Modeling New Product Diffusion under Uncertainty

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Abstract

Diffusion of new, and innovative products in a potential market has been well studied in literature, using the famous Bass Model as the primary model. Recently a few works have also focussed on the supply planning and distribution strategies for new product introduction under uncertainty. This paper aims to further our understanding on the effect of supply and demand uncertainty on the overall market share.

1.0 Introduction

Bass diffusion model (Bass, 1969) is not new in marketing literature and perhaps it has been studied extensively since decades for predicting demands of new innovative products. Most of the literature until recently have assumed product diffusion under infinite supply (Mahajan et al., 1990; Peres et al., 2010) while in reality, the constraints on supply affect the diffusion dynamics significantly. There is few literature (Ho et al., 2002; Kumar & Swaminathan, 2003; Higuchi & Troutt, 2004; Shen et al., 2011; Negahban et al., 2014; Negahban & Smith, 2016) to study Bass diffusion model and supply chain model combined. Ho et al. (2002) have considered a more generalized Bass model, where the total population is decomposed into four classes viz. Potential customer, waiting in queue, adopter, and lost-sale and studied in the presence of supply constraint. Using optimal control theory, authors established closed form expressions for the demand and sales trajectories over the product lifecycle. They further concluded that the product launch can be delayed for building up an initial inventory stockpile but demand fulfillment should not be delayed when the firm is having inventory. Kumar & Swaminathan (2003) have modified the classic Bass diffusion model in order to capture the effect of supply constraint on future period demands. They have concluded that always selling the maximum of demand and inventory may not be optimal but rather the firm should wait to build up the optimal *initial inventory level* and then start selling in order to reduce lost sales. Higuchi & Troutt (2004) have shown the importance of integration of supply chain model with product diffusion model, taking a video game product of Bandai Co. as a case study. Bandai Co. had launched a video game viz.

Tamagotchi™ in 1996 which immediately gained popularity through word of mouth and exceeded the supply capacity. Realizing such huge demand, after some time delay when Bandai Co. increased their production capacity, people lost their interest in the product and resulted in huge unsold inventory leftover for Bandai Co. Negahban et al. (2014) have developed an agent-based simulation model for managing production level after launching a new product and found effect of various factors (such as social network structure, production strategy etc.) on production/inventory cost and lost sales. They have used a classical Bass model to forecast the actual demand of future periods and dynamically update the parameters of forecasting Bass model at every decision cycle, while the actual diffusion process was modelled as a variant of Bass diffusion model with multiple customer stages. Negahban et al. (2016) have studied the impact of demand and supply uncertainties on optimal production and sales plan for new innovative products using Monte Carlo simulation. Authors have used a model similar to modified Bass model, as proposed by Kumar & Swaminathan (2003), for simulating product diffusion. They have concluded that the optimal sales and production plan for deterministic setting may not remain optimal under uncertainty of demand and supply. However, the authors have used a relatively simple ordering rule, where any production lead time is not considered. The other stream of literature (Tan, 2002; Gupta & Maranas, 2003; Li et al., 2009) in supply chain management have studied the effect of demand and supply uncertainty on customer satisfaction, production cost etc. considering demand as exogenous to the system/model and thus not valid for new product diffusion.

2.0 The Bass Model

The classical Bass model is as shown in Figure 1. The target population (N) is divided into the population of potential adopters (P) and the population of adopters (A). The total adoption rate of the new product is then defined as the sum of the adoptions resulting from advertising (and any external means) and from word of mouth (and any implicit positive feedback driven by the adopters population). The underlying equations are given as follows:

$$Adoption\,Rate, AR(t) = \underbrace{pP(t)}_{Adoptions-Advertising} + \underbrace{qP(t)A(t)/N}_{Adoptions-word-of-mouth} \tag{1}$$

Where, p is the coefficient of innovation, q the coefficient of innovation. The term pP indicates the Adoptions from advertising and qPA/N indicates the adoptions from word of mouth. The population of adopters (A) and potential adopters (P) evolve as follows:

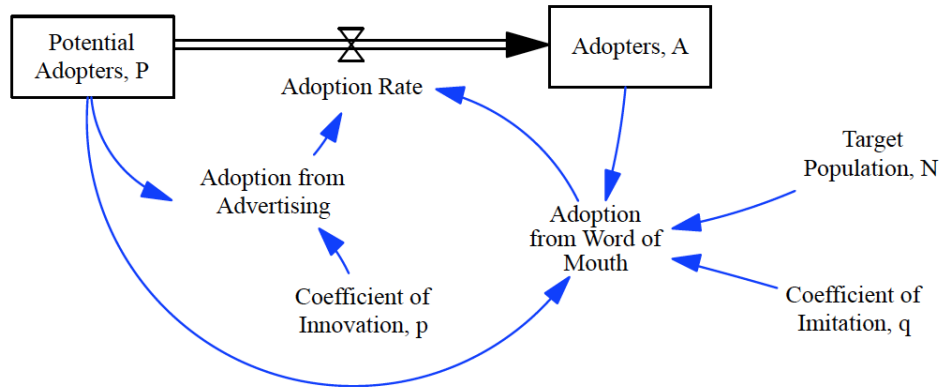$$P(t)=\int_0^t - AR(t) \quad \text{and} \quad A(t)=\int_0^t AR(t)$$

Figure 1: Stock-Flow representation of the Bass Model

The behavior of the model is characterized by the two loops. The marketing effect loop, which solves the problem of startup since the adoption from advertising is independent of the Adopter population (Sterman, 2001). The adoptions from the advertising display an accelerated delay, followed by an exponential decay. The word of mouth loop essentially drives the model to exhibit the classic S-shaped growth of Adopters. The adoptions from word of mouth grows rapidly, peaks, and then declines as the Adopters saturate the market. The steady state is reached when the entire target population become adopters, i.e., $A(t^*) = N$, or $P(t^*) = 0$. The total duration t^* can be termed as the duration of diffusion.

The above Bass model includes several assumptions:

- The target population (N) is fixed and does not change with time. This may be reasonable for diffusions happening at a small time scale. However, for models with diffusions spanning larger time scale, the effect of births, deaths, migration etc may need to be incorporated.
- The effect of word of mouth strictly results in a positive impact.

- The values of p and q remain unchanged with time.
- There is uniform mixing of the population, that is everyone can come into contact with anyone else.
- The price of the product has no effect on the size of potential adopters nor on the adoption rate.
- The adoption of the new product is considered to be independent of the adoption (or non-adoption) of other products, independent of social, economic and political conditions.
- Customer decision instantaneously changes from potential adopter to an adopter. Multi-stage models can be used to capture the evolution of customers through an 'awareness' stage and 'showing interest' stage before becoming adopters.
- There are no repeat or replacement purchases.
- Sufficient supply of new product inventory is available to meet the required adoption rate.
- Spatial or geographical spread of diffusion is not considered.

Many works have been presented in literature where the Bass model is extended by relaxing one or more of the above assumptions, as discussed earlier.

Interestingly, it is noted that for a given p and q, the behavior of the Bass model is independent of the population. That is, the fraction of adopters (A) to the target population (N) remains independent of N. This behavior is illustrated in Figure 2(a) and 2(b). The values of p and q are taken as 0.008 and 0.25. Fig. 2(a) shows the adoption rates when $N = 4,000$, 7,000 and 13,000. In Fig. 2(b), the fractional rate of adoption ($= AR(t)/ N$) is plotted. It is clearly observed that the model behavior is independent of N.

Adoption Rate

Adoption Rate : TargetPop=4000
Adoption Rate : TargetPop=13000
Adoption Rate : TargetPop=7000

fractional Rate of Adoption

fractional Rate of Adoption : TargetPop=4000
fractional Rate of Adoption : TargetPop=13000
fractional Rate of Adoption : TargetPop=7000
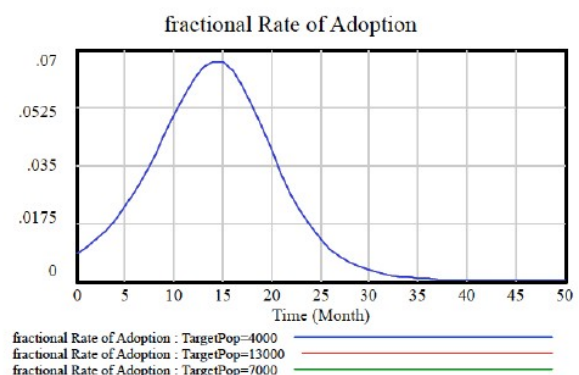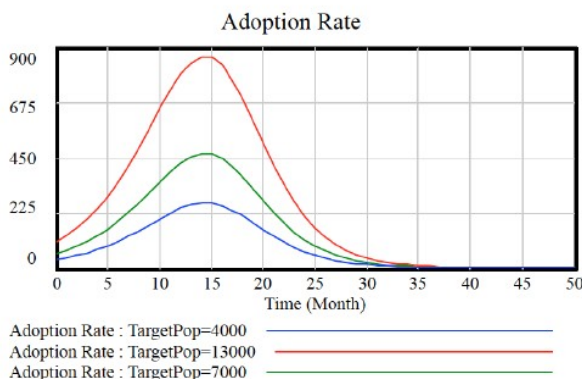
Figure 2: Dynamics with different values of target population N

Further, the model can also be expected to be robust to any demand variations. Since the populations are conserved (i.e., $A(t) = N - P(t)$), random perturbations in the adoption rates from its mean (as governed by the Bass model) will not affect the total diffusion duration t^*. Demand uncertainty can be incorporated in the model by including a perturbation term, in computing the Adoption Rate, where is a random variable with zero mean and non-zero variance, as:

$$AR(t) = pP(t) + qP(t)A(t)/N + \varepsilon \tag{2}$$

It is noted that to ensure the conservation of population ($A(t) = N - P(t)$) and prevent backward flow, it is to be ensured that the adoption rate does not exceed the current population of potential adopters, and is not negative. This is achieved by:

$$AR(t) = Min(Max(pP(t) + qP(t)A(t)/N + \varepsilon, 0), P) \tag{3}$$

Figure 3(a) and 3(b) show the dynamics of diffusion under demand uncertainty. The values of p, q and N are taken as 0.008, 0.25 and 4000. Figure 3(a) shows the adoption rates for different settings of . In Fig. 3(b), the adopters ($A(t)$) is plotted. It is clearly observed that demand uncertainty *per se* affects neither the total adopters nor the diffusion duration. These behaviors, while making the Bass model robust, also significantly overlooks the impact of supply on the spread of diffusions. This paper explores the impact of supply uncertainty on the Bass model dynamics.
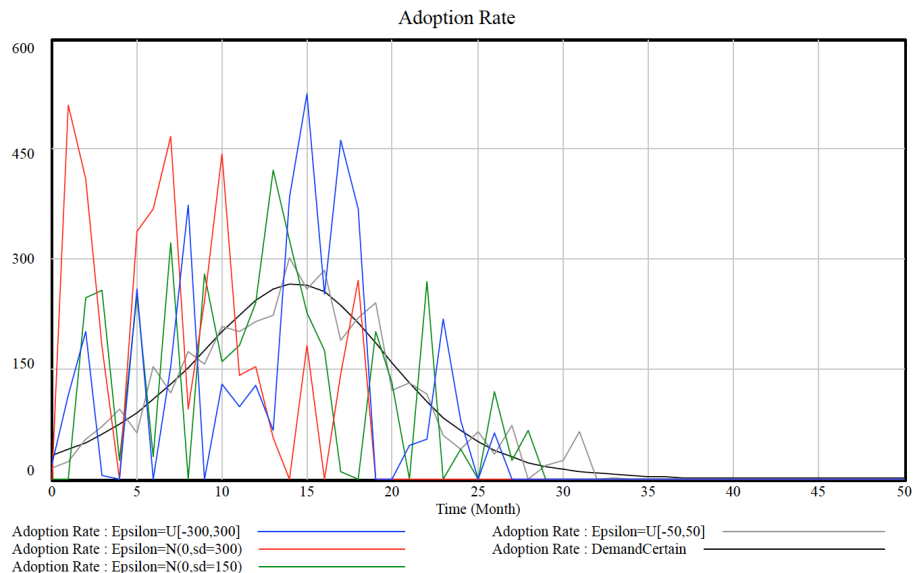

Adoption Rate

Adoption Rate : Epsilon=U[-300,300]
Adoption Rate : Epsilon=N(0,sd=300)
Adoption Rate : Epsilon=N(0,sd=150)
Adoption Rate : Epsilon=U[-50,50]
Adoption Rate : DemandCertain

Figure 3(a) Dynamics of adoption rate AR(t) under uncertain demand

Adopters, A

"Adopters, A" : Epsilon=U[-300,300]
"Adopters, A" : Epsilon=N(0,sd=300)
"Adopters, A" : Epsilon=N(0,sd=150)
"Adopters, A" : Epsilon=U[-50,50]
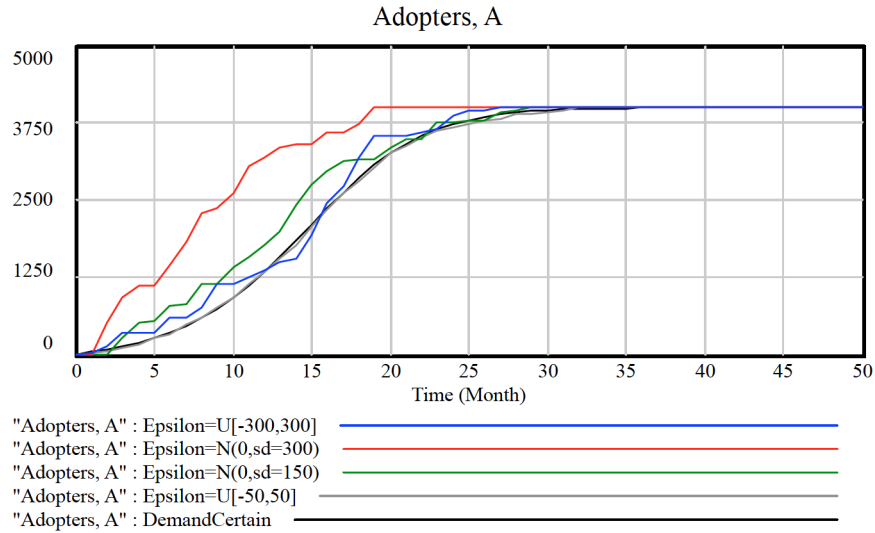"Adopters, A" : DemandCertain

Figure 3(b) Dynamics of Adopters A(t) under uncertain demand

3.0 Exploring and Extending the Bass Model

3.1 Constrained Supply

The effect of supply can be modeled as a co-flow, as shown in Figure 4. A stock of inventory is maintained, and the adoption rate can then be set to not exceed the available inventory on hand, $I(t)$. The underlying equations of the extended model are as follows:
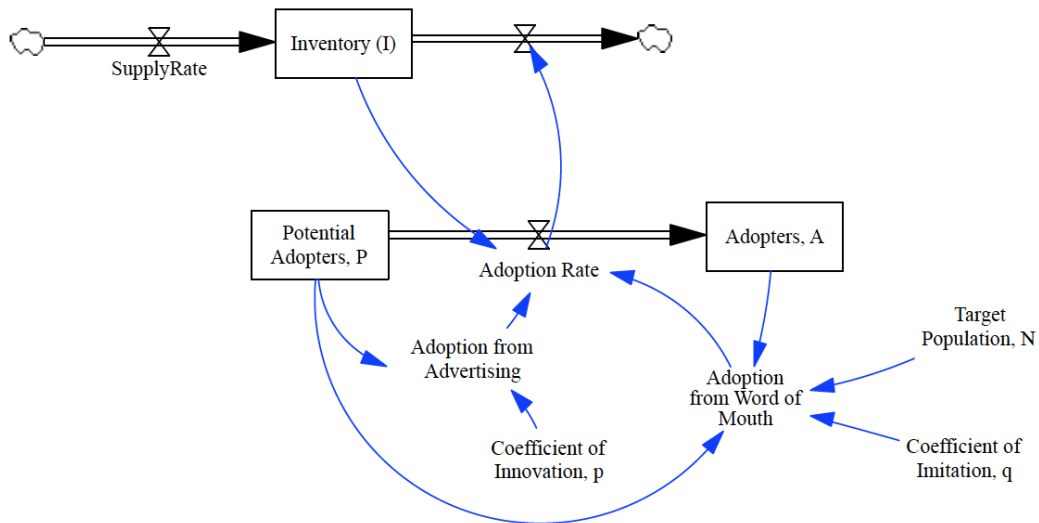
Figure 4: SFD model of extended Bass model with explicit modeling of supply

$$AR(t) = Min(\underbrace{pP(t) + qP(t)A(t)/N}_{Desired-Adoption-Rate,\,DAR(t)}, I(t)) \tag{4}$$

$$I(t) = SupplyRate(t) - AR(t) \tag{5}$$

Now, suppose the cumulative supply is more than the target population, and all potential adopters wait as-long-as-required for the product, then the dynamics are quite intuitive. In periods when the on-hand inventory exceeds the adoption rate, the dynamics will be comparable to the original Bass model. In periods when the adoption rate is constrained by the inventory ($I(t) > 0$), the system will exhibit a linear growth in adopters. Eventually, the target population of adopters will be reached, with the delay in diffusion proportional to the supply delays, i.e., the number of days inventory was insufficient to meet desired demand. Figure 5 illustrates the behavior of the system for varied patterns of supply (constant rate of supply and batch supply). When the supply is at a constant rate of 100 units/ period, the adoption rate in the first 10 periods equals the desired adoption rate. The accumulation of excess inventory in periods 1 to 5 provides to meet the demand in period 6 to 10. In periods 11 to 35, the inventory availability determines the adoption rate, and after 36 time periods, the adoption follows an exponential decay, reaching a saturation of 4000 in 50 periods. The total duration of diffusion t^* has increased when compared to the scenario of non-delayed supply. Suppose the products are supplied in a batch size of 1000 at time periods 0, 10, 20 and 30, the dynamics will be similar to the red color lines in the graphs in Fig. 5. The cumulative adopters reach 2000 (exhausting the first two deliveries) in period 15 which results in a plateau ($AR=0$) until the next lot arrives. This alternate sales and periods of idleness continue until the target is reached. It can be observed that the increase in the duration of diffusion t^* is approximately equal to the number of periods when INV is less than the desired adoption rate (in the example considered, the t^* increases by about 11 periods). Also, of the total 50 periods of diffusion, in 10 periods (i.e. 20%) the distribution does not take place (i.e., on days of zero AR(t)). This can have a serious impact on the motivation of future performance of the sales workforce, which is not considered in the current model.
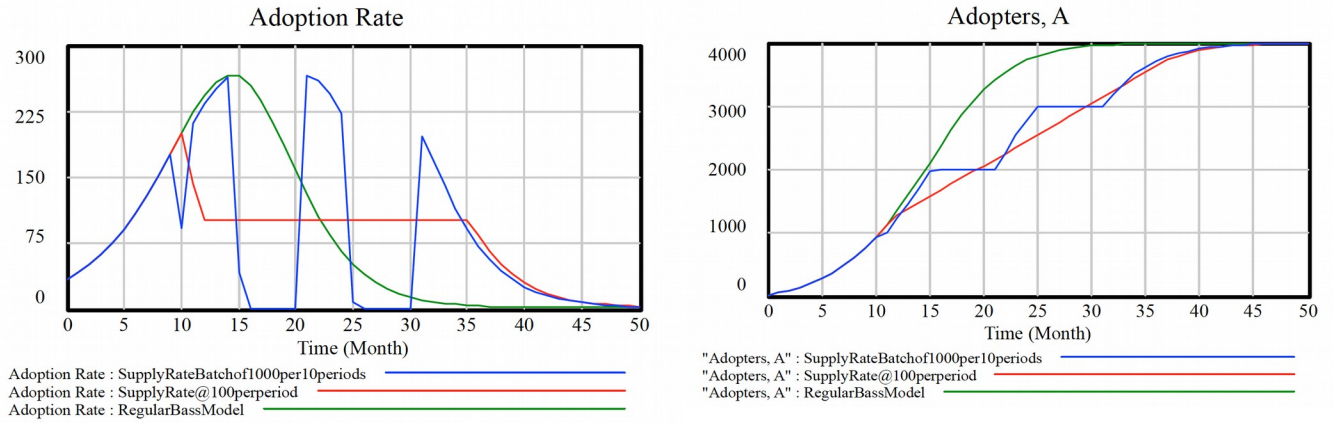
Figure 5: AR(t) and A(t) under different supply scenarios

3.2 Constrained Supply with abandonment

It will be interesting to next understand the dynamics when the potential adopters are impatient. That is, unavailability of the product (say, due to supply delay) creates a negative impression resulting in some proportion of potential unsatisfied customers abandoning the future purchase of the product (they might have purchased another equivalent product from the market or simply lost interest due to delay). This aspect is captured by including another outflow to the stock of Potential Adopters, called as Abandon Rate ($AbR(t)$). AbR(t) is a product of the abandonment fraction f and the instantaneous proportion of dissatisfied customers. Also, the total target population N is reduced by AbR in order to conserve the population. The extended SFD model that includes abandonment is shown in Figure 6.
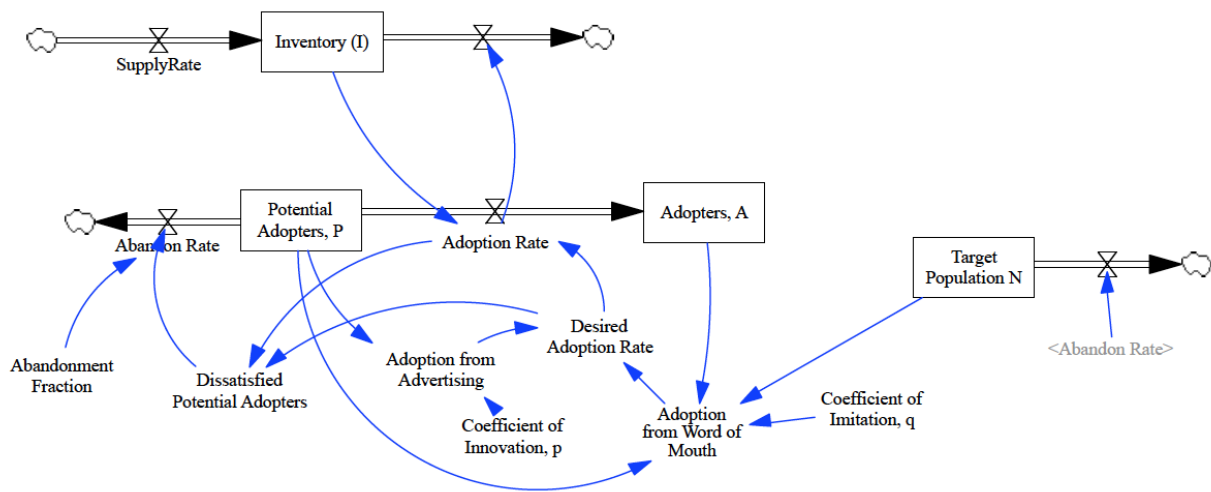

Figure 6: SFD model of extended Bass model with explicit modeling of supply & abandonment

The additional underlying equations are as follows:

$$P(t) = \int_0^t - AR(t) - AbR(t) \tag{6}$$

$$AbR(t) = f*(DAR(t) - AR(t)) = max(pP(t) + qP(t)A(t)/N(t) - I(t), 0) \tag{7}$$

$$N(t) = \int_0^t - AbR(t) \tag{8}$$

Simulation results comparing the performance with and without abandonment are presented in Figure 7. The values of *p*, *q*, *f*, and *N* are taken as 0.008, 0.25, 0.25 and 4000. Two different supply patterns are considered. When the supply is at a constant rate of 100 units/ period, it can be seen that the total adopters saturate (reach) 3550 with 450 abandoning the purchase of the product (see Figure 7(a)). However, from the manufacturer's point of view the sales rate (adoption rate) is actually quite stable until period 30 after which it decays exponentially, and saturating at 3550 adopters. Suppose the products are supplied in a batch size of 1000 at time periods 0, 10, 20 and 30 (see Figure 7(b)). The sales (adoption) first and the second batch of 1000 are identical under with and without abandonment. The third batch of 1000 is also sold (adopted) but over a longer duration (5 periods instead of 4 periods). However, the last batch of 1000, when supplied, never gets fully sold. Also, interestingly, both supply patterns results in almost the number of adopters post-abandonment. The key implication of the above results is that the manufacturers will not be aware of the potential decline in adoptions until very late, resulting in unspent inventory, additional costs and loss of market, since abandonment rates are quite difficult to estimate.

Adoption Rate

Adoption Rate : SupplyRate100perPeriodwithAbandon
Adoption Rate : RegularBassModel
Adoption Rate : SupplyRate@100perperiod

Adopters, A

"Adopters, A" : SupplyRate100perPeriodwithAbandon
"Adopters, A" : RegularBassModel
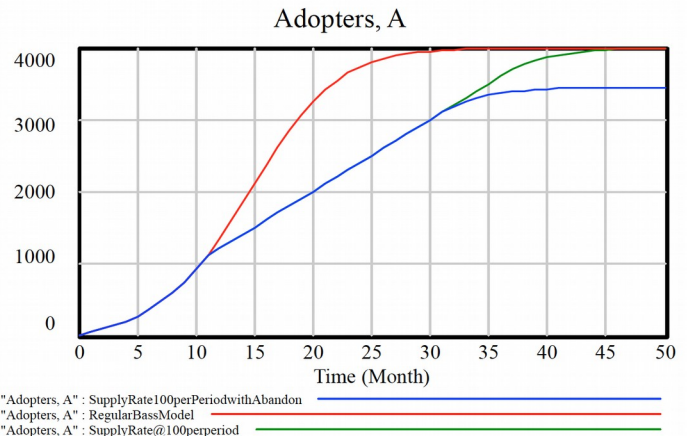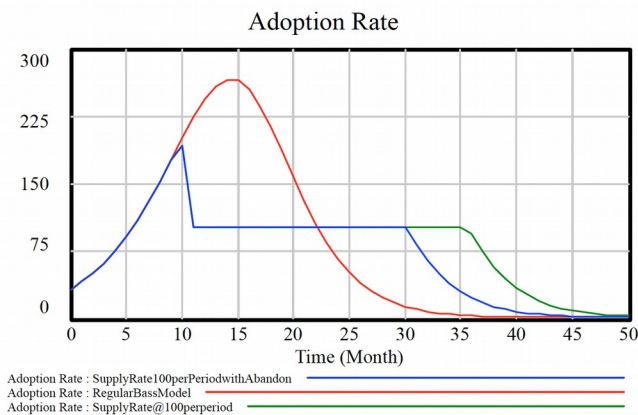"Adopters, A" : SupplyRate@100perperiod

Figure 7(a): AR(t) and A(t) when supply is a constant at 100 units per period and abandonment
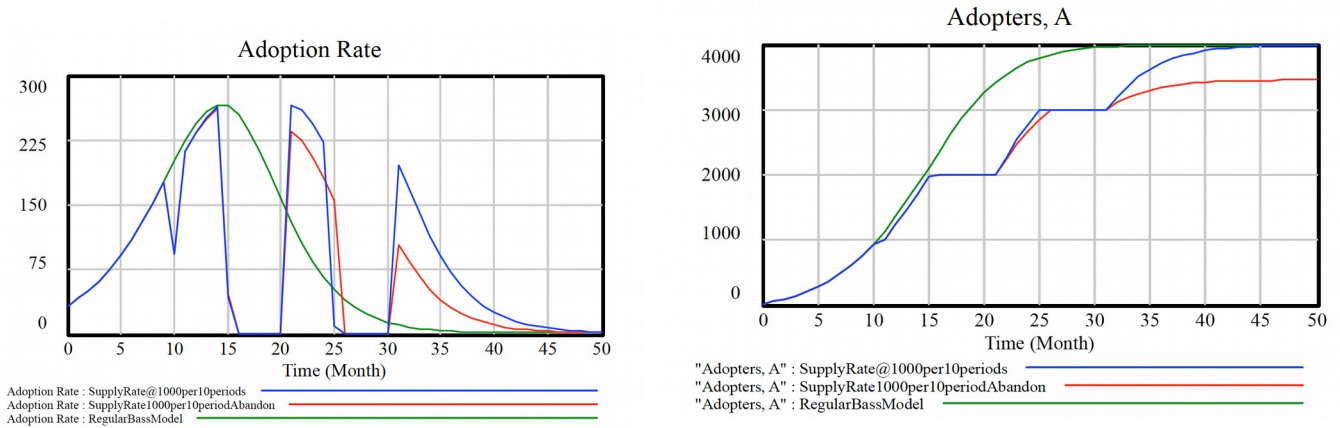

Figure 7(b): AR(t) and A(t) under batch supply of 1000 units per 10 periods and abandonment

3.3 Implications of a grand product launch

Most new products are launched in a 'grand' manner to help kick start the adoption of the product. This refers to the initial quantity of products sold (adopted) by highly concentrated efforts aimed to create awareness among the target population. However, post this, the manufacturer may choose to significantly reduce the efforts on advertising and rely more on the word of mouth effects generated by the initial set of adoptions. This is captured in the original Bass model by modifying the coefficient of innovation parameter p, such that p takes a high initial value (modeled as a PULSE() input) followed by the significantly lower value. Simulation experiments are conducted to quantify the effects of the grand product launch (p=0.008 in all periods except in period 1 when p = 0.05), under limited supply and with abandonment. The results of these experiments are presented in Figure 8.

Figure 8(a) displays the adoption rates under grand launch with infinite supply. It is observed, as expected, that there is a sharp increase in sales (~230 units) in the period since more products are adopted in the early stages, which is further reinforced by the word-of-mouth effects resulting in the overall reduction of the duration of diffusion by 3 periods. Figure 8(b) summarizes the results under a constant supply of 100 units/ period (with an initial stock of 100). Figure 8(c) summarizes the results when products are supplied in a batch size of 1000 at time periods 0, 10, 20 and 30. The total adopters for both supply patterns are lesser when the big launch is carried

out as compared with the case without a big launch. Also, it is seen that the batch supplied saturated at a lower level of adopters (~3355) as compared when to constant supply (~3370).
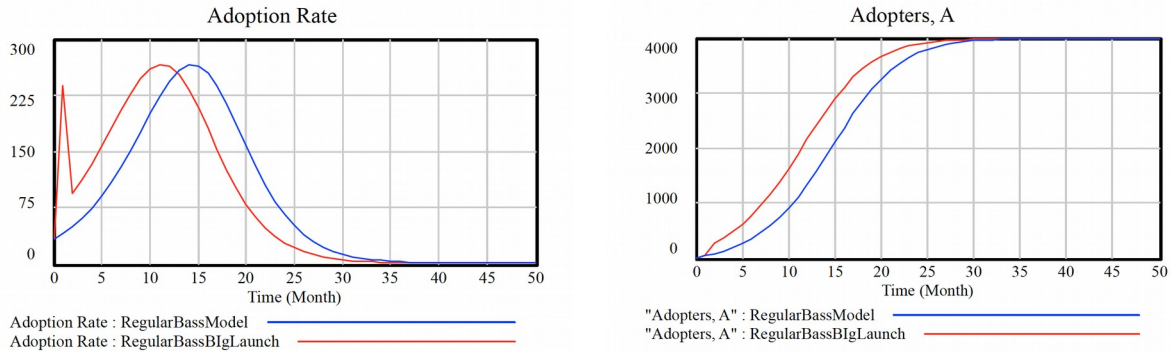

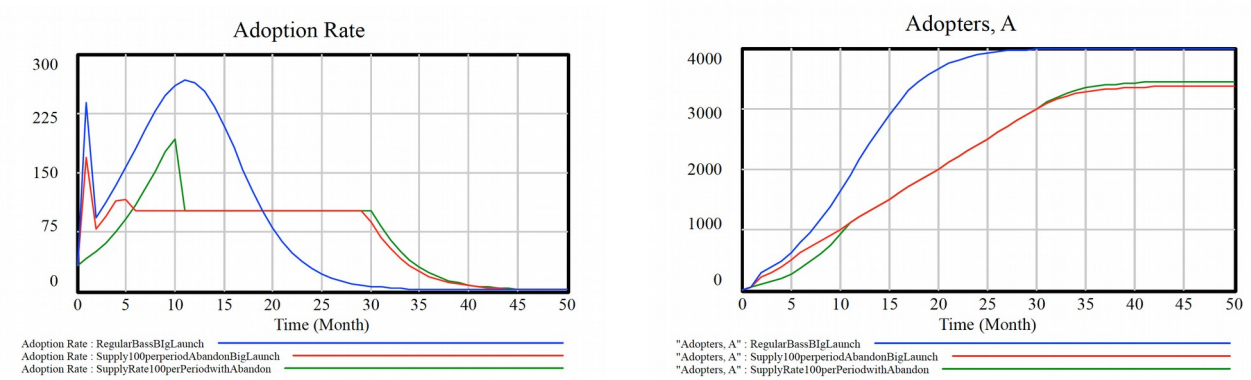Figure 8(a): Result of Bass model with and without a grand launch

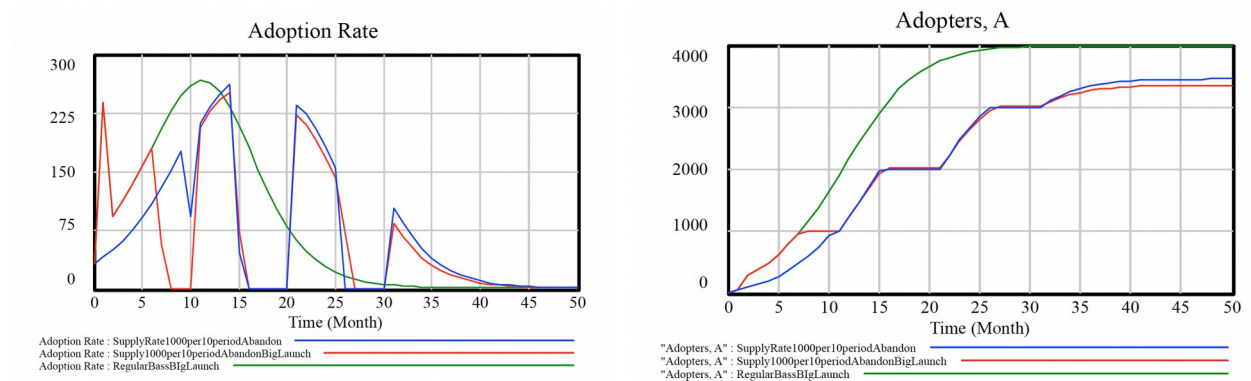Figure 8(b): Result of extended Bass model under constant supply, abandonment, grand launch

Figure 8(c): Result of extended Bass model under batch supply, abandonment and grand launch

4. Conclusions and Future Work

Simple extensions of the Bass model has been discussed in this paper to help further the understanding of the uncertainties in demand and supply on the new product diffusion dynamics. Batched supplies can cause significant periods of idleness (adoption rate equals 0) leading to loss of motivation. Suppose customers decide to abandon their purchases, then the total target population will never be reached, and more importantly, the manufacturer will not be aware of this until the very end of the diffusion period. Also, it is observed that grand launches of products, if not followed up with regular supply will result in an overall decrease in adoptions.

On going research work is being carried out to understand, model and analyse the diffusion of a new product at multiple unconnected geographical regions, but which are constrained by a common supplier of materials. Also, explicit modeling of the impact of idle periods (periods with adoption rates) on the motivation of the workers can be investigated. Further, given the (real life) adoption rates of products which includes periods of idleness (either due to supply uncertainty or demand uncertainty) the estimation of the coefficient of innovation and imitations, along with the reasons for supply delays are of interest. These investigations will help in designing a robust plan for diffusion of new products under uncertainty.

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