Better Robustly Right than Accurately Wrong

Erik Pruyt[1], Willem L. Auping, Jan H. Kwakkel, Wil A.H. Thissen

Delft University of Technology, Delft, NL

Abstract: *This paper presents a System Dynamics approach for dealing with complex issues that are characterized by deep uncertainty. Deep uncertainty refers to situations in which experts disagree on the formulation of 'the' underlying model, probabilities of inputs, and the valuation of outcomes. Instead of waiting for full information and accurate data to become available, consensus to be reached, or irrefutable scientific proof to be established, one could address such issues with approaches that enable one to simultaneously take alternative theories/models, sets of possible functions, different distributions, and distinct valuation frameworks into account. Exploratory Modelling and Analysis is such an approach: it allows for simulating sets of alternative models across vast uncertainty spaces, generating multi-model ensemble forecasts, exploring the resulting ensembles of outcomes using all sorts of machine learning techniques, identifying and selecting exemplar scenarios, performing directed searches to answer specific questions, and optimizing the robustness of potential adaptive policies that are designed to always work, especially when really needed. After introducing the approach, it will be illustrated in this overview paper with different applications for each of these typical use cases.*

Keywords: Ensemble Forecasting, Model-Based Policy Analysis, Exploratory Modelling and Analysis, EMA, Robust Decision Making, RDM

1. WHY? INTRODUCTION

Many policy issues and systems –like the economic or socio-demographic system– are not only complex, they are also characterized by many uncertainties. Their (long term) future are mostly even more uncertain, as are the possible impacts of new policies, especially of radically new policies. Although it is often possible to accurately project the short term future of complex and deeply uncertain systems/issues, it is mostly impossible to predict accurately their long term future. The divergence between projected evolutions and real world evolutions tends to be larger for higher degrees of uncertainty and complexity, and for longer time horizons. This does not mean, however, that modelling and simulating such systems/issues is not possible and that projections are not useful. To the contrary. It is often possible to develop models for severely uncertain issues/systems. Although the intended aim, expected insights, and the approach may have to change. In such situations, it may be useful to fully embrace uncertainty and adopt an exploratory approach instead of a traditional consolidative approach aimed at accuracy.

[1] Corresponding author: dr. Erik Pruyt e.pruyt@tudelft.nl

There are many reasons for embracing uncertainties in modelling and simulation: (i) it enables one to identify potential problems, (ii) it enables one to make better –albeit more uncertain– estimates, and (iii) it results in more robust decisions/policies. Reasons for not embracing uncertainties in modelling and simulation may be (i) that the analysis could become more challenging, and (ii) that it may be more difficult to communicate results and conclusions.

2. WHAT? Uncertainty, EMA, ESDMA, RDM, Scenario Discovery,…

2.1 Uncertainty

Uncertainty itself is well-known but also a rather ambiguous concept: there is no consensus about what precisely should be understood by "uncertainty". When asked to list uncertainties, most people list issue-related or contextual uncertainties, such as the uncertainty related to future oil prices. Apart from these application-related contextual uncertainties, there are other locations where uncertainties manifest themselves in model-based decision support – for example in methods, models, inputs and outputs. Uncertainty is also a multi-dimensional concept. Many authors distinguish the nature, level and location of uncertainties in model-based decision support. Most SD work is situated, in terms of the level of uncertainty, up to a level often referred to as "deep uncertainty".

Marginal Uncertainty -> Statistical Uncertainty -> Deep Uncertainty -> Recognized Ignorance -> Total Ignorance

Figure 1: Levels of Uncertainty

Deep uncertainty could be defined as the uncertainty pertaining to situations in which experts do not agree with regard to the underlying model, probabilities, and evaluations of the outcomes of models (Lempert *et al*., 2003). Note that this definition of deep uncertainty can be translated directly to the nature of uncertainty. Note also that this definition implies that, under deep uncertainty, model-based approaches could still useful, although in a different way than traditional model-based approaches. Under deep uncertainty, it is still possible to construct sets of plausible models [in plural(!)], use these models for generating plausible ensembles of plausible scenarios which could be explored and used for policy design under uncertainty.

2.2 Exploratory Modeling and Analysis[2]

Various scientific fields including the environmental sciences, transportation research, economics, and the political sciences, are involved in providing model-based decision support. In these various fields,

[2] This subsection is largely based on Kwakkel and Pruyt, (2015).

people are grappling with the treatment of deep and irreducible uncertainty while using models. A common theme across these fields appears to be a shift away from predictive model use towards more explorative model use. Exploratory Modeling and Analysis (EMA) is a research methodology that uses computational experiments to analyse complex and uncertain systems (Bankes, 1993; Lempert et al. 2003; Agusdinata, 2008).

EMA is especially useful when relevant information exists that can be exploited by building models, but where this information does not allow specifying a single model that accurately describes system behaviour. In such situations, models can be constructed that are consistent with the available information, but such models are not unique. The available information is consistent with a potentially infinite set of plausible models, whose implications for potential decisions may be quite diverse. A single model run drawn from this set provides a computational experiment that reveals how the world would behave if the various guesses this single model makes about the various irreducible uncertainties are correct. By conducting many such computational experiments, one can explore the implications of the various guesses. EMA is the explicit representation of the set of plausible models, the process of exploiting the information contained in such a set through a large number of computational experiments, and the analysis of the results of these experiments.

EMA is not focused narrowly on optimizing a (complex) system to accomplish a particular goal or answer a specific question, but can be used to address 'beyond what if' questions, such as "Under what circumstances would this policy do well? Under what circumstances would it fail?", and "what is the range of plausible future dynamic developments of a phenomenon of interest? Under what circumstances can we expect which dynamic developments?" Given this focus, EMA stimulates out of the box thinking and can support the development of adaptive plans/policies (Hamarat *et al*. 2013, 2014).

EMA is first and foremost an alternative way of using available models, knowledge, data, and information. In making policy or planning decisions about complex and uncertain problems, EMA can provide new knowledge, even where strict model validation is impossible. For example, EMA can be used for existence proofs or hypothesis generation, by identifying models that generate atypical or counterintuitive behaviour. Knowing that a system can exhibit such behaviour can change the debate or open up new directions for the design of targeted solutions. Another example is the case where there is ample data available, but also disagreement or uncertainty about which data to use. EMA can be used to identify the extent to which the choice of data influences the model outcomes and preferred ranking of policy options. Instead of debating the choice of the right data, the debate can then shift to the development of policies or plans that produce satisfying results across alternative sets of data/assumptions. Other possible uses of EMA include the identification of extreme cases, both positive and negative, in order to get insight into the bandwidth of expected outcomes, and the identification of conditions under which significant shifts in performance can be expected. All these examples rely on the fact that policy or planning debates can often be served even by the discovery of thresholds, boundaries, or envelopes that decompose the entire space of uncertainties into sub-spaces with different properties. That is, partial information can inform policymaking or planning even when prediction and optimization are not possible by using the available partial information in a systematic and transparent way. Many well-established techniques, such as sampling approaches and new types of optimization techniques can be usefully and successfully employed in the context of EMA.

In this paper, we argue that by using models differently, the challenges associated with decision-making under deep uncertainty can largely be overcome. Instead of trying to predict accurately, the models are used to explore what could happen and what policies would hold across various uncertainties. In this way, decision-making can proceed despite the presence of deep uncertainty, for decisions can be designed to be robust across the explored range of possible futures. Flavors and Names: EMA, Robust Decision-Making, Scenario Discovery, E***SD***MA,…

EMA, Scenario Discovery, Robust Decision-Making, Adaptive Robust Design, E***SD***MA refer to different strands of the Exploratory Modelling and Analysis methodology. Different groups (RAND, TUDelft, Worldbank,…) use these different labels to refer to variants of the same general approach, which are used for different aims or to refer to the use of the EMA methodology within a particular modelling field. For example, Scenario Discovery refers to exploratory approaches to identify and select exemplar scenario from ensembles of scenarios, Robust Decision-Making refers to exploratory approaches to iteratively make robust decisions, and Adaptive Robust Design refers to design adaptive robust policies,…). And ESDMA refers to the successful combination of EMA and System Dynamics modelling and simulation (Forrester, 1961; Sterman, 2000; Pruyt, 2013), which is a modelling method for simulating dynamically complex issues at the systems level. In the case, the label is used to distinguish the exploratory SD approach from the consolidative SD approach. This paper focusses particularly on ESDMA. Note, however, that it is not the first paper on ESDMA: earlier writings about ESDMA include Kwakkel & Pruyt (2013,2015), Pruyt and Kwakkel (2014), Pruyt *et al*. (2015), as well as some 50 proceedings articles. The current paper provides a more comprehensive and illustrative overview of ESDMA in terms of use cases in health policy though.

2.3 Use Cases and Steps

As indicated above, there are different use cases for EMA with distinct labels. These use cases also apply to the combination of EMA and SD. In practice, these use cases are often applied one after the other – in a logical order. First, ‘certainties’ and ‘uncertainties’ are identified in order to build sets of plausible models that are subsequently simulated. The resulting ensembles could be used as ensemble forecasts. Second, these ensembles are explored and analyzed using all sorts of algorithms (clustering, PRIM, …) during an open exploration phase. Third, interesting clusters of scenarios are identified and representative exemplars are selected. Fourth, direct searches are performed to find answers to specific questions. And fifth, policies are designed, tested and robustly optimized across the uncertainty space. The corresponding use cases are: (i) ensemble forecasting, (ii) open exploration, validation and analysis, (iii) scenario discovery, (iv) directed searches, and (v) robust decision making and/or adaptive robust policy design. These use cases are illustrated with recent cases in section 4 of this overview paper.

3. HOW? METHODOLOGY: EMA with/out the EMA Workbench

Figure 2 displays the EMA process. Different types of computational models and sampling techniques are used in EMA to generate ensembles of scenarios. Although individual scenarios could be studied, ensembles of scenarios are mainly focused on. These ensembles are subsequently explored, analyzed, and searched using all sorts of data science techniques. Different visualization techniques are used too. The exploratory process itself is quite iterative, and specific techniques like adaptive sampling and robust optimization require many iterations too.

Although it is possible to perform EMA with separate software packages, it is more convenient to use open source scripting suites that allow to perform EMA (i.e., generate, store, explore, analyze, and optimize ensembles) and to develop new algorithms to be used for EMA. TUDelft's EMA Workbench[3] is one such open source suite programmed in Python (van Rossum, 1995).

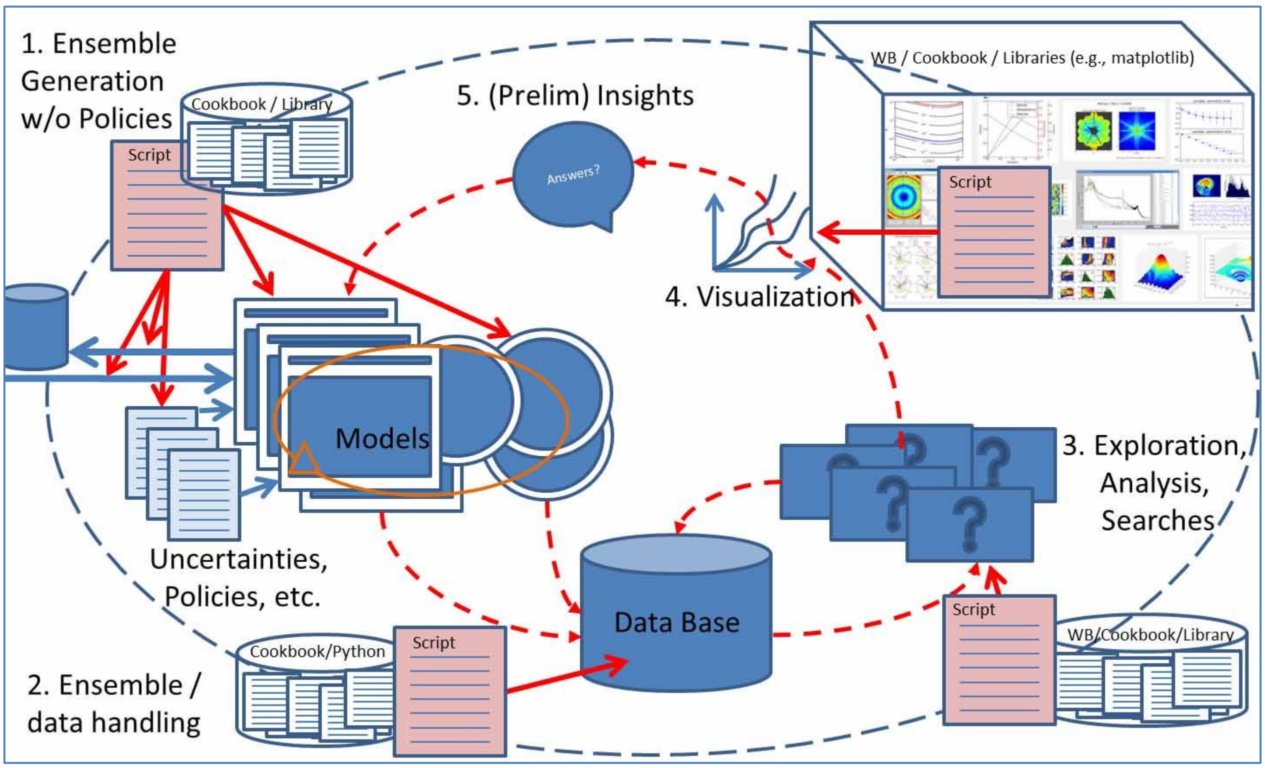


Figure 2: Exploratory Modelling and Analysis w/o the EMA Workbench

The EMA Workbench enables one to perform all steps with the same scripting language and in the same package or the same interface. And new scripts can be developed with the same language. A Cookbook with many snippets of scripts is currently being written. Only the models are still developed with modelling software packages (although python versions are currently being developed too). These models are different from traditional models in that they contain certainties *and* all sorts of *uncertainties*

[3] For the EMA workbench, see http://simulation.tbm.tudelft.nl

(different structures, functions, inputs, *et cetera*). A variety of sampling techniques is used for a variety of purposes (e.g., generating the largest variety of outcomes versus covering the input space (Islam and Pruyt, 2014)). And different types of exploration, analysis, and search algorithms are used, including algorithms to assess the importance of uncertainties or the joint causes of particular types of outcomes like undesirable futures, to cluster scenarios, to reduce the dimensionality, to search for worst possible outcomes, to robustly optimize policies on multiple dimensions, *et cetera*[4]. And large open source visualization libraries are available. These explorations, analyses and visualizations can be documented with IPython Notebooks (see below).

4. WHAT FOR? ILLUSTRATIONS OF TYPICAL USE CASES

4.1 **Ensemble Forecasting**: Ebola in West Africa

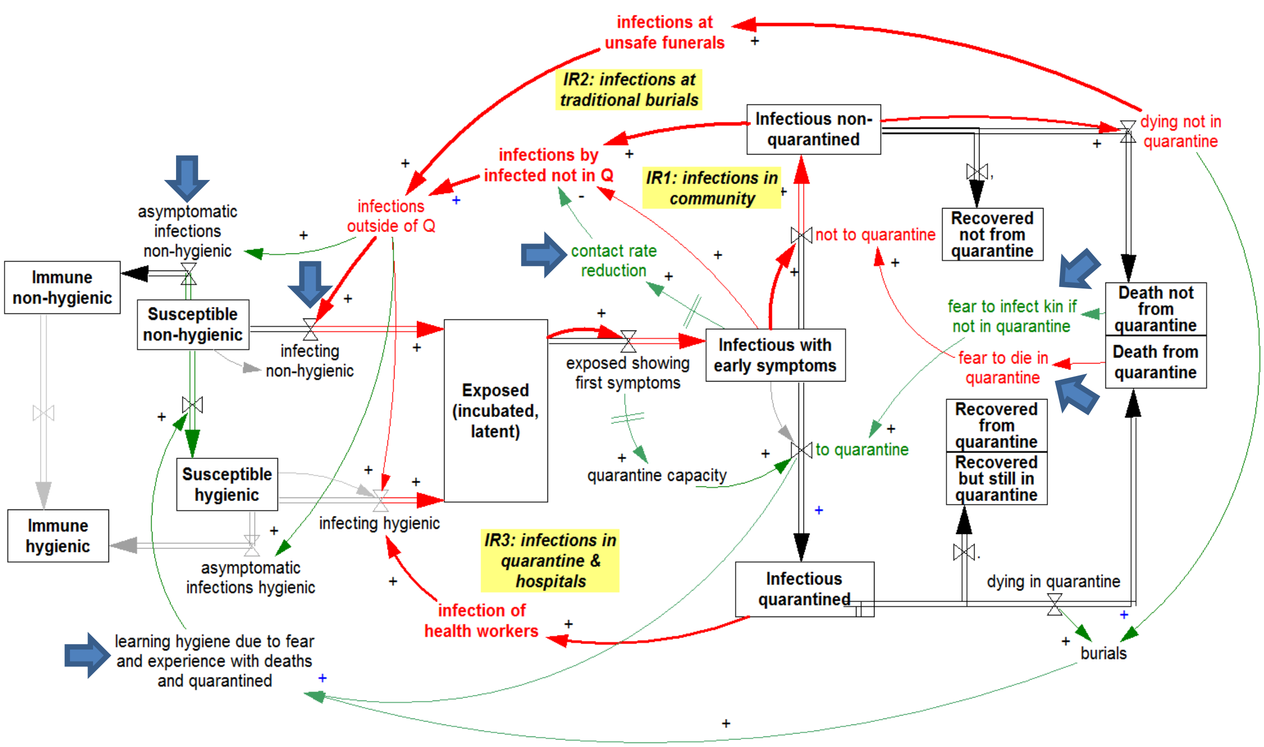


Figure 3: A suite of Ebola models with endogenous social-psychological relations

Figure 3 shows the core of a set of System Dynamics simulation models related to the recent Ebola in West Africa. Much of the data and information related to the outbreak was deeply uncertain. A multi-

[4] A more detailed discussion will be added as well as proper references to Breiman (2001), Fisher (1987), Kohavi and John (1997), McKay et al. (2000), Petitjean et al (2011), Rakthanmanon (2013), van der Maaten and Hinton (2008), Yucel (2012), and Yucel and Barlas (2011.

method EMA approach was therefore adopted to generate ensemble forecasts and test policies. We used switches (blue arrows in Figure 3) to simulate different versions of the core simulation model, sampled across a large uncertainty space, and used a soft calibration approach (see Pruyt et al. 2015).

Doing so resulted in an ensemble prediction by 4 November 2014 of between 17 thousand and 40 thousand cases by 31 December 2014 and between 18 thousand and 50 thousand cases by July 2015 (see Figure 4). Crucial for obtaining this ensemble forecast were the inclusion of uncertain endogenous social-psychological relations, of uncertain massive deployment policies, of surprises like superspreading events, and of a switch in social-psychological relations from adverse to ‘normal’. Figure 4 also shows that the ensemble forecast significantly improved between 28September and 4 November simply due to the availability of more and better real-world information.

The key take-away was that massive deployment would allow to curb the outbreak by January 2015 (and hence, that massive vaccination campaigns would not be needed). Key methodological takeaways are that embracing uncertainty leads to fewer surprises, and that including endogenous change and surprises leads to less uncertainty.

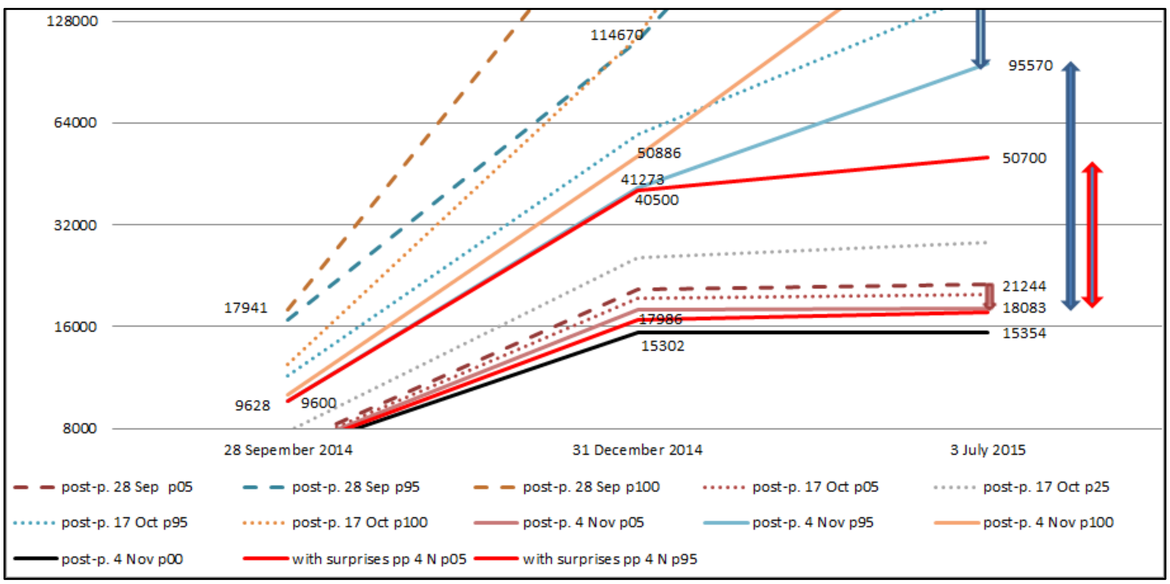


Figure 4: Ensemble forecasts of the Ebola Outbreak on 28 September, 17 October, 4 November 2014

4.2 **Open Exploration and Analysis**: Radicalization under Deep Uncertainty

Open exploration is one of the most important uses of EMA and ESDMA. Note, however, that the open exploration process itself is hardly ever reported on in academic papers. An exception is the paper by Pruyt and Logtens (2015): the appendix of that paper comprises a structured account of the systematic exploration process. Mostly, however, “outcomes” are reported on in journals, not the exploration process itself. For example, although the explorative process was the essence of the SDR paper by Pruyt and Kwakkel (2014), the authors had to rewrite the paper such that hardly anything was left of the explorative process.

A polished account of that particular explorative process is nevertheless available as a supplementary online notebook (see: http://nbviewer.ipython.org/gist/anonymous/4669f83d27d3e51b23c1). Note that the published paper only refers to one explorative technique (PRIM), while initially[5], many analytical and exploratory methods and techniques were used.

Records of explorative and analytical processes may actually be key to understanding the nature and value of explorative model-based research. Transparency and reproducibility of exploratory research then requires researchers to share models, but also data sets, scripts, and their notebooks. Publishing unpolished notebooks may in fact be a crucial addition to reporting in modelling – whether the model-based study is explorative or not.

Note that the Ebola work referred to in the previous section was published with a link to the online notebook (see http://nbviewer.ipython.org/gist/ep77/4b836f916dde743e96bf). The additional work reported on here (superspreading and shifting behaviours) are explored in another notebook (see http://nbviewer.ipython.org/gist/anonymous/c2aed2745e373a272164).

4.3 **Scenario Identification and Selection**: Shale Gas, Energy Prices, and State Stability

Scenario identification and selection is a third important use case. A good example of this particular use of EMA or ESDMA is the shale gas study by Auping et al. (2014), de Jong *et al*. (2014), and Moorlag et al. (2015). They used a first SD model to identify and select distinct future energy price scenarios. These scenarios are subsequently used in a second exploratory SD model related to state stability in order to assess the stability of important fossil fuel exporting countries when confronted with different energy price scenarios. Figure 7 visualizes the multi-model systems-of-systems architecture of this study.

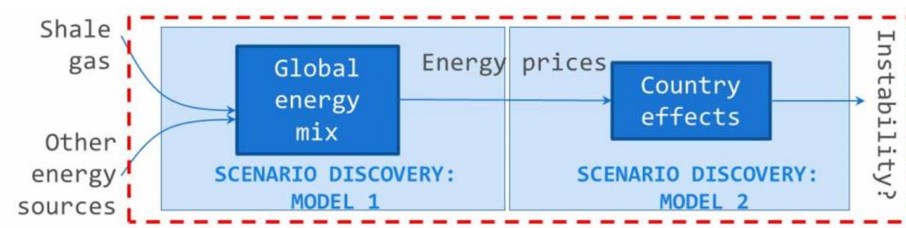


Figure 7: Scenario generation and selection for a Systems of Systems SD study

Figure 8 displays the energy price scenarios selected from the ensemble of runs generated with the first SD model that were subsequently used in country-specific versions of the second SD model.

Simulation of these scenarios under uncertainty in the one of the country-specific versions of the second SD model show or example that gradually decreasing price scenarios as well as decreasing scenarios with oscillatory patterns like scenarios 1, 2, 4, 6, 7, 8, 13, and 14 are mostly undesirable from a Russian internal stability point of view for 100% of the simulations.

[5] See http://collegerama.tudelft.nl/Mediasite/Play/ef092954b8054b068ecf9fa40115d1ca1d

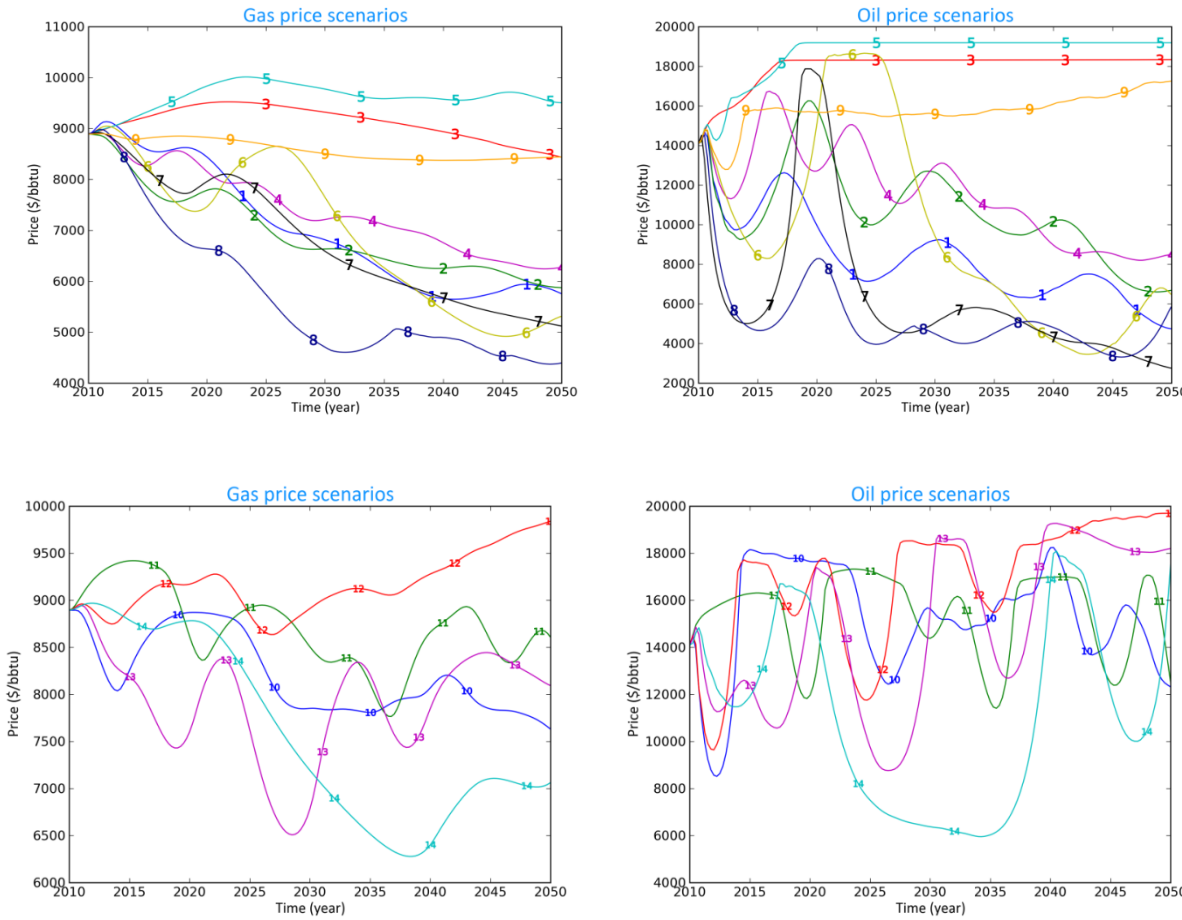
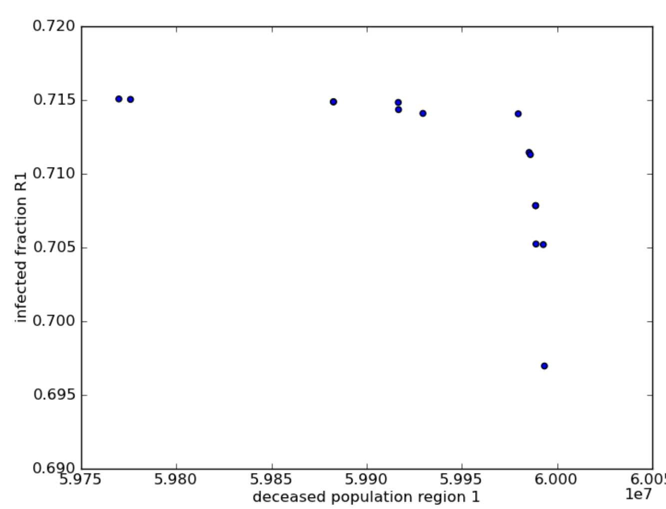


Figure 8: Selected exemplar scenarios generated with the Global Energy Mix model

4.4 **Optimization and Direct Searching**: The Worst Pandemic Flu

Direct searching under deep uncertainty is another typical EMA use case, largely inspired by Miller (1998). Optimization techniques are typically used to provide answers to particular questions or generate (ensembles of) simulation runs with particular characteristics, such as the set of worst pandemic flu scenario (see Pruyt et al. (2010, 2013)).



4.5 **Robust Policy Design**: Societal Aging in the Netherlands

The most important use case relates to Policy Design. An interesting example, at least from a policy analysis perspective, is a study related to the potential consequences of societal aging in the Netherlands and the effects of policies for dealing with the potential negative consequences of societal aging on governmental expenditure. This EMA study was performed in 2010-2011 (see Pruyt & Logtens (2015) and Auping et al. (2015)). A rather open System Dynamics model was used for this study (see Figure 5 for a subsystem diagram): many societal evolutions were taken into account with sets of exogenous evolutions (e.g. life expectancies). Figure 6 shows on the one hand that the base ensemble of this study is, due to broad uncertainty ranges, wider than comparative studies (e.g., CBS (2015)). Figure 7a shows that the most extreme outcomes of the base ensemble in terms of government spending as a fraction GDP are, due to the conscious omission of endogenous policy adaptation processes, practically impossible. Note that this was a very explicit choice.

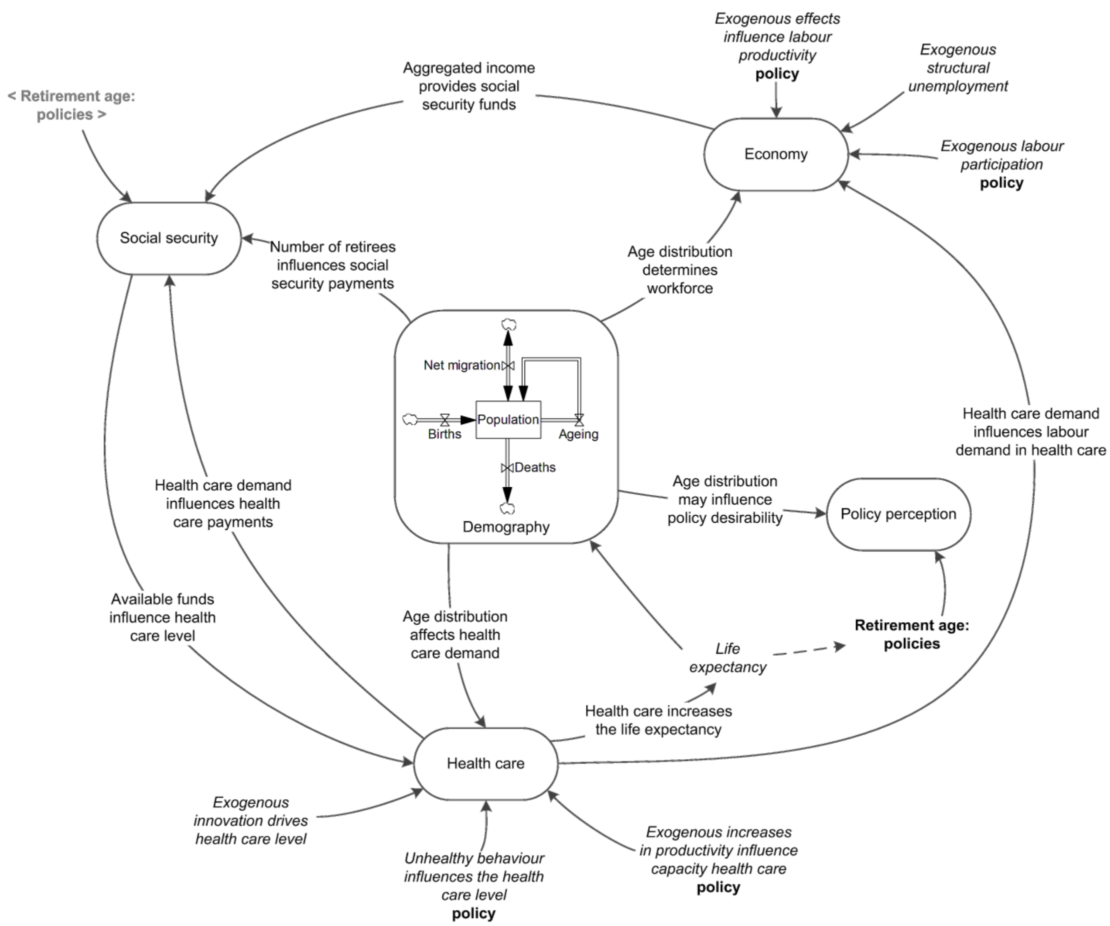


Figure 10: Sub-system diagram of the societal aging model with exogenous time series in italics and policy options in bold. Source: Auping et al. 2015 (based on Pruyt and Logtens (2015))

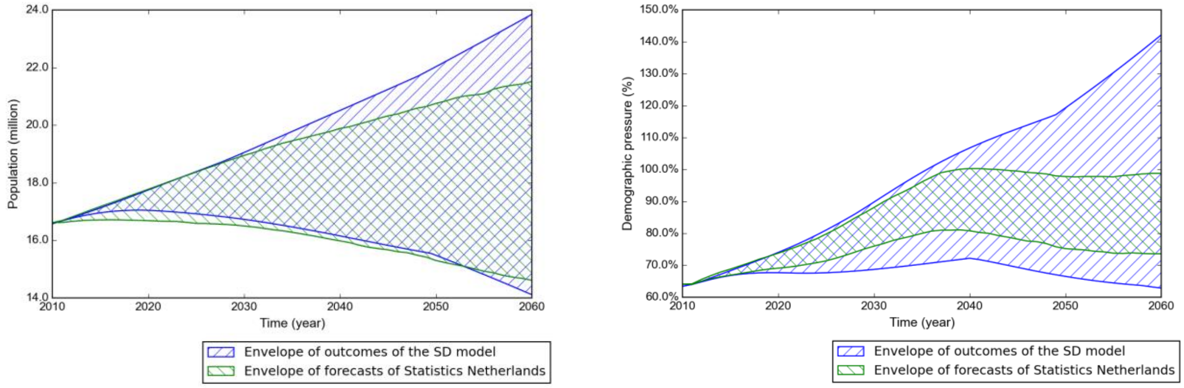


Figure 11: Ensembles for the Dutch population size (left) and the demographic pressure (right) as in Auping *et al*. (2015) and CBS (2015)

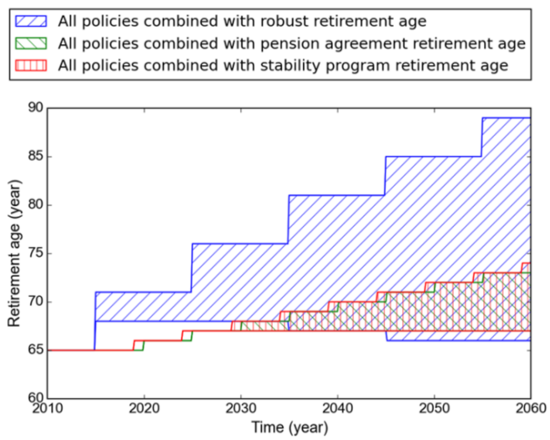


Figure 12 Three retirement age policies tested by Auping et al. 2015

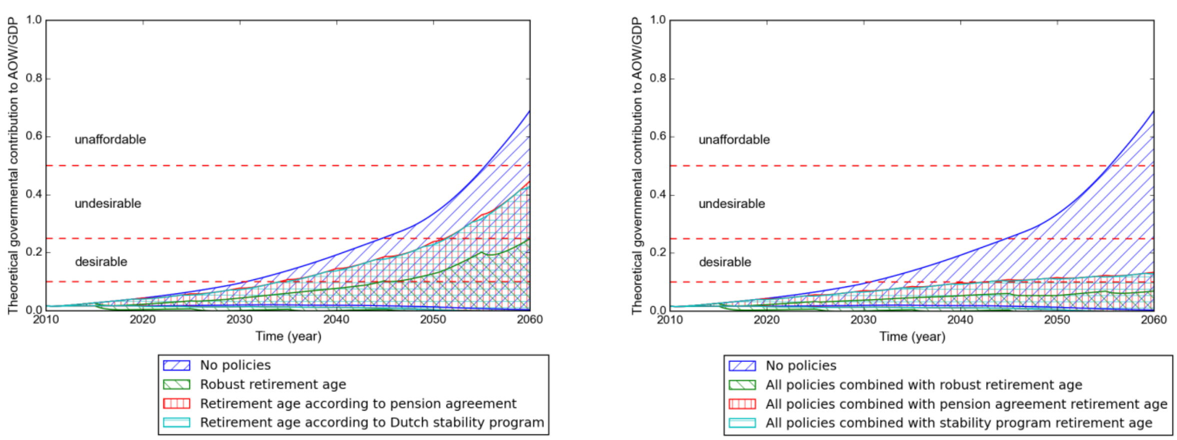


Figure 13: Effects of retirement policies on the necessary government contribution to AOW costs relative to GDP (left: without additional policies; right: with additional supporting policies)

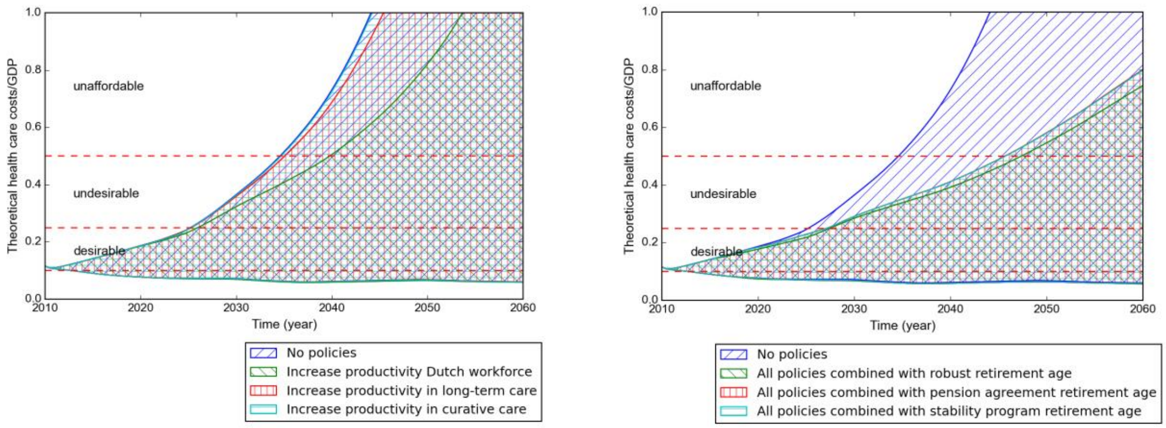


Figure 14: Effects of the retirement policies on health costs relative to GDP

That does not mean the model does not generate interesting insights. One of the many key insights in 2010 was that productivity (especially in care and cure) is both a cause of, and solution for, many problems related to societal aging (Pruyt and Logtens, 2015).

Moreover it does not mean the model cannot be used for testing policies. To the contrary, the model developed purposefully for testing policies. Figure 7b shows the three retirement age policies tested here. The “robust retirement age policy” is fully adaptive: the retirement age fully depends on the uncertain evolutions of the male and female life expectancies (hence the wide ranges of retirement ages). Figure 8 shows that additional supporting policies are necessary irrespective of the choice of retirement age policy, but also that the robust adaptive policy outperforms the other two policies in terms of ensemble predictions. Figure 8 nevertheless shows that these policies are insufficient for ensuring health costs relative to GDP remain sustainable.

Other relevant examples related to this use case are the studies by Hamarat et al. (2013, 2014). They use a process called Adaptive Robust Design to optimize the robustness of adaptive policies.

5. CONCLUDING REMARKS

This paper reviewed typical use cases of a model-based methodology for generating ensemble projections, systematically exploring the consequences of deep uncertainty, identifying and selecting exemplar scenarios, directly searching for answers to specific questions pertaining to uncertain issues or systems, and designing robust policies under deep uncertainty.

This approach is particularly useful for long term assessment and policy design in situations characterized by high degrees of uncertainty and ambiguity (i.e. the existence of rival theories/ perspectives/models). Instead of trying to reduce uncertainty and increase accuracy (with the risk of getting it precisely wrong), the EMA approach enables one to embrace uncertainty and find policies that always work especially when really needed. One of the basic principles underlying the EMA approach is that it is better to get it robustly right than precisely (or probably or probabilistically) wrong.

Typical use cases were illustrated with recent studies. The Ebola case shows that ensemble projections could be as useful as accurate predictions (which could not have been generated at the time). The radicalization case shows that The societal aging case shows that policy analysis under deep uncertainty is possible too.

Each of these use cases has particular advantages. For example, ensemble forecasting helps not to be surprised. It also enables one to include alternative/rival economic theories/models, or models with variety of plausible submodels/functions/.... Open Exploration enables one to address complex long-term issues that are deeply uncertain. Scenario Discovery enables one to develop narratives beyond the minimum / maximum / average, which very often are not the most interesting scenarios. If they are the most interesting, then direct searching enables one to identify them losing time. Finally, robust policy design enables one to develop robust adaptive policies even if highly the underlying issues or systems are deeply uncertain/ambiguous.

REFERENCES

Auping, W.L., de Jong S., Pruyt, E., and Kwakkel J.H., (2014). The Geopolitical Impact of Shale Gas: The Modelling Approach. In: Proceedings of the ISDC2014, Delft, NL.

Auping WL, Pruyt E, & Kwakkel J. (2012). Analysing the uncertain future of copper with three exploratory system dynamics models. In *Proceedings of the 30th International Conference of the System Dynamics Society*. St.-Gallen, CH, System Dynamics Society.

Auping, W.L., Pruyt, E., & Kwakkel, J.H. (2014). Dealing with Multiple Models in System Dynamics: Perspectives on the Future of Copper. *International Journal of System Dynamics Applications* **3**(4): 17-35.

Bankes, SC., 1993. Exploratory modelling for policy analysis. *Operations Research* **41**(3): 435–449.

Bankes, SC., 2002. Tools and Techniques for Developing Policies for Complex and Uncertain Systems. In *Proceedings of the National Academy of Sciences of the United States of America* **99**(3): 7263–7266.

Breiman, L. (2001). Random Forests. *Machine Learning* **45**(1): 5–32

Bryant, B.P., & Lempert R.J. (2010). Thinking inside the box: A participatory, computer-assisted approach to scenario discovery. *Technological Forecasting & Social Change* **77**(1): 34–49.

CBS 2015

de Jong S., Auping, W.L., Govers, J., (2014). The Geopolitics of Shale Gas. HCSS: The Hague

Fisher, D.H. (1987). Knowledge acquisition via incremental conceptual clustering. *Machine Learning* **2** (2): 139–172.

Ford, A. (1990). Estimating the impact of efficiency standards on the uncertainty of the Northwest electric system. *Operations Research* **38**(4):580–597

Forrester, J.W. (1961). *Industrial dynamics*. Waltham, MA: Pegasus Communications.

Forrester, J.W. (2007). System dynamics – the next fifty years. *System Dynamics Review* **23**(2–3): 359–370.

Friedman, J.H., & Fisher, N.I., 1999. Bump hunting in high-dimensional data. *Statistics and Computing* **9**(2): 123–143.

Groves, D.G., & Lempert, R.J. (2007). A new analytic method for finding policy-relevant scenarios. *Global Environmental Change* **17**(1): 73–85.

Hamarat, C., Kwakkel, J.H., & Pruyt, E. (2013). Adaptive robust design under deep uncertainty. *Technological Forecasting & Social Change* **80**(3): 408–418.

Hamarat, C., Kwakkel, J.H., Pruyt, E., & Loonen, E.T. (2014). An exploratory approach for adaptive policymaking by using multi-objective robust optimization. *Simulation Modelling Practice and Theory*. **46**: 25-39

Islam, T., & Pruyt, E. (2014). An adaptive sampling method for examining the behavioural spectrum of long-term metal scarcity. In: *Proceedings of the 30th International Conference of the System Dynamics Society*. Delft, NL, System Dynamics Society.

Kohavi, R., & John, G.H. (1997). Wrappers for feature subset selection. *Artificial Intelligence* **97**(1–2), 273–324

Kwakkel, J.H., Auping, W.L., & Pruyt, E. (2014). Comparing Behavioural Dynamics Across Models: the Case of Copper. In: *Proceedings of the 30th International Conference of the System Dynamics Society*. Delft, NL, System Dynamics Society.

Kwakkel, J.H., Auping, W.L., & Pruyt, E. (2013). Dynamic scenario discovery under deep uncertainty: The future of copper. *Technological Forecasting & Social Change* **80**(4): 789–800.

Kwakkel, J.H., & Pruyt, E. (2015). Using System Dynamics for grand challenges: The ESDMA approach. *Systems Research & Behavioural Science,* 32(3): 358-375. doi: 10.1002/sres.2225

Kwakkel, J.H., & Pruyt, E. (2013). Exploratory modelling and analysis, an approach for model-based foresight under deep uncertainty. *Technological Forecasting & Social Change* **80**(3): 419–431.

Lempert, R.J., Groves, D.G., Popper, S.W., & Bankes, S.C. (2006). A general, analytic method for generating robust strategies and narrative scenarios. *Management Science* **52**(4): 514–528.

Lempert, R.J., Popper, S.W., & Bankes, S.C. (2003). *Shaping the next one hundred years: New methods for quantitative, long-term policy analysis.* RAND report MR-1626, The RAND Pardee Center, Santa Monica, CA.

Lempert, R.J., & Schlesinger, M.E. (2000). Robust strategies for abating climate change. *Climatic Change* **45**(3–4): 387–401.

McKay, M.D., Beckman, R.J., & Conover, W.J. (2000). A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics* **42**(1): 55–61.

McKinney, W. (2012). Python for Data Analysis: Data Wrangling with Pandas, NumPy, and IPython. O'Reilly Media, p470

Miller, J. (1998). Active nonlinear tests (ANTs) of complex simulation models. *Management Science* **44**: 820–830.

Moorlag, R., Pruyt, E., Auping, W.L., & Kwakkel, J.H. (2015). Shale Gas and Import Dependency: A Multi-Method Model-Based Exploration. *International Journal of System Dynamics Applications* **4**(1).

Oreskes, N., Shrader-Frechette, K., & Belitz, K. (1994). Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences. *Science* **263**(5147): 641–647.

Petitjean, F.O., Ketterlin, A., & Gancarski, P. (2011). A global averaging method for dynamic time warping, with applications to clustering. *Pattern Recognition* **44**(3): 678–693

Pruyt, E. (2013). Small system dynamics models for big issues: Triple jump towards real-world complexity. TU Delft Library, Delft. Available at http://simulation.tbm.tudelft.nl/.

Pruyt, E. (2014). System Dynamics and Uncertainty. In: Proceedings of the ISDC2014, Delft, NL.

Pruyt, E., Auping, W.L., & Kwakkel, J.H. (2015). Ebola in West Africa: Model-based Exploration of Social Psychological Effects and Interventions, *Systems Res*earch and Behavioural Science **32**:2–14 doi: 10.1002/sres.2329.

Pruyt, E., (2015a). *Exploratory Modelling and Analysis: The Cookbook.* Forthcoming.

Pruyt, E., (2015b). From Building a Model to Adaptive Robust Decision Making Using Systems Modeling. In: M. Janssen et al. (eds.), *Policy Practice and Digital Science*. Springer International Publishing. (forthcoming April 2015)

Pruyt, E., & Hamarat, C. (2010). The Influenza A(H1N1)v Pandemic: An Exploratory System Dynamics Approach. In Proceedings of the 28th International Conference of the System Dynamics Society. Seoul, Korea, System Dynamics Society.

Pruyt, E., Hamarat, C., & Kwakkel, J.H. (2012). Integrated risk-capability analysis under deep uncertainty: An integrated ESDMA approach. In Proceedings of the 30th International Conference of the System Dynamics Society. St.-Gallen, CH, System Dynamics Society.

Pruyt, E., & Islam, T. (2014). The Future of Modelling and Simulation: Beyond Dynamic Complexity and the Current State of Science. In: Proceedings of the 30th International Conference of the System Dynamics Society. Delft, NL, System Dynamics Society.

Pruyt, E., & Kwakkel, J.H. (2014). Radicalization under Deep Uncertainty: A Multi-Model Exploration of Activism, Extremism, and Terrorism. *System Dynamics Review* **30**(1-2):1–28.

Pruyt, E., Kwakkel, J.H., & Hamarat, C. (2013). Doing more with models: Illustration of a system dynamics approach for exploring deeply uncertain issues, analyzing models, and designing adaptive robust policies. In: *Proceedings of the 31st International Conference of the System Dynamics Society*. Cambridge, MA, System Dynamics Society.

Pruyt, E., & Logtens, T., (2015). Model-Based Exploration of Societal Aging in the Netherlands. *International Journal of System Dynamics Applications* **4**(1)

Rakthanmanon, T. (2013). Addressing Big Data Time Series: Mining Trillions of Time Series Subsequences Under Dynamic Time Warping. *ACM Transactions on Knowledge Discovery from Data* **7**(3): 10:1–10:31. doi:10.1145/2510000/2500489

Sterman, J.D. (2000). *Business Dynamics: Systems thinking and modeling for a complex world*. McGraw Hill.

Van Rossum, G. (1995). *Python Reference Manual*. CWI, Amsterdam.

van der Maaten, L.J.P., & Hinton, G.E. (2008). Visualizing High-Dimensional Data Using t-SNE. *Journal of Machine Learning Research* 9: 2579–2605

Ventana Systems Inc. (2010). *Vensim DSS Reference Supplement.* Ventana System Inc.

Weaver, C.P., Lempert R.J., Brown, C., Hall, J.A., Revell, D., & Sarewitz, D. (2013). Improving the contribution of climate model information to decision making: the value and demands of robust decision frameworks. Wiley Interdisciplinary Reviews: *Climate Change* **4**(1): 39–60.

Yücel, G. (2012). A novel way to measure (dis)similarity between model behaviours based on dynamic pattern features. In: *Proceedings of the 30th International Conference of the System Dynamics Society*, St. Gallen, Switzerland, 22 July–26 July 2012.

Yücel, G., & Barlas, Y. (2011). Automated parameter specification in dynamic feedback models based on behaviour pattern features. *System Dynamics Review* **27**(2): 195–215