Disaggregation of a Stock Variable Based on Attribute Distribution

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Abstract

A co-flow structure, with a built in table function based on cumulative distribution properties, is used to disaggregate a perfectly mixed stock into two sub-stocks. This requires knowledge of the distribution of an allied co-flow attribute, presumed to be a random variable, and the specification of critical fractile threshold (Z value) for this attribute around which the stock can be split. This structure is tested for a variety of conditions. The goal of these tests is to examine whether the co-flow based partitioning is robust to variations in (i) different structural parameters (e.g. time needed for departure) and (ii) the distribution properties of the co-flow attribute. The analysis yields an approximation with less that 5% error, as long the attribute is distributed exponentially and the Z value is between 0 and 90%. Implications of the findings for comparing system dynamics models against agent based models, and for Monte Carlo simulations involving aging chains are discussed.

1. INTRODUCTION

System dynamics models are based on continuous time formulations such that the evolution of any state variable is governed by a differential equation (DE). Each state variable aggregates data. Modelers assume perfect mixing within the stock of such a state variable (Sterman 2000). This paper develops an approximate method for the disaggregation of a perfectly mixed stock into two sub-stocks based on a known distribution of an allied attribute for the relevant flow variable.

The need for disaggregation of a stock arises in certain, somewhat special, system dynamics modeling situations. In these special situations, a planner (or a modeling team) is provided with a time series dataset on an aggregated stock. Related input flow is associated with a random variable drawn from the distribution of a quality attribute. The mean and the standard deviation, as well the type (e.g. either normal or uniform) of the underlying distribution function for this quality attribute are also known. For instance, the production planners at a semiconductor firm, such as Intel, track the time series of the aggregate production volume for microprocessors. They also track a co-evolving data set on the yield in terms of the speed the microprocessors produced by this process. That is, the yield may be normally distributed with a known mean and standard deviation. The aggregate production volume is then divided into two or three different groups based on this yield. These groups are priced differently, and/or are sent into different markets based on a practice known as binning (Wu et al. 2010). Thus, the planners at Intel must disaggregate a perfectly mixed production stock into two or more groups based on its attribute (i.e. yielded speed) distribution. Table 1 lists several example situations, drawn from a variety of contexts, where such disaggregation may be necessary. This table also lists exogenous thresholds for allied disaggregation decisions.

The examples listed in Table 1 do not form an exhaustive list. There are other situations when a system dynamics model is required to disaggregate a perfectly mixed stock suitably. For instance, analytical paradigms such as agent based (AB) modeling do not aggregate data and track the state space associated with varying attributes in terms of separate agents. With the recent advent of computational capability there has been an increasing interest in comparing and contrasting the efficacy of DE and AB methods. Appropriate disaggregation and design of a testing strategy using random variables become important steps in setting up such comparisons (Rahmandad and Sterman 2008). Some applications, such as the analysis of aerospace systems, have deployed hybrid models (Mathieu et al. 2007) by combining AB and DE formulations, wherein a SD stock variable has to be disaggregated to match the modeling requirements for an AB sub-sector. The ability to disaggregate stocks in from the DE formulation into suitable sub-stocks can be a key modeling choice while building a hybrid model.

The approximate method developed in this paper disaggregates a perfectly mixed stock into two sub-stocks based on a co-flow structure. Accumulated stock is assigned to two sub-stocks based on an exogenous and known threshold level associated with the cumulative distribution properties of co-flow attributes. In the rest of this paper, these sub-stocks are termed as the Above Threshold (AT) and Below Threshold (BT) stock respectively. We conduct a systematic study of the performance of this co-flow structure using Monte Carlo simulations.

The error in assigning the co-flow sub-stocks is computed by comparing them against two reference stocks that are separated before mixing them.

Our findings indicate that for a practical range of values (i.e. thresholds associated with 0-90% range of the cumulative distribution for the attribute), the co-flow structure provides outcomes with less than 5% error, both for the AT and BT sub-stock time series, if the attribute is exponentially distributed. Outcomes show similarly low errors for the AT stock if these arrival are computed based on normal distributions and the threshold is between 0-90%. For higher thresholds, between 90-99%, a formulation based on Gumbel distribution (i.e. extreme value distributions) is recommended. The computation of BT stock using this structure, on the other hand, is associated with sizable errors unless the attribute is exponentially distributed. Implications of these findings for the comparison DE and AB models are discussed.

This study also offers insights for selecting the arrival distributions and for interpreting the findings for aging chain outputs generated through Monte Carlo simulations. In essence, Monte Carlo simulations on chains with exponential smooth should be tested using exponential input distributions. Other distributions are likely to create distortions, because a fraction of their tail gets filtered by the smoothing process, and thus the output of aging chain simulation, under random loads, must be interpreted with caution.

2. MODEL STRUCTURE

The model structure is comprised of two sectors. The basic structure, shown in Figure 1, first pipes the arrivals into a single perfectly mixed stock. Then sub-stocks are created based on a co-flow structure. A second sector, named as a reference sector in Figure 2, tracks the reference sub-stocks, without mixing them into a single stock.

2.1 Basic Structure

There are four stocks in this sector. They are named basic stock (S), attribute stock (AS), accumulated stock above threshold (AT) and accumulated stock below threshold (BT).

Basic Stock: d{S(t)}/dt = Arrival Rate(t)– (AT Diversion Rate(t) + BT Diversion Rate(t)) … (1)

AT Diversion Rate = Threshold Correction * S(t)/ Time Needed for Diversion … (2)

BT Diversion Rate = (1-Threshold Correction)*S(t)/ Time Needed for Diversion … (3)

"Arrival Rate" and "Time Needed for Diversion" are an exogenous variables selected based on the conditions in the design of an experiment described in the next section. The "Threshold Correction" is computed using a table function that is set up using the cumulative distribution function for the co-flow attribute. This function is constructed, with zero mean and unit standard deviation (e.g. in a standard normal formulation), for four different types of distributions. The input for this table function is the threshold "Z" value, i.e. a non-negative multiplier to the unit standard deviation, when the mean is zero. Values for the relevant fractile (Z) and allied CDFs, are provided in the appendix. Threshold value is set exogenously based on the design of numerical experiment described in the next section.

Attribute Stock: d {AS(t)}/dt = Random Attribute Value (t) * Arrival Rate(t) –

Co-flow Average(t) * (AT Diversion Rate(t) +BT Diversion Rate(t)) … (4)

Co-flow Average = AS(t)/ A(t) … (5)

The "Random Attribute Value" is computed using the random value generator for un-scaled mean and standard deviation. The mean and standard deviation for each distribution are selected based on the design of the experiment described in the next section. Negative values are truncated to zero.

Above Threshold Stock: d{AT(t)} = AT Diversion Rate(t) … (6)

Below Threshold Stock: d{BT(t)} = BT Diversion Rate(t) … (7)

2.2 Reference Structure

The reference structure has 4 stocks: A, B, Ref AT and Ref BT.

Stock A: d{A(t)}/dt = Arrival Rate(t)- Accumulation Rate for A(t),

for Random Generated Value > Scaled Threshold … (8)

= -Accumulation Rate for A(t), Random Generated Value ≤ Scaled Threshold

Accumulation Rate for A(t) = A(t)/ Time Needed for Diversion … (9)

Scaled Threshold = Mean + Stdev * Threshold … (10a)

= Stdev * Threshold (for exponential distribution) … (10b)

Stock B: d{B(t)}/dt = Arrival Rate(t)- Accumulation Rate for B(t),

for Random Generated Value ≤ Scaled Threshold … (11)

= -Accumulation Rate for B(t), Random Generated Value > Scaled Threshold

Accumulation Rate for B(t) = A(t)/ Time Needed for Diversion … (12)

Reference Stock A: d{Ref AT(t)}/dt = Accumulation Rate for A(t) … (13)

Reference Stock B: d{Ref BT(t)}/dt = Accumulation Rate for B(t) … (14)

2.3 Performance Measures

Performance is tracked for the basic sector as a fraction of the reference sector values.

Error Fraction Above Threshold: EFAT (t) = AT(t) / Ref AT (t) …. (15)

Error Fraction Below Threshold: BTEF(t) = AT(t) / Ref AT (t) …. (16)

3. DESIGN OF EXPERIMENT

The goal of the numerical study is to test *if the co-flow based partitioning is robust to variations in (i) different structural parameters (e.g. time needed for departure) and (ii) to the distribution properties of the co-flow attributes.* Ideally, the performance ratios computed by equations (15) and (16) should be 1.0. We set an arbitrary range of 0.95-1.05, (i.e +/-5 %) as acceptable.

Rahmandad and Sterman (2008) have explored epidemic data and find that the DE and mean AB dynamics differ for several metrics relevant to public health, including diffusion speed, peak load on health services infrastructure, and total disease burden. They have set up numerical experiments where in the key arrival attributes in an aging chain are exponentially distributed. We generalize these idea by selecting four kinds of distribution functions to setup our numerical work: exponential, normal uniform, and Gumbel (i.e. extreme value) distribution. The experimental design covers 8 cases, termed as Base Case and Conditions A through G. In the conditions A though G, either one (or two) parameters are varied systematically, as described below, while keeping the rest of the parameters at the base case settings. Conditions A, B and C test the robustness of the base case parameters, and conditions D through G are designed to compare the impact of changing the threshold (Z) for the base case (ie normal distribution) against other relevant distributions.

(i) Base case is set up for a constant arrival rate (set at unit arrival per day), with attribute values distributed normally (with mean = 100, and standard deviation of 33, i.e +/- 3 standard deviations can go through, allowing up to 99.9% variation in mean to be tracked). The critical threshold (Z) is set at “0” meaning exactly half of the arrivals are above and below this threshold respectively. The time needed for departure at 5 days, and the model is run for 200 days. The update time (dt) is 0.05 days and the simulation uses Euler (i.e stable) integration scheme. For the Monte Carlo simulation, we use a random seed (random uniform between 0.01 and 100) and average across 200 simulations.

(ii) Case A: The time needed for departure is varied from 5 day to 0.05, 0.5, and 50 days respectively. Since the time step (dt) is 0.05 days, and the simulation horizon is 200 days, 0.05 and 50 are deemed to be extreme conditions in term of allowable order of magnitude.

(iii) Case B: The arrival rate is varied systematically from unity to the following five conditions: STEP(1,50), RAMP(0.005,0), Exponential Distribution (standard deviation = 0.33, Normal distribution (with mean = 1, standard deviation = 0.33), and a PULSE (50,0.05) of height = 20.

(iv) Case C: The attribute distribution is varied systematically from N(100,33) to N(150,33), N(50,33), N(100, 66) and N(100, 16.5).

(v) Cases D through G correspond to Normal (100,33), Uniform (100,33), Exponential (100), Gumbel (100,33) distribution where the thresholds (i.e.) are selected as follow

Normal		Uniform		Exponential		Gumbel	
Z	CDF	Z	CDF	Z	CDF	Z	CDF
0	0.500	0	0.500	0	0.000	2.5	0.921
0.5	0.691	0.5	0.644	1	0.632	3	0.951
1	0.841	1	0.789	1.5	0.777	3.5	0.970
1.5	0.933	1.5	0.933	2	0.865	4	0.982
2	0.977	1.66	0.979	2.5	0.918	4.5	0.989
2.5	0.994	1.73	0.999	3.0	0.95	5	0.993

The values in grey get picked implicitly through the table functions. Since Normal and Uniform distributions are symmetric, the selected Z values correspond to cumulative distribution ranging from 50% to 99%. Since Exponential and Gumbel distributions are skewed to towards and away from the mean their values are selected to range from 0 to 95% and 92 to 99% respectively. Thus, the entire range of relevant fraction of Z values between 0 and 99% is tested for symmetric and skewed distributions.

4. RESULTS

Table 1 tabulates the results of the numerical study. The variation in the control parameters (based on the design of the experiment) is highlighted in gray. Input variables created by the test (i.e. attribute mean and standard deviation) are captured, along with the intermediate variables created by the structure (i.e. co-flow mean and standard deviation) in next four columns. The system performance is tracked in the last two columns as a fraction of the reference sector values (error fraction above threshold EFAT and error fraction below threshold EBAT). Recall that, ideally, this ratio should be 1.0 and that we have set an arbitrary range of values 0.95-1.05, (i.e +/-5 % error) as acceptable. Acceptable performance is displayed in white back ground, and performance outside the acceptable range is highlighted in white with a black background.

The first three conditions (i.e. A, B, C) show that the base case is robust in terms of varying the time needed to adjust the co-flow, the arrival rates, and the choice of distribution parameters when Z is set at 0 (i.e. when 50% of the attribute variation falls above the threshold and below the threshold respectively), even when perfect mixing and filtering based on the co-flow structures smoothed the outflow. In these data, the standard deviation of the co-flow is reduced up to 8% of the attribute co-flow value (i.e. when the attribute variance is 32.2 and comparable co-flow variance is 2.6, while the mean value of the attribute and the co-flow remain constant: 99.7). Thus, it is easy to see that the co-flow smoothing process does not affect the symmetric partitioning of the mixed stock in the base case.

The results change considerably, when the partitioning threshold is increased as shown under conditions D through G. The results for Errors below threshold (EFBT) and above threshold (ATBF) are in the acceptable range only when the Z values vary between 0 and 90% and the attributes are distributed exponentially. When the attributes are distributed normally, partitioning with Z=1 (ie CDF =84%) or beyond, creates a large error in the above threshold (in EFAT) stock. When the attributes are distributed uniformly, things get worse, and partitioning with Z= 0.5 (CDF=64%) or beyond creates a large error in the above threshold (in EFAT) stock. Gumbel distribution, that is skewed away from the mean, does poorly in most conditions for EFAT, however its performance does improve for EFBT (i.e. below threshold) when the Z value increase beyond 90%.

5. DISCUSSION

This section offers an explanation for the reported results through post-hoc examination of data. Then, implications of the findings are discussed for applications involving (i) Comparisons of SD and AB models, and (ii) Monte Carlo simulations on aging chains.

5.1 Post Hoc Analysis

In the post hoc analysis, the effect of smoothing created by the co-flow structure has been examined. In essence, the smoothing amounts to a low pass filter, that passes low-frequency signals and attenuates (reduces the amplitude of) signals with frequencies higher than the cutoff frequency. The cutoff frequency depends on the time constant (i.e. Time Needed for Departure). This works in the favor of exponential distribution that is skewed on the right hand side and thus has a lower amount of coverage at high frequency than a comparable symmetric distribution (i.e. either normal distribution or uniform distribution). In the case of symmetric distributions, the mean is preserved, but a reduction in standard deviation signals attenuation of very high and very low values of the attribute. These attenuations can be verified by comparing values of co-flow standard deviation in Table 1. It is worth recalling that in the case of exponential distribution the mean equals its standard deviation, and thus by reducing (increasing) the mean the scaled threshold is also reduced (increased). Finally, in the case of Gumbel distribution, the distribution is skewed away from the mean, and thus the CDF values do not provide a good approximation for low values of threshold (Z), and they provide a good approximation for EFBT for higher values of Z.

5.2 SD and AB Model Comparison

Yücel & van Daalen (2009) have shown that the behavior of AB systems converges to the aggregate model as the network gets denser. However, even a simple structure as shown by this study illustrates that in order to compare these two ways of modeling the reality, one must make assumptions about the distributions of relevant attributes. Rahmandad and Sterman (2008) are prescient in their analysis, because they argue that "*DE SEIR model assumes populations within each state are well mixed. Consider the recovery process (emergence is analogous). Perfect mixing implies that the hazard rate of recovery for an infectious individual (the transition from I to R) depends only on the expected duration of the infectious phase, 1/δ, and is independent of how long that particular individual has been in the I state. Consequently residence times for infectious individuals are distributed exponentially.*" While this logic is sound, this paper offers an alternative explanation for using exponential input while comparing SD and AB models, namely the possibilities of smoothing out higher frequency data due in the relevant SD structure.

5.3 Monte Carlo Simulations with Aging Chains

A related issue is Monte Carlo simulations with aging chains, either with or without attribute based co-flows. For an example of attribute based co-flow in an aging chain, see Figueiredo and Joglekar (2007). The key concern here is again that higher frequency content gets filtered out at each stage in a simple aging chain, and extreme values of attributes are either filtered or retained in a co-flow based aging chain. This means either variance will reduce across stages a regular aging chain (recall the famous Bull Whip Effect, Lee at al. 1997), or attribute distributions will get skewed in a co-flow based aging chain. Statistical inferences using a SD structure must be drawn with caution and account for this effect. The time to smooth (i.e. the cutoff frequency) in the flow becomes an important consideration.

In summary, disaggregation of a perfectly mixed stock is a desirable feature in special types of SD models. A co-flow structure, with a built in table function based on a cumulative distribution function, is has been used to disaggregate a perfectly mixed stock into two sub-stocks. This structure is tested for a variety of conditions. The analysis yields an approximation with less that 5% error, as long the attribute is distributed exponentially and the Z value is between 0 and 90%. Limitations of the approximation for a range of relevant distribution functions are documented and implications applying it for comparing system dynamics models against agent based models, and for Monte Carlo simulations involving aging chains, are discussed.

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Table 1: Examples of Co-flow Attributes & Stock Partitioning Thresholds

	Basic DE Stock	Co-Flow Attribute	Threshold	AB Formulation
1	Inventory from different production configurations	Production yield	Customer rejects a sub-stock that has yield below an agreed percentage.	Different configurations are modeled separately to understand yield in detail
2	Inventory from different production configurations	Production completion or service time	Customer rejects a sub-stock that has service time above an agreed percentage.	Different configurations are modeled separately to understand the evolution of completion time in detail
3	Inventory	Performance quality (e.g. microprocessor speed)	Planners direct the stock to different markets	Different Segments are modeled separately to forecast accurately
4	Projects in a new product pipe line	NPV of projects	Review board rejects projects with NPV below a defined threshold	Different projects (e.g. therapeutic classes) in drug discovery are modeled separately
5	Loan Applications	Underwriting risk	Underwriters reject applications above a defined risk threshold	Applicants are modeled separately by risk type
6	Job Applicants	Likelihood of meeting the job requirements	Recruiters reject applications below a defined threshold	Applicants are pooled into types based on educational needs

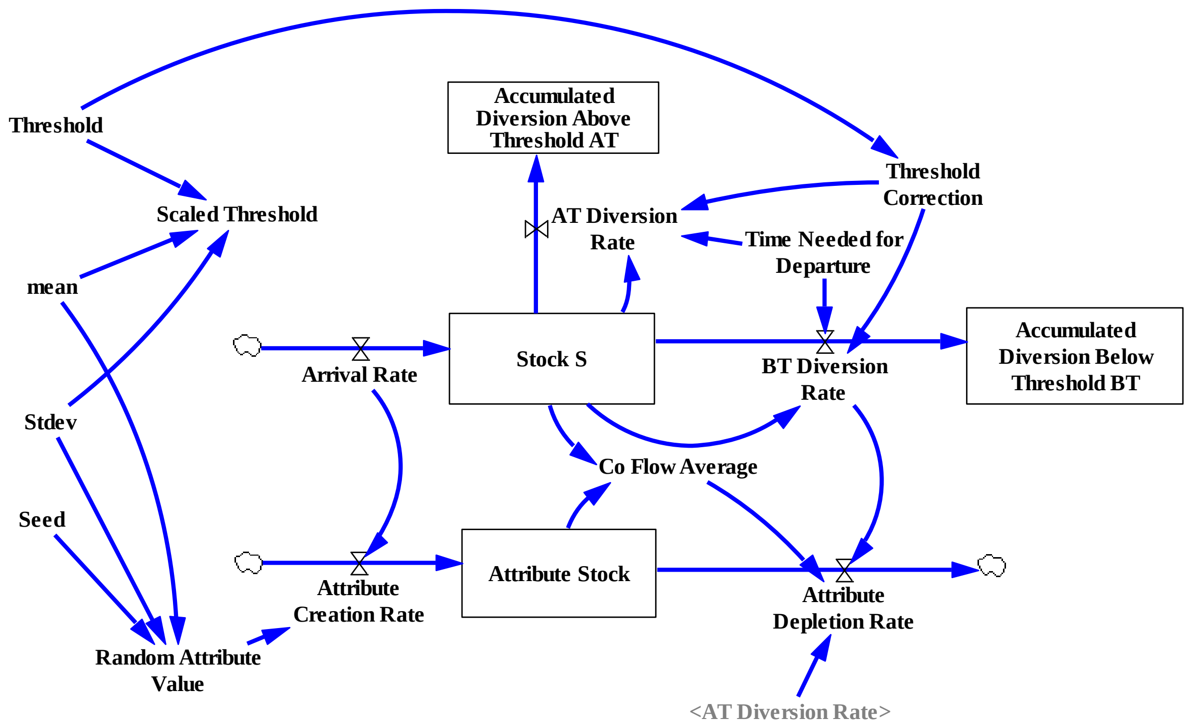

Figure 1: Basic Structure

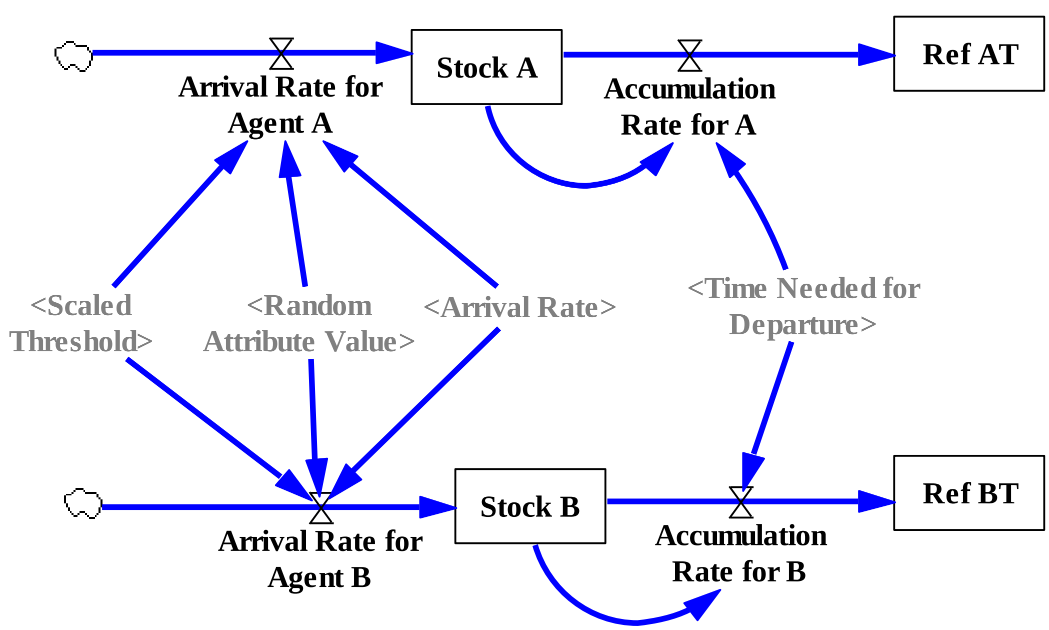

Figure 2: Reference Sub-System

Test Case	Attribute Distribution	Time Needed for Departure	Arrival Rate	Threshold	CDF	Attribute Mean	Attribute Stdev	Coflow Mean	Coflow Stdev	BTEF Mean	ATEF Mean
Base	N(100,33)	5	1 (Constant)	0	0.5	99.7	32.2	99.73	2.60	1.00	1.00
A-1	N(100,33)	0.05	1	0	0.5	99.7	32.2	99.63	32.26	1.03	1.00
A-2	N(100,33)	0.5	1	0	0.5	99.7	32.2	99.66	7.30	1.01	1.00
A-3	N(100,33)	50	1	0	0.5	99.7	32.2	99.82	1.94	1.00	1.00
B-1	N(100,33)	5	1+STEP(1,50)	0	0.5	99.7	32.2	99.71	2.61	1.00	1.00
B-2	N(100,33)	5	0.5+RAMP(0.005,0)	0	0.5	99.7	32.2	99.73	2.61	1.00	1.00
B-3	N(100,33)	5	0.33+ E(0.33,0.33)	0	0.5	99.7	32.2	99.75	2.59	1.00	1.00
B-4	N(100,33)	5	1 + N(0, 0.33)	0	0.5	99.7	32.2	99.97	2.82	1.01	0.99
B-5	N(100,33)	5	1+20*PULSE(50,0.05)	0	0.5	99.7	32.2	99.64	2.65	0.99	1.01
C-1	N(150,33)	5	1	0	0.5	150	32.2	149.72	3.13	1.00	1.00
C-2	N(100,16.5)	5	1	0	0.5	99.8	16.1	99.85	1.88	1.00	1.00
C-3	N(100,66)	5	1	0	0.5	100	48.7	100.21	4.42	1.01	1.00
C-4	N(50,33)	5	1	0	0.5	50	24.4	50.10	2.08	1.01	1.00
D-1	N(100,33)	5	1	0	0.5	99.7	32.2	99.73	2.60	1.00	1.00
D-2	N(100,33)	5	1	0.5	0.69	99.7	32.2	99.73	2.60	1.01	0.99
D-3	N(100,33)	5	1	1	0.84	99.7	32.2	99.73	2.60	0.99	1.06
D-4	N(100,33)	5	1	1.5	0.93	99.7	32.2	99.73	2.60	1.00	1.08
D-5	N(100,33)	5	1	2	0.98	99.7	32.2	99.73	2.60	1.00	1.13
D-6	N(100,33)	5	1	2.5	0.99	99.7	32.2	99.73	2.60	1.00	1.25
E-1	U(100, 33)	5	1	0	0.5	100	33.1	100.05	3.05	1.00	1.02
E-2	U(100, 33)	5	1	0.5	0.64	100	33.1	100.05	3.05	0.99	1.06
E-3	U(100, 33)	5	1	1	0.79	100	33.1	100.05	3.05	0.99	1.16
E-4	U(100, 33)	5	1	1.5	0.93	100	33.1	100.05	3.05	1.00	1.09
E-5	U(100, 33)	5	1	1.7	0.98	100	33.1	100.05	3.05	1.00	1.08
F-1	EXP(100)	5	1	0	0	98.7	92.4	98.95	7.313	1.017	1.001
F-2	EXP(100)	5	1	0.5	0.39	98.7	92.4	98.95	7.313	1.024	0.991
F-3	EXP(100)	5	1	1	0.63	98.7	92.4	98.95	7.313	1.008	0.997
F-4	EXP(100)	5	1	1.5	0.78	98.7	92.4	98.95	7.313	0.993	1.026
F-5	EXP(100)	5	1	2	0.86	98.7	92.4	98.95	7.313	1.002	0.999
F-6	EXP(100)	5	1	2.5	0.92	98.7	92.4	98.95	7.313	1	1.021
F-7	EXP(100)	5	1	3	0.95	98.7	92.4	98.95	7.313	0.994	1.181
G-1	G(100,33)	5	1	2.5	0.92	99.4	33.2	99.22	3.00	0.94	6.39
G-2	G(100,33)	5	1	3	0.95	99.4	33.2	99.22	3.00	0.96	5.41
G-3	G(100,33)	5	1	3.5	0.97	99.4	33.2	99.22	3.00	0.98	62.67
G-4	G(100,33)	5	1	4	0.98	99.4	33.2	99.22	3.00	0.98	41.90
G-5	G(100,33)	5	1	4.5	0.99	99.4	33.2	99.22	3.00	0.99	469.9
G-6	G(100,33)	5	1	5	0.99	99.4	33.2	99.22	3.00	0.99	288.0

Table 1: Effects of Varying Input Type & Distribution on Errors in AT and BT Stocks
(Black Background Highlights Output where the Error is > 5%)

Appendix I: Table Functions

Exponential Distribution		Normal Distribution		Uniform Distribution		Gumbel Distribution	
Z Value	CDF	Z Value	CDF	Z Value	CDF	Z Value	CDF
0	0.000	0	0.500	0	0.50	0	0.368
0.5	0.393	0.5	0.691	0.25	0.57	0.5	0.545
1	0.632	1	0.841	0.5	0.64	1	0.692
1.5	0.777	1.5	0.933	0.75	0.72	1.5	0.800
2	0.865	2	0.977	1	0.79	2	0.873
2.5	0.918	2.5	0.994	1.25	0.86	2.5	0.921
3	0.950	3	0.999	1.5	0.93	3	0.951
3.5	0.970	3.5	1.000	1.732	1.00	3.5	0.970
4	0.982	4	1.000	2	NA	4	0.982
4.5	0.989	4.5	1.000	3.5	NA	4.5	0.989
5	0.993	5	1.000	5	NA	5	0.993