Coordination Failures in Complex Environments: A Model for Primary Education Systems in Developing Countries

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Abstract. This article aims at improving our understanding of the process influencing attainment in primary education systems through a dynamic simulation model that incorporate local idiosyncratic complexities such as positive feedbacks and nonlinear coordinated interactions between key variables like aggregate human capital and their effects on the efficiency and vulnerability of primary education systems in developing countries. We perform a number of experiments which offer valuable insights to understand the long-term impact of large-scale events on the system's steady-state equilibrium and suggest institutional structures and social norms that may promote a sustainable transition toward universal primary education.

Keywords: Primary school, feedbacks, nonlinear effects, Latin America, Nicaragua, multiple equilibria.

Paper type Research paper

1- INTRODUCTION

Several explanations have been put forward to explain the failure of education reforms such as the quality of public education (Hallack, 2000), moral hazard problems associated to scholarships (Arcia, 2002), lack of adequate financial resources (UN, 2005), differences between urban and rural contexts (Birdsall et al., 2005), etc. However education systems in the world, mainly in developing countries, suffer not only from inadequate policies. Similarly threatening can be exogenous commotions arising from catastrophic events wreaking havoc communities and shifting the historical way toward universal primary education maintaining the country in state of low attainment. In such a circumstance it is said that a coordinated failure occurred and the system gets stuck in a bad steady-state equilibrium which is likely to condition its progress unless serious bottlenecks to further growth are removed.

In this paper we tackle the case of Nicaragua to provide illustrative evidence, using a simulation model, of the impact that these phenomena may cause in the structure and behavior of the education system and how this is translated in a long term underperforming condition. Here we focus on broad key process conditioning school decisions and its impact on education dynamics in order to keep the model tractable and observe how particular events may influence these dynamics. Therefore, it is of interest to ask to what extent education systems may become susceptible when exposed to extreme socio-economic, natural, or political manifestations and in the aftermath of the shocks how fast their effects decline with time. These events include shocks in the level of human capital in the country, the general conditions of the education system, and the conditions of the economy.

The Nicaraguan case study is interesting from an academic perspective as during the last three decades this country has experienced high-consequence events arising from political, natural, and socio-economic origins with important costs on schooling outcomes. In 1978, a Civil War ignited and remained for more than a decade which taxed its development process. One year before, in

1977, real GDP per capita in Nicaragua was US$1.533; two years later, in 1979, it was only US$980, 40 percent lower (WDI, 2010). The downward trend remained and by 1993, it ended up plunging down to US$633, just 60 percent the value before the beginning of the war. In addition to political conflicts, the country has been hit by highly devastating natural disasters like hurricanes, volcano eruptions, and earthquakes particularly in the last decades (see CRED, 2011).

As result, since the 1970s the country has experienced extensive migration waves that continue through the present. According to the World Bank (WB, 2011), by 2007, more than 700.000 Nicaraguans, 13 percent of the population, have abandoned the country, particularly those with more human capital (30 percent of population with tertiary education).

The structure of the paper is distributed as follows. In the next section we review the relevant literature which emphasizes on the multiplicity of equilibria that arises in systems dominated by positive feedbacks and their applicability to education. Then, in the third section, we propose the method of analysis and in the fourth section we present simulation experiments arising from large, exogenous, and purely temporary shocks. Results show, first, that the system reacts in a non-linear fashion to identical shocks portraying different results depending on the level achieved by key state variables; second, there is a tipping point below which coordinated policy actions are required to persist for several decades to push the education system over the critical threshold in order to get successful self-sustaining attainment. Efforts falling short of the tipping point or concurrently large catastrophes could settle the system into a long-lasting low-equilibrium condition that precludes achieving universal completion in primary education. Then, our experiments suggest the magnitude of single or combined events to drive the system from one equilibrium to the other. Discussion of their implications for policymaking is presented in section fifth and section sixth concludes.

2-BACKGROUND AND LITERATURE REVIEW

Positive feedbacks and nonlinearities are generating an increasingly vast literature in economics and other disciplines whose applications incorporate areas ranging from the evolution of social stratification to the structure of high-technology industries or regional development issues. Authors like Murphy et al. (1989) argue that investment by one firm can have a positive effect on the profitability of investment by other firms, because higher investment leads to an increase in aggregate demand, which under economies of scale raises the profitability of investment elsewhere in the economy. Arthur (1989,) discusses policy implications in technology adoption and how increasing returns act to magnify events which might drive the system to a level where everybody is worse-off. However it was after Lucas (1988), Azariadis and Drazen (1990), Romer (1990), Benabou (1993), and Dowrick (2004) that researchers applied those ideas to education emphasizing the impact of non-rivalry of education and ideas on economic growth and inequality. Multiple equilibriums are possible in systems with dominant positive feedbacks: namely low-achievement (LA) and high-achievement (HA or full completion) thresholds (Arthur, 1990; Durlauf, 1998) and specifically Azariadis and Drazen (1990) show that these kinds of thresholds are pervasive in human capital accumulation. Although at the high equilibrium everybody would be better off it is not clear how a system moves from the LA level to the HA as many argue that this condition, typically regarded as a coordination failure, is not self-correcting (Rodriguez-Clare; 2005). Usual policy options are quite vague pointing out that *some kind of intervention* is required (Rodrik, 1996) while others are too general advocating extreme operational impossibilities like *sustained* *interventions at every possible level* until the system gains momentum to achieve the higher level of performance (Mason, 2008; emphasis added). Also this kind of failures operates the other way around. A system located in high-level equilibrium may experience a downward change that may alter its position to one of permanent low-achievement state.

A reason behind the slow progress of policy guidance of coordination failures is the lack of suitable conditions ─namely large and exogenous shocks─ for empirical tests of the existence of issues precluding the shift from low to high-level performing system. Thus, the econometric literature does not provide many hints about the magnitude of these shocks. An exception is Davis and Weinstein (2002, 2008) who used (rare) data on man-made cataclysms of startling magnitude as a natural experiment but, in spite of this, these authors could not reject a unique stable equilibrium nor could establish the sufficient condition for multiple ones. This paper aims at shedding light in this regard by building a dynamic model of a primary education system and conducting experiments that rearrange its long term steady-state equilibrium. Therefore our key research question is to determine how vulnerable are primary education systems with the characteristics described in this paper, to events that could strike them at different levels of attainment. In a related line, what policy actions might be effective in changing a low performing system to one of full completion? This inquiry is of particular relevance for some countries such as those in Latin America and Africa which despite significant enhancement in their education institutions and macroeconomic environment have failed to experience sustained improvements in their primary school outcomes (Arcia, 2003; Birdsall et al., 2005). Here we focus on cases in which failures arise from the interactions of local phenomena: e.g. non-tradable intermediate inputs of the education process such as the economy's overall conditions, the state of the education system (infrastructure and personnel), and adults' literacy (human capital) in the country. [1]

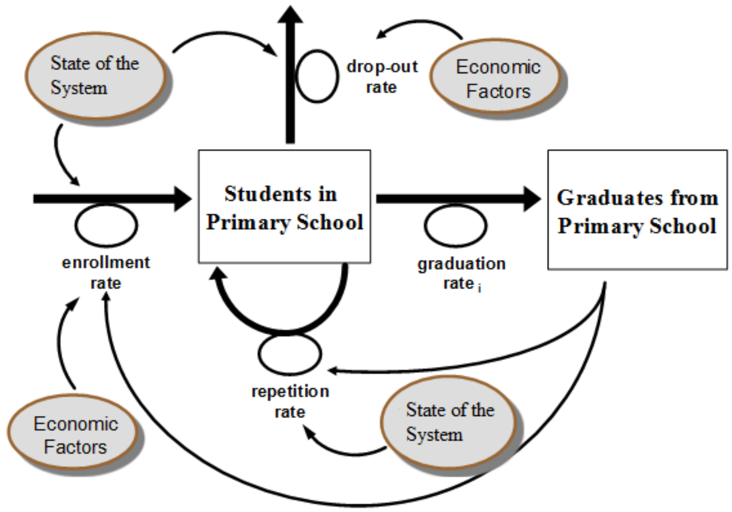
3- METHOD

This paper builds upon the model developed by Guevara (2004) and Guevara, Lopez and Zuniga (2005)[2] and extends it in two important respects: first, by formalizing the links between educational investment of adults, general school conditions, and the economic conditions of the country on children's school decisions: enrollment, repetition, and drop-out (figure 1). The second important advance over Guevara et al. (2005) is that we allow the possibility of coordination failures when these factors are jointly interrelated within a complex system.

[1] If these intermediate goods could be easily exchanged, then the efficiency of an education system that relies on such inputs would not be affected by their local availability; see Faini (1984), Rodrik (1996) and Rodriguez-Clare (2005).

[2] As in Guevara et al (2005) we chose two indicators to capture the dynamics of the education system: the gross enrollment rate (GER) and the primary completion rate (PCR) as will be shown in figure 3. The first, measures the number of children attending educational centers, regardless of age, as a proportion of the population at suitable age to attend the primary level. The primary completion rate measures the number of children who graduate from the system's last grade in a given year divided by the number of children in appropriate age for graduation.

Figure 1. Model's Overview



3.1 Model Extension

The model here depicted departs from Guevara, Lopez, and Zuniga (2005) from equation 1 to equation 5 (see appendix) and extends it to include the accumulation of graduates from primary into a stock H, as shown in figure 1 and equation 6:

$$H_{a\geq 16}(t) = \int_{t=t_0}^{T} [\pi_{6,a}(t) - D_{a\geq 16}(t)]dt + H_{a\geq 16}(0) \qquad (6)$$

$\pi_{i,a}(t)$ = promotion grade *i* at age-*a*

$D_a(t)$ = fractional death rate at age-*a*,

As equation 7 shows, we construct an index of human capital dividing the stock of graduates from equation 6 by the population from equation 1 (both at age 16 or more). We measure this index on a 0-1 scale where zero implies that there is no population with complete primary among the adults and one means that all adults have at least completed primary instruction. Therefore,

$$h_{a\geq 16} = \frac{H_{a\geq 16}}{P_{a\geq 16}}, \qquad \text{where } 0 \leq h \leq 1 \qquad (7)$$

3.2 Nonlinear Relationships.

Equation 8 and 9 show the general heuristic specifications for the three transition rates[3] and table 1 present more details on the three selected variables ─adult's literacy, per capita income, and the general state of the system─. We present in panel of look-up tables scheme in Figure 2 a) to 2 f) the value range they may accrue and the direction of the impact that they cause on the system's transitional rates. In the specification for school enrollment we included the effect of adults' literacy and the state of the system while on repetition rates only the interaction of adult's literacy and general state of the system; drop-out rates contain the joint interaction of the effect of income and

[3] There is a fourth transition rate that we define as one minus the value accrued by repetition and drop-out.

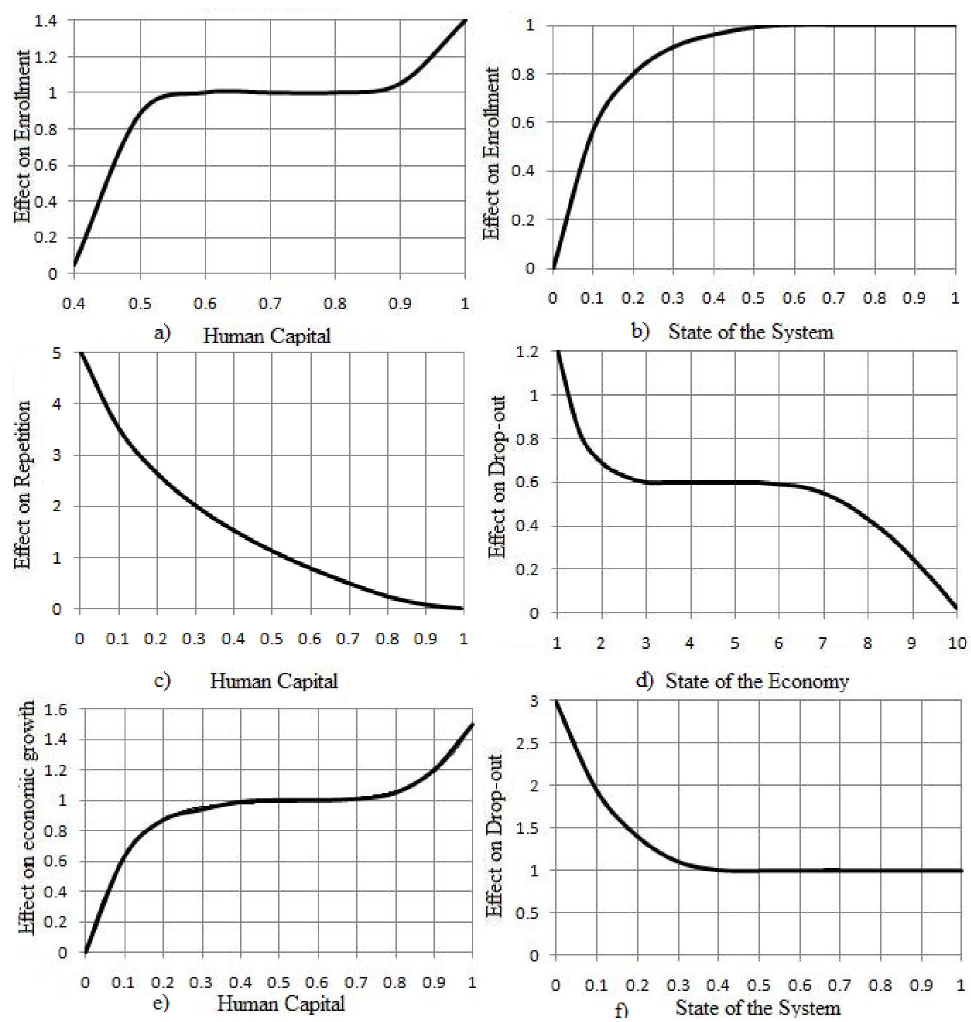
general state of the system. [4] In general these conditions say that when human capital in the country is low, the economy is not doing well, and school buildings are in disrepair and teachers are underpaid it is not unrealistic to assume that school enrollment shrinks while repetition and drop-out soar. Notice that coordination failures may arise from the multiplicative interaction specified: if one of them becomes zero it nullifies the whole interaction and, as a result, the respective transition rate in the model will be zero. Next expressions formalize previous statements,

transition rate$_i$*= transition rate initial value*$_i$**joint effect on transition rate*$_i$ *of (general state of the education system, general state of the economy, adults' literacy)*

where i=enrollment, repetition, drop-out (8)

joint effect on transition rate$_i$ *= effect of the state of the education system* state of the economy *effect of adults' literacy* *(9)*

Figure 2. Non-linear relationships



[4] These functions can be interpreted as if the probability of enrollment, repetition, and drop-out are non-linear on the effect of adult's literacy, relative per-capita income and general conditions of the education system.

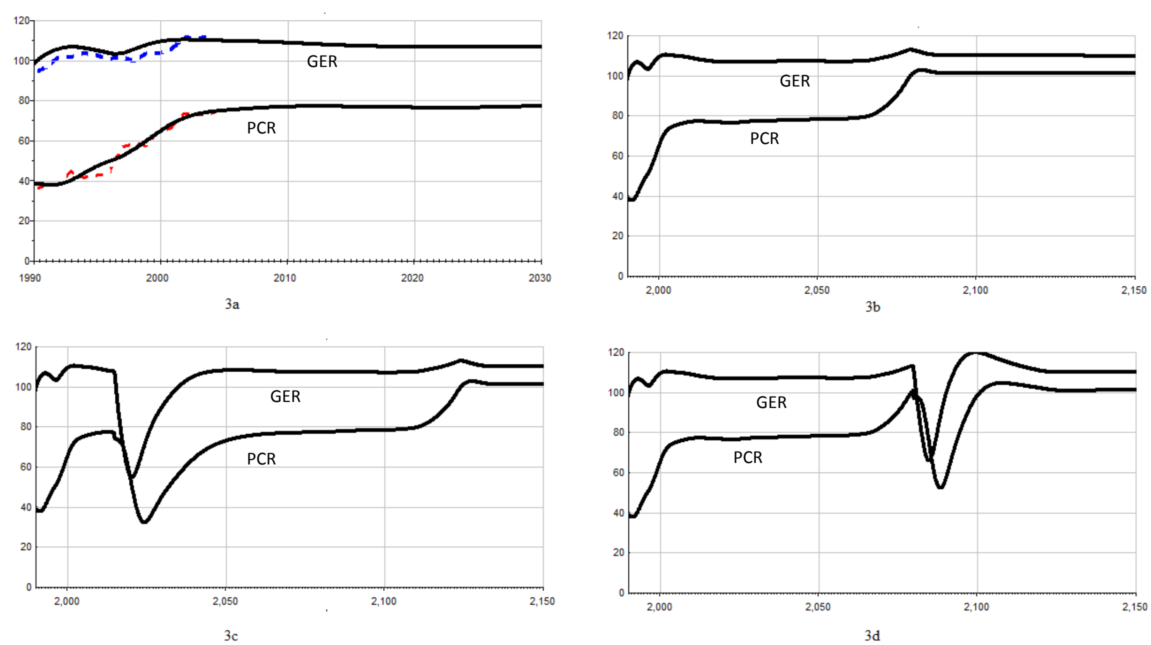
4 SIMULATIONS: BASELINE SCENARIO AND EXPERIMENTS

Baseline Scenario. Figure 3a shows the baseline scenario where simulated PCR is compared to its corresponding observed time values (dotted lines) from 1990 to 2005 while the continuous lines represent simulated series. Simulation confirms that with the model specification and assumptions, namely the interaction of parent's education, the household economy and the attractiveness of the system on enrollment, repetition, and drop-out, we can closely replicate the historical short-medium term development of the education system in Nicaragua, from 1990 to 2005.

In order to observe whether this variable will ever go beyond this threshold we extended the time frame of the base case simulation further from 2020 to 2150. After this, from figure 3b we observe three clear transitional phases in the evolution of the system like described in this paper: phase I shows convergence toward low level steady-state equilibrium below 80 percent; phase II, pose the transition to Phase III and convergence to long-term universal primary education.

There is a long period of time ranging from 2009 to 2065 where the system's PCR rests at a steady-state of approximately 78 percent (figure 3). We identify this level as the tipping point of the primary education system in Nicaragua. [5] This level represents a steady-state condition where the system will not go further, at least not before many decades of slow capital accumulation and slow economic growth. Below the tipping point, the system shows PCR centripetal convergence to low-attainment steady-state: the change in the amount of new graduates from primary is proportional to the change in the population at the graduation age. Indeed, around 2062 the primary completion rate starts a mild upward trend and by 2066 it crosses the 80 percent level and thrives. Once the threshold is surpassed, the PCR diverts its trends and increases more than 20 percent in the next two decades until it reaches the 100 percent steady-state level in the year 2080.

Figure 3. Simulation base-run and experiments



[5] This value corresponds to what that Arcia (2002) suggested as the maximum level that PCR can reach in Nicaragua.

Experiments. With the model sketched in section 2 and data from Nicaragua [see appendix and Guevara, Zuniga and Lopez, (2005)] it is possible to test different implications and explore the evolution of the primary education system in the country under alternative assumptions by applying changes to single or multiple variables. The purpose of this analysis is to get some sense about the behavioral consequences of adjustments on completion rates (PCR) arising from disruptive events like the ones that have afflicted the country in the last decades. Additionally, it is important to determine how long it takes to recover the level previous the event as well as when it crosses the tipping point and gets a full completion level.

To be able to capture short and long-term patterns all simulations cover the period 1990-2150. To our particular interest, we assess the impact on PCR of specific shocks arising from changes of key variables such as human capital once the system has already reached a plateau, either below the tipping point (at 78 percent) or at universal completion rate (100 percent). Consequently we applied shocks in years 2015 and 2080, respectively. All experiments performed here involve exogenous onetime changes.

In figures 3c and 3d we considered a large negative shock that pushes down human capital in the country. The rationale for this experiment follows from the fact that people and particularly those more educated can be easily mobilized abroad where they can search for better opportunities than the ones accrued in the country. As mentioned before Nicaragua is a paradigmatic case due to its high migration rates.

A 100 percent reduction of literate population drops down completion rate to 33 percent by 2024 (47 percent less) and this variable recaptures the 78 percent level by year 2068, about fifty years after the shock; then it crosses the tipping point in 2110, nearly hundred years later. If the same incident takes place after the country reaches full completion rates (in year 2080, figure 3d) despite the fall in PCR is similar in magnitude (47 percent) completion get back faster to the 100 percent level, it takes two decades.

To gain intuition from this exercise, consider an extreme situation in which there are no literate adults in the country. According to figures 1 and 2 this would increase repetition (figure 2c) and decrease enrollment rates at schools (figure 2a) but also economic growth stagnates in the country due to the lack of a specialized labor force in the economy (figure 2e). In this situation students cannot count on parents support to help them out with all academic activities than it would be otherwise and drop-out rates remain high. Completion rates decrease to very low levels. However given that some students still remain and finish the school, the stock of human capital and consequently the education system start a slow recovery that may last many decades before it achieves the level and dynamism previous to the event.

5 DISCUSSION

In this paper we have developed a simulation model of a primary education system to explore the behavioral dynamics that shocks might cause to its transition path toward full achievement equilibrium. It is because of the interrelationship between human capital, the state of the economy and the general state of the education sector that the system can shift smoothly from low to high-achievement under a non-intervention scenario. During this development we identified two phases of convergence to steady-state equilibrium at low and high-level achievement and a threshold in between. Once this threshold is crossed, completion rates show a fast pace to universal primary education.

However, for a country like Nicaragua, which we have used as illustration, it will take several decades to cross the tipping point due to low initial conditions in the level and growth rates of income per capita, literate population, and inefficiency of the primary education system (e.g. high drop-out and repetition). It is after long steady accumulation of human capital that this stock will get enough dimension to improve the general state of the economy and reduce drop-out substantially, allowing this positive feedback to gain self-sustained momentum until reaching the maximum-level equilibrium.

Non-linear nature of educational systems

The nonlinear and complex nature of the effect that the stock of literate adults has in the education system portrayed in this study was shown by the distinctive behavior patterns generated from identical exogenous shocks applied at different development stages of the system. Once the model reaches a critical level, a phase transition occurs which generates new behaviors. Phase transitions in our model are contingent on specific contextual factors in the Nicaraguan education system i.e. when the primary completion rate is below its tipping point, the education system is less responsive to recover from a negative shock reducing the ratio of literates in population, compared to a case when the shock occurs after the systems has crossed this threshold; that is, when the country is close to universal primary education.

6- CONCLUSIONS

As human capital still continues to be the engine of economic progress of countries in the world, increasingly, education systems and their governments are under pressure, either from non-governmental organizations or the citizens they represent, to improve their outcomes in this regard. But the fact that further education is being demanded and its rationale fully conceded, it does not ensure clarity about the ways in which this goal should be undertaken. Understanding how education systems work in reality is a complex endeavor as feedbacks and nonlinearities may preclude the full realization of the results desired under reform efforts. Slow achievement in countries needing more progress is consequent with this perspective. In this paper we have developed a behavioral and dynamic model to explore those complexities in the framework of a primary education system that is trapped in a lower than desired steady-state equilibrium.

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APPENDIX

extract from Guevara (2004) and Guevara, Zúñiga and López (2005).

The Model

Formally, the model has 3 state variables: Total Population (*P*), Latent Population (*L*), and Population in Primary School (*G*). Total Population (P) is the country's total population, and it represents the main input of the Education System. It is characterized as a continuous time structure disaggregated into stocks representing one-year cohorts from 0 to 16.[6] Equation (1) presents the Population stock which is increased by births and decreased by deaths. Equation (1a) shows the birth rate over the time domain is equal to the fractional rate of population growth, β, multiplied by the country's population (i.e., the sum of all cohorts). The death rate, equation (1b), is the stock of population multiplied by the fractional rate of death, ϕ. Aging, *A(t)*, is also included, and it represents the transition of a cohort from one age group to the next, equation (1c). These flows influence the stock of initial population, $P_a(0)$.

$$P_a(t) = \int_{t=0}^{T} \left[B(t) + A_{a-1}(t) - A_a(t) - D_a(t) \right] dt + P_a(0) \qquad (1)$$

$$B(t) = \beta \sum_a P_a \qquad 0 < \beta < 1 \qquad (1a)$$

$$D_a(t) = \phi P_a \qquad 0 < \phi < 1 \qquad (1b)$$

$$A_a(t) = P_a \qquad (1c)$$

where,

$P_a(t)$ = stock of age-a population, a= 0, 1,2,…,16.
$A_a(t)$ = aging rate
$B(t)$ = birth rate,
$D_a(t)$ = fractional death rate,

Latent population (*L*) is the portion of population not attending school. It is also represented as an array of stocks similar to the population one except that this variable provides a direct input to first grade through the intake rate ($e_a(t)$). This is shown in equation (2) where there is an additional outflow represented by those children meeting age requirements to start primary. $L_a(0)$ represents the age-*a* initial stock of latent population. By making a distinction between Population and Latent Population we can make consistent comparisons between people in the education system and people who are not attending school. Birth and death rates in this equation are adjusted to take into account Latent population only.

$$L_a(t) = \int_{t=0}^{T} \left[B(t) - e_a(t) - D_a(t) \right] dt + L_a(0) \qquad (2)$$

where $e_a(t)$ = intake rate
$L_a(t)$ = Latent population

[6] Any student is allowed to stay in the system until s/he is 15 years; we then group all the rest of the population in a single cohort representing those of 16 years or more.

Primary Education (G) represents children who are actually enrolled in school. Equation (3) shows that primary education consists of i grades represented by an equal number of stocks from the first to the i-th level, based on the official cycle length in the country, seven years. In words, $G_{1,a}$ (t) represents the population of age-a students attending the first grade. We make a distinction between $G_{1,a}$ (t) and the rest, $G_{i\geq 2,a}$ (t), in equations (3) and (3a), because intake $e_a(t)$ only occurs in the first grade and promotion only takes place as an inflow ($\pi_{i-1,a-1}(t)$) after the second grade. Once children enter school they may follow three mutually exclusive directions: passing to the next level ($\pi_{i,a}(t)$) from grade i to i+1 and growing old one year (from a to a+1); repeating the year ($r_{i,a}(t)$) just passing to the next age cohort (from a to a+1) but remaining in the same grade (i); or dropping-out the grade i at age a ($d_{i,a}(t)$).

$$G_{1,a}(t) = \int_{t=0}^{T}[e_{1,a}(t) - d_{1,a}(t) + r_{1,a-1}(t) - \pi_{1,a}(t) - r_{1,a}(t)]dt + G_{1,a}(0) \qquad (3)$$

$$G_{i\geq 2,a}(t) = \int_{t=0}^{T}[\pi_{i-1,a-1}(t) - d_{i\geq 2,a}(t) + r_{i\geq 2,a-1}(t) - \pi_{i,a}(t) - r_{i\geq 2,a}(t)]dt + G_{i\geq 2,a}(0) \qquad (3a)$$

where $G_{i,a}$ (t) = population grade $i=1,2,\ldots,6$ and age-*a*

$\pi_{i,a}(t)$ = promotion grade *i* at age-*a*

$r_{i,a}(t)$ = repetition grade *i* at age-*a*

$d_{i,a}(t)$ = drop-out grade *i* at age-*a*

All flows are the product of the vector of respective constant fractions like intake (α_a), repetition ($\tau_{i,a}$), drop-out ($\delta_{i,a}$), and promotion ($\rho_i^a \equiv (1-\delta_{i,a}-\tau_{i,a})$ rates, multiplied by the stock of people in the respective grade (latent population in the case of intake). Values considered normal drive these flows which are calculated using the fractional rate for the base year. The corresponding formulations are equations (3b) to (3e).

$$e_a(t) = e\left(L_a(t), \alpha_{1,a}\right) = \alpha_{1,a} L_a(t) \qquad (3b)$$

$$d_{i,a}(t) = d\left(G_{i,a}(t), \delta_{i,a}\right) = \delta_{i,a} G_{i,a}(t) \qquad (3c)$$

$$r_{i,a}(t) = r\left(G_{i,a}(t), \tau_{i,a}\right) = \tau_{i,a} G_{i,a}(t) \qquad (3d)$$

$$\pi_{i,a}(t) = \pi\left(G_{i,a}(t), \rho_{i,a}\right) = \rho_{i,a} G_{i,a}(t) \qquad (3e)$$

$$\alpha_a, \delta_{i,a}, \tau_{i,a}, \rho_{i,a} \in (0,1) \text{ for every } a$$

From this, the simulation model tracks desired system indicators. Gross Enrollment Rate, for instance, can be derived from equations (1), (3), (3a) and PCR from (1) and (3e), respectively:

$$GER = \left(G_{1,a}(t) + G_{i\geq 2,a}(t) \right) / P_{7-12}(t) \qquad (4)$$

$$PCR = \pi_{6,a}(t) = \pi\left(G_{6,a}(t), \rho_{6,a}\right) / \left(P_{12}(t) \right) \qquad (5)$$