The Reward System in Art Markets: A System Dynamics Approach

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Abstract

This paper shows that the tools of system dynamics theory can be methodologically useful to provide new insights into art and cultural economics. Specifically, it develops a model of how the reward system in art markets works and cause imbalance allocation of revenues and recompenses. With this, it also proves that system dynamics it is useful to model self-organizing systems.

1. Introduction

In a recent paper Buendía (2012) shows that the distribution of auction revenues of artists follows Zipf's law. That the distribution of artists' income is Zipf distributed poses a real challenge to conventional economic theory and represents a fundamental question for the nascent field of art and cultural economics. Using the tools of system dynamics, this paper address this issue and develops a system dynamics-based model of the ways in which the art market functions to allocate rewards to artists. It proceeds as follow. Next section reviews briefly the consequences of the Mathew effect and other increasing-returns mechanisms on the reward system in sciences and other economic activities. Section 3 analyzes the empirical evidence about the self-organizing nature of the distribution of revenues in art markets. Section 4 describes the relations of the actors of the art world. In the fifth section, a system dynamic model of the reward system of art markets is advanced. The paper ends with some conclusions.

2. The Matthew Effect and other Related Sources of Increasing Returns

The phenomenon of Superstars described by Rosen (1981) is that wherein relatively small number of people earn enormous amount of money and dominate the activities in which they engage. According to Rosen (1981), in certain kinds of economic activity related to the worlds of arts, sports, letters, show business, science, and many other professions, there is concentration of output among a few individuals, a marked skewedness in the associated distribution of income, and a tendency to concentrate very large rewards at the top. Rosen (1981) attaches two common elements to all of them: first, an intimate relationship between personal rewards and the size of one's own market; and second, a strong tendency for both market size and rewards to be skewed towards the most talented people in the activity.

Merton (1968) goes beyond Rosen (1981), and in his already classical article "The Mathew Effect in Science", provided an insightful *dynamic* conception about the allocation of rewards to scientists. He asserted that "eminent scientists get disproportionately great credit for their contributions to science while relatively unknown scientists tend to get disproportionately little credit for comparable contributions" (Merton, 1968: 57). Merton called the complex patterns of misallocation of credit for scientific work "the Matthew

Effect", for the Gospel According to Saint Matthew, which puts it this way: "For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken away even that which he hath". The Matthew effect, specifically, consists in the psychosocial processes that not only affect the allocations of rewards to scientists for their contributions but also the flow of scientific ideas and findings through the communication networks of science. With this, universities of demonstrated scientific excellence are allocated far larger resources for research and attract the most talented students and researchers than universities which have yet to make their mark.

The Matthew effect consists in the accumulation of recognition for particular scientific contributions to scientists with of considerable reputation and the withholding of such recognition from scientists who are not renowned. It is clear, therefore, that in the Matthew effect there are *increasing returns* working through a circular and cumulative causation between reputation and recognition that explains the unbalance allocation of rewards. The Matthew effect play an important role in the reward and communication system of science, but it can also be used as a principle of cumulative advantage that operate in many systems of social stratifications that produce the same results: the rich get richer to a rate that makes the poorer become relatively poorer.

3. Self-organization of Rewards and Increasing Returns in Art Markets

There are other studies about the uneven allocation of rewards[1], however very few empirical and theoretical studies have been done to address the rewards allocation system in art markets. Recently, however, Buendía (2012) provides empirical evidence about the stardom phenomenon by analyzing the nature of dispersion of auction income among painters. Specifically, using the publicly available data on art market trends in terms of annual auctions sales published by the firm Artprices, he shows that the distribution of income of artists by auctions is self-organizing.

In self-organizing systems, initially, there are unnoticeable differences, but over time those small differences become magnified through a process of self-reinforcement,

[1] See, for instance, Frank and Cook (1995), in their *The Winner-Take-All Society*, discuss the contemporary trend toward concentration of wealth. He argues that more and more of the current economy and other institutions are moving toward a state where very few winners take very much, while the rest are left with little. They attribute this, in part, to the modern structure of markets and technology.

which produce large-scale skewed distributed patterns. The most apparent characteristic of self-organizing systems is that their size distribution usually follows Zipf's Law. This law was introduced by the Harvard University linguistic Professor George Kingsley Zipf (1945, 1949), who asserted that the relationship between the size of data value and its rank approximately follows the law:

$$S_r r = k \tag{1}$$

where S_r is the element of the set of data ranked r in a list of elements ordered by size, beginning by the largest. To visualize how Zipf's law behaves, we take the elements of a data set and order them by size. We then draw a graph; on the x-axis we place the natural log of the size, S_r, and on the y-axis the natural log of the rank, r, of the corresponding element ($s_1 > s_2 > \ldots s_N$). If we run a regression, we obtain:

$$\ln r = \beta_1 + \beta_2 ln S_r + \varepsilon_1 \tag{2}$$

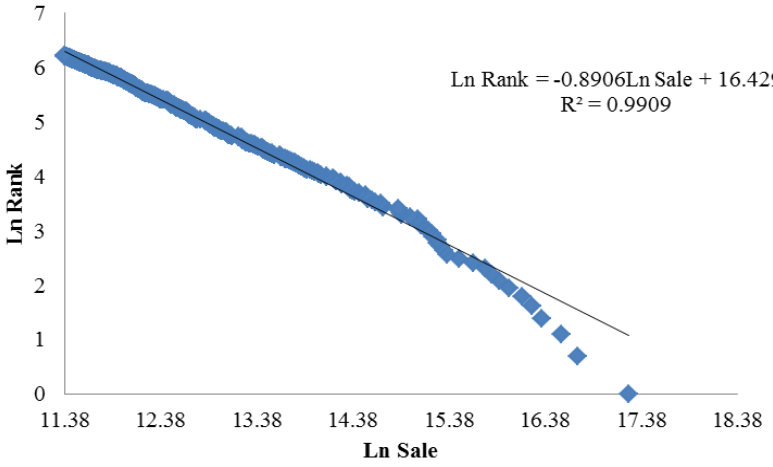
If the slope of the regression line (β_2) is close to – 1, then this data set is said to follow a Zipf distribution. If the slope of the regression line (β_2) is different from -1, it may follow a generalized Zipf distribution:

$$S_r r^q = k \tag{3}$$

As further test on the robustness of these results, Buendía (2012) repeated his analysis for annual auction sales data from 2002 to 2011 and he found that during this period the overall distribution of income of artists remained unchanged. That the distribution of income of artists by auctions is Zipf-distributed implies that artists' auction revenues are highly skew, such that small numbers of artists producing high revenues coexist alongside lager numbers of artists with smaller revenues. An example of the results that Buendía (2012) obtained is shown in Figure 1. In this figure the slope of the regression line relating the logarithm of artists' rankings and the logarithm of the revenues that the artists obtained in the 2010 auctions is close to 0.9, so it follows Zipf's law. These results

clearly show that the rewards system of art markets is subject to a process of self-reinforcement or, to put it in economic terms, strongly determined by *increasing returns*.

Figure 1. Zipf Distribution of Artists' Income in 2010 Auctions



A theoretical reference to understand why the rewards system of art markets is subject to increasing returns is provided by Buendía (2007)[2]. He argues that understanding the nature of art auctions is not enough to discover the causes of the formation of prices and the performance of art markets. In his view art auctions have to be seen as a part of a larger system where artists, galleries, dealers, museums, auction houses, art critics, art historian, experts, and other actors *interact* to try to eliminate their informational asymmetries. Therefore, the formation of prices can become evident by meticulously examining the interactions of the actors of the *art world*. In what follow a brief summary of the interaction among art actors is provided.

The artist's primary incentive to create is to satisfy himself, but he also tries to please his public and patrons to get recognition, rewards, revenues. Because of this, the artist embarks on a number of activities, in which dealers, galleries, museums, art critics and historian are involved. When well-established commercial galleries and museums — that require the abilities of professional curators— show the artist's artwork, it can achieve official and public recognition and reach specific collectors. The dealer's activity achieves recognition if the artwork is bought by important individual and institutional collectors and

[2] This description of the art markets' actors and the model of section 4 describing their interaction draw heavily form Buendía (2007).

attracts the attention of critics. National and international awards reinforce the artist's success. The artist whose work fits with the chosen trend of curators or art critics of major art journals attracts more attention.

Writing about art can take many forms, from simple labels that accompany an exhibit to historical analysis and philosophical discussions on the merits of a particular style and articles in popular art magazines and catalogues, or academic journals. Archival research and management of visual media have been recently added to the writing process. For many viewers art analysis brings structure to experiences, so writing mirrors the art and is itself part of the creative process. Objects acquire value in so far as they are inscribed in history. The work of art critics and historians is also important to both the creation and the marketing of art, because it is the equivalent of research and development process within a business organization. Their presence serves as a reminder that certain aspects of the human spirit still struggles against becoming commodity.

Another important component of the art world is the museum. Its main mission is to take art to the public. The heart of the museum from a curatorial perspective is its permanent collection, which provides it with authority, an authority that is rarely challenged. Consequently, the curators organizing the display hold the visitor hostage to their taste, standards, and objectives. Therefore, museums' caretakers and curators are very powerful players in determining public taste. The research and development essential for the commercial success of an artist, or the establishment of a style, is carried out by curators within museum setting.

Art dealers and galleries are small family businesses where everything from bookkeeping to liaising between collectors and artists is done by the owner. They make efforts to assure that collectors purchase art objects, so educating consumers is critical. Gallery owners must therefore become knowledgeable about the art object and have interest in promoting scholarship and establish art trends. Dealer or gallery owners also kindle interest in the artwork by encouraging critics and journalist to review them, and sometimes they demand the expertise of a curator and historians to further enhance their sale. Gallery owners have transformed from being artists spaces of an earlier decade into collectors

space. They mimic museums by the art they show, but their mandate is to sell art. Galleries have helped to remove the immediate connotation of art as commodity.

Dealers price art according to the following considerations: the type of art, the newness of the artist, the aesthetic innovations, and historical factors. Artwork with patina command high prices, largely based on historical evaluations, but newer works give the dealer more leeway. Strategies of survival include the following criteria: differentiating products from other sellers; assuming high risks when establishing new styles in the marketplace; promoting the international standing of the artist; and exporting the artist's work abroad. Commercial galleries may chose to promote the works of a few artists whom they cultivate over time, or they may identify potential stars and promote them extensively. They may also encourage a higher traffic of viewers by promoting exhibits space as theaters that encourage viewer involvement and learning.

A couple of decades ago high-prices art would be sold by dealers; now much of it is done by established auction houses. Sotheby's paved the way for marketing art by communicating to consumers that its business was educating them. Christie's soon adopted the same strategy for selling art. Auction houses often promulgate the view that art should not be viewed as an investment, yet they actually merchandise it. Buyers represent the auction houses' main source of income, so their strategy reflects a consumer orientation. Because of this, a large portion of their daily operations is driven by public relations, press realizes, and customer service. Sotheby's publishes the Art Market Index, which provides the novice viewer with information about the value and sales of artwork. Both Sotheby's and Christie's evaluate artwork free of charge, carefully develop marketing plans, auction catalogs, and advertising campaigns, and offer parties to their buyers.

New collectors represent a very important market segment because they are willing to pay for high-priced items. Even the reappearance of a work shortly after it is sold is no longer deemed catastrophic. New buyers who are often less knowledgeable about the object's sale history, find it a bargain, and often delight at making a purchase. Seasoned buyers, on the other hand, often demand the complete price history of an object before the actual purchase. Auction houses, like other business, know how to hedge the information to satisfy their customers. Collectors often believe that auction prices are the test for the fair

market value of an artwork and that the most quantitative information available on the art market is produced by auctions houses.

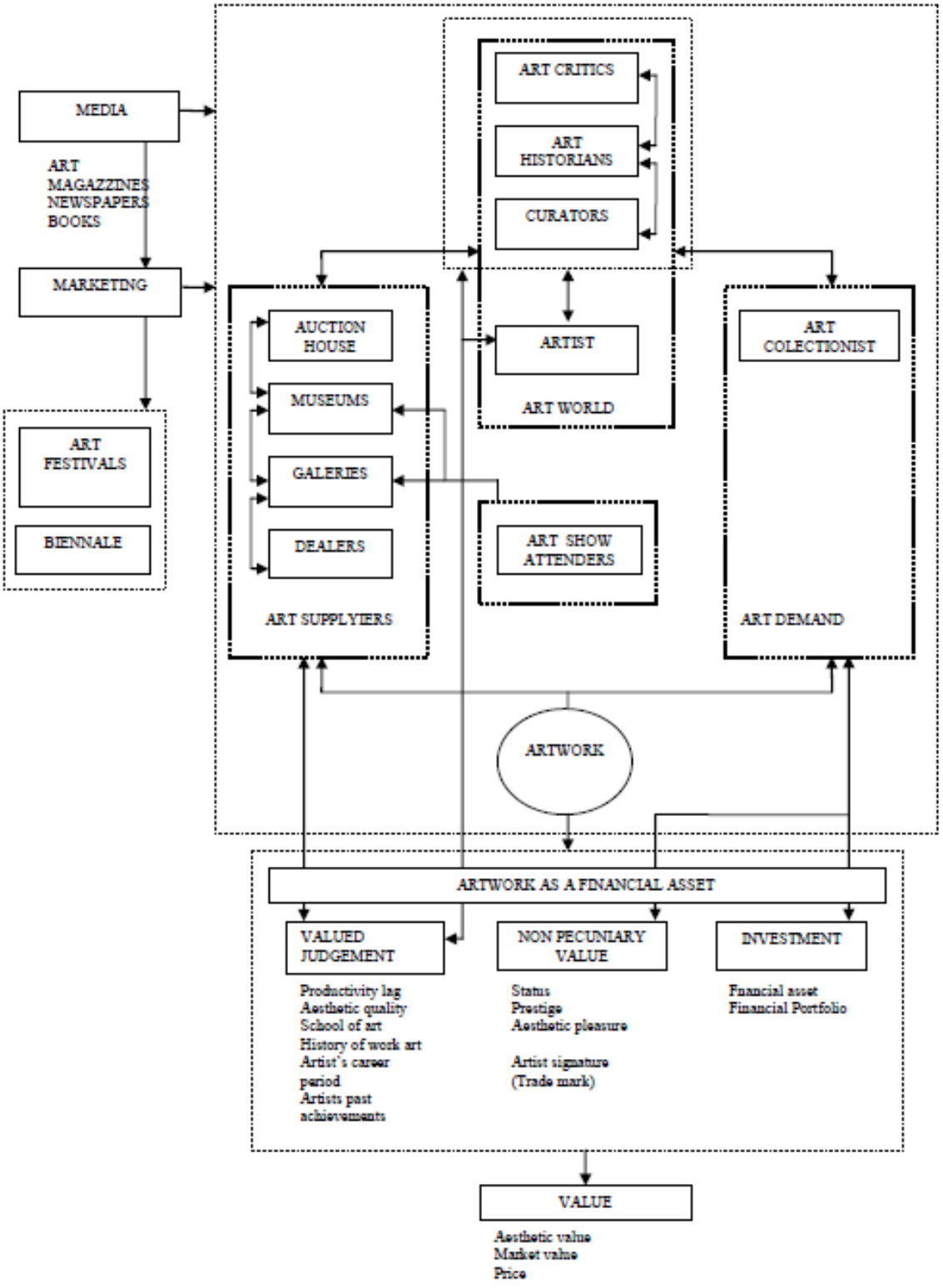
Museums, art studies, annual or biennial, international fairs, international art shows (which usually bring together artists, critics, dealers, gallery representatives, dealers, and academics from all over the world, and that are organized by artists rather than well-known curator) are the most important mechanism for delivering art to the public and render artists renown. They are rather complex events that involve the organization, export of the artworks, remuneration of the artist, costs of insurance, framing, catalogue creation and display.

A work of art will ultimately find a home with a private collector, in a museum vault or on a corporate wall. The value of an art work increases over time, and when it changes hands, this is often read as a sign of the future demand for the work. Aesthetic responses are an individual's emotional responses triggered by viewing the work. The sense of surprise, joy, or sorrow that it provides is important to both viewer and artist. The artist hopes that there is some form of dialogue between the artwork and viewers. Likewise, viewers hope to be moved by what they are viewing.

Artist responses require knowledge of artistic techniques and history of the artwork. For instance, an individual collector may not collect a significant work because of lack of knowledge of art history. A museum may collect it because it marks a certain historical transition. Aesthetic innovations and historical provenance are important criteria for museum collections. Many new collectors are economically motivated and purchase art in much the same way as they would buy stock and bonds.

Certain objects are collected by the upper class because of their monetary value and snob appeal. The elite of a society, on the other hand, may consider art collecting as a perpetual pursuit of inessential luxury goods. Viewed positively, collecting is a process of making connections, of bringing order into a disordered world, and creating the world in your own image. This activity adds excitement to life, providing individuals with a sense of mastery, expertise, and accomplishment. Collections are tangible markers of the past, and a source of meaning and knowledge of the self.

Figure 2. Actor of the Art World



The ultimate goal of many artists is to have museums collect their artwork because they can ensure their value and provide a passport to fame and a citation for posterity. The

power of museums lies in their acquisitions and in the complex narrative within which these institutions are embedded. The knowledge-based writing of curators protects any artistic work from the threat of commodification. This very knowledge production may be used to strengthen the brand image of a particular artist.

If a product is not recognized as new by other artists it will not be exhibited. If artists' work (new products) do not fit into the narrative of museum curators and art historians, they are more than likely to be bypassed. Category-defying products pose a special challenge for authenticity as well as for adoption, while proximal rather than profound difference may speed adoption or certifications, the reverse may be the case for aesthetic value. By the same token, older styles can be resuscitated and revitalized by emergent narratives long after the artist are dead. Thus, the aura of the artwork lives on.

In Figure 2 the eventual mutual dealings among the actors of the art world are summarized graphically. Such relations, as Buendía (2007) suggests, determine the incentives and behavior of each actor and are the base for his formal model of the art market.

4. The Art Market: A Formal Model

What was said in the previous sections is expressed formally by Buendía (2007), as follows. The buyers of art are wealthy collectors and art consumer who can afford to purchase a unique artwork. In any society there is a small group of wealthy individual who are interested and willing to buy artworks in the primary art market. Therefore, from the whole set of individuals of a society, $J = \{1,2,3.....N\}$, just a small subset $L = \{x \mid x$ having the property $W\}$ belong to this exclusive group. Given that W is the minimum wealth necessary to become a consumer of the primary art market, therefore the individual who has an income $W_i \geq \overline{W}$ can belong to the subset L.

The utility function of the art collector or consumer i depends on his personal wealth, W_i, but he can spend part of this wealth in artworks. What he invests in artworks can be represented by $\sum w_i^c$ where w is the price he paid by the artwork i which belongs to

the collection *c*. now the utility function of the individual depends on W_i and $\sum w_i^c$. But if $\sum V_i^c$, the future value of his collection, changes, then total wealth of individual *i* is

$$W_i^{t+1} = W_i^1 - \sum w_i^c + \sum V_i^c \quad (1).$$

This implies that if $\sum w_i^c \succ \sum V_i^c$, then the wealth of the individual *i* will decrease, but if $\sum w_i^c \prec \sum V_i^c$ it will grow. Because the motives of individual buyers in acquiring art are likely to range across a spectrum from demand for art purely as decoration to demand for art purely as asset, such a hypothesis about average differentials is likely to cover a wide variation in individual behavior. If the art buyer i has only economic motivations to acquire artwork, then his utility function is:

$$U_i = f\left[\left(W_i - \sum w_i^c + \sum V_i^c\right)\right] \quad (2).$$

If this consumer sees artworks as financial asset, he has to choose a set of artworks that assures that $\sum w_i^c \prec \sum V_i^c$ to maximize his utility, otherwise he would incur in loses and his behavior would irrational.

But the utility function of the individual *i* may also depend on R_i , the level of status and prestige, sophistication, recognition, and status he reaches with the consumption of artworks. Obviously, there is a positive relationship between R_i and $\sum w_i^c$. That is to say, the bigger the number of artworks he owns, the larger his prestige becomes. Furthermore, if the individual attaches an aesthetic benefit to the purchase of artworks, then he may obtain a benefit, $B_i\left(\sum w_i^c\right)$, so his utility function is:

$$U_i = f\left[\left(W_i - \sum w_i^c + \sum V_i^c\right), R_i\left(\sum w_i^c\right), B\left(\sum w_i^c\right)\right] \quad (3).$$

Equation (3) covers all the possible decisions of art consumers. If the consumer attaches no aesthetic value to the artworks, just a pecuniary value, then R_i and B_i are equal to zero, so his utility function depends only on $W_i^{t+1} = W_i^1 - \sum w_i^c + \sum V_i^c$. But if he attaches a non-pecuniary to the artworks he has bought, then his utility function depends on R_i and B_i , as well. Equation (3) can account for the differential between the return of art

assets and the return of other assets that we observe in the art market. This implies that if in the future $\sum w_i^c \succ \sum V_i^c$, the consumer of art *i* loses part of his wealth, but it can be compensated by the benefit he obtains from R_i and B_i.

The price of the artwork *i* of the collection *c*, w_i^c, depends on the value of previous work of the artist, w_{i-1}^{c-1}, and the productivity of the artist in the period in which he produced the artwork *i* of the collection c, P_i^c. Besides, w_i^c also depends on m_i^c / M and me_i^c / Me. The former ratio refers to the number of times (m_i^c) the artwork has been shown in the most famous museums and galleries in relation to the total number of exhibits (M). me_i^c / Me is the number of times the media, especially through art press releases, has mention the artwork with respect to the total number of press releases about art. The price of the artwork is given by:

$$w_i^c = f\left[w_{i-1}^{c-1}, P_i^c, \left(m_i^c / M \right), \left(me_i^c / Me \right) \right] \tag{4}.$$

Concerning the auction house, it tries to maximize its benefits according to the following equation:

$$\pi_s = f\left[\sum w_i^c (K, M, S, Me) - \sum C_i^c \left\{ cf, d(M, K, Me, e, G), Ar\left(n_w / a, N_i^c / M, \sum V_i^c \right) \right\} \right] \tag{5}$$

where K is the critic who has writing about the artworks, M the museum where the art pieces have been shown, S is the auctioneer, *cf* is the fixed costs of the organization of the auction, *d* is the dealer's reputation, G is the gallery where the artworks come from, *Ar* is the artist whose work is being auctioned, *a* is the age of the artist, n_m is the number of pieces the artist has produced, N_i^c is the galleries where the artwork of the artist have been shown.

Galleries have similar incentives than auction houses, so their profit function is:

$$\pi_g = f\left[\sum w_i^c (K, M, S, Me) - \sum C_i^c \left\{ cf, d(M, K, Me, e, G), Ar\left(n_w / a, N_i^c / M, \sum V_i^c \right) \right\} \right] \tag{6}$$

Museums have a little different incentive, so their reputation, E_m is given by:

$$E_m = f\left[\sum A_i^c\left(K, {n_m}/{M}, S\left(r_i^m\right), \left({n_m}/{M}\right), Au, {b_m}/{D}, U_H^i, Ar_i^m\right)\right] \tag{7}$$

where U_H^i is the utility of a high income consumer.

The artist is supposed to be more motivated by the recognition his work obtains than the pecuniary aspect of his work, so the success of artist *i*, A_i, is given by:

$$Ar_i = f\left[\left(K, {n_i^m}/{M}, {s_h^i}/{S}, S\left(Ar_i^m\right), Au_{Me}, \left({n_i^m}/{M}\right), {g_i}/{G}, U_H^i\right)\right] \tag{8}$$

where ${s_h^i}/{S}$ is the percentage of times his work has been auctioned in auction houses with respect to the total of auctions, Au_{Me} is the success of the media, $\left({n_i^m}/{M}\right)$ is the percentage of exhibits in museum in which the artist has shown his work, ${g_i}/{G}$ is the number of times the artist has sold his work in successful galleries and U_H^i is the utility of costumers.

The success of the media depends can be described by the following equation:

$$Au_{Me} = f\left[\left(K, {n_i^m}/{M}, {s_h^i}/{S}, S\left(Ar_i^m\right), \left({n_i^m}/{M}\right), {g_i}/{G}, U_H^i\right)\right] \tag{9}$$

Equations (1) through (9) describe in a general way the relationships and interactions that can take place when actors of the art world make decision to achieve their own goal. Such interaction, in turn, determine the process of the formation of prices and the performance of art markets, which —as it was said in the previous section— are characterized by a Zipf distribution of revenues.

5. The Reward System in the Art Markets as Generic System

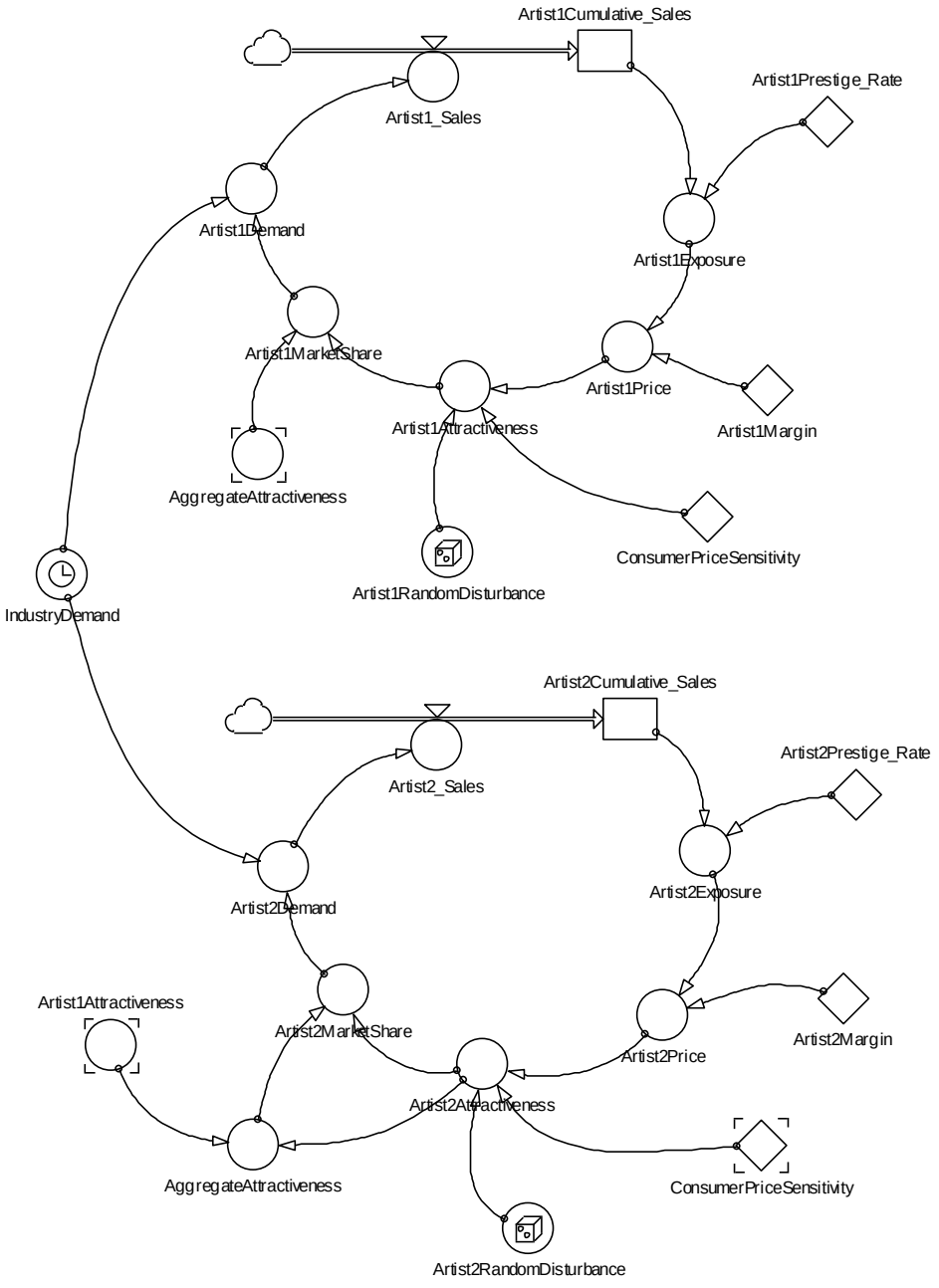
The results of the reward system in art markets follow Zipf's law and to great extend this phenomenon can be attributed to the interaction among the actors of the art world. If an artist enters into this sociological mechanism of interactions among the actors of the art

world, it is probable that this artist obtain a disproportionate recognition for his work vis-à-vis another artist producing comparable pieces of art. But it is possible to go further into the process of modeling the art market by outlining the sociological dynamic process that affect the reward system of art markets and translating such process into system dynamics model.

To put it in a simple way, the reward system in art markets consists in the increasing of recognition and revenues for particular artworks to artists of considerable prestige and the withholding of such recognition from artists who produce comparable artworks but who are not well-known. This double effect of obtaining greater revenues by famous painters and suppressing recognition to the unknown artist can be modeled through a cumulative process where there is a mutual causality between the demand for the work of an artist and the prestige she obtain for the exposure she gets from the others actors of the art world (museums, galleries, curators, historians, an do on). As figure 2 shows, this mutual causality passes through the increases in prices that the artist gets and attractiveness individual and institutional collectors perceive from her work. Attractiveness, in turn, affects the artist's market share, whose behavior is useful to prove this model reproduces the empirical evidence provided in section 3.

In terms of economic methodology, this system dynamics model of art markets can be considered a "generic structure". According to Radzicki (2003), a generic structure is a model that captures the fundamental causal relationships that appear in a variety of pattern models within a particular category. When properly parameterized, a generic structure can mimic the behavior of any pattern model within its category. System dynamic theory has identified a large number of regularities that appear in their patter models and generic structures, and/or that define accurate modeling practices. These regularities form the core of the system dynamics paradigm and can be thought of as a set of "principles of systems". The most fundamental and important of these principles is that the stock-flow-feedback-limiting factors structure of a system causes its dynamic behavior. This principle is identical to the notion of circular and cumulative causation, which is one of the core ideas of notions such as the Matthew effect.

Figure 2. Rewards Systems in Art Market as a Generic Structure

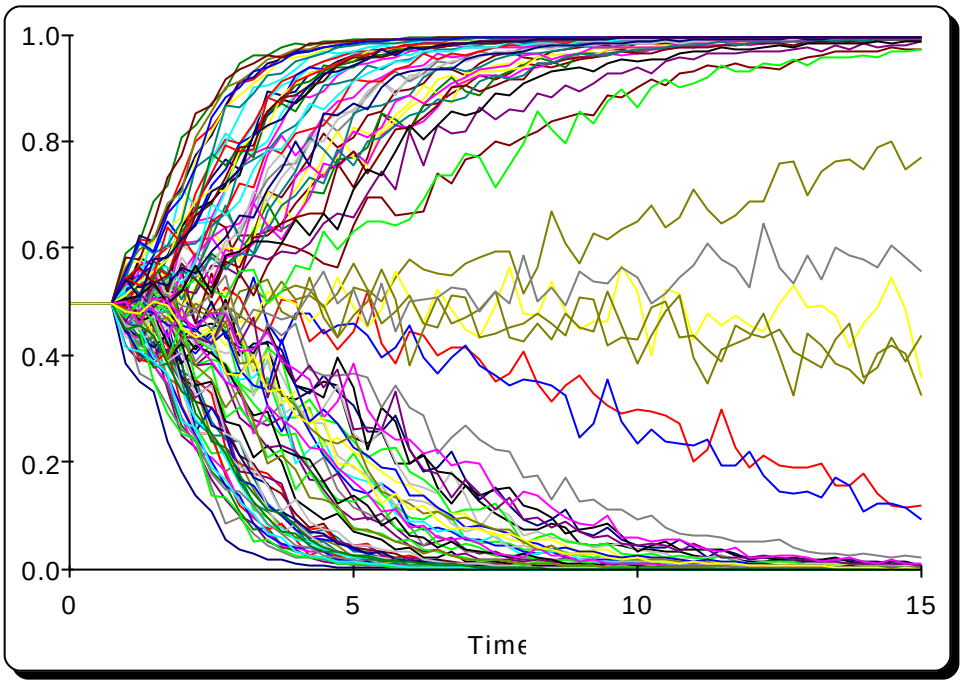


The generic structure of Figure 2 is a simple system dynamics model of the reward system in art markets that exhibits path dependence and self-organization. The model consists of two painters producing different but comparable pieces of art and competing for sales. Although figure 2 shows only two artists to simplify the analysis, this model can be generalized to an art markets where there are a large number of artists. The model has flows

which change the stocks. Artist 1 has a stock of cumulative sales that is increased by her sales. This stock of sales is increased by the exposure Artist 1 has obtained, which has a positive effect on the prices of her artworks. This is obviously a positive loop that is connected to her artworks attractiveness, market share, demand, and flow of sales. The second loop is similar to the first one.

Figure 3 shows 50 simulations of the evolution of market shares for two artists and that in many of the simulation the results are very close to the empirical evidence provided in section 3. In these simulation it not clear who is absolute winner, but what it is clear it is that the results are path dependent and self-organizing. This model is a good approximation to the functioning of the reward system of art market, for it provides outcomes similar to the ones observed in real art markets. This model also proves that the application of system dynamics to art and cultural economics can provide more realistic explanations of challenging empirical evidence, such as the self-organizing nature of artists' revenues.

Figure 3. Market Share evolution in 50 simulations from the System Dynamic Model of Art Markets



6. Conclusions

Art and cultural economics is a new field of economics. This new economic branch can benefit from new empirical and theoretical studies. One important empirical fact that art and cultural economics has to explain is why the distribution of revenues of artists follows Zipf's law. This is not only a major challenge to conventional economics but also a fundamental question for art and cultural economics. This paper discusses the relevance of the tools of system dynamics theory to develop a model of the reward system of art markets capable to explain the self-organizing nature of artists' rewards. The amazing aspect of this model is that it reproduces very well the distribution of rewards as we observe it in reality.

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