

# Hybrid Models in Developing System Thinking

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## Abstract

*Hybrid models, consisting of the interacting continuous and discrete processes, form an emerging field in science and engineering. Such models, combining stock-flow and state-diagram representations, should serve a part of the system education curriculum. The present paper reports an experiment in which undergraduate students were asked to choose between a hybrid and a continuous solution for a number of predefined problems. Results of the experiment show that the hybrid approach is more preferred by students with relatively low ability of dynamic thinking. We discuss the meaning of the results in terms of analyzing and improving mental models, and also discuss several contexts in which the hybrid approach enriches system thinking.*

Keywords: System thinking, hybrid models

## Introduction

One of the fundamental presuppositions of System Dynamics is that models should have the continuous (or analog) nature (Richardson, 1991). The canonic books of System Dynamics recommend avoiding discrete variables as much as possible (Forrester, 1961; Forrester, 1968; Sterman, 2000). The use of discrete models runs contrary to common practice. Discrete variables are considered both inadequate to describe the continuous nature of reality, and more difficult to understand from a cognitive perspective.

It therefore seems out of place to suggest including hybrid models comprising both the discrete portion and the continuous portion into the system dynamics curriculum. However, we have significant reasons to believe that hybrid models have pedagogical advantages, and breaking the taboo on discrete variables is worth considering. This paper aims at giving some theoretical justification to our belief, and comprises empirical data to support it.

### *Historical Perspective*

Continuous models, based on linear differential equation and describing dynamics of changes in nature and society are "among the greatest successes stories of mathematical modeling" (Maler, 2001). Starting with the birth of the modern physics in the 18 century, the great success of continuous models based on infinitesimal calculus led to the belief that this is the only modeling language of reality (Bunge, 1974). From the physical

sciences, these models rolled to classical control theory and further to descriptions of social change (Richardson, 1991).

In applying the system approach to social issues, System Dynamics kept the continuous thread. Forrester and others inspired to lead the social research "one step behind the events", to receive a deeper perspective of change (Forrester, 1961). According to this tradition, discrete events and local decisions are insignificant details in the general picture. The real processes are described by continuous flows.

The rival cybernetic approach (Wiener, 1948) – with its focus on messages and events - was considered inadequate to describe reality. Lacking classical mathematical beauty, it was also considered inferior to the rich beautiful theory of continuous change. Until recently, the two threads were rarely combined both in the natural and social sciences (Branicky, 1995; Richardson, 1991).

However, the invention of the digital computer followed by extensive informatization of society led to vast developments in discrete control theory, as new and sophisticated mathematical models have been developed. Furthermore, a new discipline in engineering emerged to study hybrid models, in which continuous and discrete systems interact.

### *What are Hybrid Models?*

Hybrid models describe systems where continuous and discrete sub-systems interact. In the context of control theory, the discrete element is usually a controller of a continuous process. These models have vast applications in computer embedded systems, where a physical or technological process is controlled by a digital unit (Johansson, 2000).

Such models combine continuous representations with representation typical to discrete mathematics. For educational use, we propose to use the stock-flow representation to describe the continuous part of the system, and a state diagram to describe the discrete element. We believe that this combination has a unique educational value as part of the system approach (Levin et al, 2001; Levin & Levin, 2002). We have two main reasons to that belief.

1. The hybrid structure can simplify the description of systems in certain cases. Simplifying models for educational purpose is important, as research has shown that various students – including elite students - have poor understanding of some of the most basic concepts of system dynamics (Sweeney and Sterman, 2000; Sterman and Sweeney, 2002; Kainz and Ossimitz, 2002; Jensen and Brehmer, 2003; Sterman, 2010). As a reconstruction of equivalent stock-flow models with many feedback loops, hybrid models are simple. Moreover, they provide a good enough approximation (Levin et al, 2001). The hierarchical structure divides the system in a clear way, and emphasizes the logic of control. We are aware of the opinion that conditional statements of if-then-else considered difficult to understand (Forrester, 1961; Sterman, 2000). However our personal experience as educators shows that the state machines notation may provide a friendly alternative representation of the logic.

2. Hybrid models may be similar to hidden modes of thinking, which are common among students. One of the goals of teaching the system thinking is improving mental models (Doyle and Ford, 1998). Mental models are some primitive representation – usually unconscious – that guides us in our thinking and decision making (Forrester, 1971). In the

process of model construction, we expose hidden mental models, and explore them critically through discussion and computer simulation.

A basic hybrid model resembles a stage in the moral development of a child, where his response to reality is regulated by a machine comprising two states: allowed and not-allowed (Piaget, 1971). Such type of thinking may develop an abstract form, as some young people tend to believe they are controlled by inner machines (Fried and Agassi, 1985). Such models may later be repressed, but they still act unconsciously, especially as response to unpleasant situations. The use of hybrid models may help exposing them, and to analyze them critically towards a more rational and realistic approach.

The current research studies our intuitive hypotheses about the simple structure and hidden mental models in the empirical context.

### **Rational of Research**

As part of the general research of control concepts, we tried to measure the attitude of participants toward hybrid models. Participants were asked to choose between discrete and continuous control strategy in solving a number of given problems that are based on continuous processes.

We looked for correlation between the choices participants made on the one hand and their academic background and system thinking skills on the other hand. The above correlation enables to figure out, to what extent the simplicity and the existing mental models play a role in their preference.

We suppose that: 1) the chosen approach corresponds to the level of understanding the problem and 2) hidden mental models may be a good starting point for forming a new knowledge.

### **Method**

We presented the subject with three stories describing dynamics. The stories described various types of dynamics: change in rabbits' population, change in the amount of money in a bank account and change in the temperature of a room. The type of dynamic behavior in each case is unique: linear, exponential and logarithmic.

For each case, we presented questions that require the recognition of patterns of behavior related to the given structure. The questions measured the student's understanding of the dynamics. The questions were multi-valued. Each question had three possible answers. The distractors are based on common mistakes in the previous research.

We then presented a control goal for each story that was specified in terms of optimum and min-max values. Two methods of control were given to reach the goal, and we asked the students to choose between them, based on several criteria: a) it is more user friendly b) it is within the minimal requirements c) it yields optimal results d) it would be recommended for implementation e) it would recommended if numeric values can be changed.

Table 1: Input and output for the controlling element

<i>Case</i>	<b>Input</b>	<b>Output</b>
<b>Rabbits</b>	Number of rabbits	Number of hunters permitted
<b>Bank</b>	Sum of money in the account	Sum deposited for saving
<b>Room</b>	Room temperature	Volume of heater

## **Subjects**

One hundred and twenty one undergraduate students participated in the study. Fifty nine of them were natural sciences and engineering students (49%), and sixty two were social science students (51%); 36% were first year students, 43% were second year students, 16% were third year, and the rest were in their fourth year in the university. The students had no prior formal experience in learning the system approach.

We approached the students around the campus and distributed the questionnaires. The students were asked to fill the questionnaire on their free time, and to call us when finished. Most students called us within 24 hours, and 6% did not return it due to various reasons. When collecting the questionnaire we paid them the equivalent of 7\$.

The majority of the students (approximately 90%) were interviewed on returning the questionnaires. Each student was interviewed alone. They were then given a chance to change their choices, which they rarely did (less than 3% ).

## **Reliability and Validity**

We have taken several steps to increase the reliability and validity of the research. A pilot test was given to ensure that students understand the questions correctly. After the pilot test we've made several changes, especially in the wording and graphical presentation of the answers.

Questions addressing the dynamics behavior were similar to questions given in prior researches (Sweeney and Sterman 2000; Kainz and Ossimitz 2002). The main difference between our test and those mentioned is the closed multiple-choice structure of the answers as compared to open answers. Our distracters were based on common mistakes exposed by previous research. Two experts secured the validity of the questions in terms of correctness and equivalence of solutions. The case studies were given in random orders, while the questions, in each case study, were in a fixed order to keep the inner logic consistency.

The results were tested for inner consistency as will be shown below.

## Results

### *Continuous approach preferred*

We measured the variable on a scale from 0 to 1, where 0 represents favoring of the discrete logical control strategy, 1 represents favoring of the continuous control strategy, and 0.5 means that both of them are the same for a participant. In general, students prefer the continuous type of control.

The statistical analysis yields a value of 0.56. The standard deviation is 0.18. An inner consistency test yields a sufficient value of Alfa Cronbach,  $\alpha = 0.72$

The continuous approach was conceived much better in terms of optimality (.7245), and considered more appropriate for recommendation (.5923). The students consider both strategies as nearly equal in terms of “keeping within constraints” (.5138), while the logical approach was considered more friendly and simple to use (.3595).

### *Difference by case*

The total average score in the questions on dynamic thinking was 71 (SD=0.2). The results were significantly lower in the questions on changes in rabbits population (50), as compared to the cases of bank account (82) and room temperature (80).

The continuous approach was favored both in the case of heating a room (0.65) and managing a bank account (0.57). The hybrid approach was favored in the case of controlling rabbits' population through hunting (0.45).

Table 2: Results by case

	<b>Dynamic thinking</b>	<b>Continuous over hybrid</b>
<b>Rabbits</b>	50	0.45
<b>Bank</b>	82	0.57
<b>Room</b>	80	0.65

### *Influence of background*

A correlation has been found between the background of the subjects and their preferred modeling approach:

1. Students with relatively low dynamic thinking ability tend to prefer the hybrid type of modeling. Students with relatively high dynamic thinking ability tend to prefer the continuous type of solution.
2. Females prefer the hybrid approach more than males.
3. Social sciences students prefer the hybrid approach more than natural science and engineering students.

The regression analysis yields a total value of R square= 0.14 (Sign F< 0.001 ). That means that the linear dependence of the dependent variable on the independent ones explains 14% of its variation. Table 3 shows the proportional contribution of each of the independent variables to the variation in the preference of the controlling type.

Table 3: results of regression analysis

<i>Variable</i>	<i>Relative weight Beta</i>	<i>Correlation with preferring variable</i>	<i>Reliability by T- Test</i>
Dynamic thinking ability	0.18	0.25	0.002
Gender	0.22	0.21	0.008
Faculty	0.18	0.24	0.003

## Discussion

The research measures preferences in selecting control strategies, as an indication of the pedagogical potential of the hybrid systems approach. The results show that students preferred the continuous solution over the hybrid one for a number of problems. Based on the research rationale, we should have concluded that continuous models might be more suitable for teaching than hybrid ones.

Nevertheless, the closer analysis implicates another interpretation. Let us consider the criteria by which each of the approaches is preferred. The continuous approach is preferred in terms of (owing to the criterion of) optimality, while the hybrid approach is preferred in terms of “friendliness”. It may be the case that the continuous solution is preferred since the problems are relatively simple. Therefore, more weight is given to the optimality, while the friendliness is underappreciated.

Furthermore, the hybrid approach was preferred by participants with low system thinking skills and students from the social sciences and humanities faculties. Supposing the lack of understanding was the reason for the choice, it may be linked to some familiarity with existing hidden mental models. In that case, the hybrid model serves as a starting point to overcome complexity of the task.

Since the research is a pioneering one in its field, additional data is required to determine whether hybrid models make systems simpler to understand. However our study provides a significant insight as to what can be learned from hybrid models. It seems that hybrid models convey some deep-seated beliefs, whose exposure may help improving the mental models.

### *Regression to archaic models*

To make our point clear, let us focus on a certain exemplary problem concerning the policy of hunting rabbits. This is the only portion of the questionnaire where the hybrid approach was preferred. Two probable causes may explain it.

The first cause is the fact that the dynamic behavior in this case was considered as complicated. This is the only case that describes the exponential growth and decay, which the previous research showed difficult to comprehend (Wagenaar and Sagaria, 1975). Indeed, the students received much lower scores of dynamic understanding in this story in comparison with two other stories.

The second cause is based on remarks made by several participants during the interviews. They mentioned that their choice in this case was affected by some moral considerations. The discrete control seemed to them less harmful to the rabbits, and better for the environment.

Both explanations follow a classical pattern of regression (Freud, 1917): when facing a confusing and unpleasant feeling – whether of a cognitive, moral or other origin– there is some regress to archaic modes of thought. We believe that two state machines may imitate a rigid and immature way of thinking. They may represent a primitive superego that either allows something or not, but leaves no space for a middle ground.

The regression to archaic models affects not only actual decisions, but also exposes a tacit knowledge theory (Polanyi, 1964) concerning what decisions are. It therefore creates an educational opportunity to construct new personal knowledge on the subject.

### *A constructivist approach to decision making*

In hybrid models, control unit is located outside the system (is not a part of the system), obeying their own inner logic of transitions. This is in contrast to the classical system dynamics, where decision rules are represented as continuous variables within the models (Forrester, 1998). The inclusion of decision rules within the model enables realistic description of gradual changes in policy.

We do not challenge the assumption that the continuous approach is more adequate to model social processes. However we do argue that many people experience their own decisions as discrete units outside the system. The difference may be the matter of resolution. The continuous approach looks at reality from 10,000 foot, while the discrete approach is near the ground level (Richmond, 2001).

The resolution of everyday life is that of the ground level. It is more intuitive for many people to locate their decisions outside the natural continuous process, with discrete logic of their own. Their view of the subject may lack perspective and sophistication, but it is authentic in expressing subconscious mental models.

From a constructivist point of view, the coexisting of opposing views on a subject is not an obstacle to learning (Piaget, 1971; Papert, 1980). A shift between perspectives is also considered a good educational practice. Therefore, the dialectic use of the two types of models concerning decision-making may serve to overcome misconceptions, and to construct meaningful and long lasting knowledge.

Furthermore, the ability to see oneself both from inside and outside of the system is an educational achievement. The idea that players both affect and are affected by the system is central to the system approach. Awareness of the mutual dependence serves one of the main goals of the system education: to develop individual responsibility for the future of the environment in which he/she lives (Forrester, 1994).

### *Values vs. Norms*

There is also a moral dimension to the difference between continuous and discrete models. Continuous models may be considered as describing a human's behavior from a social perspective and thus according to norms. They describe the way a human's action

is governed by economical and sociological laws. To modify the behavior within the model, one has to find ways to intervene indirectly, preferably through leverage points in the system.

The hybrid structure helps to distinguish between the actual and the moral, and thus between norms and values. In contrast to the accumulative nature of norms, the application of values is modeled as switches between states of a discrete state machine. Given some input from the environment, the state machine calculates its state according to its transition function that can be considered as a certain moral code, and acts by transmitting an output. A modification of the behavior is reflected by the change of a state, or attitude.

For example, heroes in western movies resemble such state machines. As individuals with integrity, they refuse to obey to social norms, and aspire to do the right thing according to their own moral code. The conflict with the society is inevitable, usually after certain thresholds are surpassed. Their individuality is achieved through avoidance of social assimilation, and through loyalty to a clear and distinct set of values.

On a larger scale, the Jewish law (Halacha) may be described as a model of interaction between the moral world and the reality. Given a social context, it dictates specific instructions how to act. However, the influence is usually found in two ways, as the Law itself has to adjust itself to changes in the social environment, according to the principle of the dominant Beit Hillel school. The Jewish heritage may therefore be considered as a hybrid entity, in the sense that it constantly mediates between particular rules of action and understanding of the social dynamic. This hybrid mode of existence may be one of the reasons for the strength and vitality of Jewish intellectual life throughout history.

### *A dualistic worldview*

To conclude, the results of the research show that hybrid models constitute a useful concept for the development of system thinking. It is not clear yet whether they can serve as a practical methodology for modeling and simulation of social systems. But analysis of the results demonstrates that hybrid models have a unique explanatory power as a concept for qualitative modeling.

Not everybody might accept usefulness of hybrid models. For people who view the systems from above – like scientists and managers - the continuous approach may be more appropriate. Such methods as stock-flow and feedback loops constitute a coherent description of systems that both seem friendly, and offer handy tools for analysis and communication.

By contrast, hybrid models may appeal more to outsiders, who feel independent from or alienated by the society. Failing to identify their role within the systems, they fear that assimilation in dynamics contradicts their inner feeling of self-identity (Ericsson, 1968). Like the hero of Charlie Chaplin from the famous movie “Gold Rush”, they join the others in the rush for gold and success, but actually they are looking for something else. The hybrid structure is a metaphor of their existential stance.

System thinking connects elements from various origins and disciplines. Under the continuous paradigm, the connection is possible through a unified law, and is therefore

monistic. The hybrid approach is more dualistic in nature: it recognizes two distinct types of systems that obey different kinds of dynamic rules, but still try to interact.

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