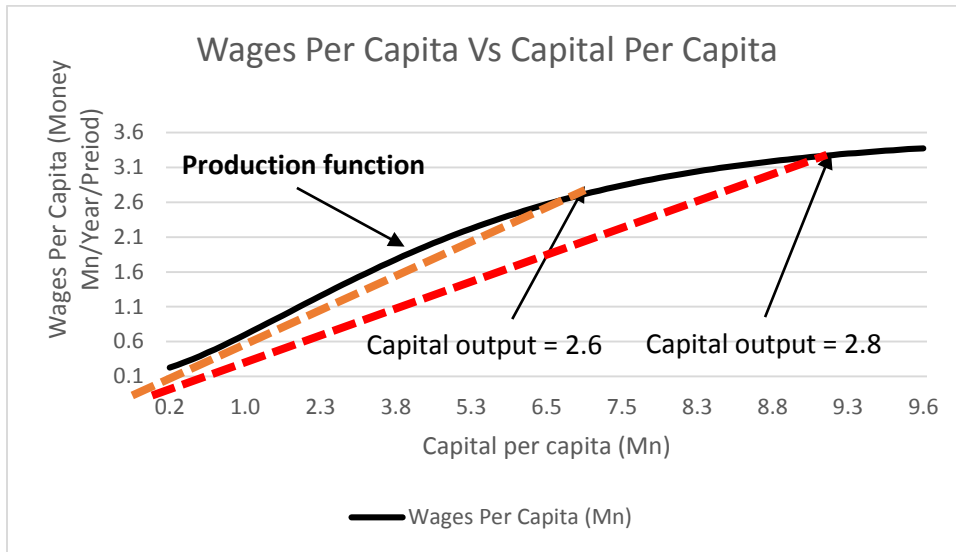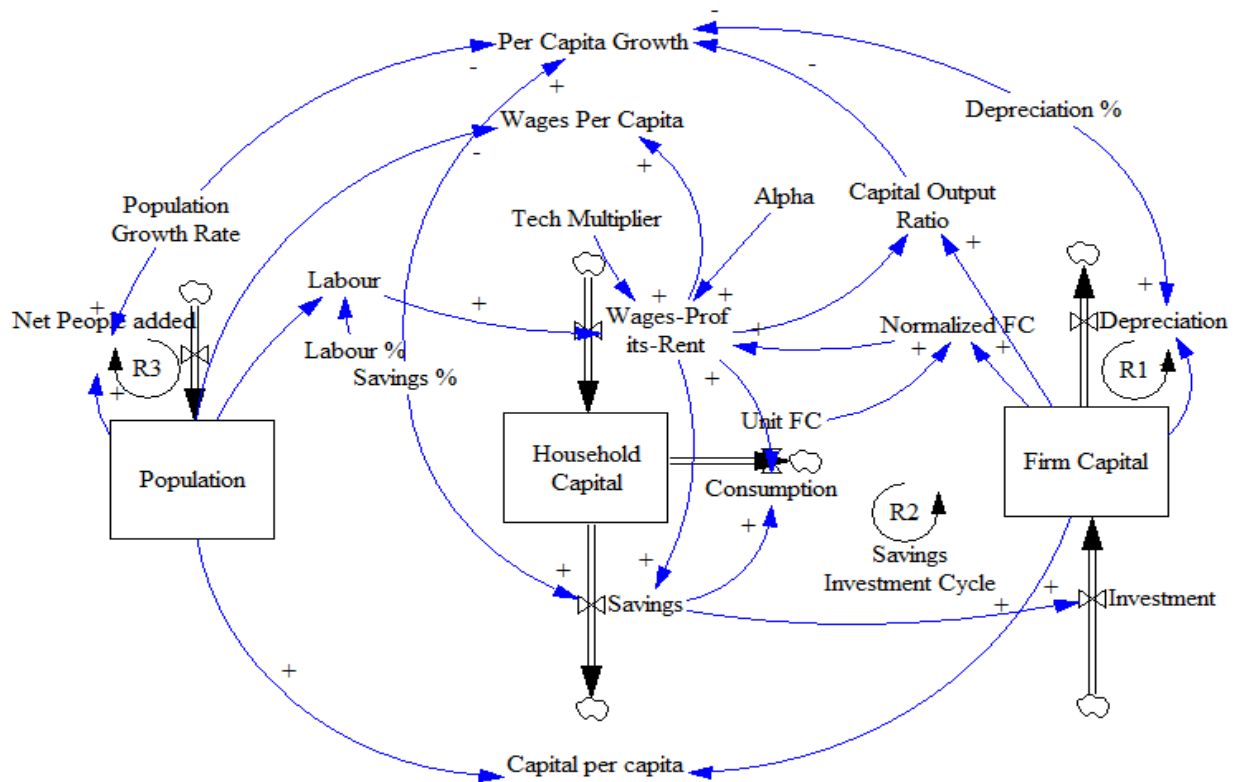

Solow's extension to the Harrod Domar model
Population Growth=1.35%, Alpha=0.7, Savings=35%, Depreciation=10%

Figure 12

4.3 Changes required for implementing Solow's extension

Figure 13 shows the modified System Dynamics representation of the Solow's extension to the Harrod Domar model. Figure 13 is the enhanced version of the per capita Harrod Domar model discussed in Figure 8. This enhanced version, in line with equations, 13 & 14, now has the capital output ratio as a variable dependent on labor participation, the ratio of capital income to total income, level of technology and firm capital.

Solow's extension to the Harrod Domar model
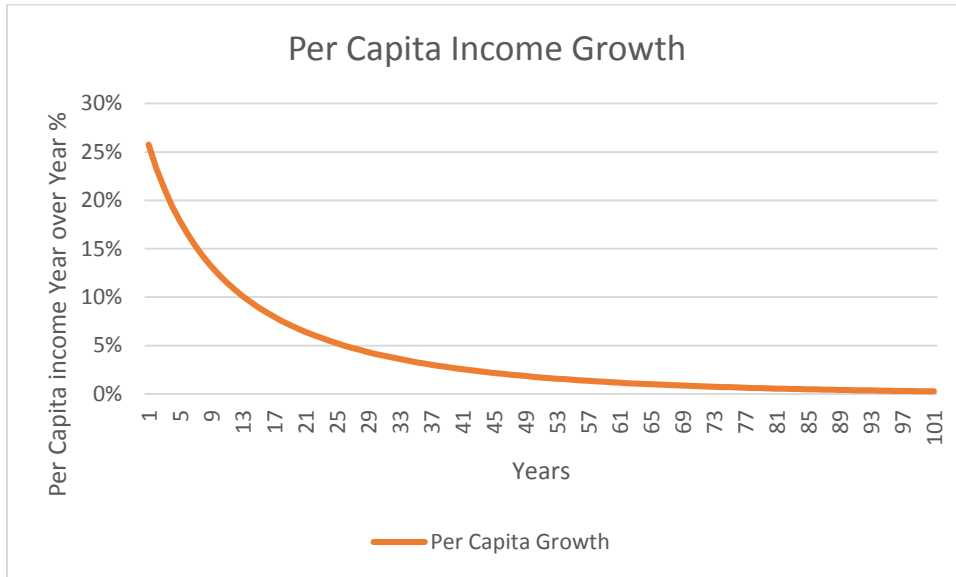*Figure 13*

4.4 Endogeneity of the Capital Output Ratio

The range of values that the capital output ratio takes to changes in per capita income has been shown in Figure 14. The capital output ratio becomes relatively insensitive to the higher levels of per capita income (as well as per capita capital as shown in Figure 12). Although the output per person continues to rise, due to a relative shortage of labor, the ratio of output to capital used in production falls.
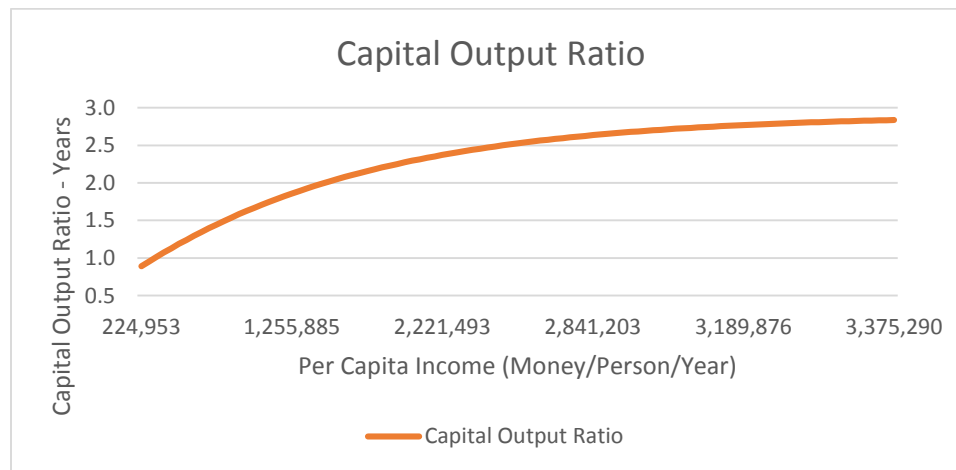
Change in Capital Output ratio to changes in per capita income
Figure 14

Growth in Solow's extension loses momentum if capital is unable to grow fast relative to labor. This is shown in Figure 15

Per Capita Growth in Solow's extension

Figure 15

4.5 Discussion

Solow's extension allowed for modeling the important concept of diminishing return to capital. System Dynamics tools added insights to the modeling process by incorporating the endogeneity of the capital output ratio to list of other endogenous variables. Solow's extension introduced additional new variables that include: labor participation, level of capital income to total income, level of technology.

Section 5

Discussion

Successive additions to the simple Harrod Domar, four variable world view show how a simple model extends itself beyond its original boundaries to incorporate additional perspectives. The Harrod Domar model demonstrates both the usefulness and limitations of modelling. Simple models are useful as they provide a clear and quick understanding based on limited number of important variables. But the limitations too become clearer as a simple model is validated against a range of inputs. Among others, equations supporting consumption & savings links between households and firms are missing in the original Harrod Domar model. Additional variables and changing nature of the variables (from being exogenous to endogenous), limits the usefulness of the original model.

Starting from the first Harrod Domar model (Figure 2), successive extensions incorporated increasing detail. The nature of variables changed from absolute to per capita. Interconnectedness among the variables provided a completely different dimension. System Dynamics tools provided insights by seamlessly incorporating the endogeneity of variables. As part of future work, it will be useful to test & tune the models in this paper to real inputs based on macroeconomic variables of different countries.

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