ERM quantitative risk analysis methods and techniques applied to a small commercial bank

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Since modeling and risk management must be viewed as a tool to improve business performance, data and modeling tools are required to support financial risk quantification and capital allocation. Market developments and enhanced regulations require new techniques in order to improve Asset and Liability Management. Despite the importance of risk evaluation, lack of reliable public information, the singular probabilistic behavior of the return (or the loss) of market and credit risks and the underlying nature of the business leads to a complexity that is difficult to handle without a combination of methods and techniques that could together give a systemic view of the problem. Based on a research over a 10 year data base, a methodology will be detailed to quantify financial risks based on the combination of methods and techniques such as parametric VaR, historical and monte carlo simulations, Bayesian inference and game theory. The aim of the paper is to put together the techniques and describe the usefulness of each one in order to develop a SD policy model that can use many insights and informations from them.

1 Introduction
The primary goal of a financial institution is to maximize the outcomes considering the difference between the return obtained from the investments and the cost in fund raising as well as the revenue from direct intermediations to take a good liquidity and solvency over time. In order to achieve this purpose the institutions take risks, singling out the Market Risks, Credit Risk and Operational Risk, which belong to Pillar I of the Basel Accord II.

If on one side competitive forces have pushed financial institutions towards more aggressive investment strategies, on the other side, although there is a wide variety of definitions, risk management is generally related to assuring good solvency and liquidity behavior according to Asset-liability management (ALM) approach.

More frequently, ALM terminology is generally used by Banks and Insurance Companies to prevent the exposure to fluctuations in the ratio beyond prescribed limits whichever they are: GDP, interest rate, stock market index, inflation rate and so on. These random variables require the modeling process to consider stochastic methods and techniques to give them a systemic approach in order to better manage inherent risks and uncertainties that financial institutions might face.

The assets of a company should be more than the liabilities in its book. The liability could be current ones (payables to manage & run business) or just the exposure that could follow due to an incident.

2 Background
Because financial institutions are interconnected to others worldwide, an international standard can help protect the international financial system from the types of problems that might arise if a major bank or a series of banks collapse.
Issued by the Basel Committee on Banking Supervision (BCBS), the purpose of Basel II Accord was to create an international standard that banking regulators can use when creating regulations concerning how much capital banks need to put aside to guard themselves against the types of financial and operational risks they face.

Among the several reasons for the existence of banking regulations, we single out the following:

- Financial mediation – the main role of banks operating in the connection between the saver agents and the resource taker agents;
- Bank racing – lack in credibility that a certain bank will honor its commitments, making the clients withdraw amounts greater than its capacity to honor the short term withdrawals;
- Contagion effect – the regulation tries to avoid situations in which difficulties in a certain institution pose a risk to the entire financial system, the so-called systemic risk;
- Protection to depositors – comes from the necessity of the regulation organ ensuring protection to depositors by intervening in a bank that shows difficulty in honoring its commitments.
- Soundness in the payment system – looks for reliability in the banking system to ensure the liquidation of all operations.

The first coordinated step taken by the regulatory organs to impose a more strict risk administration came up in 1974 with the creation of the Basel Committee, which was made up of financial authorities from G-10 countries, where issues related to the bank industry would be discussed with the aim of establishing behavior patterns, improving the quality of banking supervision, ensuring stability, and tightening the security of the international banking system.

In July/2008 after a long discussion process the Basel Accord was signed. The Basel Accord established and standardized the use of risk factors for assets and the requirement of minimum capital compatible with the credit risk taken by the banking institutions of the signatories in their operations.

The main objective of the accord was to ensure the stability and the soundness of the international banking system through the establishment of a minimum standard for capital requirement, therefore creating homogeneous conditions of competition among international banks by harmonizing the rules.

The concept of capital of a financial institution is set by the Accord of 1998, which is known as Basel I, as follows:

- Level 1 Capital or Main – Shareholders Capital + Reserves (retained profits)
- Level 2 Capital or Supplementary – Other Reserves (not published, revaluations, etc.) + general provisions + hybrid tools of capital and subordinate debts.

It cannot exceed 100% of the level 1 capital and the subordinate debts are limited to 50% of the level 1 capital, since hybrid tools and the subordinate debt are banking products with both characteristics of debt and capital.

Subsequently, in 1996, an amendment to the Basel Accord additionally determined the requirement for capital adequacy related to the level of exposure to market risk taken by the banks from the members.
Thus in January/1996, an addendum to the Basel I, called Amendment to the capital Accord to incorporate Market Risks was published. It has the following relevant aspects:

- Increase the control over risks taken by banks;
- Extend the requirements for the definition of minimum capital (or regulatory), embodying market risk;
- possibility of using the internal models in the measurement of risks as long as they are approved by the regulatory organ;
- creation of level 3 capital, which corresponds to the bonds of subordinate debt with maturity under 2 years;

In 1994, the Central bank of Brazil published the Resolution 200/94, which established the minimum values of capital as well as the obligation of maintenance by the banking institutions of minimum assets compatible with the credit risk of the active operations through the use of deliberation factors.

In 1997 was created the Credit Central and through the Resolution 2.399 of the National Monetary Council, capital for covering the credit risk in swap operations was required.

In 1998 was established the implementation of internal controls of the activities of financial institutions.

In 1999, the requirement of capital for covering exchange and gold (resolution 2606) was established. It was set the maximum level of usage of Reference Asset (RA) related to the permanent fixed asset establishing that the financial institutions should classify the credit operations in a decreasing order and also verify the provision for credits with doubtful liquidation (Resolution 2682).

The Circular 2.972/00 established criteria and conditions for the verification of the asset quota required for covering the interest rate risk because of the exposure of remunerated operations based on pre-fixed rates.

The System of Credit Information was created, thus replacing the Central of Credit Risk. The criterion to control liquidity risk was also established.

In 2001 the Resolution 2.837 defined the Reference Asset (RA) as the sum of levels 1 and 2;

In June/2004 the Committee published the New Basel Capital Accord – Basel II, which is structured under 3 pillars:

- Pillar 1: Capital – requirement of capital for the Market Risk, Credit Risk, Market/Liquidity Risk and Operational Risk.
- Pillar 2: Revision through Supervision
- Pillar 3: Market Discipline


Despite of having registered an advance related to the settlement of a capital requirement standard to Brazilian institutions, so far the demand of capital included only the credit risk and the market risk associated with pre-fixed positions and indexed to currency exchange or to gold. The positions coupled with inflation rates and other indexers, notwithstanding implying in relevant risks were not considered. In order to eliminate such deficiency, the Central Bank published a set of Resolutions and Circulars, which came into effect in July 2008.

In addition to the improvement in the calculus of the minimum capital requirement for market risk, the Central Bank of Brazil determined the implementation by the financial institutions of management structure of market risk, adapting the Brazilian Financial System to the recommendations approved by the New Basel Accord.

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According to FSA (2012),

the crisis in financial markets over 2008 and 2009 prompted a strengthening of the Basel rules to address the deficiencies exposed in the previous set of rules. The Basel III proposals sought to strengthen the regulatory regime applying to credit institutions in the following areas.

- Enhancing the quality and quantity of capital.
- Strengthening capital requirements for counterparty credit risk (and in CRD III for market risk) resulting in higher Pillar I requirements for both.
- Introducing a leverage ratio as a backstop to risk-based capital.
- Introducing two new capital buffers: one on capital conservation and one as a countercyclical capital buffer.
- Implementing an enhanced liquidity regime through the Net Stable Funding Ratio and Liquidity Coverage Ratio.

The Basel III proposals are a long-term package of changes that are due to commence on 1 January 2013 and, based on the Commission’s timetable, the transition period is expected to run until 2021. (FSA, 2012).

Therefore, in Brazil, the financial institutions must comply with the rules and procedures established by the normative guidelines as in appendix I.

3 ERM risks

Once risks are voluntarily taken by banks with a view to benefiting from the exposures, ALM could be defined as the ongoing process of formulating, implementing, monitoring, and revising strategies related to assets and liabilities in an attempt to achieve financial objectives for a given set of risks, uncertainties and constraints.

Because of the wide range of risks and uncertainties they face, the lack of reliable public information, the singular probabilistic behavior of the return (or the loss) of market and credit risks and the underlying nature of the business convey a complexity that is difficult to be handled by the analyst without a combination of methods and techniques when he is observing many aspects of the sample.
The traditional risk paradigm views risks in terms of frequency and severity; the higher the severity and frequency, the more critical the risk. It tends to be a reactionary model in which the risk manager handles the claim and evaluates its impact because it denies the analyst the ability to fully analyze the event. This kind of model views risks as being synonymous with loss while ERM considers risks not merely as a problem but also as an opportunity.

An enterprise risk management program is frequently based on management software and computer modeling programs that provide the opportunity to track and analyze losses both on a historic basis and in a hypothetical setting.

Under an ERM model, risk is viewed in a holistic fashion where everything in your risk universe is connected to everything else and the risk manager's activities would be much more comprehensive, including evaluating the activity in question and the potential risks associated with it; refining job descriptions and requirements based on the anticipated risks of the job; and exploring how to modify activities and policies providing the facility to avoid future injury.

The growing complexity of financial products and the greater promptness of businesses in the markets made necessary the adoption of market risk management practices by the financial institutions, aiming the improvement of results as well as the reduction of losses.

It is important to highlight the recommendation made by the Basel Committee of Banking Supervision regarding the determination of minimum capital requirement compatible with the risks taken, including the market risk, thus making the financial institutions calculate the level of risk exposure.

It is important to single the Market Risk out, which might happen due to the possibility of losses resulting from the fluctuation in the market values of positions held by a financial institution, oscillations which might happen in the interest rates, exchange rates, stock prices, or in the price commodities.

Structured models for the calculation of minimum capital requirement for market risk are based on VaR according to the recommendation of the Basel Committee of Banking Supervision. Since it is easy to understand, VaR is used by most financial institutions as the main tool for measuring and limiting the market risk.

Despite the easiness for interpretation and use, VaR is less adequate for risk measurement under severe instability in the markets, as in crises for instance. Thereto, stress tests are adopted aiming at inferring the possibility of losses from abrupt oscillations in the prices of assets, allowing the adoption of preventive measures. Such tests are based on the use of scenarios, which might be deterministic or based on simulation techniques.

Despite the advances made concerning the modeling and the measurement of market risks, it is necessary to pursue the improvement of techniques so that the process of risk management minimizes the losses from adverse market movements, increasing the probability of generating foreseen flows.

4 Financial modeling dynamics
In order to make rational decisions, it is necessary to quantify uncertainty through a risk analysis model such as a factor model based in many variables that interact to each other and with information that can be obtained by available data or by expert opinion.

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To Pézier (2004), the financial models fall into three main categories:

(i) probabilistic/statistical models describing uncertainties about the future values of market factors; (ii) pricing models relating the prices and sensitivities of instruments to underlying market factors; and (iii) risk aggregation models evaluating the corresponding uncertainties on the future values of portfolios of financial instruments.

In the first category are the stochastic processes commonly used to describe the evolution of market factors: geometric Brownian motion, stochastic volatility models, GARCH models etc. A prime example of the second type is the Black–Scholes option pricing model which, for a given choice of dynamics for the underlying asset price, adds some efficient market assumptions and a hedging argument to yield a risk-free option price. The third type is exemplified by value-at-risk (VaR) models which, with the help of a few simplifying assumptions, produce a probability distribution (or at least some statistics) on the future value of a static portfolio at a chosen future time. (PÉZIER, JACQUES. in ALEXANDER & SHEEDY, 2004).

Anderson & Johnson (1997, p. 6-10) state that variables could be analyzed on an event, a pattern or a structure way. While events are pictures of a single moment in time, patterns are trends, or changes in events over time, while “thinking at the structural level means thinking in terms of causal connections. For the authors, actions taken at the structure level are creative, because they help to shape a different future, but they recommend that the best actions remain focused on the present at the event level. For them, “the art of thinking at the systemic structure level comes with knowing when to address a problem at the event, pattern, or structural level, and when to use an approach that combines the three”.

<table>
<thead>
<tr>
<th>ACTION MODE</th>
<th>TIME ORIENTATION</th>
<th>WAY OF PERCEIVING</th>
<th>QUESTIONS YOU WOULD ASK</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVENTS</td>
<td>React!</td>
<td>Present</td>
<td>Witness event</td>
</tr>
<tr>
<td>PATTERNS</td>
<td>Adapt!</td>
<td>↓</td>
<td>Measure or track patterns of events</td>
</tr>
<tr>
<td>STRUCTURE</td>
<td>Create Change!</td>
<td>Future</td>
<td>Causal loop diagrams and other systems thinking tools</td>
</tr>
</tbody>
</table>

Figure 1: Levels of understanding
Source: Anderson & Johnson (1997, p. 9)

Observed data may come from many sources which may be combined: scientific experiments, surveys, computer databases, literature searches, computer simulations. It is assumed that the analyst reviews the available data and the characteristics of the variable that is to be modeled.

5 The methodology

Empirically it is possible to define the fitted theoretical distributions that of observed data by an intuitive approach or by using techniques such as the maximum likelihood estimators, goodness-of-fit statistics and plots. Most risk analysis models are

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Quantitative risk analysis requires several techniques that could aid to explain data, particularly Bayesian inference and Monte Carlo simulation.

In order to assimilate the various uncertainties of a problem or to produce a realistic view of the problem’s total uncertainty, Bayesian statistics are employed to determine posteriori distributions of a variable by combining a priori opinion with observed data. The strong law of large numbers is the principle upon which Monte Carlo simulation is built, e.g. the larger the sample size, the closer their distribution will be to the theoretical distributions.

After computing the value of the t statistic from two samples, Monte Carlo simulation is used when there is interest in learning the true significance level for the t statistic when the populations don’t follow the standard assumptions of normality and equal variances as the true significance level will depend on

- the stated level of significance $\alpha$
- the shape of the populations (normal, skewed, heavy-tailed, etc.)
- the spreads of the two populations as measured by the two standard deviations
- the sample sizes $m$ and $n$

To Albert (2007), given a particular choice of $\alpha$, shape, spreads, and sample sizes, we wish to estimate the true significance level given by

$$\alpha_T = P(|T| \geq tn+m-2,\alpha/2).$$

A simulation algorithm to compute $\alpha_T$ could be:
- Simulate a random sample $x_1, ..., x_m$ from the first population and $y_1, ..., y_n$ from the second population.
- Compute the t statistic $T$ from the two samples.
- Decide if $|T|$ exceeds the critical point and $H_0$ is rejected.

One repeats steps 1-3 of the algorithm $N$ times and estimates the true significance level by the formula:

$$\hat{\alpha}_T = \frac{\text{number of rejections of } H_0}{N}$$

It can be used to explore the pattern of the true significance level $\alpha_T$ for different choices of sample sizes and populations.

The Process of Measurement of Market Risk proposed is based on the use of widely spread tools established by the best practices of market risk management, including all the Bank positions. The process includes the following procedures:

(a) Market to Market of the positions (MTM)
(b) Calculation of VaR (Value-at-Risk), standard deviation, portfolio quality ratio, etc.
(c) Measurement of values of the positions sensitivities concerning the fluctuation of interest rates.

(d) Stress Tests;

(e) Verification of gaps

(f) Backtesting; and

(g) Calculation of capital allocation for market risk, that is, required reference asset.

Market to Market is the process through which a financial asset is evaluated at market value, that is, at the probable value of purchase/sale where the buyer and the seller are independent, not being a compulsory trading or resulting from a liquidation process.

Market is understood as the one where the asset is traded ordinarily and frequently. It must be selected among the available markets where the greater trading volume of the asset occurs.

The Market Risk is directly related to the fluctuations of trading values of financial assets, which may be measured according to the verification of the volatility of their prices.

Besides being a crucial element in the process of risk measurement, market to market allows the verification of real exposure of a position of a specific factor of risk or market.

The appropriate market to market method to be used depends on the intrinsic characteristics to each asset, especially concerning the related risk factors and optionality, if it is the case.

Financial tools with high liquidity must be market to market based on the trading prices effectively seen in the market.

The following primary sources was used as reference of prices and rates:

- Federal Government Bonds – ANBIMA (www.anbima.com.br);

- Private Bonds – Debentures – ANBIMA;

- Private Bonds – Other – Own Methodology;

- Stocks, options over net stocks, stock terms – BOVESPA (www.bmfbovespa.com.br);

- Future contracts, swaps, commodities (agribusiness) – BM&F

Financial tools with low liquidity must be market to market based on the use of market parameters observed for tools with high liquidity, of at least nature, term, risk and similar indexers.

For tools with no liquidity in the market, the market to market process will take place according to the present value in the future cash flows to be obtained, adjusted based on the expectations of valid market interest rates, on the day of the precification.
The following sections thoroughly describe the methodologies used in the market to market process.

(A) PRE-FIXED TOOLS: The unit price at market value of a pre-fixed financial tool on a t date corresponds to the nominal value on the maturity date (redemption value), deducted by the pre-fixed market interest rate defined according to trading performed in the future exchanges DI of 1 day, from date t until maturity, as follows:

\[
PUMTM_t = \sum_{i=0}^{T} \frac{VN \cdot C_i}{(1 + r_i)^{\frac{du}{365}}} + \frac{VN}{(1 + r_i)^{\frac{du}{365}}}
\]

PUMTMt: Unit price market to market on date t,
VN: Nominal Value
T: Maturity date of the bond
T: Calculation date
Ci: Interest coupon paid periodically, if it is the case
du: Number of business days between the calculation date and the maturity date
ri: Pre-fixed interest rate applied to the term I;

(B) TOOLS INDEXED TO CDI/SELIC

The unit price concerning market to market on date t - PUMTMt - will be defined by the projected redemption value, considering the CDI/CELIC rate of 1 day, on annuaL basis, published by CETIP/BACEN (www.cetip.com.br) until the calculation date – t – and from the calculation date until the maturity of the asset, on date T, the CDI/CELIC rate of 1 day, on annual basis is defined according to the trading in BM&F in the future exchanges of DI interest rate of 1 day, calculated with the respective issuing percentage, deducted by a new factor created from the trading date until the maturity with the market percentage traded and the expectation of future interest rates.

On the trading date the percentage of DI/SELIC is defined based on market price, which includes the quota of credit spread and the daily calculation of the asset must reflect the variations of these percentages according to the market/liquidity and the quality of credit of the issuer/asset.

Methodology for calculation
Definition of the PU of the asset curve on trading date t

\[
PU_t = FATOR_{acum} \cdot VN
\]

Where
PUt: Unit price market to market on date t,
VN – Nominal Value of issue (generally R$ 1,000.00)

\( FATOR_{acum1} \) - Accumulated factor 1 – daily percentage factor of accumulated CDI/SELIC from the issue date until the business day immediately before the calculation date. The calculation of the accumulated \( FATOR_{acum1} \) is made using the following formula:

\[
FATOR_{acum1} = \prod \left\{ \left[ \left( 1 + \frac{DI_i}{100} \right)^{\frac{1}{252}} - 1 \right] * (X \%) + 1 \right\}
\]

Where:

daily CDI/SELIC (rate expressed once a year published by CETIP), or daily TMS (rate expressed once a year published by BACEN) – based on the asset issue rate.

\( X \%) \) percentage defined for the Bond on issue;

\[
\prod \text{ the sum of the product of daily factors from date 1 (issue) until date } t
\]

(business day immediately before the calculation date)

Definition of accumulated factor 2

\( FATOR_{acum2} \) - accumulated factor 2 - daily percentage factor of accumulated CDI/SELIC from date \( t \), of calculation – until date \( T \) – maturity of the asset using the projected rate of CDI or SELIC based on traded rates in the futures market of DI interest of 1 day of BM&F. The calculation of the accumulated \( FATOR_{acum2} \) is made using the following formula:

\[
FATOR_{acum2} = \prod \left\{ \left[ \left( 1 + \frac{DI_t}{100} \right)^{\frac{1}{252}} - 1 \right] * (X \%) + 1 \right\}
\]

Where:

\( DI_t \) - daily CDI/SELIC or daily TMS – based on the future expected interest rate.

\( X \%) \) percentage defined for the bond on issue;

\[
\prod \text{ the sum of the product of daily factors from date } t \text{ (calculation) until date } T
\]

(business day immediately before the redemption date)

The unit price concerning the market to market effect on date \( t \) - \( PUMTM_t \), will be defined by the redemption value projected for date \( T \) based on the nominal value of issue – VN accrued to accumulated factor 1 - \( FATOR_{acum1} \) - and accumulated factor 2 - \( FATOR_{acum2} \) previously defined, deducted by the calculated accumulated factor from the
date of calculation \( t \) until the redemption date \( T \), considering the new percentage of DI/SELIC – \( Y\% \) - of market, according to the following formula:

\[
PUMTM_t = \left( \frac{FATOR_{ACUM_1} \cdot FATOR_{ACUM_2} \cdot VN}{FATOR_{ACUM_3}} \right)
\]

Where:

\[
FATOR_{acum_3} = \prod_{t}^{T} \left( \left( 1 + \frac{DI_t}{100} \right)^{\frac{1}{252}} - 1 \right) \cdot \left( Y\% \right) + 1
\]

(D) TOOLS INDEXED AT 100% OF CDI/SELIC + \( \alpha \% \) A YEAR

The unit price concerning market to market on date \( t \) - \( PUMTM_t \) - will be defined by the projected redemption value, considering the CDI published by CETIP or SELIC published by BACEN, from issuing date until the calculation date – \( t \) – and from the calculation date until the maturity date \( T \), the expected CDI/SELIC according to additional spread of issue \( \alpha \% \) a year, deducted by the new factor constructed from the trading date until the maturity with the projected CDI/SELIC adding the spread \( \beta \% \) a year, percentage of trading in market on date \( t \); 😊

On the trading date the spread \( \beta \% \) a year is defined according to the market price, which includes the quota of credit spread and the daily calculation of the asset must reflect the variations of these percentages based on the market/liquidity and the quality of credit of the issuer/asset.

Methodology for calculation

Definition of the PU of the asset curve on trading date \( t \)

\[
PU_t = FATOR_{acum_1} \cdot VN,
\]

Where

- \( PU_t \) - Unit price on date \( t \)
- \( VN \) – Nominal Value of issue (generally R$ 1,000.00)
- \( FATOR_{acum_1} \) - Accumulated factor 1 – daily percentage factor of accumulated CDI/SELIC from the issue date until the business day immediately before the calculation date

The calculation of the accumulated FATOR 1 is made using the following formula:
\[
FATOR_{acum} = \prod_{i=1}^{t} \left( \left( 1 + \frac{DI_i}{100} \right)^{\frac{1}{252}} \right)^*(\alpha\% + 1)^{\frac{1}{252}}
\]

Where

\( DI_i \): daily CDI (rate expressed once a year published by CETIP), or daily TMS (rate expressed once a year published by BACEN)

\( \alpha\% \): spread in percentage a year defined on the issue of the bond

\( \prod_{i=1}^{t} \): the sum of the product of daily factors of CDI/SELIC from date 1 (issue) until date t (business day immediately before the calculation date t)

Accumulated factor from calculation date until the redemption date

\( FATOR_{acum2} \) - accumulated factor 2 – given by the projected average rate of CDI/SELIC based on tradings performed in the futures market of DI interest rate of 1 day of BM&F accrued to the credit spread \( \alpha\% a year based on market tradings \)

The calculation of the accumulated FATOR 2 is made using the following formula:

\[
FATOR_{acum} = \prod_{i=1}^{t} \left( \left( 1 + \frac{DI_i}{100} \right)^{\frac{1}{252}} \right)^*(\alpha\% + 1)^{\frac{1}{252}}
\]

Where

\( DI_T \) - daily CDI/SELIC or daily TMS – based on the future expectations

\( \prod_{i=1}^{t} \): the sum of the product of daily factors of CDI/SELIC from date t (calculation) until date T (business day immediately before the redemption date)

The unit price concerning market to market on date t - PUMTM\_t - will be defined by the projected redemption value for the date T based on the nominal value of issuing – VN, accrued to accumulated factor 1 - \( FATOR_{acum1} \) to accumulated factor 2 - \( FATOR_{acum2} \) previously defined, deducted by the calculated accumulated factor from the date of calculation t until the redemption date T, considering the new market spread \( \beta\% \)

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a year, given based on the trading in market on date \( t \), according to the following formula:

\[
\text{PUMTM}_t = \left( \frac{\text{FATOR}_{\text{ACUM}_1} \cdot \text{FATOR}_{\text{ACUM}_2} \cdot \text{VN}}{\text{FATOR}_{\text{ACUM}_3}} \right)
\]

Where

\[
\text{FATOR}_{\text{ACUM}_m} = \prod_{t} \left\{ \left[ \left( 1 + \frac{\text{DI}_t}{100} \right)^{\frac{1}{252}} \right] \cdot \left( \beta \% + 1 \right)^{\frac{1}{252}} \right\}
\]

(E) TOOLS INDEXED TO PRICE INDEXES (IPCA/INPC/IGP-M)

The market value of a tool indexed to price index corresponds to the issuing price (Nominal Value), updated by the correction factor related to the variation of the index of the issue date until the moment of evaluation, deducting the present value by the discount rate regarding the inflation coupon related to the remaining term of the tool, as follows:

\[
\text{MTM} = \sum_{i=0}^{T} \left( \frac{\hat{\text{ÍNDICE}}_{i} \cdot \text{VN} \cdot C_i}{(1 + r_i)^{\frac{du}{252}}} \right) + \left( \frac{\hat{\text{ÍNDICE}}_{i_0} \cdot \text{VN}}{(1 + r_i)^{\frac{du}{252}}} \right)
\]

**MTM:** Market to Market

**VN:** Nominal Value;

**t:** Calculation date

**T:** Maturity date of the bond

**\( C_i \):** Interest coupon paid periodically, if it is the case

**\( r_i \):** Reference rate for the inflation coupon for term \( t \);

**\( du \):** Number of business days between the calculation date and the maturity date

**\( \hat{\text{ÍNDICE}}_{i_0} \):** Index rate on the issue date of the bond

**\( \hat{\text{ÍNDICE}}_{i} \):** Index rate on the calculation date considering the projection for the current month

(F) TOOLS INDEXED TO DOLLAR

The market value of a tool indexed to dollar corresponds to the issue price (Nominal Value), updated by the correction factor corresponding to the exchange fluctuation of the issue date until the moment of evaluation, being

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deducted at present value by the discount rate related to the exchange coupon
to the remaining term of the tool, as follows:

\[
MTM = \sum_{i=0}^{T} \left( \frac{PTAX_{i} \cdot VN \cdot C_i}{(1 + r_i)^{du_{i}}} \right) + \left( \frac{PTAX_{i} \cdot VN}{(1 + r_i)^{du_{i}}} \right)
\]

\(MTM\): Market to Market;
\(VN\): Nominal Value;
\(PTAX_{i-j}\): Dollar Ptax of the date immediately before the agreement date of the
bond;
\(PTAX_j\): Dollar Ptax of the date of calculation, for clean coupon ;
\(t\): Calculation date;
\(T\): Maturity date of the Bond;
\(C_i\): Interest coupon paid periodically, if it is the case;
\(r_i\): reference rate for the dollar coupon for term \(t\);
\(du\): number of business days between the calculation date and the maturity
date;

(G) TOOLS INDEXED TO TR
The market value of a tool indexed to TR corresponds to the issuing price
(Nominal Value), updated by the correction factor corresponding to the TR fluctuation of
the issuing date until the moment of evaluation, being deducted at present value by the
discount rate related to the TR coupon with the remaining term of the tool, as follows:

\[
MTM = \frac{VNA \cdot (1 + i)^{dcT_{i}}}{(1 + r_i)^{dcT_{i}}},
\]

\[
VNA = VN \cdot \left(1 + TR_{Acumulada}\right) \cdot \left(1 + TR^{(dcu/dep)}\right)
\]

\(PU\): Market unit price;
\(VNA\): updated nominal value;
\(TR\): Reference Rate for the period between the last and the next bond
anniversary date;
\(TR_{Acumulada}\): Accumulated reference rate from the issuing date until the last
bond anniversary date;
\(i\): prime lending rate;
\(r_i\): Reference rate coupon for term \(t\);
dc: Number of consecutive days between the calculation date and the maturity date;

dcT: Number of consecutive days between the issuing date and the maturity date;

dca: Number of consecutive days between the last anniversary date and the calculation date;

dep: Number of consecutive days between the last and the next anniversary date;

(H) TOOLS INDEXED TO TJLP

The market value of a tool indexed to TJLP corresponds to the issuing price (Nominal Value), updated by the correction factor corresponding to the TJLP fluctuation of the issuing date until the moment of evaluation, being deducted at present value by the discount rate related to the TJLP coupon with the remaining term, as follows:

\[
PU = \frac{VNA \cdot (1+i)^{\frac{dcT}{360}}}{(1+r_i)^{\frac{dc}{360}}}
\]

\[
VNA = VN \cdot (1 + TJLP_{acumulada})
\]

MTM: Market to Market value;
VNA: updated nominal value;
TJLP_{acumulada}: Accumulated TJLP from the issue date until the verification date;
i: prime lending rate;
r_i: TJLP coupon for term t;

dc: Number of consecutive days between the calculation date and the maturity date;

dcT: Number of consecutive days between the issuing date and the maturity date;

dca: Number of consecutive days between the last anniversary date and the calculation date;

dep: Number of consecutive days between the last and the next anniversary date.

(H) STOCK OPTIONS

Concerning market to market the options are daily valued based on the quotations published on the file “Daily Report” (BDIN), made available on Bovespa website daily.

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If the option becomes illiquid or stops being published by BOVESPA, we will adopt as an alternative pricing model the Black & Scholes model, as follows:

\[
\begin{align*}
\text{CALL} & : C = S \cdot N(d_1) - X \cdot e^{-rt} \cdot N(d_2) \\
\text{PUT} & : P = X \cdot e^{-rt} \cdot N(-d_2) - S \cdot N(-d_1) \\
\end{align*}
\]

\[
\begin{align*}
& d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(\frac{r + \sigma^2}{2}\right)t}{\sigma \sqrt{t}} \\
& d_2 = d_1 - \sigma \sqrt{t}
\end{align*}
\]

Where:

- \( C \): Price of the buy option (call);
- \( P \): Price of the sell option (put);
- \( S \): Price of underlying stock;
- \( X \): exercise price of the option;
- \( r \): interest rate (risk free interest);
- \( \sigma \): volatility of the asset object;
- \( N(.) \): standard normal distribution function.

In the case of a CALL (call option), the model calculates the intrinsic value of the option, when deducting the cash price (S) weighed by the probability (\( N(d_1) \)) of having this value, the present value of the strike price \( X \cdot e^{-rt} \), weighing the probability \( N(d_2) \) that this value happens.

In the case of a PUT (put option), the model calculates the intrinsic value of the option, deducting the present value of the strike price \( X \cdot e^{-rt} \), weighing the probability \( N(d_2) \) that this value happens, from the cash price of the asset (S), weighed the probability (\( N(d_1) \)) of having this value.

In the calculations (\( d_1 \)) and (\( d_2 \)) the factors of volatility and time are taken into consideration, that is, the premise of the model is that these values happen at maturity.

9. CURRENCY OPTIONS

The options are market valued based on the BM&F reference prices - Option Premium, published on BM&F website daily.

When there is no disclosure of the quotation by BM&F, the value of the quotation for this day will correspond to the same value of the day before.

If the market continues showing liquidity for a specific stock, they must be market to market based on the use of the Garman-Kolhagen model, as follows:
\[ C = S \cdot e^{-rt} \cdot N(d_1) - X \cdot e^{-rt} \cdot N(d_2) \]
\[ P = -S \cdot e^{-rt} \cdot N(-d_1) + X \cdot e^{-rt} \cdot N(-d_2) \]
\[ d_1 = \frac{\ln(S/X) + (r - c + \frac{1}{2} \sigma^2) \cdot t}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{t} \]

Where:

\( C \): Price of the buy option \((\text{call})\);
\( P \): Price of the sell option \((\text{put})\);
\( S \): Price of underlying stock;
\( X \): Exercise price of the option;
\( r \): Interest rate (risk free interest);
\( \sigma \): Volatility of the asset object;
\( N(.): \) Standard normal distribution function.

The model described by the formula above allows the pricing of exchange options based on the behavior of the cash price of the currency and the exchange coupon rate.

10. FUTURES OPTIONS

Concerning the market to market the options are valued daily based on the BM&F reference prices - Option Premium, published on BM&F website daily.

If the option becomes illiquid or stops being published by BM&F, we will adopt as an alternative pricing model the Black-76 model, as follows:

\[ C = e^{-rt} \left[ S \cdot e^{rt} \cdot N(d_1) - X \cdot N(d_2) \right] \]
\[ P = e^{-rt} \left[ X \cdot N(-d_2) - S \cdot e^{rt} \cdot N(-d_1) \right] \]
\[ d_1 = \frac{\ln \left( \frac{S}{X} \right) + \left( \frac{\sigma^2}{2} \right) \cdot t}{\sigma \sqrt{t}} \]
\[ d_2 = d_1 - \sigma \sqrt{t} \]

Where:

\( C \): Price of the buy option \((\text{call})\);
\( P \): Price of the sell option \((\text{put})\);
\( S \): Price of underlying stock;
\( X \): Exercise price of the option;
\( r \): Interest rate (risk free interest);
\( \sigma \): Volatility of the asset object;

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\( N(\cdot) \): standard normal distribution function.

Once the paths typically diverge, moving randomly in their ‘laws of motion’ as given in the above equations and there is a tendency for the stock prices to ‘drift’ upwards. The degree of dispersion of the simulated stock prices - the extent to which they move away from each other over time – is governed by the volatility \( \sigma \).

The bigger is it the more dispersed the stock prices will be at any point in the simulation.

5 the methodology applied to a small commercial bank

The methodology was applied to a small commercial bank that invests mostly in rural credit. As an analytical approach, VaR provide the simplest and most easily implemented method to estimate based on market data histories that can be obtained from commercial suppliers or gathered internally as part of the daily mark-to-market process.

Figure 1 Var Consolidated diary

<table>
<thead>
<tr>
<th>GROUP</th>
<th>POSITION</th>
<th>RISK</th>
<th>% VAR/POSITION</th>
<th>$ MARGIN</th>
<th>DURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid capital</td>
<td>-77,326,209,33</td>
<td>162,35</td>
<td>0,00</td>
<td>-40,32</td>
<td>1,00</td>
</tr>
<tr>
<td>Credit Cards</td>
<td>35,093,961,78</td>
<td>1,791,50</td>
<td>0,01</td>
<td>753,95</td>
<td>15,96</td>
</tr>
<tr>
<td>CDI Rural</td>
<td>-1,356,294,687,62</td>
<td>584,067,41</td>
<td>0,04</td>
<td>-244,263,70</td>
<td>108,72</td>
</tr>
<tr>
<td>Rural Product Note(CPRF)</td>
<td>22,976,558,31</td>
<td>8,410,06</td>
<td>0,04</td>
<td>3,786,15</td>
<td>95,31</td>
</tr>
<tr>
<td>Rural credit</td>
<td>789,575,418,01</td>
<td>439,593,84</td>
<td>0,06</td>
<td>337,061,64</td>
<td>167,35</td>
</tr>
<tr>
<td>Abroad deposits</td>
<td>463,242,34</td>
<td>8,708,79</td>
<td>1,88</td>
<td>-854,43</td>
<td>0,00</td>
</tr>
<tr>
<td>Own emissions Pos</td>
<td>-2,274,563,802,51</td>
<td>4,775,69</td>
<td>0,00</td>
<td>-1,186,15</td>
<td>170,12</td>
</tr>
<tr>
<td>Own emissions pré</td>
<td>-89,559,240,99</td>
<td>21,300,61</td>
<td>0,02</td>
<td>-9,628,51</td>
<td>63,96</td>
</tr>
<tr>
<td>Equalizable Assets</td>
<td>387,061,572,05</td>
<td>190,007,95</td>
<td>0,05</td>
<td>105,367,61</td>
<td>130,99</td>
</tr>
</tbody>
</table>

Work supported by Fundacao de Apoio à Pesquisa do Distrito Federal – FAPDF
As we can see at figure 1, the bank made operations in rural credit (Rural credit group) at fixed rates and with a term longer than their funding (CDI Rural group), and also preset and with a shorter period. Its market risks reside mostly there and are concentrating on the interest rate risk because of the mismatch between assets and liabilities operated at fixed rates.

In the column RISK it is possible to see the major contributions of the financial instruments used in the bank, highlighting the active and passive RURAL CREDIT and CDI RURAL groups. Column $MARG$ shows the marginal effect of adding the position of the financial instrument to the portfolio.

VaR is an estimate of the loss from a fixed set of trading positions over a fixed time horizon that would be equalled or exceeded with a specified probability. The value of any VaR estimate will depend on the stochastic process that is assumed to drive the random realizations of market data.

The use of VaR involves two arbitrarily chosen parameters – the holding period, generally ten days or one month and the confidence level that depends mainly on the purpose to which the risk measures are being put. Once the bank is using risk measures to set capital requirements and wish to achieve a low probability of insolvency or a high credit rating, the confidence levels required for these purposes can be higher than those needed to meet regulatory capital requirements and is fixed at 99% of confidence.

Figure 1 shows that the one day VaR with 99% of confidence is just R$ 618 thousand, denoting a risk aversion behavior. The bank uses the management limit of 1% of its net worth to this metric.

Currently, it is in the house of $ 300 million and the limit is therefore R$ 3 million. It is noteworthy that at the date of the position shown in Figure 1 the rural portfolio was not completely done, and has an asset (R$ 789.5 million) less than the corresponding
liability (R$ 1.356 billion). With more two months of new hires rural credit the bank will reach full funding of the CDI RURAL group.

The bank determines the VaR separately to allow the treasury management actions solely within that area. Figure 2 shows the credit Portfolio.

Figure 2 – Daily Var of the treasury management

<table>
<thead>
<tr>
<th>GROUP</th>
<th>POSITION</th>
<th>RISK</th>
<th>% VaR/POSITION</th>
<th>$ MARG.</th>
<th>DURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hibrid capital</td>
<td>-77.326.209,33</td>
<td>162,35</td>
<td>0,00</td>
<td>-29,33</td>
<td>1,00</td>
</tr>
<tr>
<td>Rural Product</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Note(CPRF)</td>
<td>22.976.558,31</td>
<td>8.410,06</td>
<td>0,04</td>
<td>7.842,09</td>
<td>95,31</td>
</tr>
<tr>
<td>Abroad deposits</td>
<td>463.242,34</td>
<td>8.708,79</td>
<td>1,88</td>
<td>1.810,42</td>
<td>0,00</td>
</tr>
<tr>
<td>Own emissions Pos</td>
<td>2.274.563.802,51</td>
<td>4.775,69</td>
<td>0,00</td>
<td>-862,64</td>
<td>170,12</td>
</tr>
<tr>
<td>Own emissions pré</td>
<td>-89.559.240,99</td>
<td>21.300,61</td>
<td>0,02</td>
<td>19.480,81</td>
<td>63,96</td>
</tr>
<tr>
<td>Receivangel Investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Funds (FIDCS)</td>
<td>22.947.394,96</td>
<td>17.025,29</td>
<td>0,07</td>
<td>3.510,87</td>
<td>0,00</td>
</tr>
<tr>
<td>Transfers assets</td>
<td>64.999.769,35</td>
<td>90,07</td>
<td>0,00</td>
<td>24,65</td>
<td>1,00</td>
</tr>
<tr>
<td>OVER LIABILITIES</td>
<td>-242.110.185,05</td>
<td>508,34</td>
<td>0,00</td>
<td>-91,82</td>
<td>1,00</td>
</tr>
<tr>
<td>Liabilities transfers</td>
<td>-33.848.877,88</td>
<td>40,52</td>
<td>0,00</td>
<td>-4,27</td>
<td>1,00</td>
</tr>
<tr>
<td>Term</td>
<td>557.205.565,28</td>
<td>50.028,66</td>
<td>0,01</td>
<td>48.543,13</td>
<td>28,57</td>
</tr>
<tr>
<td>Actives post titles</td>
<td>3.196.651.250,61</td>
<td>10.458,24</td>
<td>0,01</td>
<td>3.510,87</td>
<td>0,00</td>
</tr>
<tr>
<td>Active pré titles</td>
<td>752.090.103,11</td>
<td>40.483,64</td>
<td>0,01</td>
<td>39.173,43</td>
<td>17,80</td>
</tr>
<tr>
<td>Securities liabilities pos</td>
<td>1.530.684.527,29</td>
<td>3.213,84</td>
<td>0,00</td>
<td>-580,52</td>
<td>220,94</td>
</tr>
<tr>
<td>Securities Liabilities pré</td>
<td>-3.021.938,01</td>
<td>147,51</td>
<td>0,00</td>
<td>-119,80</td>
<td>13,60</td>
</tr>
</tbody>
</table>

The daily VaR calculated was R$ 85.595,70, a very modest cash reflecting the fact that the bank keeps its assets and liabilities closely related in terms of maturities and indices. The negative sign in column "$ MARG" indicates whether it is active or passive. Note that the major assets and liabilities are of the post-fixed (indexed to the CDI) and therefore do not generate much interest rate risk. The bank raises nearly 2.27 billion of its network customer’s variable rates in CDI and invests about $ 3.19 billion in bonds also post fixed in CDI. Figure 3 shows the credit portfolio.

Figura 3: Credit Portfolio

<table>
<thead>
<tr>
<th>GRUPO</th>
<th>POSIÇÃO</th>
<th>RISCO</th>
<th>% VAR/POSIÇÃO</th>
<th>$ MARG.</th>
<th>DURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit Cards</td>
<td>35.093.961,78</td>
<td>1.791,50</td>
<td>0,01</td>
<td>627,64</td>
<td>15,96</td>
</tr>
<tr>
<td>CDI Rural</td>
<td>-1.356.294.687,62</td>
<td>584.067,41</td>
<td>0,04</td>
<td>-185.865,62</td>
<td>108,72</td>
</tr>
<tr>
<td>Rural Credit</td>
<td>789.575.418,01</td>
<td>439.593,84</td>
<td>0,06</td>
<td>304.272,66</td>
<td>167,35</td>
</tr>
<tr>
<td>Equalizable Assets</td>
<td>387.061.572,05</td>
<td>190.007,95</td>
<td>0,05</td>
<td>87.574,70</td>
<td>130,99</td>
</tr>
<tr>
<td>Asset operation savings</td>
<td>127.103.980,28</td>
<td>106.253,02</td>
<td>0,08</td>
<td>44.681,05</td>
<td>137,27</td>
</tr>
<tr>
<td>OP active rp pos</td>
<td>41.364.335,56</td>
<td>57,32</td>
<td>0,00</td>
<td>20,22</td>
<td>362,10</td>
</tr>
<tr>
<td>Active Pre OP RP</td>
<td>196.114.004,45</td>
<td>263.753,12</td>
<td>0,13</td>
<td>256.237,95</td>
<td>380,65</td>
</tr>
<tr>
<td>Rural Savings</td>
<td>-597.377.322,04</td>
<td>17.962,53</td>
<td>0,00</td>
<td>-5.514,96</td>
<td>8,37</td>
</tr>
<tr>
<td>Transfers assets</td>
<td>907.117.266,76</td>
<td>659.824,45</td>
<td>0,07</td>
<td>613.464,19</td>
<td>534,84</td>
</tr>
<tr>
<td>Liabilities transfers</td>
<td>-850.298.173,29</td>
<td>553.105,62</td>
<td>0,07</td>
<td>-511.727,90</td>
<td>514,08</td>
</tr>
</tbody>
</table>

Work supported by Fundacao de Apoio à Pesquisa do Distrito Federal – FAPDF
The calculated VaR was R$ 603,769,93, showing that the majority of market risk (arising from the mismatch of term lending and borrowing positions pre-determined) is due to credit portfolio. Note also that the funding rural savings rate which is operated post-fixed in TR (TR + pre-fixed coupon) is distributed between the active and equalizable assets. The latter is active in operating the fixed rate with farmers and receive supplemental government that subsidizes rural credit. The final rate is obtained by specific formula which provides the bank with a performance compatible with the CDI.

Figura 4 presents the parametric of ten working days, calculated at 99% confidence for the positions being considered with banking services. BCB Resolution 3464 indicated how institutions should classify their financial positions between banking and trading. In the bank, the vast majority is comprised of banking positions, which carries up to maturity. The rules of the Bank recommend special care with positions denominated trading, or that can be sold before maturity. BACEN issued specific rules stating standard templates and formulas for calculating the interest rate risk positions classified as trading. The formulas are a variant of the traditional method of calculating parametric VaR where the Bank supervise parameters such as volatility and correlations.

<table>
<thead>
<tr>
<th>GRUPO</th>
<th>POSIÇÃO</th>
<th>RISCO</th>
<th>% VAR/POSIÇÃO</th>
<th>$ MARG.</th>
<th>DURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid capital</td>
<td>-77.326.209,33</td>
<td>856,85</td>
<td>0,00</td>
<td>220,66</td>
<td>1,00</td>
</tr>
<tr>
<td>Credit Cards</td>
<td>35.093.961,78</td>
<td>3.775,36</td>
<td>0,01</td>
<td>-907,38</td>
<td>15,96</td>
</tr>
<tr>
<td>CDI Rural</td>
<td>-1.356.294.687,62</td>
<td>1.390.271,11</td>
<td>0,10</td>
<td>173.676,04</td>
<td>108,72</td>
</tr>
<tr>
<td>Rural Product</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Note(CPRF)</td>
<td>22.976.558,31</td>
<td>17.881,24</td>
<td>0,08</td>
<td>-2.645,29</td>
<td>95,31</td>
</tr>
<tr>
<td>Rural credit</td>
<td>789.575.418,01</td>
<td>833.255,61</td>
<td>0,11</td>
<td>345.620,06</td>
<td>167,35</td>
</tr>
<tr>
<td>Abroad deposits</td>
<td>463.242,34</td>
<td>29.346,47</td>
<td>6,34</td>
<td>1.497,36</td>
<td>0,00</td>
</tr>
<tr>
<td>Own emissions Pos</td>
<td>-2.274.563.802,51</td>
<td>25.204,32</td>
<td>0,11</td>
<td>6.490,61</td>
<td>170,12</td>
</tr>
<tr>
<td>Own emissions pré</td>
<td>-89.559.240,99</td>
<td>52.118,37</td>
<td>0,06</td>
<td>3.382,93</td>
<td>63,96</td>
</tr>
<tr>
<td>Equalizable Assets</td>
<td>387.061.572,05</td>
<td>409.333,81</td>
<td>0,11</td>
<td>12.132,61</td>
<td>130,99</td>
</tr>
<tr>
<td>Asset operation savings</td>
<td>127.103.980,28</td>
<td>166.734,24</td>
<td>0,13</td>
<td>-13.372,16</td>
<td>137,27</td>
</tr>
<tr>
<td>OP active rp pos</td>
<td>41.364.335,56</td>
<td>162,23</td>
<td>0,00</td>
<td>-118,04</td>
<td>362,10</td>
</tr>
<tr>
<td>Active Pre OP RP</td>
<td>196.114.004,45</td>
<td>531.531,36</td>
<td>0,27</td>
<td>517.166,52</td>
<td>380,65</td>
</tr>
<tr>
<td>OVER ASSETS</td>
<td>64.999.769,35</td>
<td>254,93</td>
<td>0,00</td>
<td>-185,48</td>
<td>1,00</td>
</tr>
<tr>
<td>OVER LIABILITIES</td>
<td>-242.110.185,05</td>
<td>2.682,81</td>
<td>0,00</td>
<td>690,88</td>
<td>1,00</td>
</tr>
<tr>
<td>Rural Savings</td>
<td>-597.377.232,04</td>
<td>65.462,11</td>
<td>0,01</td>
<td>8.520,96</td>
<td>8,37</td>
</tr>
<tr>
<td>Transfers assets</td>
<td>907.117.266,76</td>
<td>1.491.751,46</td>
<td>0,16</td>
<td>1.348.725,31</td>
<td>534,84</td>
</tr>
<tr>
<td>Liabilities transfers</td>
<td>-884.147.051,17</td>
<td>1.306.081,32</td>
<td>0,15</td>
<td>1.105.133,09</td>
<td>494,43</td>
</tr>
<tr>
<td>Actives post titles</td>
<td>3.196.651.250,61</td>
<td>55.622,69</td>
<td>0,00</td>
<td>1.345,70</td>
<td>628,35</td>
</tr>
<tr>
<td>Active pré titles</td>
<td>752.090.103,11</td>
<td>81.038,04</td>
<td>0,01</td>
<td>-19.594,08</td>
<td>17,80</td>
</tr>
<tr>
<td>Securities liabilities pos</td>
<td>-1.530.684.527,29</td>
<td>16.981,43</td>
<td>0,00</td>
<td>4.367,91</td>
<td>220,94</td>
</tr>
<tr>
<td>Securities Liabilities pré</td>
<td>-3.021.938,01</td>
<td>509,91</td>
<td>0,02</td>
<td>95,70</td>
<td>13,60</td>
</tr>
</tbody>
</table>

Portfolio VaR 10 days
R$ 1,281,977,73

In figure 5 below we calculate the VaR of trading positions. Central Bank of Brasil (BACEN) called portions of interest rate risk (Pjur) for the VaR calculated by formula.
and for positions classified as trading. The total market risk (which is all due to interest rate) is around R $ 4.33 million, being the most results from the trading positions (U.S. $ 3.05 million). In Figure 5 one can see the calculation for the vertices (salaries standardized methodology contained in the Central Bank of Brazil.) Note that there is enough leeway in relation to the total VaR limit allocated for market risk. In one case we have permission to take a maximum risk of $ 11.9 million.

In figure 6 below we list the results of simulation of loss in condition of stress or a stress scenario. The scenario is based on historical simulation for the past ten years, ie, the system returns the portfolio over the past ten years, noting the worst loss the portfolio would have a day to day based on happened in the last ten years.

<table>
<thead>
<tr>
<th>VÉRTICE</th>
<th>POSITION (PJUR)</th>
<th>RISK</th>
<th>Int.rate (a.a.)</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>231.885.812</td>
<td>74.180</td>
<td>8,66%</td>
<td>0,00000521</td>
</tr>
<tr>
<td>42</td>
<td>136.829.325</td>
<td>87.543</td>
<td>8,96%</td>
<td>0,00000521</td>
</tr>
<tr>
<td>63</td>
<td>70.098.304</td>
<td>67.273</td>
<td>9,19%</td>
<td>0,00000521</td>
</tr>
<tr>
<td>126</td>
<td>6.164.092</td>
<td>10.809</td>
<td>9,85%</td>
<td>0,00000476</td>
</tr>
<tr>
<td>252</td>
<td>444.939</td>
<td>1.560</td>
<td>10,86%</td>
<td>0,00000476</td>
</tr>
<tr>
<td>2520</td>
<td>22.947.395</td>
<td>967.131</td>
<td>12,28%</td>
<td>0,00000572</td>
</tr>
<tr>
<td>TOTAL</td>
<td>468.369.867</td>
<td>1.208.497</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Composição</th>
<th>Total Risk</th>
<th>Limit R$</th>
<th>% PR</th>
<th>Freedom</th>
<th>%PR</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR Banking + Trading</td>
<td>4.336.380</td>
<td>11.949.966</td>
<td>4,60%</td>
<td>7.613.586</td>
<td>2,93%</td>
</tr>
</tbody>
</table>

PR February (R$) : 259.781.873

<table>
<thead>
<tr>
<th>Treasury</th>
<th>Position</th>
<th>Losses due to stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid capital</td>
<td>-77.326.209,33</td>
<td>1.435,24</td>
</tr>
<tr>
<td>Rural Product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Note(CPRF)</td>
<td>22.976.558,31</td>
<td>-227.201,06</td>
</tr>
<tr>
<td>Abroad deposits</td>
<td>463.242,34</td>
<td>9.151,79</td>
</tr>
<tr>
<td>Own emissions Pos</td>
<td>-2.274.563.802,51</td>
<td>42.217,90</td>
</tr>
<tr>
<td>Own emissions pré</td>
<td>-89.559.240,99</td>
<td>574.921,28</td>
</tr>
<tr>
<td>Receivable Investment Funds (FIDCS)</td>
<td>22.947.394,96</td>
<td>-43.488,77</td>
</tr>
<tr>
<td>OVER ASSETS</td>
<td>64.999.769,35</td>
<td>-1.206,45</td>
</tr>
<tr>
<td>OVER LIABILITIES</td>
<td>-242.110.185,05</td>
<td>4.493,78</td>
</tr>
</tbody>
</table>

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Figure 6 shows the simulation for each portfolio (cash and credit) and the result of consolidated loss in the order of R $ 14.33 million. Note, again, that the credit portfolio is that it would suffer more with a stress on the rate of interest (in this case, a surge of about 6% interest rate, which occurred in mid-2002).

Finally, Figure 7 presents the graph of backtest the model in the last 252 business days, the case for a lambda (the decay factor in the EWMA model) of 0.89.

Figure 7: backtest graf with lambda equal to 0.89.

Verified the existence of eight errors (situations where the loss was greater than the VaR calculated by parametric model). The test of Kupiec (used to assess the maximum number of errors allowed in the number of evaluation days in the case, 252) indicates the limit of five errors. The test indicated the need to change the parameter lambda to 0.94 giving more weight to past data.

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Figure 8 presents the results of the test chart showing that five situations extrapolation of the loss, i.e., it is within the limit of five errors defined in Kupiec test.

Figura 8 backtest graf with lambda equal to 0.94.

7 SD Modelling

A computer model of a financial system is a way to better understand its behavior in order to foresee its future performance and to get insights concerning the impact of some variables over other variables that together might impact the system. The combination of scientific methods and computing tools make it possible to get new forms of analysis and even desirable results and thus support a trend towards more sophisticated modeling techniques.

Risks must be quantified in order to manage it. To understand a risk model, risk managers require knowledge of probability distributions, simulation methods and a host of other mathematical and statistical techniques. Market model is assessed by mapping portfolios to their risk factors and forecasting the volatilities and correlations of them. A credit model can be assessed using firm-value models that are based on the theory of options, or statistical and/or macro-econometric models. Probability distributions are applied to operational risks that are particularly very difficult to quantify because the data are sparse and unreliable and because generally estimations are made based on expert opinions.

Financial systems are complex because it requires the modeling of the outcomes of a money’s allocation process, the calculation of the liquidity, solvency and so forth in order to discover emergent behaviors considering their systemic impacts upon one variable over others.

Using computer simulation is primarily a way to modeling and understanding social processes for experiment, proof or discovery of social phenomena. To be useful, simulation must be related to one or more the following primary goals: to explain the reality, to predict the reality, to develop new tools to substitute for human capabilities, to training, to entertainment, or, to assist in discovery and formalization. A frequent used intention of a model in insurance or finance institution is to simulate population changes.

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in order to predict or foresee the size and age structure of a populational set of individuals or to foresee the impacts of using a particular populational table over the costs of the pension fund or bank. See Gilbert & Troitzsch (2002, p. 1-13)

This models could aggregate statistics to be calculated or used as estimates of the future characteristics of the population.

Based on the case described, the following SD Model can give For backtesting and model validation, relatively lower confidence levels are

If we wish, we can estimate the VaR of the stock price by simulating a large number of terminal stock prices S(T). We then read the VaR from the histogram of S(T) values so generated. In addition, Monte Carlo simulation can easily handle problems with more than one random risk factor. Monte Carlo methods have many applications in market risk measurement and would be the preferred method in almost any ‘complex’ risk problem.

We might be dealing with underlying risk factors that are ‘badly behaved’ in some way (e.g. because they jump or show heavy tails) or we might have a mixture of heterogeneous risk factors. For example, we might have credit-related risk factors as well as normal market risk factors, and the credit risk factors cannot be modelled as normal. We might have a portfolio of options. In such cases, the value of the portfolio is a nonlinear (or otherwise difficult) function of underlying risk factors, and might be impossible to handle using analytical methods even if the risk factors are themselves ‘well behaved’.

We might be dealing with instruments with complicated risk factors, such as mortgages, credit derivatives, and so forth. We might have a portfolio of heterogeneous instruments, the heterogeneity of which prevents us from applying an analytical
approach. For example, our portfolio might be a collection of equities, bonds, foreign exchange options, and so forth. Monte Carlo simulation has many advantages over analytical approaches to calculating VaR, once it can capture a wider range of market behaviour. Below, a general model to put all the thinghs together.

7 Conclusions

For backtesting and model validation, relatively lower confidence levels are desirable to get a reasonable proportion of excess-loss observations. For limit-setting,
most institutions prefer confidence levels low enough that actual losses exceed the corresponding VaR estimate somewhere between two and twelve times per year (implying a daily VaR confidence level of 95% to 99%). This forces policy committees to take the size of the limit seriously, since losses over that limit can occur with a reasonable likelihood.

For the above reasons, among others, the ‘best’ choice for these parameters depends on the context. What is important is that the choices be clear in every context and be thoroughly understood throughout the institution so that limit-setting and other risk-related decisions are made in light of this common understanding.

Appendix I: Brazil normative guidelines

Brazilian financial institutions must comply with the rules and procedures established by the following normative guidelines:

(a) Resolution 3.464/07: Concerns the implementation of the structure of market risk management.
(b) Circular 3.464/07: Establishes the minimum criteria to classify the operations in Trading Book.;
(c) Resolution 3.490/07: Concerns the verification of the Required Reference Asset, which must be calculated based on the following equation:

\[
PRE = PEPR + PCAM + PJUR + PCOM + PACS + POPR
\]

- **PEPR** = Quota concerning the exposures weighed by the weight factor to risk imputed to them
- **PCAM** = Quota concerning the risk of exposures in gold, foreign currency and in operations subjected to exchange fluctuation.
- \[ \sum_{i=1}^{N} PJUR_i \] , quota concerning the risk of operations subjected to interest rates fluctuation and classified in the Trading Book according to the Resolution 3.464/07, in which \( n \) is the number of different quotas related to the risk of operations subjected to interest rates fluctuation.
- **PCOM** = Quota concerning the risk of operations subjected to the fluctuation of commodities prices.
- **PACS** = quota concerning the risk of operations subjected to the fluctuation of the price of stocks classified in the Trading Book according to the Resolution 3.464/07.
- **POPR** = Quota concerning the operational risk

(d) Circular 3.361/07: Establishes the procedures for the calculation of the quota of the Required Reference Asset concerning the exposures subjected to the fluctuation of pre-fixed interest rates denominated in real (PJUR [1]), dealt in the Resolution n. 3.490, 2007.

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(e) Circular 3.362/07: establishes the procedures for the calculation of the quota of the Required Reference Asset concerning the exposures subjected to the fluctuation of the foreign currencies coupons (PJUR [2]), dealt in the Resolution n. 3.490, 2007.

(f) Circular 3.363/07: establishes the procedures for the calculation of the quota of the Required Reference Asset concerning the exposures subjected to the fluctuation of the price index coupons (PJUR [3]), dealt in the Resolution n. 3.490, 2007.

(g) Circular 3.364/07: establishes the procedures for the calculation of the quota of the Required Reference Asset concerning the exposures subjected to the fluctuation of the interest rates coupons (PJUR [4]), dealt in the Resolution n. 3.490, 2007.

(h) Circular 3.365/07: Concerns the measurement of interest rates risk in the operations not classified in the Trading Book.

(i) Circular 3.366/07: establishes the procedures for the calculation of the quota of the Required Reference Asset concerning the risk of the exposures subjected to the fluctuation of stock prices (PACS), dealt in the Resolution n. 3.490, 2007.

(j) Circular 3.367/07: Consolidates the procedures for the calculation and the formulation of the information concerning the follow-up and the control of exposure in gold, foreign currencies and in assets and liabilities subjected to exchange fluctuation, on consolidated bases.

(k) Circular 3.368/07: establishes the procedures for the calculation of the quota of the Required Reference Asset concerning the exposures subjected to the fluctuation of commodities prices (PCOM), dealt in the Resolution n. 3.490, 2007.

Appendix II: Equations

Accumulated_inflation(t) = Accumulated_inflation(t - dt) + (Readjustments) * dt

INIT Accumulated_inflation = 1

INFLOWS:
Readjustments = PULSE (Inflation/100,12,12)*Accumulated_inflation
Accumulated_inflation_2(t) = Accumulated_inflation_2(t - dt) + (withdrawl) * dt
INIT Accumulated_inflation_2 = 1

INFLOWS:
withdrawl = if time <Months_of_application then 0 else (pulse (Inflation/100,(Months_of_application+12),12))*Accumulated_inflation_2
Liquid_Assets_Values_________l[ Poupança](t) =
Liquid_Assets_Values_________l[ Poupança](t - dt) + (Dividends[ Poupança] +
Total_Investments___________\[ Poupança] - Withdrawl_1[ Poupança]) * dt

INIT Liquid_Assets_Values_________l[ Poupança] = 0

Liquid_Assets_Values_________l[Fundo_Curto_Prazo](t) =
Liquid_Assets_Values_________l[Fundo_Curto_Prazo](t - dt) +
(Dividends[Fundo_Curto_Prazo] +
Total_Investments___________\[Fundo_Curto_Prazo] -
Withdrawl_1[Fundo_Curto_Prazo]) * dt

INIT Liquid_Assets_Values_________l[Fundo_Curto_Prazo] = 0

INFLOWS:

Dividends[ Poupança] =
Liquid_Assets_Values_________l[ Poupança] * (Interest_rates_2[ Poupança] -
Mensal_Inflation)

Dividends[Fundo_Curto_Prazo] =
Liquid_Assets_Values_________l[Fundo_Curto_Prazo] * ((Interest_rates_2[Fundo_Curto_Prazo] -
Mensal_Inflation)) * 0

Total_Investments___________\[Poupança] = IF TIME>Months_of_application
THEN 0 ELSE Adjustment__of_the_deposit[ Poupança]

Total_Investments___________\[Fundo_Curto_Prazo] = IF
TIME>Months_of_application THEN 0 ELSE
Adjustment__of_the_deposit[ Fundo_Curto_Prazo] * 0

OUTFLOWS:

Withdrawl_1[ Poupança] = IF TIME<Months_of_application THEN 0 ELSE
Withdraw_Value[ Poupança]

Withdrawl_1[Fundo_Curto_Prazo] = IF TIME<Months_of_application THEN 0 ELSE
Withdraw_Value[ Fundo_Curto_Prazo] * 0

Actual_Age = 40

Adjustment__of_the_deposit[ Poupança] = if Deposits_corrected_by_inflation=1 then
(Deposit_initial_value[ Poupança] * Accumulated_inflation) else
Deposit_initial_value[ Poupança]
\[
\text{Adjustment of the deposit[Fundo Curto Prazo]} = \text{if Deposits corrected by inflation=1 then Deposit initial value[Fundo Curto Prazo]*Accumulated_inflation else Deposit initial value[Fundo Curto Prazo]*0}
\]

\[
\text{Age = Actual Age+(TIME)/12}
\]

\[
\text{Alert = IF Total Payments+1 <Intended Withdraw AND TIME >Months of application THEN 0 ELSE 1}
\]

\[
\text{Amount alert = Monetary Assets Accumulation and Maturity- Withdrawl_1[Poupança]+Dividends[Poupança]}
\]

\[
\text{Annual inflation = 12.6825}
\]

\[
\text{Deposits corrected by inflation = 0}
\]

\[
\text{Deposit initial value[Poupança] = 1000}
\]

\[
\text{Deposit initial value[Fundo Curto Prazo] = 0}
\]

\[
\text{Difference = DIVIDENDO_TOTAL-Total Payments}
\]

\[
\text{DIVIDENDO TOTAL = ARRAYSUM(Dividends[*])}
\]

\[
\text{Inflation = NORMAL(Annual inflation, Inflation volatility/100*Annual inflation)}
\]

\[
\text{Inflation volatility = 10}
\]

\[
\text{Intended Withdraw = 1000}
\]

\[
\text{Interest rates[Poupança] = 0}
\]

\[
\text{Interest rates[Fundo Curto Prazo] = 0}
\]

\[
\text{Interest rates 2[Poupança] = NORMAL (Interest rates[Poupança]/100, (Interest rates volatility Savings of[Poupança]/100)*(Interest rates[Poupança]/100 ))}
\]

\[
\text{Interest rates 2[Fundo Curto Prazo] = NORMAL (Interest rates[Fundo Curto Prazo]/100, Interest rates volatility Savings of[Fundo Curto Prazo]/100)*(Interest rates[Fundo Curto Prazo]/100))}
\]

\[
\text{Interest rates volatility Savings of[Poupança] = 0}
\]

\[
\text{Interest rates volatility Savings of[Fundo Curto Prazo] = 0}
\]

\[
\text{Mensal Inflation = (((Inflation/100)+1)^((1/12))-1)
\]

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Monetary_Accumulation_and_Maturity = ARAYSUM(Liquid_Acuments_Values[*])

Month = time

Months_of_application = 120

Notice_of_the_amount = MAX(1,
(Monetary_Accumulation_and_Maturity/(Difference+0.001)))

Savings_Withdraw = 100

Total_Payments = ARAYSUM(Withdrawl[*])

withdrawl_updated_by_inflation = 0

Withdraw_Value[Poupança] = if Time<Months_of_application then 0 else if
withdrawl_updated_by_inflation=0 then Intended_Withdraw*(Savings_Withdraw/100)
else Intended_Withdraw*(Savings_Withdraw/100)*Accumulated_inflation_2

Withdraw_Value[Fundo_Curto_Prazo] = if time< Months_of_application then 0 else if
withdrawl_updated_by_inflation=0 then Intended_Withdraw*(1-Savings_Withdraw/100)
else Intended_Withdraw*(1-Savings_Withdraw/100)*Accumulated_inflation_2
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FSA (Finantial Service Authority). Available at: http://www.fsa.gov.uk/about/what/international/basel.


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Ricardo Matos Chaim obtained a PhD degree in Information Science at the University of Brasilia, Brazil. Nowadays, he is an Associate Professor at the Software Engineering Department of the University of Brasilia. He also worked for 19 years at a Brazilian Governamental Social Insurance Company. His current scientific interests center upon information management, risk management, Systems Engineering, information technologies applied to Insurance and methods to model risks, uncertainty and imprecision in financial organizations and Pension Funds.

Murilo Castellano, MsC, is an Associate Professor of the Getúlio Vargas Foundation, Brasilia, specialized in market and credit risks, worked as an Auditor and Risk manager at Banco do Brasil. He is the risk manager of Bancoob. His current scientific interests center upon derivatives market, enterprise risk management, quantitative methods applied to finance, mathematical analysis and other financial and stock market themes.