# Control Heuristics for Soft Landing Problem[1]

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***Abstract***
*In this paper, we developed two different control heuristics for the soft landing problem. The first heuristic is adapted from the mass spring damper model using the similarity of the equations of the soft landing model given in this paper to the equations of the mass spring damper model; both models can be reduced to a second order linear differential equation. The second one is a bang-bang heuristic that first allows the spacecraft to fall freely, but after a critical point is reached, it uses the reverse force thruster at its maximum power until the touchdown. Bang-bang heuristic minimizes the time needed to land. However, it may crash the spacecraft in the presence of an error in the parameter estimates, or an error in the velocity or height readings, or an overlooked factor such as a delay in changing the level of the force created by the reverse force thruster, which is known as actuator delay. The mass spring damper based control heuristic requires a longer landing time, but it is more robust compared to the bang-bang control heuristic in the sense that it is less sensitive to the errors in parameter values, errors in readings, and presence of an actuator delay.*

**Keywords:** soft landing; spacecraft; control heuristic; mass spring damper; bang-bang; error in parameter estimates; error in readings; actuator delay.

# 1. Introduction

In some cases, landing on the surface of a celestial body is a part of a space exploration program. In such cases, soft landing becomes a problem to be addressed. A reasonable landing process requires a control heuristic that will ensure *the safety of the spacecraft*, which practically means a soft touchdown of the spacecraft to the surface of the celestial body at the end of the landing process. Note that the crash force (the force created at the time of touchdown) is a complex result of the crash velocity (the velocity with which the spacecraft touches the surface), the landing gear parameters of the

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spacecraft, the mass of the spacecraft, and the gravitational force. Out of these four important components that determine the crash force, a control heuristic can only have an effect on the crash velocity. Moreover, this effect is indirect. Control heuristic determines the control force, control force results in the net force, net force determines the acceleration, acceleration gradually adjusts the velocity, and the value of velocity at the time of touchdown becomes the crash velocity. Therefore, it’s not an easy task to manage the crash velocity at around a desired level. Moreover, the heuristic that will be employed should also manage the length of the time needed to land at a reasonably low value because a long landing duration requires extensive fuel usage. The two criteria, minimizing the crash velocity and minimizing the length of the time needed to land, are contradictory, which makes the soft-landing problem a challenging task. A control heuristic aiming to satisfy the two criteria, should allow the vehicle descend to the surface rather quickly, but make it decelerate safely to low velocity values before the instant of landing (Liu, Duan, and Teo, 2008; Zhou et al., 2009). The landing dynamics of Apollo 15 is an example of this strategy (Figure 1).

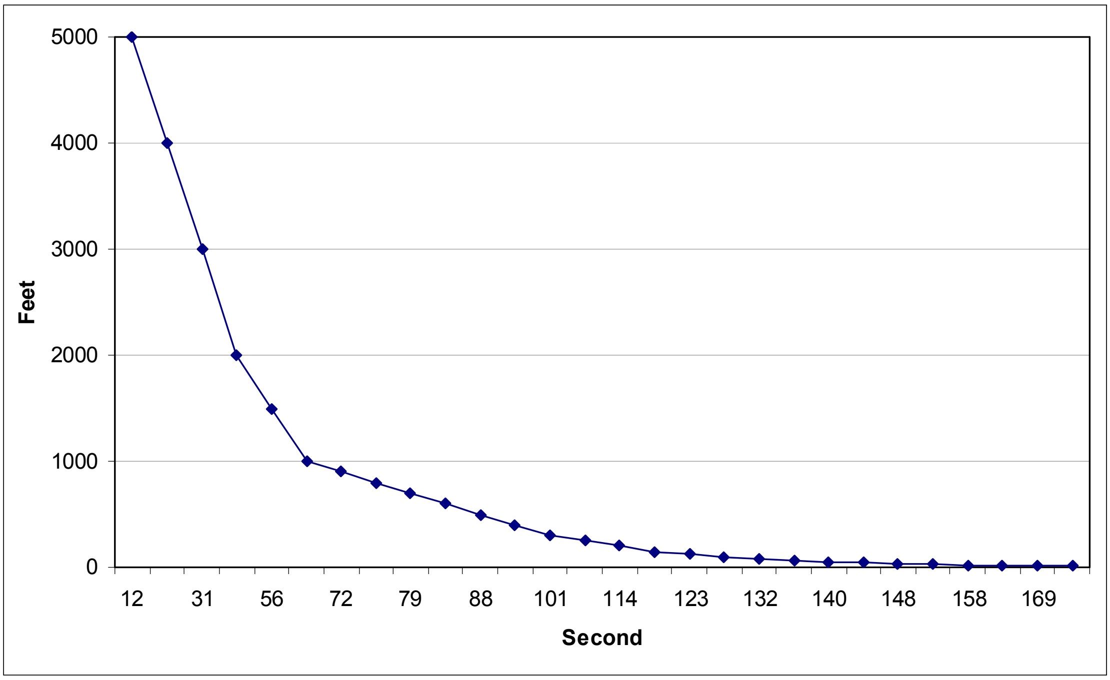


Figure 1: The landing dynamics of Apollo 15

In plotting the dynamics observed in Figure 1, we connected to the Apollo 15 entry of the Wikipedia website (http://en.wikipedia.org/wiki/Apollo_15; accessed on 16 September 2011) and time coded the landing video on the page (http://en.wikipedia.org/wiki/File:Apollo_15_landing_on_the_Moon.ogg; accessed on 16 September 2011). Note that Apollo 15 was the fourth to land on the Moon (30 July 1971).

In this paper, we first presented the stock-flow diagram and the equations of the soft landing model. Later, we developed two control heuristics; a mass spring damper based control heuristic and a bang-bang control heuristic. The mass spring damper based control heuristic is adapted from the mass spring damper model using the similarity of the equations of the soft landing model given in this paper to the equations of the mass spring damper model; both models can be reduced to a second order linear differential equation. The bang-bang heuristic dynamically calculates a critical point. It first allows the spacecraft to fall freely, but after the critical point is reached, it uses the reverse force thruster at its maximum power until the touchdown.

The behaviors obtained from the two control heuristics are also presented and discussed in the paper. Bang-bang heuristic minimizes the time needed to land under the assumed conditions. However, this aggressive management of the time needed to land may make it crash the spacecraft under problematic conditions. We tested the performances of the two heuristics in the presence of an error in the parameter estimates; in the presence of an error in the velocity or height readings; and in the presence of an overlooked factor such as a delay in changing the level of the force created by the reverse force thruster, which is known as actuator delay. The mass spring damper based control heuristic requires a longer landing time, but it is more robust compared to the bang-bang control heuristic in the sense that it is less sensitive to the errors in parameter values, errors in readings, and presence of an actuator delay.

## 2. The Model Structure and Equations

In this study, we first constructed a stock-flow model of the soft-landing problem, which is given in Figure 2. This diagram represents only the physical structure of the problem described in the previous section; it does not represent the controller (e.g. a human decision maker, a computer). *Height* and *Velocity* are the two stock variables in the model. *Velocity*, which is a stock variable, is at the same time the one and only flow of *Height*. *Velocity* has a single flow too; *Acceleration*. *Height* is controlled via *Velocity*, *Velocity* via *Acceleration*, *Acceleration* via *Net Force*, and *Net Force* via *Control Force* (equations 1-7)[2]. The control feedback loop also includes the controller (Figure 3), which determines *Control Force* of the reverse force thruster via *Desired Control Force*. Note that the natural inputs to the controller are *Height* and *Velocity*.

$$Height_0 = 1000 \quad [m] \tag{1}$$

[2] One Newton amounts to the force needed to increase the velocity of a one kilogram body of mass by one meter per second in one second ( $\mathrm{N = kg \cdot m / s^2}$ ).

$$Height_{t+DT} = Height_t + Velocity_t \cdot DT \quad [m] \tag{2}$$

$$Velocity_0 = -10 \quad [m/s] \tag{3}$$

$$Velocity_{t+DT} = Velocity_t + Acceleration \cdot DT \quad [m/s] \tag{4}$$

$$Acceleration = Net\ Force / Mass \quad [m/s^2] \tag{5}$$

$$Mass = 1000\ [kg] \tag{6}$$

$$Net\ Force = -Gravitational\ Force + Damping\ Force + Control\ Force \quad [N] \tag{7}$$

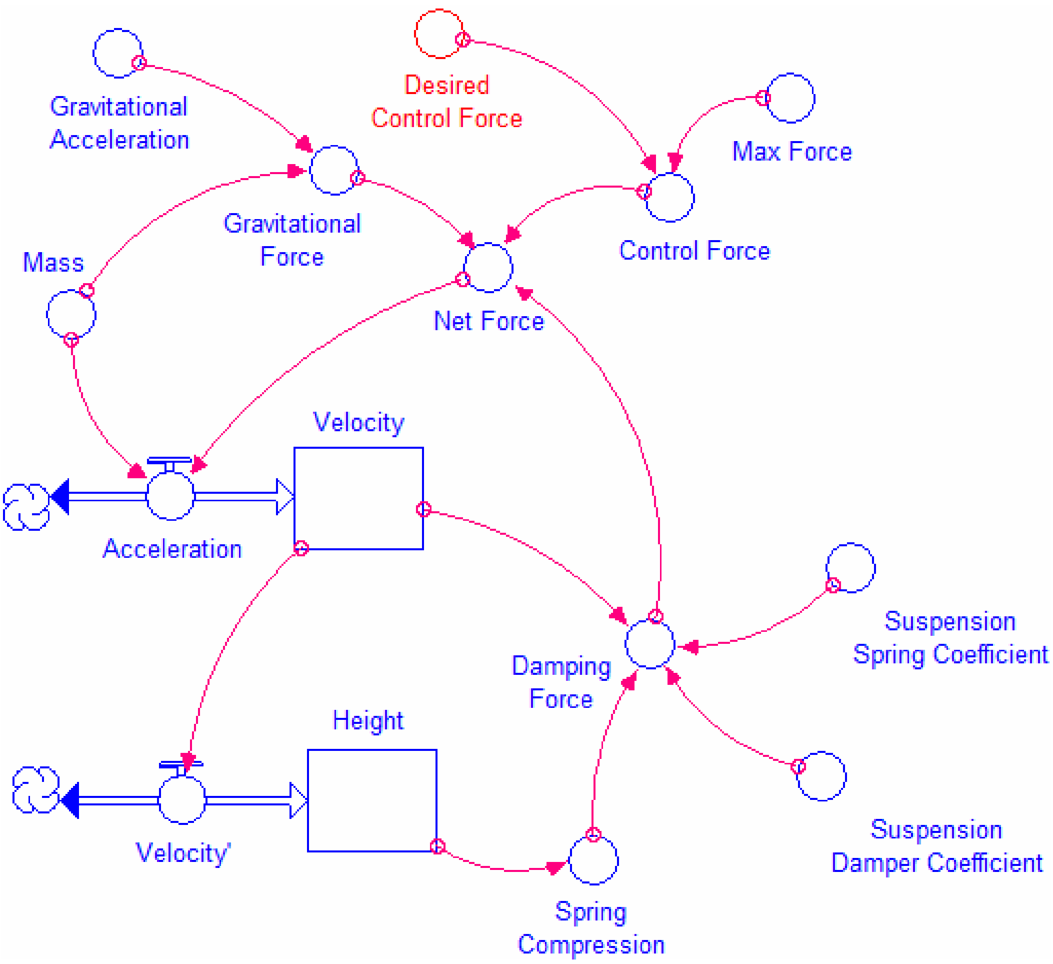


**Figure 2**: Simplified stock-flow diagram of the soft landing model

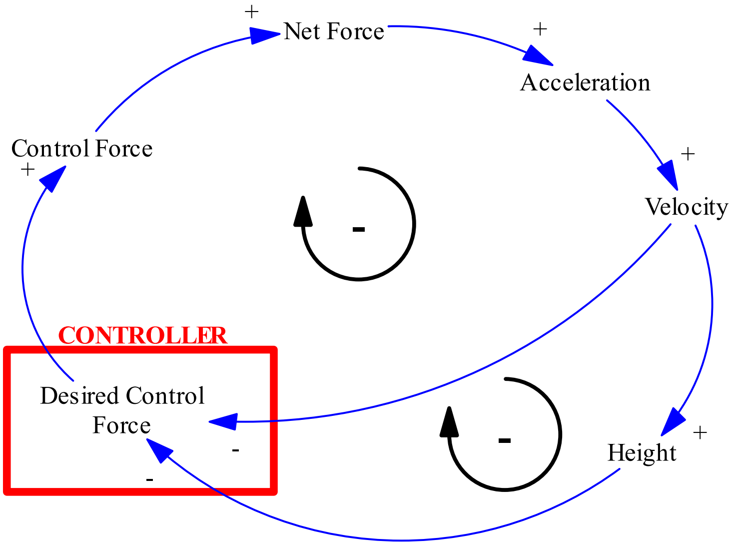


**Figure 3**: Causal-loop diagram of the control feedback loop structure

**The simplifying model assumptions are given below:**

- The movement of the spacecraft in the horizontal axes is not modeled. Spacecraft is assumed to move only vertically.
- Positive *Height*, *Velocity*, *Acceleration*, and force directions are upward from the surface.
- There is no atmosphere in the landing area, thus no air friction exists that would cause a drag force on the vehicle.
- *Gravitational Acceleration* is assumed to be constant during landing, it does not change with the distance to the surface.
- *Mass* is a constant, the change in the mass due to fuel consumption is ignored.
- There are no delays caused by actuators; *Desired Control Force* generated by the controller affects *Control Force* without a time lag.
- Information flow from the system to the controller is perfect and instantaneous; There are no errors or delays caused by measurement processes.
- Upon touching the ground, the thruster is off and is not switched on again. The simplified model diagram in Figure 2 and Equation 8 do not reflect this assumption.

The aim of this paper is not to discuss the modeling process. Moreover, by giving the simplified version of the model in Figure 2, we aim to improve the readability of the manuscript and prevent digression. If more details are needed, see Yasarcan and Tanyolaç (2012).

The rest of the model equations follow:

$$\textit{Control Force} = \begin{Bmatrix} 0, & \textit{Height} \leq 0 \\ \textit{Desired Control Force}, & \begin{cases} \textit{Height} > 0, \\ \textit{Desired Control Force} \leq \textit{Max Force} \end{cases} \\ \textit{Max Force}, & \text{otherwise} \end{Bmatrix} [N] \quad (8)$$

$$\textit{Max Force} = 30{,}000\ [N] \quad (9)$$

$$\textit{Gravitational Force} = \textit{Mass} \cdot \textit{Gravitational Acceleration}\ [N] \quad (10)$$

$$\textit{Gravitational Acceleration} = 8.87\ [m/s^2] \quad (11)$$

$$\textit{Damping Force} = \begin{Bmatrix} 0, & \textit{Spring Compression} = 0 \\ \begin{pmatrix} \textit{Suspension} \\ \textit{Spring} \\ \textit{Coefficient} \end{pmatrix} \cdot \begin{pmatrix} \textit{Spring} \\ \textit{Compression} \end{pmatrix} - \begin{pmatrix} \textit{Suspension} \\ \textit{Damper} \\ \textit{Coefficient} \end{pmatrix} \cdot \textit{Velocity}, & \text{otherwise} \end{Bmatrix} [N] \quad (12)$$

$$\textit{Spring Compression} = \begin{pmatrix} 0, & \textit{Height} \geq 0 \\ -\textit{Height}, & \text{otherwise} \end{pmatrix} [m] \quad (13)$$

$$\textit{Suspension Spring Coefficient} = 17{,}740 \quad [N/m] \quad (14)$$

$$\textit{Suspension Damper Coefficient} = 2{,}803 \quad \left[\frac{N \cdot s}{m}\right] \quad (15)$$

# 3. A Mass Spring Damper Based Control Heuristic

The stock-flow model given in Figure 2 represents only the physical structure of the soft landing problem. However, the simulated behavior discussed in the previous section is generated by the model including the suggested mass spring damper based control heuristic, which is assumed to be used by the controller (Figure 3) in producing the values for *Desired Control Force*. The aim of this section is to present the formulations of this heuristic.

Yasarcan and Barlas (2005) uses a procedure in developing control heuristics for control problems involving information delay or indirect control via a secondary-stock. This procedure adapts a well known successful heuristic for control problems involving material supply line delay, using the similarity of the differential equations of control problems involving different types of delay structures. The model presented in this paper can be reduced to a second order linear differential equation because it contains two stock variables, which are defined by approximate integral equations (Equation 2 and Equation 4). The mass spring damper model is well studied and it is known how to obtain a certain behavior by adjusting the model parameter values. Furthermore, it can also be represented by a second order linear differential equation. Utilizing an approach similar to the approach of Yasarcan and Barlas (2005), we developed a heuristic based on the similarity of the differential equations of the mass spring damper model and the model presented in this paper[3].

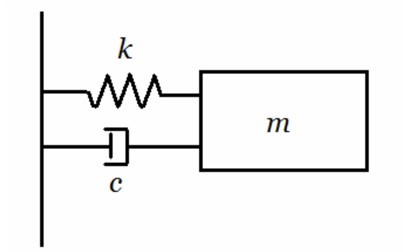


**Figure 4**: Mass spring damper schematic

The schematic given in Figure 4 is a well known one. The differential equation of a non-driven (i.e. $F_{external} = 0$) mass spring damper model with mass *m*, spring constant *k*, and damper coefficient *c* is given below:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = 0 \qquad (16)$$

In Equation 16, $x$ represents displacement, $\dot{x}$ represents velocity, and $\ddot{x}$ represents acceleration. This equation can be described by using stock-flow concepts, $x$ and $\dot{x}$ being the stocks and their associated flows being $\dot{x}$ and $\ddot{x}$ respectively. Note that $\dot{x}$ is a flow and a stock at the same time. As a further clarification, $-k \cdot x$ is the spring force ($F_{spring}$) and $-c \cdot \dot{x}$ is the damper force ($F_{damper}$). The net force applied on the body of mass is the sum of these two forces ($F_{net} = F_{damper} + F_{spring} = -c \cdot \dot{x} - k \cdot x$). According to Newton's second law of motion mass times acceleration is equal the net force acting on the body ($F_{net} = m \cdot \ddot{x}$). Therefore, mass times acceleration is equal to the sum of the spring force and damper force. Hence, Equation 16 is obtained.

[3] The authors of this paper acknowledge that it is Dr. I. Emre Köse who suggested us to use the mass spring damper model for this purpose.

The damping ratio ζ of the mass spring damper model defined by Equation 16 is:

$$\zeta = \frac{c}{2 \cdot \sqrt{m \cdot k}} \tag{17}$$

The dynamics of the mass spring damper model can be underdamped, overdamped, or critically damped depending on the value of the damping ratio ζ. For ζ values under 1, the dynamic behavior is underdamped and for ζ values over 1, it is overdamped. The case where the damping ratio ζ is exactly 1 is called critically damped. When the dynamic behavior is underdamped, the spring dominates the movement and the body oscillates. In the critically damped case, the body asymptotically approaches the rest condition without an overshoot. In the overdamped case, the damper dominates the dynamics and the body approaches the rest condition slower compared to the critically damped case (Åström and Murray, 2008). As a summary, the importance of ζ is that determining its value determines the dynamics of the mass spring damper model.

The suggested control heuristic is adapted from the mass spring damper model that is defined by Equation 16. *Height*, *Velocity*, *Acceleration*, and *Mass* in our model corresponds to $x$, $\dot{x}$, $\ddot{x}$, and $m$ in Equation 16, respectively. In the heuristic, we named $k$ as *Height Coefficient* and $c$ as *Velocity Coefficient*. Thus, Equation 16 becomes:

$$Mass \cdot Acceleration + \begin{pmatrix} Velocity \\ Coefficient \end{pmatrix} \cdot Velocity + \begin{pmatrix} Height \\ Coefficient \end{pmatrix} \cdot Height = 0 \tag{18}$$

Utilizing Newton's second law of motion, the following can be written:

$$Desired\ Net\ Force = -\begin{pmatrix} Velocity \\ Coefficient \end{pmatrix} \cdot Velocity - \begin{pmatrix} Height \\ Coefficient \end{pmatrix} \cdot Height \quad [N] \tag{19}$$

The reverse force thruster should also counteract *Gravitational Force*. Hence, *Desired Control Force*, which is the output of the heuristic and an input to *Control Force* (see Equation 8 and Figure 2), can be given as:

$$Desired\ Control\ Force = Desired\ Net\ Force + Gravitational\ Force \quad [N] \tag{20}$$

The parameters of the adapted heuristic, *Height Coefficient* and *Velocity Coefficient* values are set to 10 $[N/m]$ and 200 $[N \cdot s/m]$, respectively. Consequently, the damping ratio ζ for our model becomes:

$$\zeta = \frac{Velocity\ Coefficient}{2 \cdot \sqrt{Mass \cdot Height\ Coefficient}} = \frac{200}{2 \cdot \sqrt{1000 \cdot 10}} = 1 \tag{21}$$

The value of the damping ratio means that the suggested control heuristic produces a critically damped behavior for the height of the spacecraft.

### 3.1. Selection of the Controller Parameters

Decreasing *Control Force Damping Factor* (CFDF) shortens the landing duration and increases the final velocity (See Figure 5). Long landing durations and also great final velocity values should be avoided. Therefore, a CFDF value with a reasonable landing duration and final (crash) velocity should be selected.

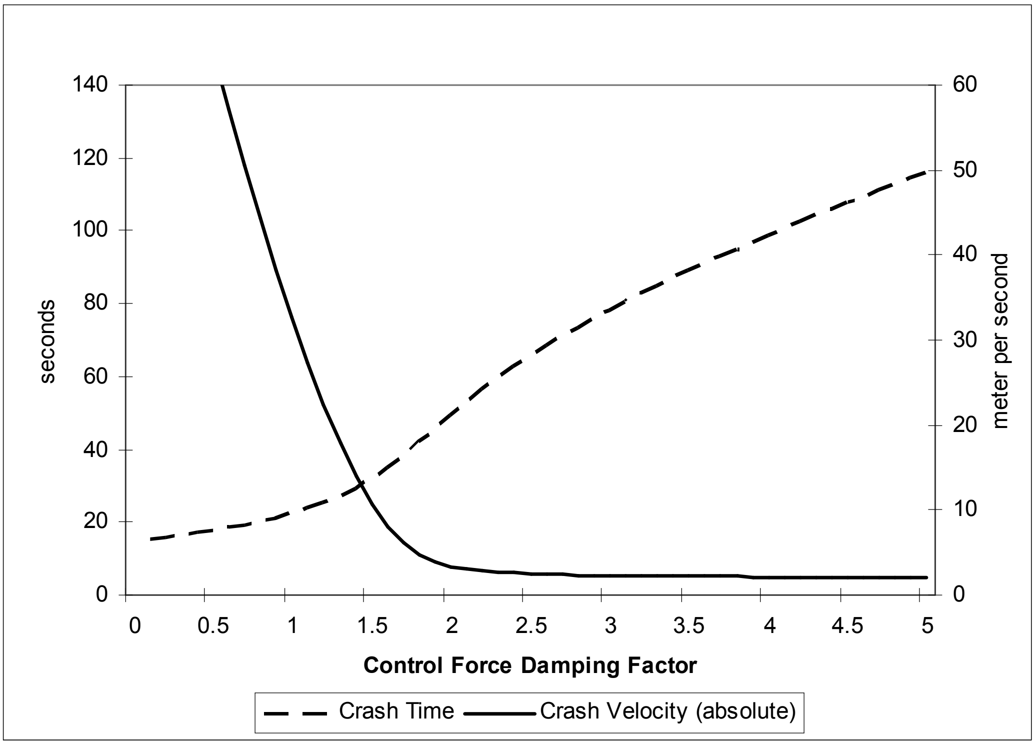


**Figure 5**. *Crash Time* (the time of touchdown) and the absolute value of *Crash Velocity* (the velocity at the time of touchdown) variation with different *Control Force Damping Factor* values

The final velocity should be less than -10 m/s to be able to obtain a safe landing, so CFDF should minimally be 1.6. The CFDF value 2 has a special mathematical significance; it is the minimal value that makes the vehicle asymptotically[4] seek the ground level and is not affected by the initial conditions. Due to this mathematical property, CFDF is taken as 2.

### *3.2. Adjustment to the Mass Spring Damper Based Heuristic*

Two adjustments to the heuristic is necessary:

- As mentioned in the previous sub-section, there is a problem with the asymptotical approach; the vehicle continues to hover on the ground with a very small distance away from the surface. We corrected this problem by adding *Desired Final Velocity* to the heuristic. Note that Equation 19 implicitly assumes that the heuristic seeks *Velocity* = 0. Therefore, we replaced Equation 19 with Equation 22. The existence of a negative *Desired Final Velocity* makes the vehicle approach the ground level with an acceptable velocity. Hence, the problem of the infinite landing duration due to the asymptotical seek is avoided.
- The heuristic should stop engines at the time of first touchdown. We replaced Equation 20 with Equation 24 so that upon touching the ground, the thruster is off and is not switched on again. The variable *Landing State* is given in equations 25-26. Note that *Landing State* equations are valid for the bang-bang control heuristic too.

$$Desired\ Net\ Force = -\begin{pmatrix} Velocity \\ Coefficient \end{pmatrix} \cdot \left( Velocity - \begin{pmatrix} Desired\ Final \\ Velocity \end{pmatrix} \right) - \begin{pmatrix} Height \\ Coefficient \end{pmatrix} \cdot Height \quad [N] \tag{22}$$

$$Desired\ Final Velocity = -1.2 \quad [m/s] \tag{23}$$

$$\begin{pmatrix} Desired \\ Control \\ Force \end{pmatrix} = \begin{cases} \begin{pmatrix} Desired \\ Net\ Force \end{pmatrix} + \begin{pmatrix} Gravitational \\ Force \end{pmatrix}, & Landing\ \ State = 0 \\ 0, & Landing\ \ State = 1 \end{cases} \quad [N] \tag{24}$$

$$Landing\ State_0 = 0 \quad [\text{dimensionless}] \tag{25}$$

[4] To be mathematically correct, asymptotical seek of the goal takes indefinite time. This issue will be addressed in the next sub-section.

$$Landing\ State_{t+DT} =$$
$$Landing\ State_t + \begin{pmatrix} 1/DT, & Landing\ State_t = 0, Height_t < 0 \\ 0, & \text{otherwise} \end{pmatrix} \quad [\text{dimensionless}] \qquad (26)$$

### 3.3. Dynamic Behavior of Landing Obtained by Using the Mass Spring Damper Based Heuristic

As described in the previous section, *Height* is controlled via *Velocity* (Equation 2), *Velocity* via *Acceleration* (Equation 4), *Acceleration* via *Net Force* (Equation 5), and *Net Force* via *Control Force* (Equation 7). The control feedback loop also includes the controller, which determines *Control Force* applied by the reverse force thruster via *Desired Control Force*. In order to obtain a reasonable value for *Desired Control Force*, the controller should consider the system state variables (i.e. *Height* and *Velocity*). Only by doing so is it possible to reach the aim of landing the spacecraft as gently and as fast as possible. Even under the simplifying assumptions listed in the previous section, the control task remains a challenging one because it is quite difficult to appropriately consider the system state information in the decisions. The main reason for the difficulty is that the control task requires simultaneous control of *Height* and *Velocity*, which –due to the physical structure of the problem– can only be indirectly affected by the reverse force thruster; *Height* and *Velocity* have inertia; their values do not change instantaneously (see Figure 2 and equations 1-7).

The stock-flow model given in Figure 2 and defined by equations 1-9 describes the structure of the soft landing problem excluding the controller. The formulations of the heuristic suggested for the controller is explained in the next section. The dynamic behavior presented in figures 6-4 is generated by simulating the model including the controller with the proposed heuristic for 60 seconds (equations 1-13 and equations 19-20).

The dynamic behavior of *Height* is given in Figure 6. Initially, the change in *Height* (i.e. *Velocity*) is relatively fast and, as the spacecraft approaches to the surface, the change in *Height* slows down. Hence, the behavior obtained by the control heuristic is a reasonable one; by a fast initial decline, the heuristic tries to decrease the time to land; by a slow final approach, it keeps the impact force well below harmful values. At the instant of touchdown, the value of *Velocity* is -2.04 meters per second (-7.35 km/h) creating a maximum impact force of circa 14,782 Newton, approximately 1.67 times the weight of the spacecraft on the target celestial body (8,870 Newton). The weight corresponds to the model variable *Gravitational Force*, which is the force that the landing gear must bear when the spacecraft is standing still on the ground.

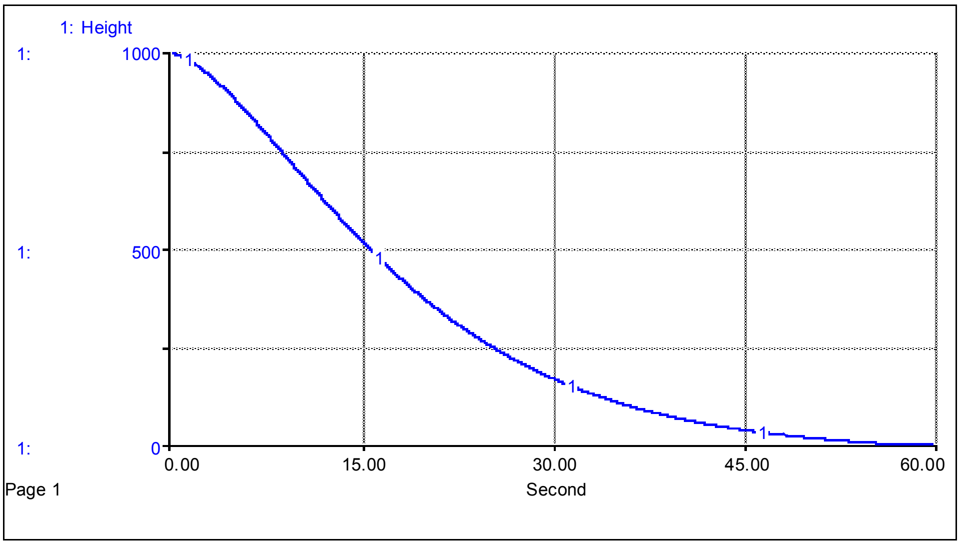


**Figure 6**: Dynamic behavior of *Height*

The dynamic behavior of *Velocity* and *Net Force* acting on the vehicle during landing are given in figures 7 and 8, which further explain the dynamic behavior obtained by the control heuristic. At first, the heuristic allows the spacecraft to accelerate in the negative direction towards the landing surface (see Figure 7, approximately within the time range of 0-10 seconds) by keeping *Net Force* negative (i.e. *Control Force* less than *Gravitational Force,* see figures 8 and 9). Aiming to decrease the duration of landing, *Velocity* continues to increase during this initial period. After this initial phase, *Velocity* decreases until the vehicle touches the surface (see Figure 7, approximately within the time range of 10-55 seconds). In this later phase, the heuristic produces more *Control Force* than *Gravitational Force* (Figure 9) resulting in a positive *Net Force* (Figure 8). At the moment of landing, *Control Force* is turned off and *Damping Force*, which is zero throughout the simulation up to this point, takes over and stops the vehicle (see figures 8 and 9, approximately around 55 seconds).

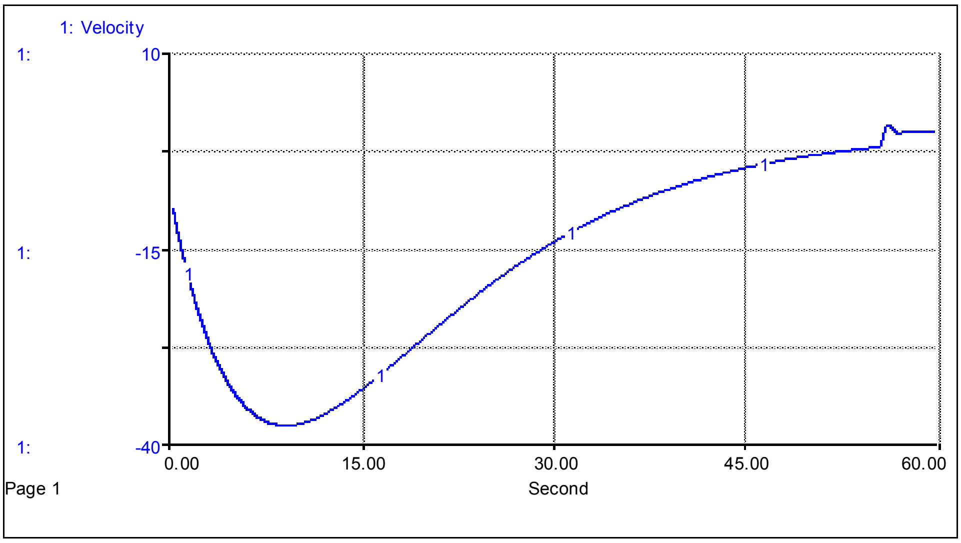


**Figure 7**: Dynamic behavior of *Velocity*

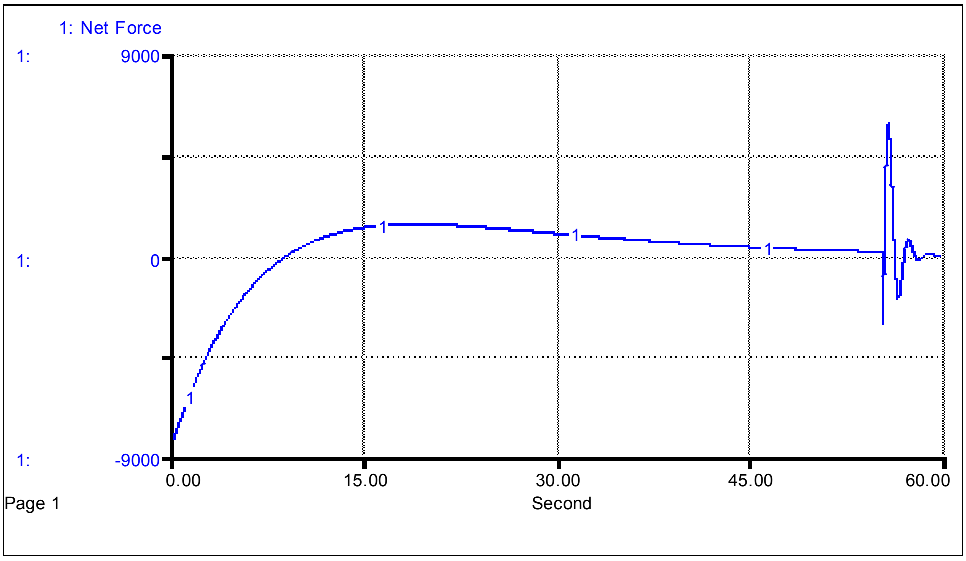


**Figure 8**: Net force acting on the vehicle during landing

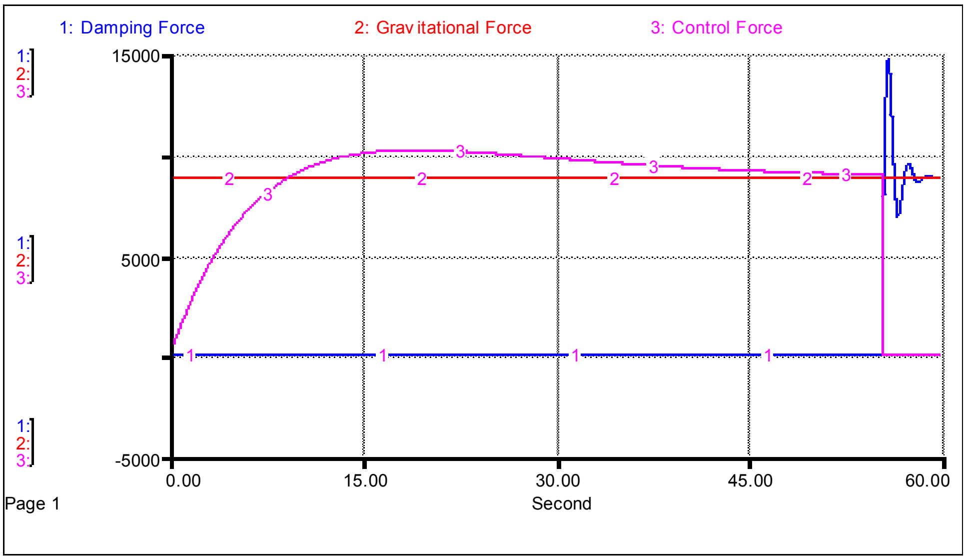


**Figure 9**: Absolute values of the forces acting on the vehicle during landing[5]

# 4. A Bang-Bang Control Heuristic

The bang-bang principle relies on the fact that a system can be controlled in minimal time using properly all available power throughout the whole control (LaSalle, 1960). Based on this principle, we developed a bang-bang control heuristic for our model. The purpose is to let the vehicle descend with only the effect of *Gravitational Force* in an accelerating fashion up to a point in time, and then apply the maximum possible force until touchdown. Note that; in our model, *Control Force* is in the positive *Height* direction and *Gravitational Force* is the only force in the negative direction that can pull the vehicle towards the ground. The maximum force generated by the reverse force thruster creates *Maximum Acceleration* (Equation 27). Actually, during the time the maximum force is applied, the spacecraft is moving towards the celestial body (in the negative direction). Therefore, *Maximum Acceleration* decelerates the negative speed of the vehicle to a desired level (Equation 28).

$$\begin{pmatrix} Maximum \\ Acceleration \end{pmatrix} = \left(Max\ Force - Gravitational\ Force\right) / \ Mass \ \left[m/s^2\right] \qquad (27)$$

$$Desired\ Final\ Velocity = -2 \quad \left[m/s\right] \qquad (28)$$

[5] In order to ease the comparison of the different forces acting on the vehicle, the directions of the forces are ignored on this diagram.

We call the time that *Max Force* is first applied as *Deceleration Start Time*. *Acceleration* in the positive direction (or deceleration in the negative direction) is a constant and equal to *Maximum Acceleration* (Equation 27) between *Deceleration Start Time* and *Crash Time* (the time of touchdown). The bang-bang heuristic dynamically decides when to use the reverse force thruster at its maximum power, by looking at current *Velocity*, current *Height*, *Maximum Acceleration*, and *Desired Final Velocity* values. In order to be able to bring the current *Velocity* to *Desired Final Velocity* at the time of touchdown, there should be a sufficient remaining distance between the vehicle and the surface of the planet (i.e. *Height*) for the given *Maximum Acceleration* and current *Velocity* values (Equation 29).

$$\textit{Desired Net Force} = \left\{ \begin{array}{ll} \textit{Max Force}, & \textit{Velocity}^2 - \begin{pmatrix} \textit{Desired} \\ \textit{Final} \\ \textit{Velocity} \end{pmatrix}^2 \geq 2 \cdot \textit{Height} \cdot \begin{pmatrix} \textit{Maximum} \\ \textit{Possible} \\ \textit{Acceleration} \end{pmatrix} \\ -\textit{Gravitional Force}, & \text{otherwise} \end{array} \right\} \; [N] \tag{29}$$

## *4.1. Derivation of Desired Net Force Equation*

Equation 29 is equivalent to Equation 30, where current *Velocity* corresponds to $v_0$, *Desired Final Velocity* to $v_1$, current *Height* to $\Delta x$, and *Maximum Acceleration* to $a$. Equation 31 is a natural result of conservation of mechanical energy (Serway and Faughn, 1989) and Equation 30 is obtained by dividing Equation 31[6] by mass.

$$v_o^2 - v_1^2 = 2 \cdot a \cdot \Delta x \quad \left[ m / s^2 \right] \tag{30}$$

$$\frac{1}{2} m \cdot v_o^2 - \frac{1}{2} m \cdot v_1^2 = m \cdot a \cdot \Delta x \; [J] \tag{31}$$

## *4.2. Dynamic Behavior of Landing Obtained by Using the Bang-Bang Heuristic*

The dynamic behavior of *Height* is given in Figure 10. Initially, the change in *Height* (i.e. *Velocity*) is relatively fast and, as the spacecraft approaches to the surface, the

[6] One Joule amounts to the work done by applying a force of one Newton through a distance of one meter. ( $\mathrm{J = N \cdot m = kg \cdot m^2 / s^2}$ ).

change in *Height* slows down. At the instant of touchdown, the value of *Velocity* is -3.28 meters per second (-11.81 km/h) creating a maximum impact force of circa 17,869 Newton, approximately 2.01 times the weight of the spacecraft on the target celestial body (8,870 Newton).

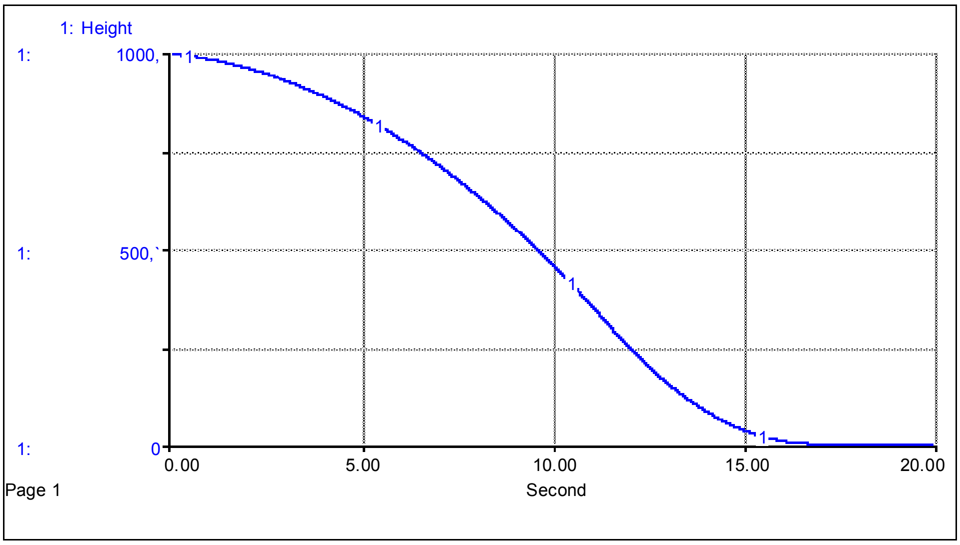


**Figure 10**. Dynamic behavior of *Height* in the Bang-Bang Heuristic

The dynamic behavior of *Velocity* and *Net Force* acting on the vehicle during landing are given in figures 11 and 12, which further explain the dynamic behavior obtained by the control heuristic. At first, the heuristic allows the spacecraft to accelerate with the effect of *Gravitational Acceleration* in the negative direction towards the landing surface (see Figure 12, approximately within the time range of 0-11 seconds) by keeping *Net Force* equal to *Gravitational Force* (see figures 12 and 13). Aiming to decrease the duration of landing, *Velocity* continues to increase during this initial period. After this initial phase, *Velocity* decreases until the vehicle touches the surface (see Figure 11, approximately within the time range of 11-17 seconds). In this later phase, the heuristic produces *Control Force* equal to *Max Force* (Figure 12) resulting in a positive *Net Force* (Figure 13). At the moment of landing, *Control Force* is turned off and *Damping Force*, which is zero throughout the simulation up to this point, takes over and stops the vehicle (see figures 12 and 13, approximately around 17 seconds).

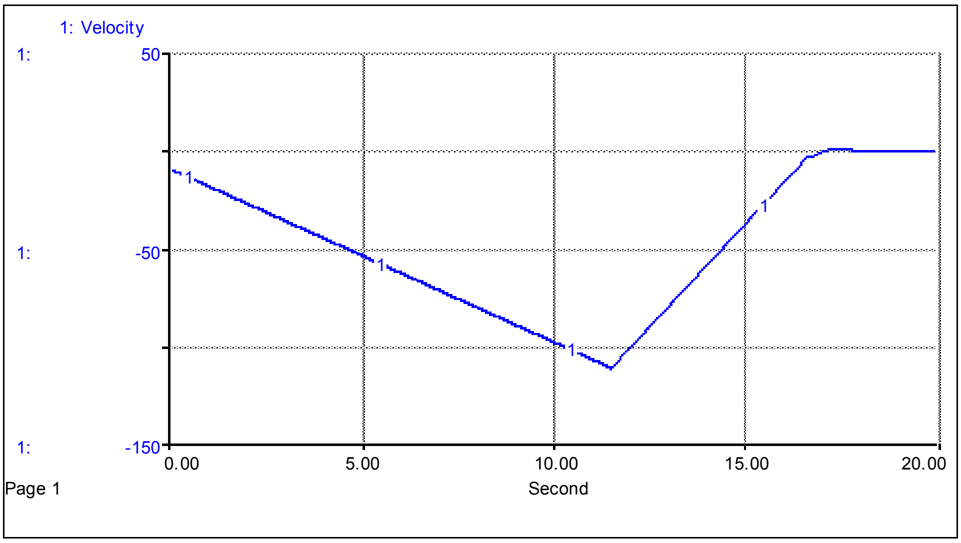


**Figure 11**. Dynamic behavior of *Velocity*

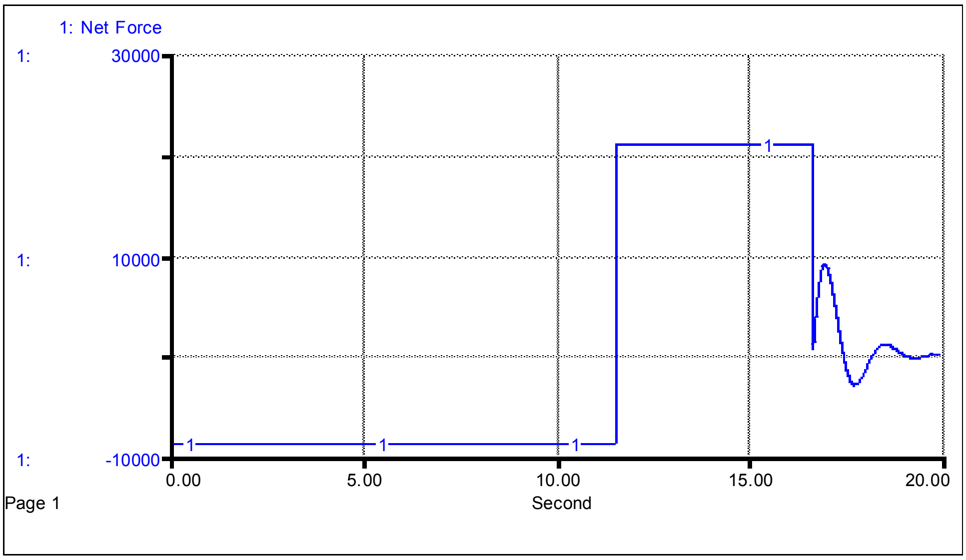


**Figure 12**. Net force acting on the vehicle during landing

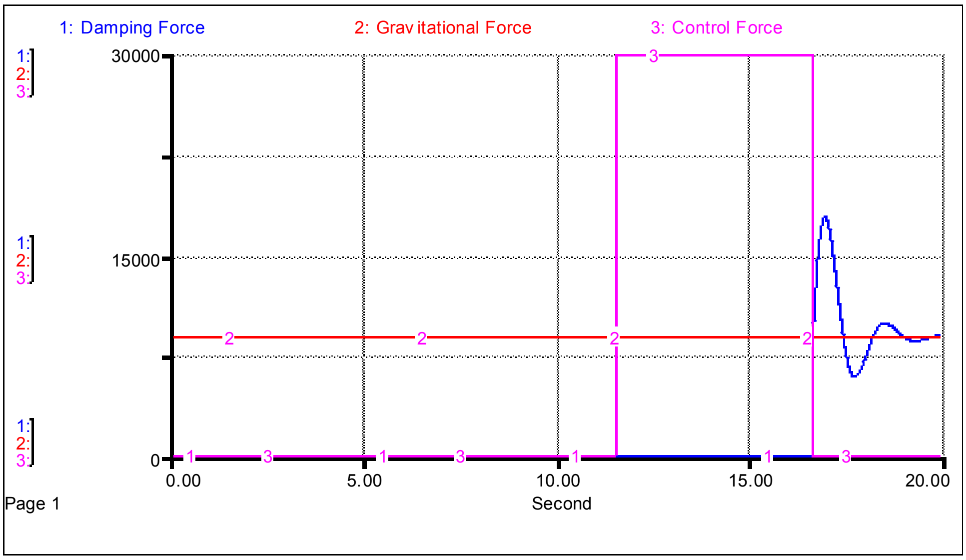


**Figure 13**. Absolute values of the forces acting on the vehicle during landing

## 5. The Comparison of the Two Control Heuristics

The mass spring damper heuristic (MSD) and the bang-bang heuristic presented in the previous sections have different characteristics. The differences between the two heuristics and the difference in the resulting behavior is explained in this section and a summary is provided in Table 1.

Table 1. Comparison of Mass Spring Damper and Bang Bang Heuristics

| | **MSD** | **Bang-Bang** |
|---|---|---|
| Changes in Control Force | smooth | catastrophic |
| Crash Velocity [m/s] | -2.04 | -3.28<br>(Numerical error is big. Theorethically, it should be -2.) |
| Max Landing Force [N] | 14782 | 17869<br>(Numerical error is big. Theorethically, it should be close to the result generated by the MSD based heuristic.) |
| Crash Time [s] | 55.46 | 16.66 |
| Sensitivity to errors in parameters | low | high |
| Sensitivity to variable readings | low | high |
| Sensitivity to Actuator Delay | low | high |

The qualitative comparison of the velocity figures 7 and 11 gives a preliminary insight to the difference in the smoothness of the control. Furthermore, the comparison of the net force figures 8 and 12 reveals that the bang-bang control heuristic makes a sudden jump in the force, whereas the mass spring damper heuristic changes force gradually. *Max Instantaneous Change in Force* (Equations 42-43)[7] quantify the momentary difference in force as 30,000 Newton in the bang-bang heuristic. In the mass spring-damper heuristic, however, *Max Instantaneous Change in Force* is very small and only exists due to the discrete nature of the simulation; it approaches zero as *DT* (simulation time step) goes to zero[8]. Therefore, changes in Control Force is smooth with the mass spring damper based control heuristic and catastrophic with the bang-bang heuristic.

*Crash Velocity* is another important criterion like *Crash Time,* and equations 34 and 35 are necessary for monitoring it. According to our simulation runs, the bang-bang heuristic and the mass spring damper heuristic landed with velocities of -3.28 m/s and -2.04 m/s, respectively. It is worth noting that *Crash Velocity* of the bang-bang heuristic should theoretically be equal to -2 m/s, which is the value of *Desired Final Velocity* (see Equation 28). Extreme forces used by the bang-bang heuristic results in bigger simulation errors compared to the mass spring damper based heuristic. As a summary, the two heuristics land the vehicle at the same speed.

The variable *Crash Time* (i.e. the duration of the landing process) captured by the performance measure equations 32 and 33 are about 55 seconds for the MSD heuristic and about 17 seconds for the bang-bang heuristic. This result was not a surprising one as the bang-bang heuristic is the minimum-time solution for our problem. Considering that minimizing the time to land is one of the main criteria, bang-bang heuristic seems very successful. However, the aggressive management of the time needed to land may make the bang-bang heuristic crash the spacecraft under problematic conditions.

### 5.1. An Error in one of the Parameter Values

To be able to compare the deterioration in the results, we assumed that the estimate of *Mass* used in the heuristics is 950 kg instead of 1000 kg. The dynamic behavior generated by the two heuristics in the presence of this error is given in figures 14 and 15. The *Crash Velocity* values for the MSD and bang-bang heuristics deteriorate to -2.21 m/s and -25.59 m/s, respectively. These values suggest that, in the case of a parameter estimation error, a great deterioration in the bang-bang heuristic occurs, whereas MSD succeeds in making a reasonable landing.

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[7] The performance measure equations are given in the appendix.

[8] In our simulations, we set DT (simulation time step) equal to $2^{-9}$ (1/512) seconds.

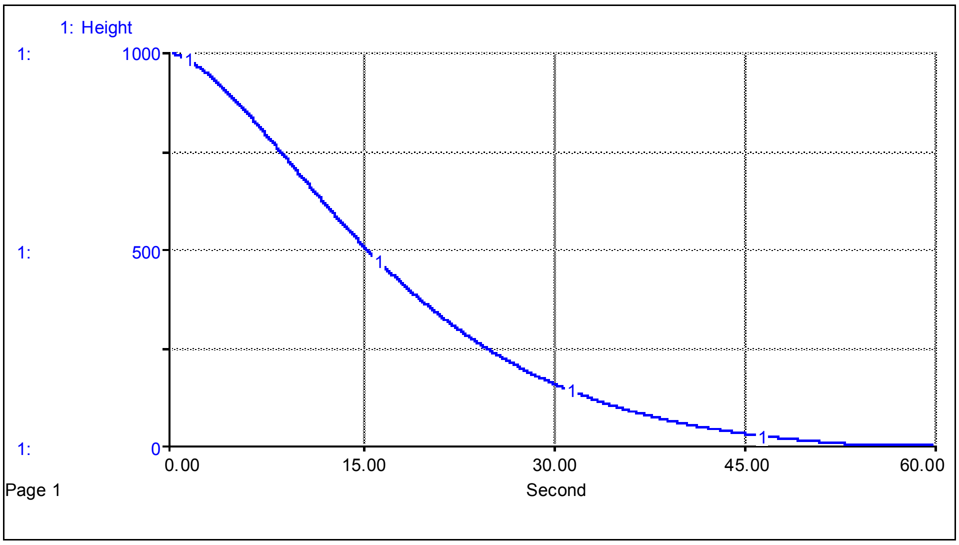


**Figure 14**. Landing behavior generated by the MSD heuristic in the presence of an error in the *Mass* estimate

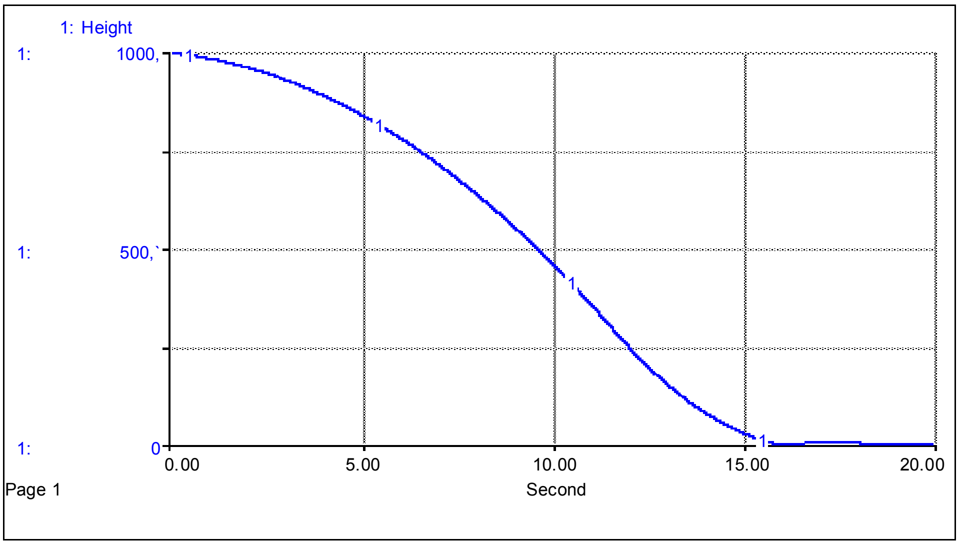


**Figure 15**. Landing behavior generated by the bang-bang heuristic in the presence of an error in the *Mass estimate*

### *5.2. An Error in Height Readings*

We assumed that there is an error in *Height* readings; it is read as 10 meters more than it actually is at all times during the simulation. The dynamic behavior generated by the two heuristics in the presence of this error is given in figures 16 and 17. The *Crash Velocity* values for the MSD and bang-bang heuristics deteriorate to -2.86 m/s and -20.76 m/s, respectively. Similar to the case with an error in the parameter estimates, the behavior generated by the bang-bang heuristic deteriorates in a qualitatively significant manner.

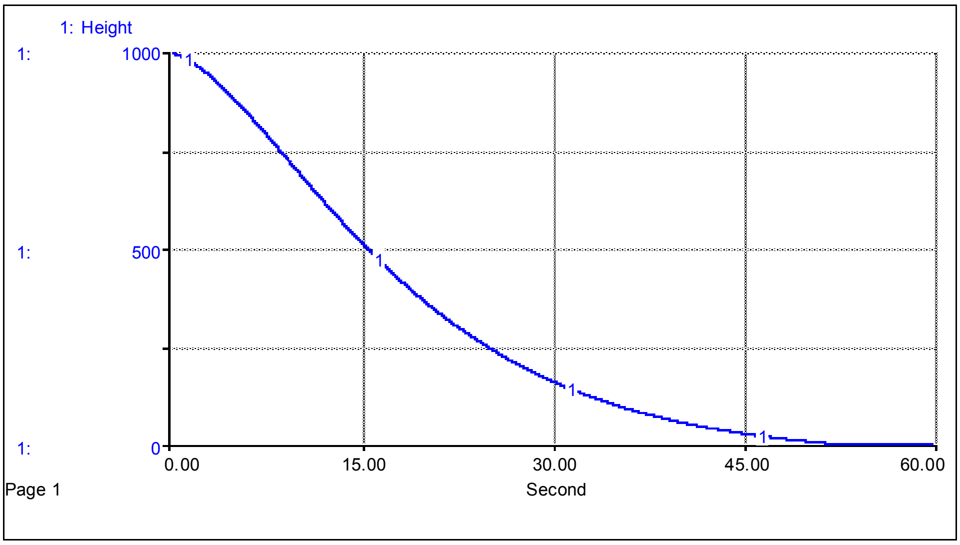


**Figure 16**. Landing behavior generated by the MSD heuristic in the presence of an error in *Height* readings

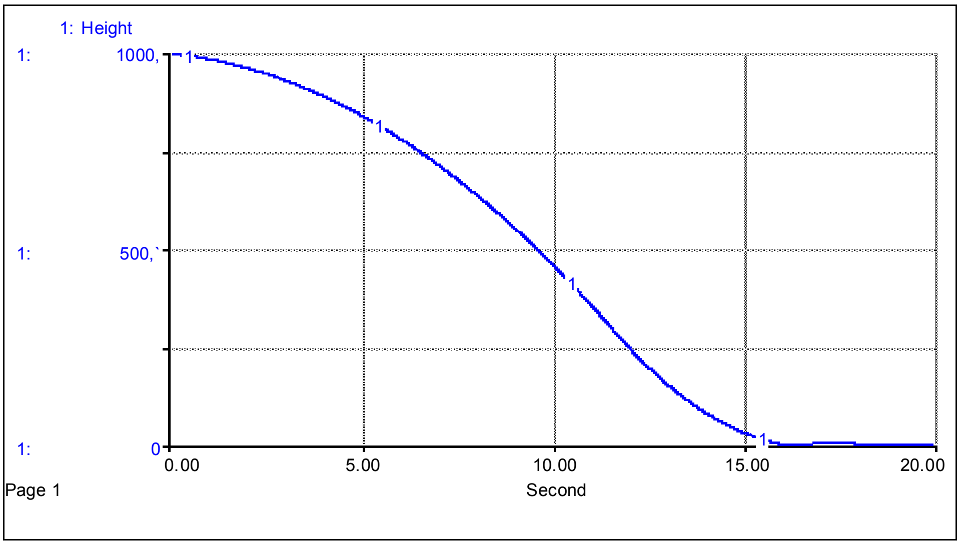


**Figure 17**. Landing behavior generated by the bang-bang heuristic in the presence of an error in *Height* readings

### *5.3. The Presence of an Actuator Delay*

In this sub-section, we assumed that there is an overlooked factor present in the model, an actuator delay (i.e. a delay in changing the level of the force created by the reverse force thruster) of 2 seconds. The presence of this delay creates no significant change in the behavior generated by the MSD heuristic (Figure 18) and the new *Crash Velocity* value generated by this heuristic is -1.75 m/s. However, a huge deterioration in the behavior generated by the bang-bang heuristic is observed (Figure 19). The new *Crash Velocity* value generated by this heuristic is -120.48 m/s. Similar to the cases with an error in the parameter estimates and with an error in readings, MSD heuristic manages a safe landing, proving its robustness. However, bang-bang heuristic is quite unreliable in the presence of aforementioned issues.

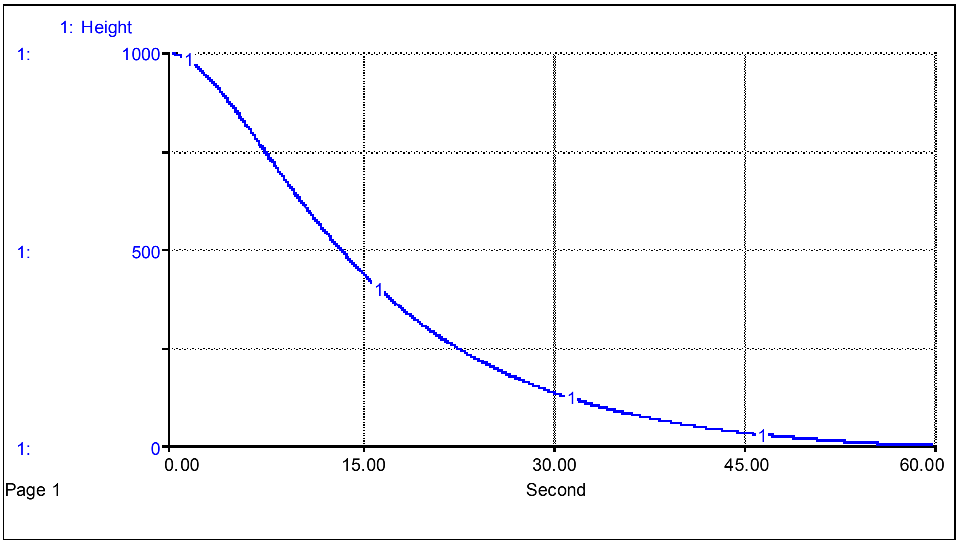


**Figure 18**. Landing behavior generated by the MSD heuristic in the presence of actuator delay

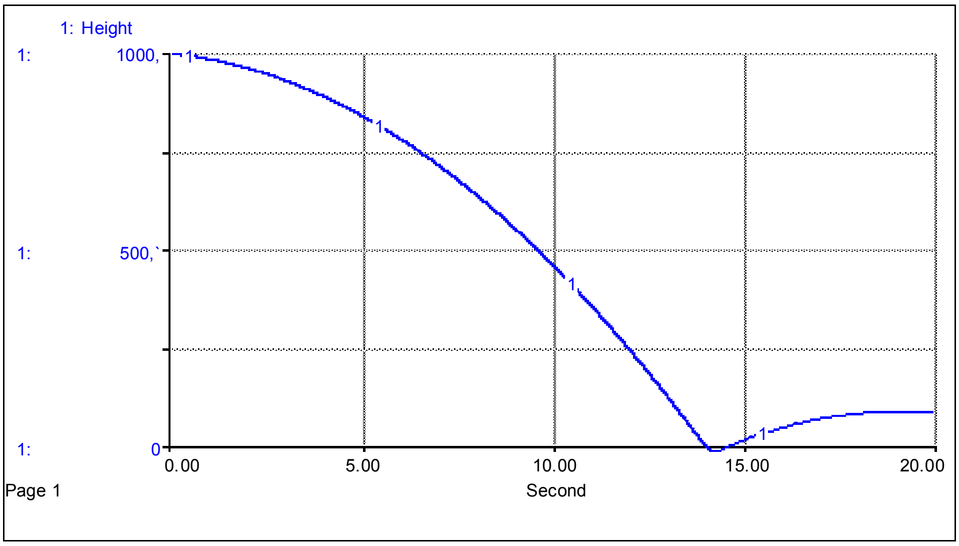


**Figure 19**. Landing behavior generated by the bang-bang heuristic in the presence of actuator delay

# 6. Conclusions and Future Research

In this study, we first developed a soft landing model using System Dynamics methodology. The soft landing challenge can simply be summarized as trying to land a spacecraft on the surface of a celestial body as gently and as fast as possible. The main reason for the challenge is that the control task requires simultaneous control of the height and velocity of the spacecraft, which have inertia and can only be indirectly affected by the reverse force thruster. We also presented two control heuristic. The first one is adapted from the mass spring damper model and the second one is a bang-bang heuristic. According to the initial simulation runs that we obtained, the bang-bang heuristic quickly lands the spacecraft at the end of a very brief landing period. However, it is not robust in the sense that it is over sensitive to the presence of errors in the parameter estimates and errors in the velocity or height readings. It is also very sensitive to the presence of an actuator delay (a delay in changing the level of the force created by the reverse force thruster). On the other hand, the mass spring damper based control heuristic requires a longer landing time, but it is more robust compared to the bang-bang control heuristic in the sense that it is much less sensitive to the aforementioned problems.

Note that, a longer actuator delay may make the mass spring damper control heuristic create problematic behaviors too. In the continuation of this study, we plan to focus on addressing this issue by further improving the mass spring damper based control heuristic. In order to overcome the possible problematic behaviors, we plan to adapt and use the heuristics developed by Yasarcan and Barlas (2005) and Yasarcan (2011), which are specifically suitable for this kind of control problems. It is also possible to develop a soft landing game based on the model as a platform for learning and dynamic decision making experimentation.


# Acknowledgement

We presented an earlier draft of this work in the paper titled "*A Soft Landing Model and a Mass Spring Damper Based Control Heuristic*" at the 29th International Conference of the System Dynamics Society, 24-28 July 2011, Washington DC – USA. We improved that previous work and divided it into two parts. One part is this paper and the other part is the paper titled "*A Soft Landing Model and an Experimental Platform as an Introductory Control Design Tool*", which is also presented at this conference.

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## Appendix: Performance Measure Equations

Performance measure equations are used to evaluate the landing. *Crash Time* gives the duration of the landing beginning with the initial conditions until the moment of touchdown (equations 32-33). *Crash Velocity* is the velocity value at the moment of touchdown (equations 34-35). *Max Landing Force* reports the maximum force that is generated by the landing gear after touchdown (equations 36-37). *Force Ratio* gives a scale of *Maximum Landing Force* compared to *Gravitational Force* (Equation 38). Note that at static equilibrium the landing gear withstands *Gravitational Force*. Thus, *Force Ratio* = 1 is the theoretical minimum. *Max Acceleration* gives the maximum acceleration of the vehicle during landing (equations 39-40). *Instantaneous Change in Net* Force is the absolute value of the change in *Net Force* between two consecutive time steps. It is necessary for the calculation of *Maximum Instantaneous Change in Net Force*, which reports the maximum of the changes in *Net Force* between two consecutive time steps. The value of the *Maximum Instantaneous Change in Net Force* is a measure for the smoothness of the control (equations 41-43).

$$CrashTime_0 = 0 \quad [s] \tag{32}$$

$$CrashTime_{t+DT} = CrashTime_t + \left\{ \begin{array}{ll} t/DT, & CrashTime_t = 0,\ Height_t < 0 \\ 0, & \text{otherwise} \end{array} \right\} \quad [s] \tag{33}$$

$$Crash\ Velocity_0 = 0 \quad [m/s] \tag{34}$$

$$\begin{pmatrix} Crash \\ Velocity \end{pmatrix}_{t+DT} = \begin{pmatrix} Crash \\ Velocity \end{pmatrix}_t + \left\{ \begin{array}{ll} Velocity_t/DT, & CrashVelocity_t = 0,\ Height_t < 0 \\ 0, & \text{otherwise} \end{array} \right\} [m/s] \tag{35}$$

$$Max\ Landing\ Force_0 = 0 \quad [N] \tag{36}$$

$$Max\ Landing\ Force_{t+DT} = \begin{pmatrix} Max \\ Landing \\ Force \end{pmatrix}_t + \left\{ \begin{array}{ll} \dfrac{\begin{pmatrix} Damping\ Force_t \\ - Max\ Landing\ Force_t \end{pmatrix}}{DT}, & \begin{pmatrix} Damping \\ Force \end{pmatrix}_t > \begin{pmatrix} Max\ Landing \\ Force \end{pmatrix}_t \\ 0, & \text{otherwise} \end{array} \right\} [N] \tag{37}$$

$$Force\ Ratio = \frac{Maximum\ Landing\ Force}{Gravitational\ Force} \qquad [\text{dimensionless}] \tag{38}$$

$$Max\ Acceleration_0 = 0 \quad [m/s^2] \tag{39}$$

$$Max\ Acceleration_{t+DT} = Max\ Acceleration_t + \begin{cases} \dfrac{\begin{pmatrix} Acceleration_t \\ - Max\ Acceleration_t \end{pmatrix}}{DT}, & Acceleration_t > \begin{pmatrix} Max \\ Acceleration \end{pmatrix}_t \\ 0, & \text{otherwise} \end{cases} \quad [m/s^2] \tag{40}$$

$$\begin{pmatrix} Instantaneous \\ Change\ in\ Net\ Force \end{pmatrix}_{t+DT} = ABS(Net\ Force_{t+DT} - Net\ Force_t) \quad [N] \tag{41}$$

$$Max\ Instantaneous\ Change\ in\ Net\ Force_0 = 0 \quad [N] \tag{42}$$

$$Max\ Instantaneous\ Change\ in\ Net\ Force_{t+DT} = Max\ Instantaneous\ Change\ in\ Net\ Force_t + \begin{cases} \dfrac{\begin{pmatrix} Instantaneous \\ Change\ in \\ Net\ Force \end{pmatrix}_t - \begin{pmatrix} Max \\ Instantaneous \\ Change\ in \\ Net\ Force \end{pmatrix}_t}{DT}, & \begin{pmatrix} Instantaneous \\ Change\ in \\ Net\ Force \end{pmatrix}_t > \begin{pmatrix} Max \\ Instantaneous \\ Change\ in \\ Net\ Force \end{pmatrix}_t \\ 0, & \text{otherwise} \end{cases} \quad [N] \tag{43}$$