# SENSITIVITY ANALYSIS OF SYSTEM DYNAMICS MODELS BY BEHAVIOR PATTERN MEASURES

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## Abstract
Parameters of system dynamics models are subject to uncertainty, so sensitivity analysis is an important task for the reliability of simulation results. Since system dynamics is a behavior-oriented simulation discipline, sensitivity of behavior pattern measures, such as equilibrium level or oscillation amplitude to the model parameters should be evaluated in order to explore the effects of parameter uncertainty on the behavior patterns. In this study, a procedure for pattern sensitivity analysis of system dynamics models is suggested. Then this procedure is applied to the tipping point project management model (Taylor and Ford, 2006) and two oscillatory system dynamics models, namely the generic supply line and inventory-workforce models (Sterman, 2000), using regression method. Our results indicate that project scope and total staff parameters are important for the project management model. We also conclude that inventory adjustment time is most critical in oscillatory models. Experiments with our procedure show that regression analysis on behavior pattern measures constitutes a useful approach for sensitivity analysis of system dynamics models.

Key Words: *sensitivity analysis, behavior pattern sensitivity, regression analysis*

## INTRODUCTION
In simulation models, assumptions represent uncertain information that cannot be collected from real life observations. These model assumptions may be about parameter values, feedback loops or decision heuristics of system manager. The effect of input uncertainty on the model output is analyzed by sensitivity analysis that searches how the "variation in output can be apportioned to different sources of variation" (Saltelli, Chan, Scott, 2000). Furthermore, sensitivity analysis not only determines the effect of variations in assumed information on the model output, but "it also helps to develop intuition about model structure and it guides the data collection efforts" (Sterman, 2000). Modelers may spend much time on estimation of possibly unimportant model parameters. On the other hand, parameters significantly affecting output behavior should be chosen as candidates for additional data collection (Sterman, 2000).

The first step of sensitivity analysis is *one-variate* sensitivity analysis which is conducted with "*one-at-a time approach"* (Saltelli et al., 2000). The changes in model output, which stem from perturbation in each parameter value, are analyzed separately and the most influential parameters are estimated roughly. On the other hand, in nonlinear and complex models, *one-variate* sensitivity analysis is insufficient for a comprehensive study of the model. Simultaneous changes in more than one parameters' values may create an unexpected output change because of the nonlinear relationships among different model components (Sterman, 2000, p.561). Therefore, one-variate analysis should be followed by *multi-variate sensitivity* analysis.

In system dynamics literature, different methods are proposed for multi-variate sensitivity analysis of models. A good example for sensitivity analysis of system dynamics models can be found in Ford (1990), in which the key model parameters are identified using partial correlation coefficients. Furthermore, an extension to the Ford's study (1990) is provided in 2005 by Ford and Flynn. They propose the use of simple correlation coefficients instead of the partial ones in order to determine important model parameters. This approach is dubbed as *screening* in the study of Ford and Flynn (2005). They state that calculation of simple correlations is easier, so this method is more appropriate for the screening of system dynamics models. In screening method, correlation coefficients between each parameter and values of output variables at each time point are calculated and plotted against the simulation time. This plot provides information about the dynamic importance of model parameters during simulation. On the other hand, as will be seen below, use of individual output values in correlation coefficients is the main weakness of screening method in the analysis of oscillatory models (Ford and Flynn, 2005).

In system dynamics, the behavior *patterns* of model variables are more important than their numerical values. Namely, if a model exhibiting s-shaped growth is being considered, the exact value of the variable at a specific time point is not important for the analyst. Instead, the inflection points, equilibrium level or time to equilibrium deserve the attention of system dynamics researcher. For oscillatory system dynamics models, sensitivities of behavior *pattern* measures such as equilibrium levels, oscillation periods or amplitudes should be evaluated.

In system dynamics literature, several researchers consider pattern measures for sensitivity analysis purposes. Moizer et al. (2001) propose a *one-variate sensitivity analysis* procedure using performance indexes suggested by Coyle (1978). Performance indexes are defined as "a single number summarizing the whole performance of a run on the model" (Coyle, 1978). Furthermore, Ozbas et al. (2008) analyze the pattern sensitivity of oscillatory models using amplitude and period of oscillation. In our study, on the other hand, a general analysis procedure for behavior pattern sensitivity is proposed and this procedure is applied to different system dynamics models. Specifically, in the third and forth sections of the article, behavior *pattern* sensitivity and the analysis procedure is discussed in detail. In the remaining sections, the applications of behavior *pattern* sensitivity to a tipping-point model and two oscillatory models are presented.

**BEHAVIOR PATTERN SENSITIVITY**

In system dynamics methodology, the dynamic problem and related policy suggestions are discussed through the characteristics of behavior patterns. In problem conceptualization phase, some specific patterns of the system behavior are considered as the symptoms of the dynamic problem. Moreover, after the completion of model building, different policy options are tried on the model in order to analyze their effect on the problematic behavior patterns of the system. In short, the specific characteristics of behavior patterns, such as equilibrium levels, periods and amplitudes of oscillations constitute the main interest for system dynamics researcher. Thus, sensitivity analysis of system dynamics models should focus on the behavior patterns' sensitivity to various model structures or different parameter values. In this study, this type of sensitivity is called *behavior pattern sensitivity*.

In system dynamics literature, *numerical sensitivity*, *behavior mode sensitivity* and *policy sensitivity* are discussed as sensitivity types in various references. *Numerical sensitivity* evaluates the sensitivity of output values to the change in model assumptions, whereas *behavior mode sensitivity* tries to determine sensitivity of output behavior to the alterations in the model (Sterman, 2000). *Policy sensitivity*, on the other hand, is defined as the change in desirability or suitability of an existing policy when there is a change in model parameters or structure (Moxnes, 2005). *Behavior pattern sensitivity* should be added to the list of sensitivity types in order to cover the effect of changing inputs on output patterns of the model. In the following section, methodology for *behavior pattern sensitivity* analysis is discussed in detail.

## METHODOLOGY FOR SENSITIVITY ANALYSIS

Sensitivity analysis may provide important information to the researcher. The results of sensitivity analysis may allow the model builder to determine which of the model parameters are more important for the simulation output. The parameters, to which model output is sensitive, necessitate more data analysis in order to decrease the uncertainty in the parameter value.

As a first step of sensitivity analysis, the distribution function and range of each parameter are determined by using the information and sampled data obtained from the real system. Typically, ± 20% of the parameter value are used as the distribution ranges (Sterman, 2000). These parameter ranges and distributions can be entered to Vensim's Sensitivity Simulation module which is explained in detail in the study by Ford and Flynn (2005).

Selection of a sampling strategy comes after the determination of parameter ranges. There are many different sampling strategies such as random sampling, stratified sampling, Taguchi method or latin hypercube sampling that are discussed in the literature. Among these different sampling strategies, latin hypercube sampling (LHS) is the most appropriate one for simulation models (McKay et. al, 1976). Furthermore, Clemson et al. (1995) compare Taguchi method with LHS and conclude that latin hypercube sampling is an effective technique when it is impossible to use Taguchi method because of nonlinearity in model structure or too many model parameters. In this study, LHS in Vensim's Sensitivity Simulation Module is used in order to take samples from distributions of model parameters.

Sensitivity simulation data includes the values of model variable and parameters for each simulation run. After importing this data into an Excel spreadsheet, one should diagnose the behavior of each run graphically in order to separate behavior patterns from each other. Usually different feedback loops play an active role for different behavior modes. Moreover, each behavior mode has different output measures that are of interest in behavior pattern sensitivity analysis. Therefore, one should separate different output modes from each other at the beginning of analysis.

After separation of the different behavior modes, their pattern measures should be determined. In each dynamic problem, different characteristics of the output behavior are relevant. Interested behavior characteristics depend on the dynamic hypothesis of the problem. For instance, according to the results of the famous World1 model, world population will follow a boom and bust behavior in future. Typically, the peak point and, if they exist, equilibrium levels are important output measures for the boom and bust behavior. Therefore, if a researcher would want to make behavior sensitivity analysis to the World model, possible behavior measures would be peak level, peak time, equilibrium level and time to reach equilibrium.

The determination of the behavior measures of interest should be followed by estimation procedures of each pattern measure. The complexity of estimation procedure differs for each behavior measure. Specifically, the peak point of boom and bust behavior can be obtained by a simple maximum function in Excel; however, the log-amplitude slope of a damping oscillation necessitates a calculation algorithm including estimation of oscillation period and amplitudes. In this study, BTSII, a validation testing software developed by Barlas et al. (1997), is used in order to estimate period and amplitude of oscillations. The estimation procedures for the different measures of several behavior patterns are explained in Barlas (1989) in great detail.

At the end of the estimation process, we have behavior measures for each simulation run and the parameter values that are used in these simulations. In order to calculate the sensitivity of behavior measures to the model parameters, different statistical procedures such as regression or partial correlation coefficient can be used. In this study, the linear regression method is utilized to comparatively assess the importance of each parameter for different pattern measures using standardized regression coefficients and individual F tests. Furthermore, within regression methodology the analyst has the ability to check whether the linear functional relationship is appropriate for the data set at hand.

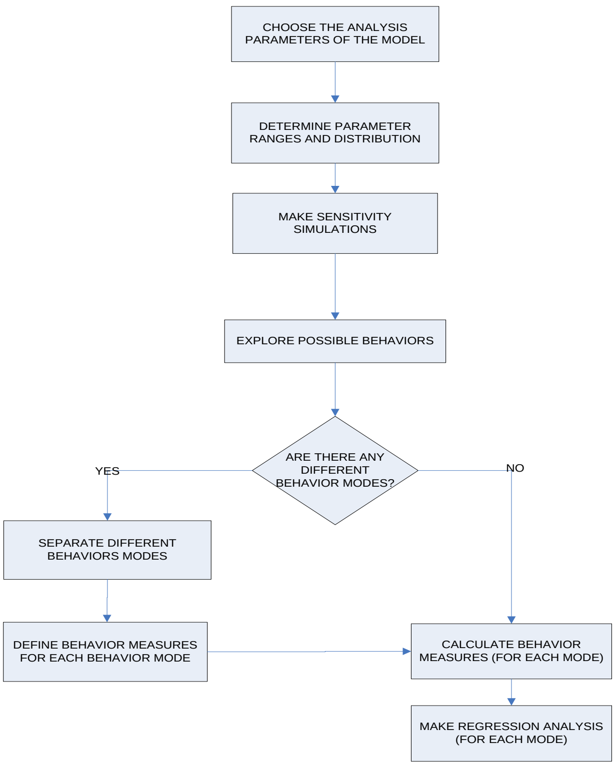



Figure 1: Behavior Sensitivity Analysis Algorithm.

Latin Hypercube Sampling procedure of Vensim's Sensitivity Simulation assumes independency of model parameters and makes sampling from given parameter ranges under this assumption. Therefore, the regressor variables in regression equation would be statistically independent from each other and the absolute values of standardized regression coefficients can be used as "importance ranking criteria" for sensitivity analysis purpose (Saltelli et al. 2000). In addition to the magnitudes of standardized regression coefficients, their signs indicate the direction of correlation between the model parameters and behavior measures. Individual F tests may also be used in order to assess significance of each parameter in the regression equation.

Nevertheless, analysts should be aware of the variances of the regression coefficients. Since variances may be significant in some circumstances, the importance ranking obtained using the standardized regression coefficients may be erroneous. Therefore, one should repeat the analysis with different seed values and for different numbers of sensitivity simulations. The importance rankings obtained from different sensitivity experiments should be compared in order to verify the sensitivity results.

In regression method, it is assumed that the residual terms are normally distributed with zero expectation and a constant variance. This assumption of the regression can be easily checked with residual plots. If the residual terms are distributed randomly around zero, the linear model is appropriate for the data set at hand. On the other hand, if the regression assumptions are not fulfilled, different procedures may be applied to obtain a suitable regression model. For instance, a linear model including regressors can be augmented with the products of significant regressors, to represent the interactions of the regression variables.

In short, sensitivity analysis of system dynamics models starts with the determination of parameter ranges and distributions. Then, estimation of behavior measures and regression analysis should be performed sequentially. This procedure is illustrated in Figure 1 above. Among different statistical procedures, regression methodology can indicate parameter importance, direction of correlation and appropriateness of linearity assumption. These inferences provide insight about the model structure and leverage parameters which can be used for policy design purposes.

## ANALYSIS OF A PROJECT MANAGEMENT (TIPPING POINT) MODEL

Project management is a dynamic problem that must be monitored by project manager continuously. The project progress may slow down or stop before the completion of all tasks or the total cost of project may go over the budgeted costs. Project failure may stem from different factors such as an unrealistic goal, project scope change or lack of expertise etc. Since in each project management problem different factors or their combinations are dominant, identifying a common reason for project failure is difficult (Taylor and Ford, 2006). On the other hand, quick completion of project tasks brings some quality problems which may affect even completed tasks by means of design changes. Such "secondary or tertiary effects" on productivity, quality or even the task sequence of the project is defined as ripple effect (Taylor and Ford, 2006). Taylor and Ford (2006) focus on rework and ripple effect triggered by the deadline stress. In order to model the effect of deadline stress which appears through ripple effect, they use a tipping point structure.

Tipping point structure represents systems that can present two different behaviors under different circumstances. Sterman (2000) defines the tipping point as the "conditions below which the system remains stable". In project management problems, the tipping point indicates the threshold circumstances before which the project is completed successfully. Ford and Taylor (2006) use tipping point structure in their system dynamics model in order to analyze the effect of deadline stress on the project success. They state that project progress may follow either s-shaped growth or tipping point behaviors according to the conditions of the project. If project remains below the tipping point, the project progress follows s-shaped growth and it reaches to equilibrium at 100 percent which represents the success of the project. On the other hand, if the project passes beyond the tipping point, the ripple effect becomes dominant and project progress follows tipping point behavior. A brief discussion of the model structure is given in the following section.

**Description of the Project Management Model**

There are many feedbacks and nonlinear relationships between fundamental components of the project management problem such as productivity, project deadline, quality of workforce etc. Among these different factors schedule pressure, which affects the quality of completed tasks and workforce motivation, is common in almost all project management problems. As the fixed deadline of the project approaches, schedule pressure on the project staff increases. They tend to increase their pace of work and possibly complete the project tasks incorrectly. Taylor and Ford (2006) model the schedule stress on project staff through ripple effect and rework.

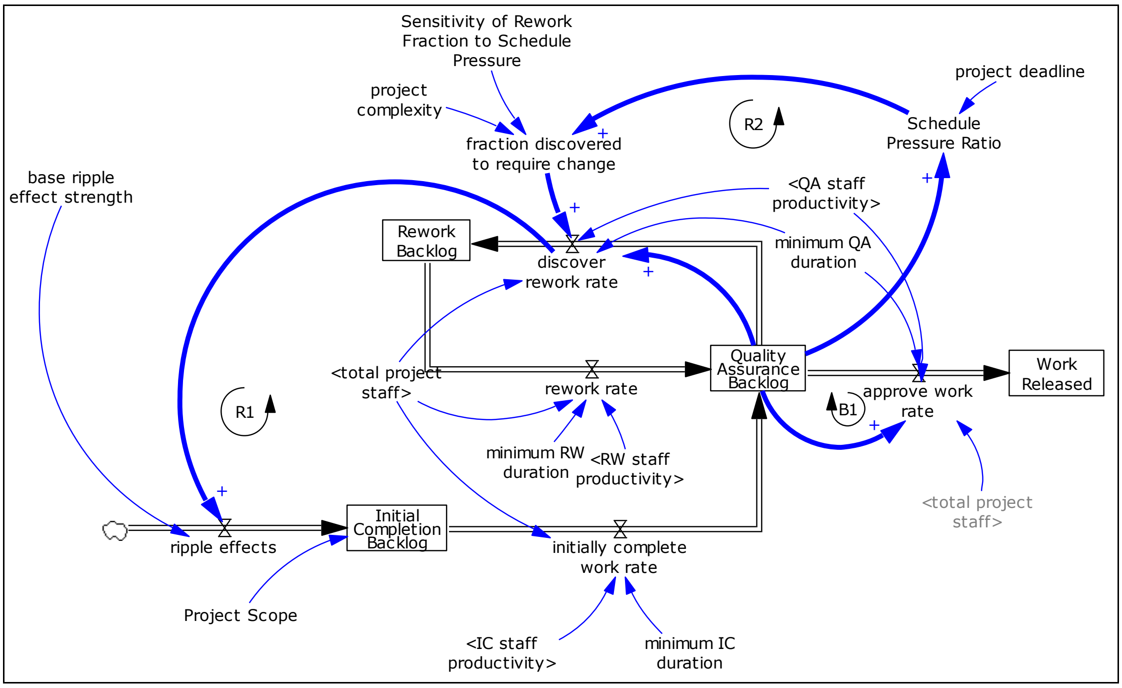


Figure 2: Stock Flow Structure of Tipping Point Project Management Model by Taylor and Ford (2006)

Important model components are given in Figure 2. In this model, project tasks flow from one stock to another. At the beginning of the simulation all project tasks exist in initial completion backlog stock. Completed tasks go to the quality assurance backlog stock that represents the quality control process of a project. Work released stock is the one that keeps the completed work. The ratio of the completed tasks to initial completion backlog is the measure that determines the performance of the project. The project tasks that require rework is kept in rework backlog stock whose inflow represents the discovery of completed tasks that need reprocessing. This inflow, called discovery rework rate, is determined by the *fraction discovered to require change*. This fraction, which consists of the project complexity and schedule pressure components, indicates the percent of project tasks requiring reprocessing. As deadline pressure of the project increases, discovery rework rate increases since the effect of deadline stress grows. Formulation of schedule sensitivity and the rework fraction is described in detail in Taylor and Ford (2006).

In this model there are three fundamental coupled feedback loops which can be seen in Figure 2. The loop named R1 adds new tasks to the initial completion backlog stock. Increasing rework backlog stock increases the initial completion backlog which is described as ripple effect above. On the other hand, increasing quality assurance backlog (QA Backlog) increases the quality assurance rate (QA rate) which strengths the B1 negative feedback loop. If this negative feedback loop dominates R1, the system follows s-shaped growth and the project becomes successful. On the other hand, dominance of R1 loop creates tipping point behavior that represents project failure. These two output behaviors of the model can be seen in Figure 3 and Figure 4 below.

As explained above, behavior *pattern* characteristics are considered in behavior sensitivity analysis of the system dynamics models. Therefore, the behavior measures of two resulting patterns are determined as a first step of sensitivity analysis. The behavior measures of s-shaped growth behavior are listed below.

- Equilibrium Level
- Inflection Point Level
- Inflection Time
- Time to reach equilibrium

In S-Shaped growth patterns, one of the important behavior measures is the point at which the system reaches equilibrium. However, in tipping point project management model, all S-Shaped growth behavior runs reach equilibrium at 100%. Therefore, only the last three output measures are relevant in this study.

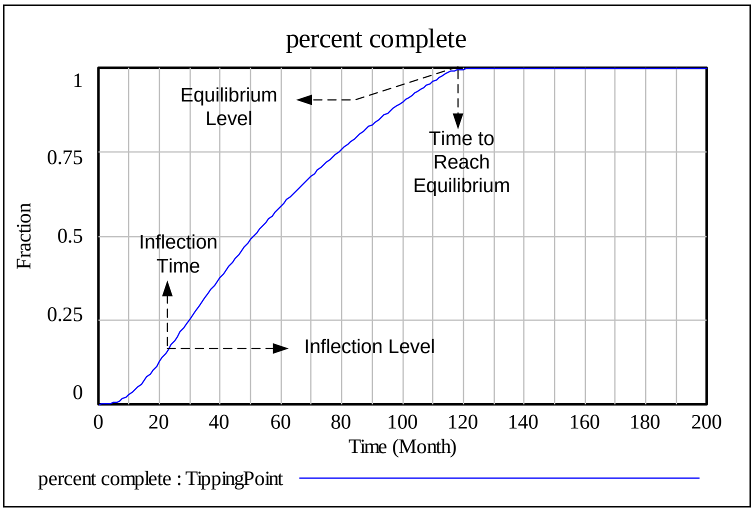



Figure 3: S-Shaped Growth Behavior of Tipping Point Project Management Model

The *inflection point* of a s-shaped growth can be defined as the point at which the second derivative equals to zero. Up to this point the system follows exponential growth which means the positive feedback loops are dominant in the system. After the inflection point, controlling (negative) feedback loop becomes dominant in the system. Inflection point level of the behavior is related to the initial strength of the positive feedback loop. Similarly, *inflection time* indicates the time when negative feedback loops become dominant in the system and both measures are very similar to each other. Finally, *time to reach equilibrium*, which provides idea about the strength of the negative feedback loop, is another measure of s-shaped growth. This behavior measure also indicates the time at which all the tasks of the project are completed successfully.

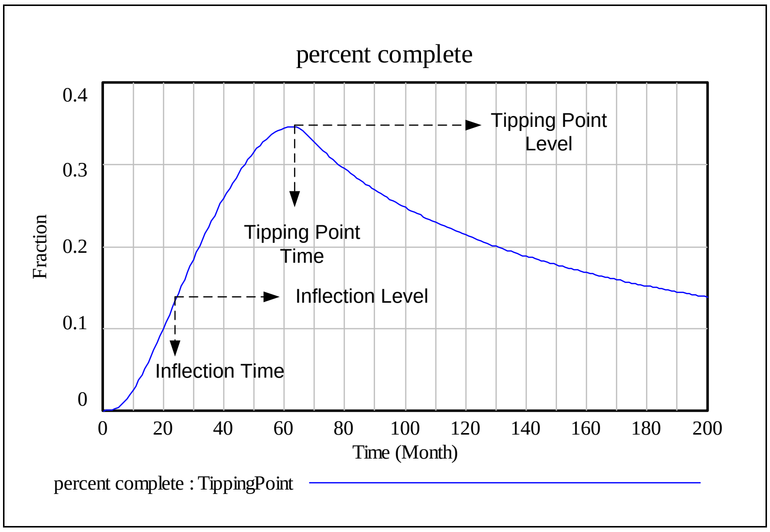



Figure 4: Tipping Point Behavior of Tipping Point Project Management Model

Second type of behavior pattern of this model is tipping point behavior, which represent unsuccessful projects. This behavior pattern follows S-shaped growth initially but does not reach completion equilibrium. After the tipping (peak) point, the output variable, *percent complete*, begins to decline i.e. the project cannot be completed successfully. Typically, this behavior pattern has 4 different measures that can be used for analysis purposes. These are;

- Inflection level
- Inflection time
- Peak Point Level (Tipping Point Level)
- Time of peak

Inflection level and inflection time are explained in previous behavior mode above. Further typical pattern measures of tipping point behavior are *peak point* and *time of peak*. These measures are related to the relative strengths of the feedback loops B1 and R2. B1 loop is the controlling loop of the model and it plays critical role since it includes the completion of project tasks. On the other hand, as the project continues, the rework and schedule pressure create additional project tasks which represents the ripple effect. This reinforcing loop determines the peak level and peak time; when it becomes dominant against the B1 loop, the project tasks begin to increase.

**Sensitivity Analysis of Tipping Point Project Management Model**

In behavior sensitivity analysis, the first step is identification of input parameters and their distribution functions and ranges. Parameters of tipping point project management model and their distribution functions are given in Table 1. These distribution ranges are also used in screening analysis of the model conducted by Taylor et al. (2007, 2010).

After completion of the sensitivity simulations, tipping point and S-Shaped behaviors are separated from each other. Since these two behaviors have different dynamics and behavior measures, their separation is necessary in order to make sensitivity analysis for each behavior pattern respectively. Each behavior patterns' measures provide different information about the model structure. Therefore, analyst should perform sensitivity analyses for each measure in order to have complete information about the system. In this study, the sensitivity of two pattern measures are analyzed since complete sensitivity analysis of the model is out of scope. Specifically, sensitivity analysis for *inflection point* of S-shaped growth and *peak* of tipping point behavior are discussed in the following sections of this article.

Table 1: Parameter Distributions of Tipping Point Project Management Model

| **Parameters** | **Range** | **Distribution** |
|---|---|---|
| Project Complexity | [0.24- 0.36] | Uniform |
| base ripple effect strength | [0.8 - 1.2] | Uniform |
| Project Deadline | [240 - 360] | Uniform |
| Scope Initial | [28000 - 42000] | Uniform |
| Sensitivity to Schedule Pressure | [0.32 - 0.48] | Uniform |
| Total Staff | [1200 - 1800] | Uniform |
| Staff Adjustment Time | [3.2 - 4.8] | Uniform |
| IC staff Productivity | [0.8 - 1.2] | Uniform |
| RW staff Productivity | [0.8 - 1.2] | Uniform |
| QA staff Productivity | [0.8 - 1.2] | Uniform |
| Minimum IC Duration | [0.8 - 1.2] | Uniform |
| Minimum RW Duration | [0.8 - 1.2] | Uniform |
| Minimum QA Duration | [0.8 - 1.2] | Uniform |
| Release Productivity Adjustment | [0.4 - 0.6] | Uniform |

**Sensitivity Analysis of Inflection Level of S-Shaped Behavior**

Inflection point of s-shaped growth indicates the initial performance of the project. The dynamics of s-shaped growth behavior can be explained as follows: At initial stages of the project, tasks are completed and sent to quality control. Eventually, growing *quality assurance stock* increases the strength of controlling loop B1 and initial exponential growth becomes an s-shaped growth. So, inflection point of s-shaped growth is the instant at which the balancing loop B1 becomes dominant in the model and this pattern measure provides information about the initial completion speed of project tasks

Mathematically, inflection point is the time at which first derivative of the variable is maximum. Since this is a cumbersome process, inflection points of each behavior pattern are estimated using subtraction of two consecutive output variables. These subtractions are treated like first derivative of the behavior pattern and their maximum value is used as inflection point estimate of s-shaped growth behavior.

At the beginning of the sensitivity analysis, the regression model including 14 regressors are built in order to observe the effects of each parameter on the behavior measure. Since the linearity assumption of the regression is found to be acceptable, the results of regression model are considered for sensitivity purposes. Namely, the most significant parameters of the regression are *total staff* and *project scope*. As discussed above, inflection points of s-shaped growth behaviors are affected by the initial completion speed of project tasks so, two parameters affecting this speed are concluded as the most significant in the regression equation. Furthermore, other important parameters of the regression equation are *project deadline* and *project complexity*. These two parameters are strongly related with schedule pressure and ripple effect loops (R1 and R2) of the model. Postponement of project deadline releases the stress factor, so the project progresses rapidly. On the other hand, inflection points occur earlier in more complex projects since the quality assurance backlog grows rapidly because of rework and ripple effect. The results of the regression model are given in Table 2 below.

Regression results not only indicate the most influential parameters of the model through standardized regression coefficients, but also the signs of coefficients indicate the direction of the correlation between the parameter and inflection point. Namely, if there is great amount of project tasks, the inflection point of the s-shaped growth will take place earlier than the smaller projects' progress behavior. On the other hand, increasing the project staff will affect the completion speed of the tasks and the inflection point will occur later. Since the higher *quality assurance stock* means more powerful B1 feedback loop, the inflection level of the S-Shaped growth would appear more quickly in larger projects with inadequate team size.

To sum up, the *inflection point* of S-Shaped growth behavior indicates the time point at which the controlling loop B1 becomes dominant, in other words rapid progress rate of the project begins to slow. The sensitivity analysis of inflection point indicates which parameters play important role in S-shaped growth behavior. Namely, total staff, project scope, project deadline and complexity are concluded as the most influential parameters for inflection point of s-shaped growth. (All eight significant parameters are summarized below in Table 4). These components are good leverage points in order to control the initial completion speed of the project before the progress behavior turns into goal-seeking behavior from exponential growth. Other parameters are found insignificant, as seen in Table 2.

**Sensitivity Analysis of Tipping-Point Behavior**

Tipping point behavior is the other output pattern of this project management model. In this pattern, project progress follows s-shaped growth until it crosses the tipping point after which total amount of uncompleted tasks begin to increase. This behavior pattern represents unsuccessful projects which cannot reach 100% completion rate.

Furthermore, tipping point behaviors that are obtained from the sensitivity simulation module of the Vensim are given in Figure 5. These runs indicate that, some projects may begin to decline at very beginning stages of project while others may turn failure although great portion of project tasks are completed. In dynamic models, tipping point behavior appears if the reinforcing loop dominates the controlling loop (Sterman, 2000). Tipping points are the instants at which reinforcing loops (R1 and R2) become dominant to the controlling loop (B1).

Table 2: Regression Results for the Inflection Level Measure

| Coefficients | | | | | | |
|---|---|---|---|---|---|---|
| | | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | | B | Std. Error | Beta | t Test | Sig. |
| 1 | (Constant) | -0.211 | 0.176 | | -1.201 | 0.233 |
| | Project Scope | -1.26E-05 | 0 | -0.645 | -9.202 | 0 |
| | Total Staff | 0 | 0 | 0.588 | 8.155 | 0 |
| | Complexity | -0.996 | 0.174 | -0.416 | -5.711 | 0 |
| | Deadline | 0.003 | 0.001 | 0.34 | 5.196 | 0 |
| | IC Productivity | 0.17 | 0.038 | 0.298 | 4.491 | 0 |
| | QA Productivity | 0.114 | 0.038 | 0.205 | 3 | 0.003 |
| | RW Productivity | 0.117 | 0.043 | 0.187 | 2.738 | 0.007 |
| | Sense2Pressure | -0.221 | 0.102 | -0.145 | -2.162 | 0.033 |
| | Ripple Strength | -0.079 | 0.043 | -0.128 | -1.848 | 0.068 |
| | Min IC Duration | 0.056 | 0.04 | 0.091 | 1.426 | 0.157 |
| | Min RW Duration | 0.051 | 0.041 | 0.084 | 1.233 | 0.221 |
| | Min QA Duration | 0.042 | 0.04 | 0.07 | 1.061 | 0.291 |
| | Time2Actual Prod | 0.059 | 0.082 | 0.048 | 0.717 | 0.475 |
| | Staff Adjustment | 0.003 | 0.01 | 0.021 | 0.311 | 0.756 |

Sensitivity of tipping points to the model parameters indicates the parameters that are important for ripple effect and deadline stress. Such information is useful while developing policies for management of large projects in which ripple effect and schedule stress may appear quickly. In this part of study, therefore, the sensitivity of peaks points to the parameter values are analyzed through regression. Results of regression analysis are given in Table 3.

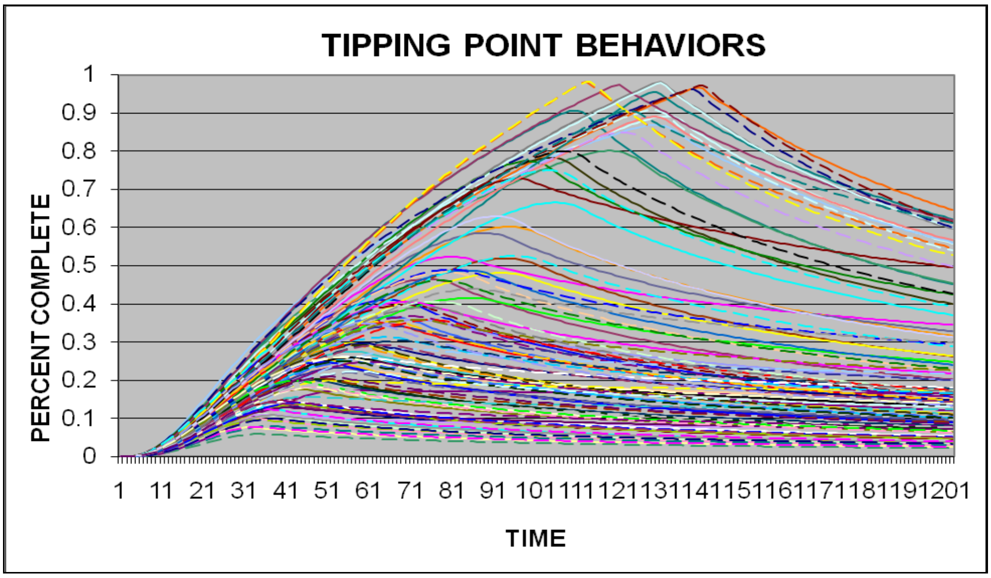



Figure 5: Sensitivity Graph for Tipping Point Behaviors

Like inflection point of s-shaped growth, *total staff* and *project scope* are again the most important parameters in the regression equation. These parameters affect the schedule pressure on the project team; therefore their important effects are consistent with the intuition about the project management problems. Furthermore, *initial completion productivity* and *project deadline* are the third and forth important parameters for tipping points. High productivity of initial completion staff causes early domination of controlling loop and decreases the schedule stress at the end of the project duration. *Project deadline,* on the other hand, determines the project duration which is the main factor of schedule pressure. However, total staff is a more effective leverage point since it affects all feedback loops of the system. We can conclude that increasing number of project staff is a more efficient policy than the deadline postponement which is usually decided once schedule stress emerges.

Table 3: Regression Results for Levels of the Tipping Point Measure

| Coefficients | | | | | | |
|---|---|---|---|---|---|---|
| | | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | | B | Std. Error | Beta | T | Sig. |
| 1 | (Constant) | -1.5 | 0.612 | | -2.45 | 0.016 |
| | Total Staff | 0.001 | 0 | 0.669 | 10.691 | 0 |
| | Project Scope | -4.87E-05 | 0 | -0.622 | -10.243 | 0 |
| | IC Productivity | 0.992 | 0.132 | 0.432 | 7.503 | 0 |
| | Deadline | 0.012 | 0.002 | 0.398 | 7.016 | 0 |
| | Complexity | -3.658 | 0.608 | -0.38 | -6.017 | 0 |
| | Ripple Strength | -0.447 | 0.149 | -0.18 | -3.008 | 0.003 |
| | Time2Actual Prod | 0.769 | 0.285 | 0.158 | 2.699 | 0.008 |
| | QA Productivity | 0.319 | 0.132 | 0.143 | 2.411 | 0.018 |
| | Sense2Pressure | -0.757 | 0.357 | -0.123 | -2.123 | 0.036 |
| | RW Productivity | 0.223 | 0.149 | 0.089 | 1.5 | 0.137 |
| | Min QA Duration | 0.166 | 0.138 | 0.069 | 1.201 | 0.233 |
| | Staff Adjustment | 0.032 | 0.034 | 0.055 | 0.954 | 0.343 |
| | Min RW Duration | 0.108 | 0.144 | 0.044 | 0.753 | 0.453 |
| | Min IC Duration | 0.066 | 0.138 | 0.027 | 0.477 | 0.634 |

In linear regression models, it is assumed that the variability in independent variable can be explained with a formulation of variables in additive form. In other words, linear functional relationship between independent variable and regressors are assumed to be true. Moreover, residual terms of regression are assumed to have normal distributions with zero mean and constant variance. In fact, normality assumption is highly critical for regression analysis since the power of statistical tests is best under this assumption. Therefore, after regression coefficients are calculated, the residual terms are plotted to check the zero expectation and constant variance assumptions. If there is any non-random pattern in these plots, the linear model assumption should be suspicious. Explanations about non-random patterns and their implications are discussed in great detail in Draper and Smiths’ reference book (2000).

In this study, regression assumptions are checked in every regression model. Residual plots indicate that all regression models' assumptions can be accepted except the ones used for tipping points. Residual terms of regression models of tipping point levels show a non-random pattern which can be seen in Figure 6. This indicates that the linear model is not so appropriate for the sensitivity analysis of tipping point levels and the results obtained from this regression model should be considered with caution.

Once the regression assumptions are concluded as invalid, sensitivity analysis using correlation based methods may yield erroneous results as well. Correlation based methods measure the strength of linear relationship among variables. However, if linearity is not a good assumption, the results obtained using these techniques may give wrong results about model structure. In such cases, transformation on dependent or independent variables (Montgomery, 2001) or rank transformation (Saltelli et al., 2000) or graphical methods can be used in order to assess significance of model parameters. However, applications of these different procedures are out of scope for this article. (See Hekimoğlu, 2010)

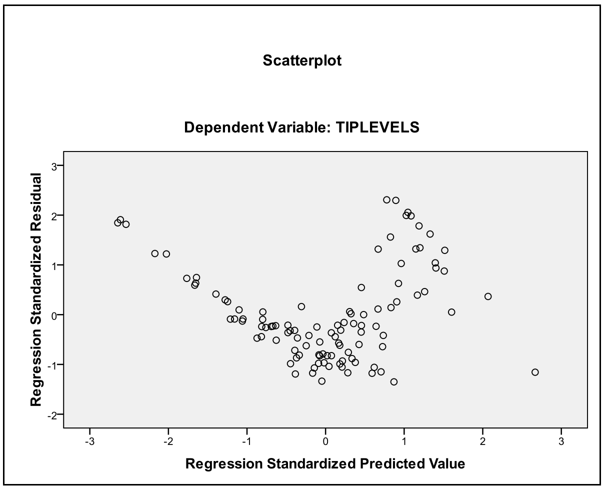



Figure 6: Residual Plot of Regression Model for Tipping Points

Results of sensitivity analysis of measures of tipping point and S-shaped growth behaviors are summarized in Table 4. Total staff and project scope parameters appear as the most influential parameters of the model. Furthermore, the signs of regression coefficients are included in the table since they indicate the direction of relationship between model parameter and behavior measure. Namely, increasing total staff parameter would result in higher tipping point and inflection point level in the output behaviors.

The sensitivity analysis of models that make S-shaped growth behavior and tipping point behavior can be made using behavior measures. Relating these measures with the feedback loops provides information about the model structure and significant parameters. On the other hand, sensitivity analysis becomes more difficult when the model includes the interactions of several delays. In such situations, the intuition about the model structure is usually weaker. In addition to this lack of intuition, the common behavior of models including interaction of several delays is oscillation which is very difficult to analyze with correlation based techniques (Ford and Flynn, 2005). Therefore sensitivity analysis of oscillatory models is almost impossible by standard tools of sensitivity analysis; behavior pattern sensitivity becomes more important for such kind of models. In the next section sensitivity analysis of a small oscillatory model will be presented.

Table 4: Summary of Sensitivity Analysis of Project Management Model by Taylor and Ford (2006)

| RANKING | Tipping Point Level | Inflection Level of S-Shaped Growth |
|---|---|---|
| 1 | Total Staff (+) | Project Scope (-) |
| 2 | Project Scope (-) | Total Staff (+) |
| 3 | Initial Completion Productivity (+) | Project Complexity (-) |
| 4 | Project Deadline (+) | Project Deadline (+) |
| 5 | Project Complexity (-) | Initial Completion Productivity (+) |
| 6 | Ripple Effect Strength (-) | Quality Assurance Productivity (+) |
| 7 | Time to Switch Actual Productivity (+) | Rework Completion Productivity (+) |
| 8 | Quality Assurance Productivity (+) | Sensitivity to Schedule Pressure (-) |
| 9 | Sensitivity to Schedule Pressure (-) | |

**SENSITIVITY ANALYSIS OF OSCILLATORY MODELS**

Oscillation is one of the most difficult and interesting behavior modes, which may have stable, unstable or limit cycle characteristic. Damping oscillations have stable characteristic since they asymptotically approach their equilibrium. On the other hand growing oscillations are examples of unstable systems since they tend to oscillate with growing magnitudes. Finally, limit cycles is a special type of borderline oscillations which settle into a closed trajectory and remain on it in state space.

Statistical sensitivity analysis of oscillatory models is a very challenging task because of non-linear and cyclic behavior patterns. In non-oscillating systems, statistical analysis may be conducted using numerical values of a variable at a specific time point. In oscillatory systems, on the other hand, correlation based methods do not work when they are applied to individual numerical values of any variable (Ford and Flynn, 2005). Therefore, use of behavior pattern measures is a useful approach in order to derive information from oscillatory systems. The difficulty of information derivation is also seen once the sensitivity graphs of oscillatory systems are inspected. Specifically, a sensitivity graph, which consists of behaviors of all simulation runs, can be obtained in Vensim. In non-oscillating systems, these graphs represent the possible output behaviors of the model when its parameters remain in the pre-specified range. In fact, in some sensitivity analysis studies sensitivity graph is used to obtain tolerance interval of output behavior of the model (Ford, 1990). In oscillating systems, however, any meaningful interpretation of output behaviors is too complex for visual inspection. In Figure 7, sensitivity graphs for oscillatory and non-oscillatory models are given.

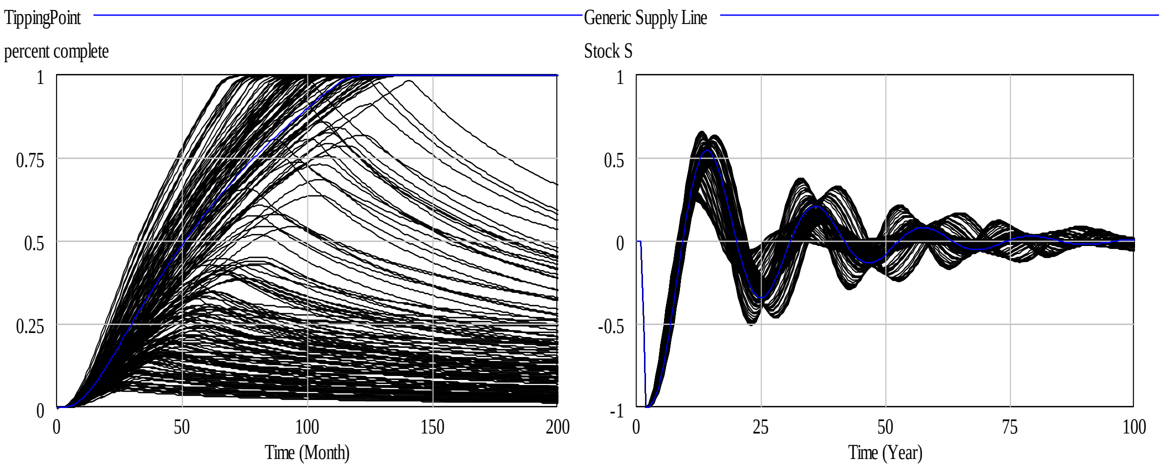



Figure 7: Vensim Sensitivity Graphs of Oscillatory and Non-Oscillatory Models

In damping oscillations, the system state converges to its equilibrium as time proceeds. The behavior measures of damping oscillations, which is depicted in Figure 8, may be listed as follows:

- Period
- Maximum amplitude
- Amplitude Slope

Period of an oscillation can be defined as the time between two peak points (Figure 8). There are different ways, to estimate oscillation period, such as autocorrelation and spectral density functions. These estimation procedures are discussed in Barlas (1990). In this study, the periods of oscillatory behaviors will be estimated by using autocorrelation function in BTS II, a behavior validity software that estimates the behaviors measures of the simulation output in order to compare them with real data.

Another behavior measure of damping oscillations is maximum amplitude which is the distance between first peak and first trough in damping oscillations (Figure 8). It indicates how much system overshoots its goal after it is perturbed by an exogenous force. Once an exogenous perturbation arrives, system tries to adjust the discrepancy between the stock and its desired level. Because of inherent delays in negative feedback loops, however, the state of the stock overshoots its desired level.

The last behavior measure of damping oscillation is the slope of amplitudes. Amplitude of an oscillation can be defined as the measure of the change between peak and trough. The amplitudes of successive oscillations can be easily estimated using the procedure in the study of Barlas et.al. (1997). On the other hand, calculation of slope of a straight line that is fit to successive amplitude is problematic because of nonlinear pattern of amplitudes, as shown in Figure 8. In order to deal with this problem natural logarithm of amplitudes are taken and slope of regression line that fits to log-amplitudes is calculated.

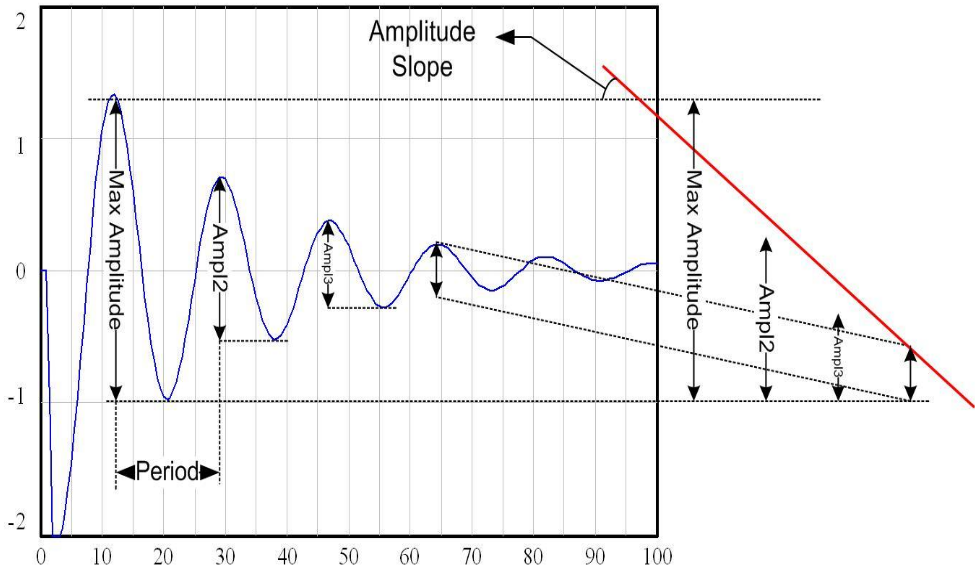
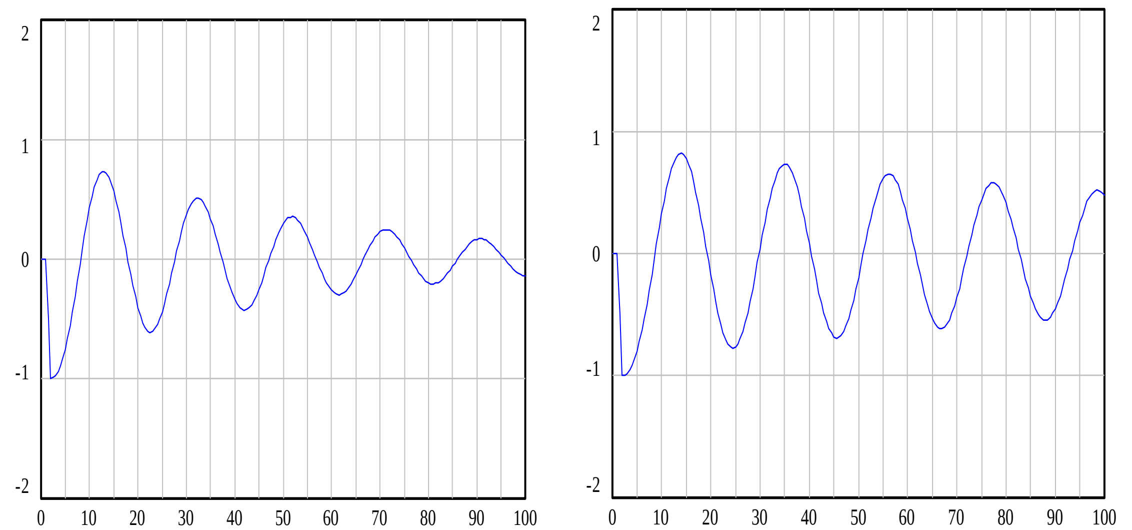



Figure 8: Period, Maximum Amplitude and Amplitude Slope of Damping Oscillation

Log-amplitude slope indicates the stability character of the oscillatory behavior. Namely, negative and high amplitude slope represents a more stable oscillation while low amplitude slope implies less stable one. In other words, amplitude slopes imply how fast the system will reach its equilibrium when it is disturbed by a shock. For instance, two oscillatory behaviors of the supply line model are presented in Figure 9. Obviously, the right hand side oscillation tends to reach its equilibrium more rapidly, i.e. more stable than the other one. This stability character can be attributed to the slope of the oscillation amplitudes.



Resulted Amplitude Slope= 0.36189　　Resulted Amplitude Slope= 0.116169

Figure 9: Comparison of Amplitude Slopes for Two Oscillatory Behaviors

Another oscillation type which can be observed in dynamic systems is growing oscillations. Among behavior measures of stable oscillations, the maximum amplitude is invalid for unstable oscillations since it is a function of simulation time in this behavior mode. But instead, minimum amplitudes can be used in order to analyze the first response of the system to the new coming shocks.

The third type of oscillation behavior is limit cycle in which the “non-linear system structure limits amplitudes of the growing (or damping) oscillations. In such systems, system remains in certain ranges and follows a particular orbit in state space” (Sterman, 2000). There are two measures for limit cycles which are;

- Amplitude
- Period

Like growing oscillations, period is a useful pattern measure for limit cycles. Moreover, amplitude height remains eventually constant and its estimation procedure is the same with other oscillation types discussed above.

In short, behavior pattern measures are critical information for sensitivity analysis of oscillations. In fact, sensitivity analysis of oscillatory systems is only possible if their behavior pattern measures are used as dependent variable in statistical analysis. In this study, sensitivity analyses of two oscillatory system dynamics models are conducted using behavior measures and regression methodology. In the following section, a generic supply line model is analyzed using sensitivity analysis.

**ANALYSIS OF BASIC OSCILLATORY STRUCTURES: GENERIC SUPPLY LINE MODEL**

Supply line structures consist of negative feedback loops and time delays since their fundamental purpose is keeping the stock at the desired level. Systems including negative feedback loops with inherent time lags are prone to oscillations. Many processes in business world include different supply line structures coupled with each other. Namely, employment process is closely related with domestic unemployed labor pool which is another supply line structure. Therefore, models including supply line structures are good cases to analyze oscillatory behaviors. In order to understand dynamics and leverage points of the oscillatory systems, simple supply line models should be analyzed first (Sterman, 2000 and Yaşarcan 2003). A simple generic supply line model[1], which is given in Sterman (2000), is used for analysis of pattern sensitivity of oscillatory behaviors.

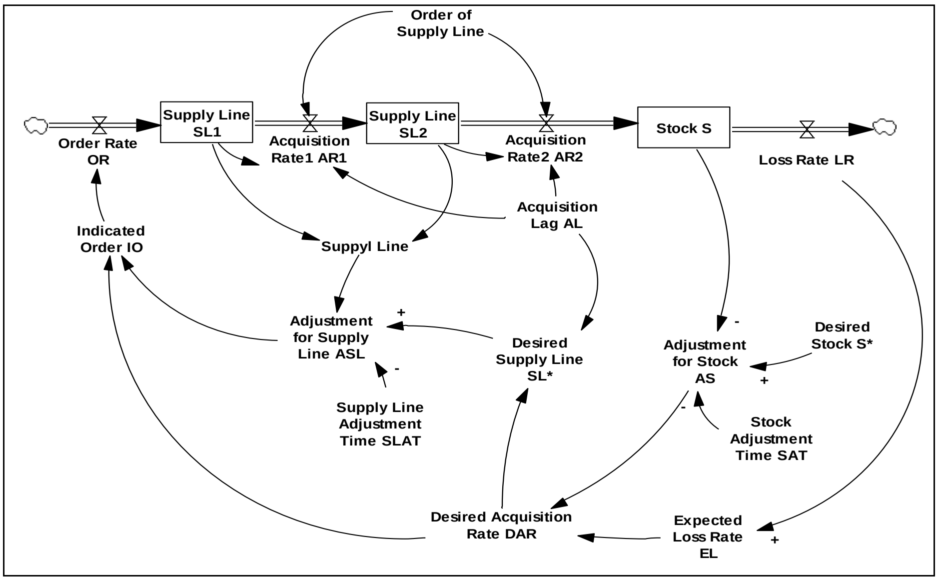



Figure 10: The Stock Flow Structure of Generic Supply Line Model

[1] Second order generic supply line model in Sterman’s book (2000) is modified with one additional supply line stock in this study in order to obtain more oscillatory model.

In this model a hypothetical supply line structure is represented with generic names (Figure 10). Specifically, *desired tock S** is the level at which the model aims to keep the *stock*. In order to keep the stock at the desired level, the amount, which looses through outflow (*Loss Rate LR*), should be added into the *indicated order rate,* after it is adjusted with discrepancy between desired and actual stock values. The summation of these two variables constitutes *desired acquisition rate* which is multiplied with *acquisition lag* in the formulation of *desired supply line*. Therefore, resulting *order rate (OR)* contains adjustment for stock, adjustment for supply line and expected loss rate. Furthermore, in order to keep the model simple, the *expected loss rate* is assumed to be equal to the *loss rate*. We obtain oscillatory behaviors from this model: Supply line model is started in equilibrium at the beginning of the simulations and this condition is disturbed by a pulse function in an artificial outflow of the *stock*. This artificial shock stimulates oscillatory behaviors.

Table 5: Parameter Distributions of Generic Supply Line Model

| **PARAMETER NAME** | **MODEL VALUE** | **Range** | **Distribution** |
|---|---|---|---|
| Stock Adjustment Time (SAT) | 2.5 | [2 - 3] | Uniform |
| Supply Line Adjustment Time (SLAT) | 3.75 | [3 - 4.5] | Uniform |
| Acquisition Lag (AL) | 11 | [8.8 - 13.2] | Uniform |

There are three stocks and parameters in this model and in the sensitivity analysis we use the behavior of *stock* variable. The distribution functions and ranges of model parameters, which are used in sensitivity simulations, are given in Table 5. The sensitivity simulations of this model are performed with the assumption that the parameter values are uniformly distributed within ±% 20 range of base values. Among sensitivity simulation runs, it is possible to observe all three types of oscillations discussed above

After separation of different behaviors, oscillation periods are estimated using autocorrelation function in BTSII software. These estimated periods and model parameters which are supply line adjustment time (SLAT), stock adjustment time (SAT) and acquisition lag (AL), constitute sensitivity data subject to regression analysis. Results of regression analysis for oscillation period are given in Table 6. According to the regression results, stock adjustment time and acquisition lag are the most significant parameters for oscillation period.

Table 6: Regression Results for Oscillation Period

| **Coefficients** | | | | | | |
|---|---|---|---|---|---|---|
| | | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | | B | Std. Error | Beta | t | Sig. |
| | (Constant) | -0.242 | 0.342 | | -0.706 | 0.481 |
| | StockAdjst | 6.971 | 0.08 | 0.694 | 87.099 | 0 |
| | AcqLag | 1.492 | 0.018 | 0.654 | 82.051 | 0 |
| | SupLineAdjst | 2.157 | 0.053 | 0.322 | 40.494 | 0 |

*Stock adjustment time* affects order rate variable directly. Higher values of this parameter represent a decision maker who is reluctant to respond demand changes. Positive sign of the regression coefficient indicates that as the system manager responds less, the period of oscillation increases. Furthermore, higher values of *supply line adjustment time* and longer time delays yield larger period oscillations which are more desirable than the high frequency ones.

Other behavior measure of damping oscillation is log-amplitude slope which indicates the stability character of oscillatory behavior. As discussed above, natural logarithms of successive amplitudes are taken before slope estimation. The calculated log-amplitude slopes are subject to regression analysis of which the results are presented in Table 7. According to these results, the most important parameter for amplitude slope is *stock adjustment time*. Greater stock adjustment time values lead to reduced response to the pulse function, so more stable oscillations emerge. In other words, the absorption effect of stock adjustment time is the most dominant factor in this model.

Table 7: Regression Results for Log-Amplitude Slopes

| Coefficients | | | | | |
|---|---|---|---|---|---|
| | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | B | Std. Error | Beta | t | Sig. |
| (Constant) | 0.612 | 0.02 | | 30.885 | 0 |
| StockAdjst | 0.709 | 0.005 | 0.808 | 154.589 | 0 |
| AcqLag | -0.092 | 0.001 | -0.459 | -87.813 | 0 |
| SupLineAdjst | -0.193 | 0.003 | -0.33 | -63.228 | 0 |

Moreover, one can observe from Table 7 that *acquisition lag (AcqLag)* is negatively correlated with the log-amplitude slope. In other words, increasing time delay makes the system more oscillatory. Therefore, a manager designing a system including inherent delays should keep the delay times as small as possible in order to avoid oscillatory behaviors. Another interesting result of regression equation is *supply line adjustment* parameter. According to the regression results, increasing supply line adjustment time makes the model less stable.

Maximum amplitude of the damping oscillations is analyzed in this sensitivity analysis study as the last behavior measure of generic supply line model. Regression results for the maximum amplitude of damping oscillation are given in the Table 8. Like other behavior measures, stock adjustment time parameter is concluded as the most significant parameter. Furthermore, positive correlation coefficient of supply line adjustment time indicates that systems, which have large stock adjustment time and small supply line adjustment time, are more robust to the incoming shocks. These results are also verified with larger size sensitivity simulations.

Sensitivity analysis results for generic supply line model are summarized in Table 9. *Stock adjustment time* has the greatest impact on different features of oscillation. As discussed above, this parameter represents the responsiveness of the decision maker to the changes in stock. So, its value directly affects the oscillatory characteristic of output behavior. Furthermore, *acquisition lag*, which represents the inherent delay of the supply line structures, is the second important component of the model with respect to all measures. Decreasing time delays increases the stability of the system. Finally, *supply line adjustment time* parameter is found to be the least important parameter for all behavior measures.

Table 8: Regression Results for Maximum Amplitude of Damping Oscillations

| Coefficients | | | | | |
|---|---|---|---|---|---|
| | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | B | Std. Error | Beta | t | Sig. |
| (Constant) | 2.299 | 0.042 | | 54.51 | 0 |
| StockAdjst | -1.088 | 0.01 | -0.807 | -111.454 | 0 |
| AcqLag | 0.144 | 0.002 | 0.47 | 64.926 | 0 |
| SupLineAdjst | 0.278 | 0.007 | 0.31 | 42.757 | 0 |

Table 9: Results Table of Sensitivity Analysis of Generic Supply Line Model

| | Importance Ranking of Parameters for Different Behavior Measures | | |
|---|---|---|---|
| **RANK** | **PERIOD** | **LOGARITHM of AMPLITUDE SLOPE** | **MAXIMUM AMPLITUDE** |
| 1 | Stock Adjustment Time (+) | Stock Adjustment Time (+) | Stock Adjustment Time (-) |
| 2 | Acquisition Lag (+) | Acquisition Lag (-) | Acquisition Lag (+) |
| 3 | Supply Line Adjustment Time (+) | Supply Line Adjustment Time (-) | Supply Line Adjustment Time (+) |

In short, sensitivity analysis by using behavior measures and regression methodology seems to be successful for detecting important parameters of small size oscillatory models. On the other hand, analysis of larger system dynamics models is more difficult due to larger number of parameters and nonlinear feedback loops. Sensitivity analysis of a more complex oscillatory model is presented in the following section.

## SENSTIVITY ANALYSIS OF A MEDIUM SIZE MODEL: INVENTORY-WORKFORCE OSCILLATIONS

Manufacturing process is a common example of supply chain structures in the business world. Procurement of raw material, production process and shipment of finished goods are the major phases of this process. The main goal of the production manager is keeping the inventory at the desired level through production and shipment rates. Moreover, production process of the manufacturing firm is affected by other sub-systems, such as finance labor management, which consist of supply chain structures.

In the comprehensive reference book of Sterman (2000), a system dynamics model which describes the relationship between production and workforce is presented. Workforce is one of the most important constraints on the production process. Number of employees in the firm can be modeled with a stock in order to represent delays in human resource management processes (Sterman, 2000). There is a supply line structure that can be modeled with a single stock, which is called *vacancies*. Vacancy for a new employment is opened and advertised by the human resource department of the firm after new workforce requirement emerges (Sterman, 2000).

*Desired labor* is the initiating factor of the workforce supply chain. This variable is a function of the *desired production start rate* coming from the manufacturing part of the model. Namely, the division of desired production start rate to *productivity* and *standard workweek* determines the *desired labor*. In workforce sector of the model, *desired labor* yields *desired hiring rate* and *desired vacancy creation rate* after appropriate adjustments for stocks (See Figure 11). The adjustment parameters are *labor adjustment time* and *vacancy adjustment time.* Furthermore, if immediate changes in demand appear, the managers may want to stop employment processes, so *vacancy cancellation* outflow is added to the model. In fact, vacancy cancellation takes some time that is represented with *vacancy cancellation time* parameter in the model. Discussion of the workforce sector (Figure 11) of the model is given in great detail in (Sterman, 2000).

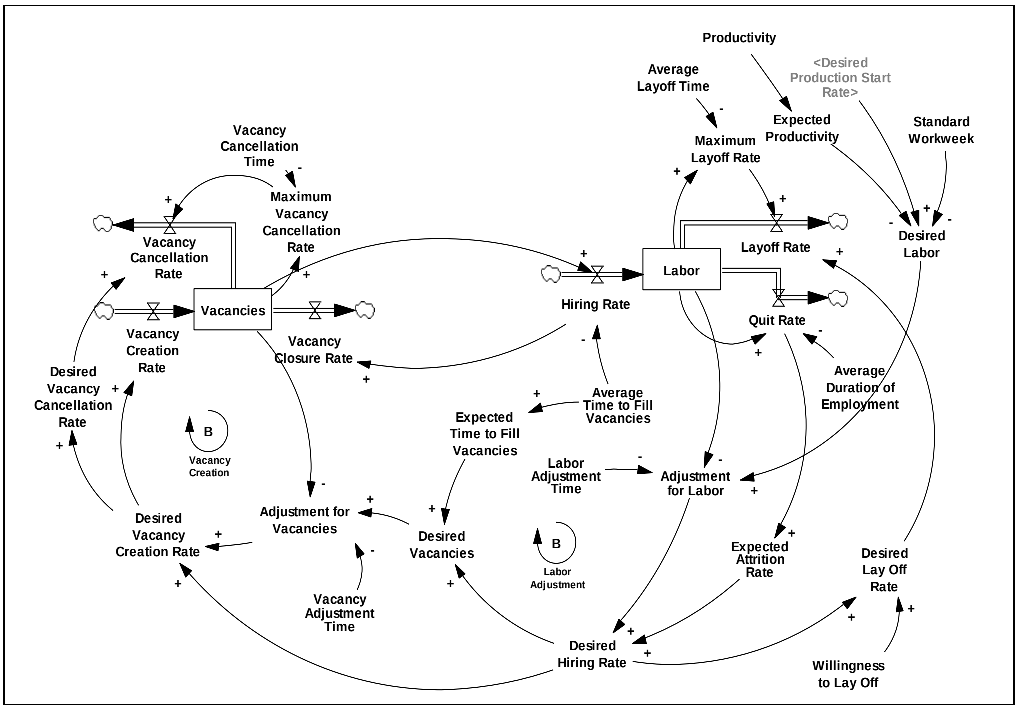



Figure 11: Inventory-Workforce Model (Workforce Sector) (Sterman, 2000)

Other part of inventory-workforce model is manufacturing sector in which p*roduction start rate* is a function of *labor*, *productivity* and workweek[2] (Figure 12). Therefore, feedback loop between *desired production start rate* and *production start rate* is closed through labor structure of the model. *Desired production* rate is the analogous with the *desired acquisition* rate of the generic supply line model discussed above. In fact, the remaining formulations of the model are similar to the equations of generic supply line model except the desired inventory structure, one of the key points of the model.

In the whole inventory–workforce model there are 14 exogenous parameters and 5 stocks. A researcher may focus on the behavior of each stock respectively according to his/her interest. In this study, we will analyze the sensitivity of inventory behavior to the parameter uncertainties.

[2] This workweek represents actual workweek, which is different than the standard workweek used in workforce part of the model.

In simple generic supply line model, it is easy to relate the parameters with the feedback loops and statistical analysis indicates the relative strengths of the feedback loops. On the other hand, in inventory workforce model, there are many feedback loops and each parameter takes role in at least two or three of them. Therefore it is difficult to assess importance of the feedback loops through parameter sensitivities. However, the results of sensitivity analysis point out the potential leverage points of the system in order to control different behavior measures of the system and the reliability of model outputs against changing model parameters.

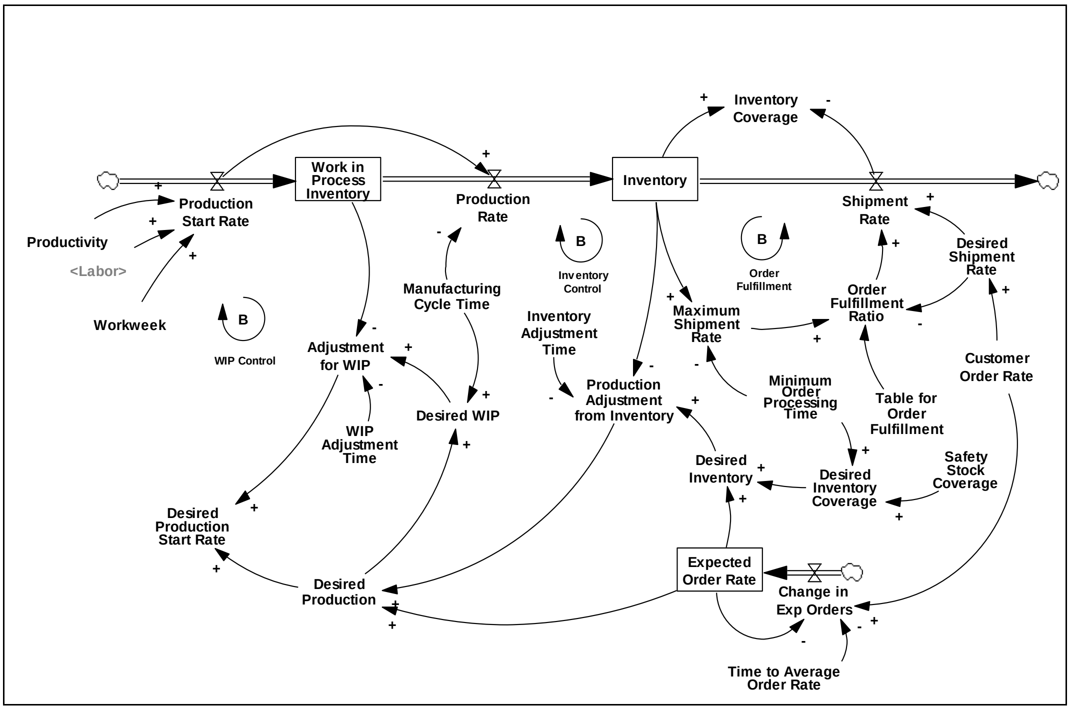



Figure 12: Inventory-Workforce Model (Manufacturing Sector Part)

**Behavior Pattern Sensitivity of Inventory-Workforce Model**

In the inventory-workforce model, there are 14 exogenous parameters which are assumed uniformly distributed within the range of ±20% of its base values. Distributions and their ranges are given in the Table 10. At the end of sensitivity simulations, the inventory values are exported to Excel sheet and their behaviors are inspected visually.[3] Except several growing oscillations, 95% percent of 200 simulation runs are in stable characteristic. Therefore, only pattern measures of damping oscillation are analyzed in this study.

**Sensitivity Analysis of Oscillation Period**

Period of any oscillatory behavior implies how frequently the system oscillates. In some sense shorter period oscillations have more devastating effects on the systems, like heart arrhythmia in human body, than the systems

[3] Visual inspection of behavior patterns may be used even for medium size models. However, large size models' behavior patterns may be much more complex than to be classified visually. Therefore, advanced pattern recognition techniques may be necessary for behavior separation process. An example of such technique is proposed in Barlas and Kanar (1999).

oscillating with longer periods. Therefore, the parameter set resulting with high frequency oscillations is more preferable for the system designer.

Table 10: Parameter Distributions for Inventory – Workforce Model

| PARAMETER NAME | Actual Value | Range | Distribution |
|---|---|---|---|
| Productivity | 0.25 | [0.2 - 0.3] | Uniform |
| WIP Adjustment Time | 6 | [4.8 - 7.2] | Uniform |
| Manufacturing Cycle Time | 8 | [6.4 - 9.6] | Uniform |
| Inventory Adjustment Time | 12 | [9.6 - 14.4] | Uniform |
| Minimum Order Processing Time | 2 | [1.6 - 2.4] | Uniform |
| Safety Stock Coverage | 2 | [1.6 - 2.4] | Uniform |
| Time to Average Order Rate | 8 | [6.4 - 9.6] | Uniform |
| Vacancy Cancellation Time | 2 | [1.6 - 2.4] | Uniform |
| Average Layoff Time | 8 | [6.4 - 9.6] | Uniform |
| Standard Workweek | 40 | [32 - 48] | Uniform |
| Average Duration of Employment | 100 | [80 - 120] | Uniform |
| Average Time to Fill Vacancies | 8 | [6.4 - 9.6] | Uniform |
| Labor Adjustment Time | 13 | [10.4 - 15.6] | Uniform |
| Vacancy Adjustment Time | 4 | [3.2 - 4.8] | Uniform |

In order to explore which parameter explains more variability in behavior measures, a regression model, including 14 model parameters, is utilized. Regression results are given in Table 11. Like other regression results in this paper, regressors are ordered according to the magnitude of standardized regression coefficients. 200 sensitivity simulation runs are used in regression analysis and probability of type 1 error ($\alpha$) assumed to be 0.05. According to analysis results, 8 of the 14 parameters are not significant in the regression model. In other words, some model parameters have very limited effect on the oscillation period. Interestingly, productivity is one of these insignificant parameters (Table 11). This is a counterintuitive result of sensitivity analysis of inventory workforce model.

Furthermore, insignificant *time to average order rate* implies that amount of smoothing applied to demand perturbations does not change oscillation period of the behavior. Oscillation period is most related with the internal time delays in negative feedback loops and smoothness of exogenous input does not affect this behavior measure.

*Manufacturing cycle time* and *WIP adjustment time* parameters are equivalent to *acquisition lag* and *supply line adjustment time* parameters in generic supply line model. According to the results of generic supply line model, the *acquisition lag* is more important than the supply line adjustment time. However in the analysis of inventory workforce model, *manufacturing cycle time* parameter is insignificant while WIP adjustment time plays significant role in regression equation. This contradictory result is also checked via Vensim's Synthesim Module. We can conclude that the parameter sensitivities of generic supply line model and inventory workforce model does not always coincide with each other because of additional nonlinearity in the latter model.

Moreover, according to the regression results the most critical parameters for the oscillation period are *labor adjustment time, WIP adjustment time* and *standard workweek*. *Labor adjustment time* parameter represents how much responsive is the top management of the firm in their hiring policy. Positive sign of the regression coefficient of this parameter implies that more aggressive human resource management policy would result with lower oscillation period which indicates a more dangerous situation.

Sensitivity of oscillation periods is analyzed through regression methodology above. Results of the analysis indicate that *labor adjustment time* and *work in process adjustment time* are the most influential parameters for controlling the period of inventory oscillations. In economic and biological systems, if oscillations are inevitable, we seek large period, small amplitude and more stable oscillations. The stability character of oscillations is measured with amplitude slope that is discussed in the following section.

**Sensitivity Analysis of Log-Amplitude Slope**

In addition to the oscillation period, another important behavior measure is the slope of the amplitudes. This behavior measure indicates how fast the system reaches its equilibrium. In inventory oscillations, amplitudes follow exponentially decaying pattern, likewise generic supply line model (Figure 13). Therefore, we used the slopes of log-amplitudes as the dependent variable in the regression equation. Furthermore, logarithm is monotonically increasing function, so it does not cause any additional monotonicity problem while explaining the relationship between amplitude slope and model parameters.

Table 11: Regression Results for Oscillation Periods of Inventory – Workforce Model

| Coefficients | | | | | |
|---|---|---|---|---|---|
| | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | B | Std. Error | Beta | t | Sig. |
| (Constant) | 38.468 | 4.286 | | 8.976 | 0 |
| StandardWorkweek | -0.854 | 0.038 | -0.53 | -22.345 | 0 |
| LaborAdjstTime | 1.617 | 0.076 | 0.522 | 21.137 | 0 |
| WIPAdjst | 2.928 | 0.16 | 0.436 | 18.266 | 0 |
| SafetyStockCoverage | 3.542 | 0.499 | 0.176 | 7.1 | 0 |
| MinimumOrderProcssing | -1.539 | 0.473 | -0.076 | -3.256 | 0.001 |
| VacancyAdjstTime | 0.547 | 0.244 | 0.054 | 2.241 | 0.026 |
| InvntAdjstTime | 0.14 | 0.08 | 0.042 | 1.752 | 0.081 |
| Time2AvgOrderRate | -0.196 | 0.119 | -0.039 | -1.641 | 0.102 |
| Time2FillVacancy | 0.132 | 0.123 | 0.026 | 1.08 | 0.281 |
| MnfctrngCycleTime | 0.129 | 0.122 | 0.026 | 1.057 | 0.292 |
| AvgLayoffTime | 0.094 | 0.123 | 0.019 | 0.767 | 0.444 |
| VacancyCancelTime | -0.29 | 0.481 | -0.014 | -0.603 | 0.547 |
| Productivity | 1.18 | 3.887 | 0.007 | 0.304 | 0.762 |
| AvgDuratnEmploy | -0.003 | 0.01 | -0.006 | -0.262 | 0.793 |

Additionally, as we discussed above, presentation of oscillatory behaviors in one graph does not provide so much information because of the repeating oscillatory pattern of the behavior (Figure 7). But instead, we can present amplitudes of all behaviors in one graph in which we can consider spread of the oscillatory behavior patterns (Figure 13). This plot is called *amplitude graph* in this study. In fact, presentation of all simulation behaviors in one graph gives insight about the possible output behaviors and their characteristics. Therefore, *amplitude graph* is a useful tool for sensitivity analysis of oscillatory systems.

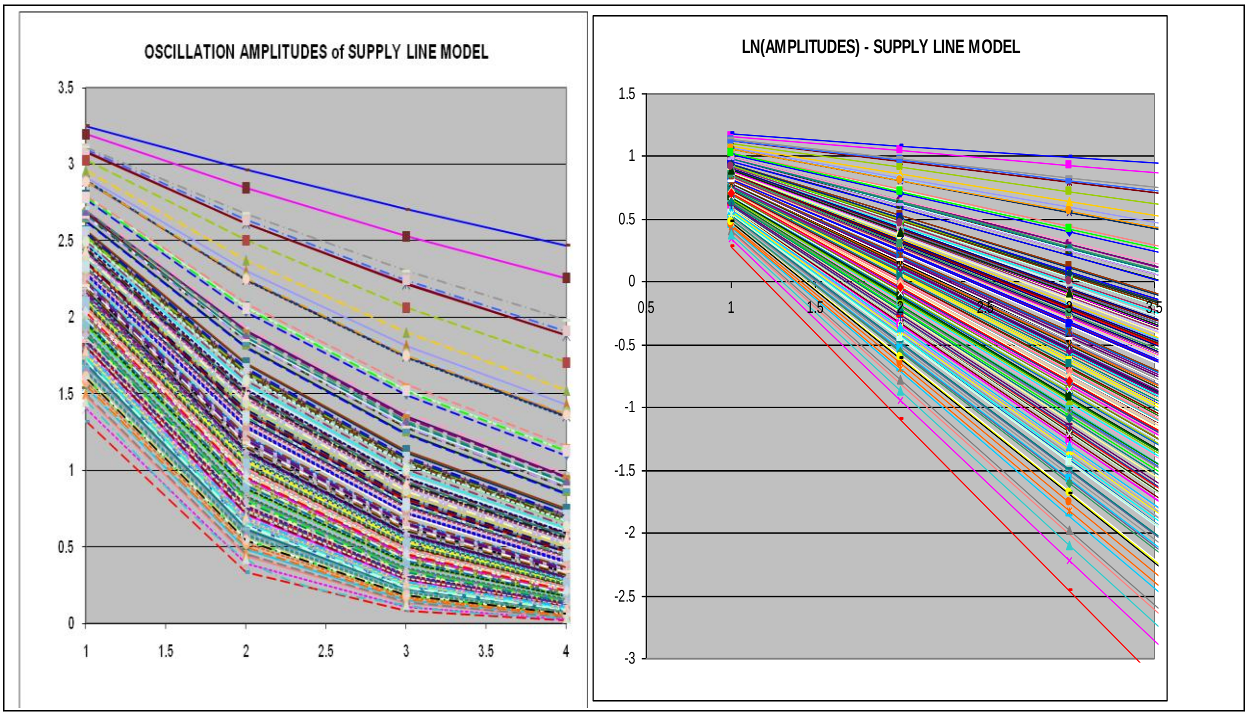


Figure 13: Amplitude Graphs for Oscillatory Behaviors (Standard and Log plots)

Regression results for the log-amplitude slope are given in Table 12. The insignificance of the *average duration of employment* and *average layoff time* indicate that the changes in flows of labor stock do not affect log-amplitude slope. Furthermore, *labor adjustment time* parameter seems insignificant in the regression equation. This observation implies that the aggressive hiring policy of the firm is irrelevant for the stability of inventory oscillations.

For log-amplitude slopes, the most important parameters are *inventory adjustment time* and *standard workweek*. Standard workweek parameter affects the desired labor of the firm according to the desired production rate while inventory adjustment time parameter, which is the analog of *stock adjustment time* parameter in generic supply line model, determines the responsiveness of the decision maker to the immediate demand perturbations. The sign of regression coefficient of inventory adjustment time implies that reluctant approach of the production manager to the immediate demand changes results with less oscillatory manufacturing system. This result also matches with the results of generic supply line model discussed above.

In short, sensitivity analysis through log-amplitude slope indicates that inventory adjustment time and manufacturing cycle time are the most influential leverage points for controlling the stability character of the oscillations. Systems which are less responsive and have shorter time delays will present more stable oscillations once exogenous perturbation arrives to the system. In fact, as discussed above, stability character can be described with the equilibrium reaching pace and first response to the incoming perturbation. The analysis of latter system feature, which is maximum amplitude of damping oscillation, is analyzed in the following section below.

Table 12: Regression Results for Log–Amplitude Slope of Inventory–Workforce Model

| Coefficients | | | | | |
|---|---|---|---|---|---|
| | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | B | Std. Error | Beta | t | Sig. |
| (Constant) | -2.077 | 0.319 | | -6.519 | 0 |
| InvntAdjstTime | 0.206 | 0.006 | 0.668 | 34.854 | 0 |
| StandardWorkweek | 0.076 | 0.003 | 0.512 | 26.745 | 0 |
| MnfctrngCycleTime | -0.218 | 0.009 | -0.47 | -23.958 | 0 |
| VacancyAdjstTime | -0.173 | 0.018 | -0.187 | -9.542 | 0 |
| WIPAdjst | 0.083 | 0.012 | 0.134 | 6.968 | 0 |
| SafetyStockCoverage | -0.192 | 0.037 | -0.103 | -5.172 | 0 |
| Time2FillVacancy | -0.046 | 0.009 | -0.099 | -5.037 | 0 |
| LaborAdjstTime | 0.009 | 0.006 | 0.03 | 1.495 | 0.137 |
| Productivity | -0.444 | 0.289 | -0.03 | -1.536 | 0.126 |
| AvgDuratnEmploy | 0 | 0.001 | -0.021 | -1.108 | 0.269 |
| MinimumOrderProcssing | 0.038 | 0.035 | 0.02 | 1.073 | 0.285 |
| AvgLayoffTime | -0.008 | 0.009 | -0.018 | -0.918 | 0.36 |
| VacancyCancelTime | -0.006 | 0.036 | -0.003 | -0.166 | 0.868 |
| Time2AvgOrderRate | 0.001 | 0.009 | 0.001 | 0.065 | 0.948 |

**Sensitivity Analysis of Maximum Amplitude**

The last behavior measure that will be discussed here is maximum the amplitude of the damping oscillations. This behavior measure indicates the first response of the system to the incoming shocks. One should be aware that highly responsive systems are very dangerous and difficult to manage since “noise in demand never ends for real manufacturing firms” (Sterman, 2000). The firm’s response to these disturbances may be devastating with the oscillations coming after the first shock. Like other analyses, sensitivity of maximum amplitude is analyzed with regression model including 14 model parameters. The regression analysis results for the maximum amplitude measure are given in Table 13. Since standardized regression coefficients indicate the importance of the parameter in the regression, regressors are ordered according to the magnitudes of standardized regression coefficients after the result table is obtained from SPSS.

In the regression model, *productivity*, *vacancy cancellation time*, *WIP adjustment time* and *average layoff time* are found to be insignificant. *WIP adjustment time* represents the willingness of the production manager to consider the work in process inventory of the firm and insignificance of this parameter is a very counter-intuitive result which should be evaluated with caution. Specifically, regression implies that the uncertainty in WIP adjustment time parameter does not explain important amount of variability in maximum amplitude.

There are three important parameters for maximum percent amplitude of the oscillations. These are *inventory adjustment time*, *time to average order rate* and *manufacturing cycle time*. *Inventory adjustment time* is the most important parameter that affects the maximum amplitude of the damping oscillations. This parameter determines the amount of corrective action that the system takes as the result of the change in demand. *Time to average order rate* is another important parameter that smoothes the demand changes. This parameter represents that how much time the decision maker looks back while forecasting incoming demand i.e. larger values of this parameter make the immediate demand perturbations smoother. Furthermore, *manufacturing cycle time* indicates that the system gives higher responses if there is longer time delay in the supply line.

Table 13: Regression Results for Maximum Amplitude of Inventory – Workforce Model

| Coefficients | | | | | |
|---|---|---|---|---|---|
| | Unstandardized Coefficients | | Standardized Coefficients | | |
| Model | B | Std. Error | Beta | t | Sig. |
| (Constant) | 0.71 | 0.053 | | 13.35 | 0 |
| InvntAdjstTime | -0.026 | 0.001 | -0.523 | -26.049 | 0 |
| StandardWorkweek | -0.011 | 0 | -0.445 | -22.156 | 0 |
| MnfctrngCycleTime | 0.03 | 0.002 | 0.406 | 19.75 | 0 |
| Time2AvgOrderRate | -0.022 | 0.001 | -0.304 | -15.115 | 0 |
| SafetyStockCoverage | 0.075 | 0.006 | 0.253 | 12.083 | 0 |
| MinimumOrderProcssing | -0.07 | 0.006 | -0.238 | -11.966 | 0 |
| LaborAdjstTime | 0.007 | 0.001 | 0.151 | 7.206 | 0 |
| VacancyAdjstTime | 0.018 | 0.003 | 0.124 | 6.034 | 0 |
| Time2FillVacancy | 0.006 | 0.002 | 0.075 | 3.641 | 0 |
| AvgDuratnEmploy | 0 | 0 | -0.06 | -2.984 | 0.003 |
| AvgLayoffTime | 0.001 | 0.002 | 0.008 | 0.388 | 0.698 |
| Productivity | -0.016 | 0.048 | -0.007 | -0.341 | 0.733 |
| VacancyCancelTime | 0.002 | 0.006 | 0.007 | 0.364 | 0.716 |
| WIPAdjst | 0 | 0.002 | 0.005 | 0.224 | 0.823 |

Analysis results for inventory-workforce model are summarized in Table 14. We can generally state that the labor structure is not as critical as the manufacturing portion of the model. This result can be attributed to the fact that in this study the oscillation measures of inventory stock are subject to statistical analysis. In other words, sensitivity analysis that focuses on the labor behavior may result with different parameter sensitivity. In all regression equations, irrelevancy of *productivity* parameter implies that increasing labor productivity of the firm is a useless policy in order to deal with the inventory oscillation. On the other hand, *inventory adjustment time* and *manufacturing cycling time* are the parameters that determine the stability character of the system. The importance of these two parameters is consistent with the results of generic supply line model.

Interestingly, standard workweek parameter is resulted as the one of the most effective parameters of the model. This indicates that the effect of standard workweek on the model dynamics should be analyzed in greater detail. Perhaps, deeper analysis may point out some unreasonable formulations, which indicates a potential validation problem.

Table 14: Summary Results of Pattern Sensitivity Analysis of Inventory Workforce Model

| RANKING | PERIOD | LOGARITHM of AMPLITUDE SLOPE | MAXIMUM AMPLITUDE |
|---|---|---|---|
| 1 | Standard Workweek (-) | Inventory Adjustment Time (+) | Inventory Adjustment Time (-) |
| 2 | Labor Adjustment Time (+) | Standard Workweek (+) | Standard Workweek (-) |
| 3 | Work In Process Adjustment Time (+) | Manufacturing Cycle Time (-) | Manufacturing Cycle Time (+) |
| 4 | Minimum Order Processing Time (-) | Vacancy Adjustment Time (-) | Time to Average Order Rate (-) |
| 5 | Vacancy Adjustment Time (+) | Work In Process Adjustment Time (+) | Safety Stock Coverage (+) |
| 6 | Safety Stock Coverage (+) | Safety Stock Coverage (-) | Minimum Order Processing Time (-) |
| 7 | | Time to Fill Vacancy (-) | Labor Adjustment Time (+) |
| 8 | | | Vacancy Adjustment Time (+) |
| 9 | | | Time to Fill Vacancy (+) |

**CONCLUSION**

Sensitivity analysis is a critical tool for evaluating the reliability of model outputs. Since the typical outputs of system dynamics models are the behavior *pattern*s rather than point-by-point numerical values, some characteristics' of output patterns should be analyzed against the uncertainty in model parameters. Example characteristics of behavior patterns are oscillation periods, amplitudes and equilibrium levels. In this study, a pattern-oriented sensitivity analysis procedure for system dynamics models is proposed and it is applied to different system dynamics models by using regression methodology.

Regression is very useful for sensitivity analysis of system dynamics model since it can be used for multi-variate analysis without too much computational effort. Furthermore, residual plots, which are very common in regression literature, provide information about fulfillment of linearity assumption of the regression method. In this study, sensitivity analyses of the project management model (Taylor and Ford, 2006), a generic supply line model, and inventory workforce model (Sterman, 2000) are conducted using regression methodology.

Sensitivity analysis of project management model reveals that *total staff* and *project scope* parameters are the most influential parameters of the model. These parameters have important effects on the *peak of tipping point* and the *inflection point* measures of S-shaped growth. Moreover, *project complexity* and *deadline* parameters are other important factors of this model. These parameters are concluded to be the most important ones among fourteen model parameters analyzed. Therefore, a project manager trying to control progress of the project may use these parameters as intervention points to the system. Employment of new staff or postponement of the project deadline are found as influential policies for projects in which the failure risk emerges because of the deadline stress.

Another simulation model that is analyzed against parameter uncertainty is the generic supply line model (Sterman, 2000). The analysis of this model is the first example for sensitivity analysis of oscillatory models published in system dynamics literature. Oscillations are one of the most interesting and difficult behavior patterns in dynamic systems theory. Since supply line structures, which exist in many real processes, include negative feedback loop and delays, they are good platforms to analyze oscillatory behaviors. In generic supply line model there are three parameters: stock adjustment time, acquisition lag and supply line adjustment time. The sensitivity analysis of this simple model indicates that *stock adjustment time* is the most important parameter. Stock adjustment time indicates the responsiveness of the decision maker to the changes in conditions. Higher values of this parameter imply a decision maker who is slower in adjusting his decision in response to the changes in exogenous factors. This kind of policy produces oscillations that are relatively more stable. Furthermore, *acquisition lag parameter* is concluded to be more important than the *supply line adjustment time* that represents the weight that the manager gives to the condition of the supply line in making the decision.

Finally, our sensitivity analysis procedure is tried on a medium size oscillatory model, the Inventory- Workforce Model by Sterman (2000). This model includes 14 parameters and 5 stock variables related with workforce and manufacturing dynamics of a firm. Sensitivity analysis of this model is conducted on the oscillations in inventory stock. Behavior measures of these oscillations are tested against the uncertainty in the parameters of the model.

In summary, *inventory adjustment time* and *manufacturing cycle time* parameters of the model are found to be the most influential parameters. This result coincides with the findings of the smaller generic supply line model summarized above. Manager's decision heuristic parameters that determine the production orders in response to the changes in demand are the most important factors for such oscillatory models. A counter intuitive finding of this analysis is the irrelevancy of the productivity parameter for the oscillatory behavior. Increasing the labor productivity of the firm does *not* influence the stability of the model.

In concluding, for system dynamics models, behavior pattern measures should be used for sensitivity analysis because of the pattern oriented nature of SD methodology. At the end of three different sensitivity applications, we conclude that regression analysis is an appropriate methodology to analyze sensitivity of output pattern measures to the model parameters.

In future research, more effort should be spent on cases in which the sensitivity relationships between dependent and independent variables are non-linear and non-monotonic. In such cases, modified and more advanced versions of regression analysis is necessary in order to attribute the output variability to model parameters. Another interesting research topic is the behavior *mode* sensitivity. Model parameters and structures that change the output behavior modes of the system should be analyzed in order to obtain complete sensitivity information of a system dynamics model.

**REFERENCES**

Barlas, Y. 1989, Multiple Tests for Validation of System Dynamics Type of Simulation Models. European Journal of Operational Research 42(1):59-87.

Barlas, Y., 1990, An Autocorrelation Function Test For Output Validation. Simulation, 55(1): 7-16.

Barlas, Y., Topaloğlu, H. and Yılankaya, S. 1997, A Behavior Validity Testing Software (BTS), Proceedings of the 15th International System Dynamics Conference, Istanbul
Barlas, Y., Kanar, K., 1999, Proceedings of 4th Systems Science European Congress, Valencia, Spain, pp: 269-286

Clemson, B., Yongming, T., Pyne J., Unal, R., 1995, Efficient Methods for Sensitivity Analysis, System Dynamics Review 11(1): 31-49

Coyle R.G., 1978, An Approach to the Formulation of Equations for Performance Indices, Dynamics 4(2): 62-81

Draper, N.R., Smith, H. 1998, Applied regression analysis, New York : Wiley

Ford, A., 1990, Estimating the Impact of Efficiency Standards on the Uncertainty of the Northwest Electric System, Operations Research, 38(4): 580-597

Ford, A., 2003, SENSITIVITY ANALYSIS, in System Dynamics, [Ed. Yaman Barlas], in Encyclopedia of Life Support Systems (EOLSS), Developed under the Auspices of the UNESCO, Eolss Publishers, Oxford ,UK, [http://www.eolss.net]

Ford A., Flynn H., 2005, Statistical screening of system dynamics models, System Dynamics Review, 21(4): 273-302.

Hekimoğlu, M., 2010, Sensitivity Analysis of Oscillatory System Dynamics Models, The 28th International Conference of the System Dynamics Society, Seoul

Kleijnen, J.P.C. 1995, Sensitivity analysis and optimization of system dynamics models: regression analysis and statistical design of experiments. System Dynamics Review, 11(4): 275-288

McKay, M. D., W. J. Conover, and R. J. Beckman, 1976, A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code. Technometrics,22 (May): 239.

Moizer J.D., Arthur D., Moffatt I., 2001, A Formal but Non-Automated Method to Test the Sensitivity of System Dynamics Models, , Proceedings of the 2001 International Conference of the System Dynamics Society, Atlanta, Georgia

Montgomery D.C. 2001, Introduction to linear regression analysis, Third Edition, Wiley: New York.

Moxnes, E. 2005, Policy sensitivity analysis: simple versus complex fishery models, System Dynamics Review, 21(2): 123–145.

Ozbas B., Ozgun O., Barlas Y., 2008, Sensitivity Analysis of Real Estate Price Oscillations Model, Proceedings of the 2008 International Conference of the System Dynamics Society, Athens, Greece

Saltelli, A., Chan, K. and Scott, E.M. (ed), 2000, Sensitivity Analysis, John Wiley & Sons, Chichester

Sterman, J. D. 2000. Business Dynamics: Systems Thinking and Modeling for a Complex World. New York, Irwin. McGraw-Hill. 982 pp.

Taylor T., Ford D., 2006, Tipping point failure in single development projects. System Dynamics Review 22(1): 51–71.

Taylor T, Ford D, Ford A. 2007. Model analysis using statistical screening: extensions and example applications, In Proceedings of the 25th International Conference of the System Dynamics Society, Boston, MA, 29 July–2 August.

Taylor T, Ford D, Ford A., 2010, Improving model understanding using statistical screening, System Dynamics Review **26(1**): 73–87

Taylor T, Ford D, Johnson S. 2005. Why good projects go bad: Managing development projects near tipping points, In Proceedings of the 23th International Conference of the System Dynamics Society, Boston, MA, 17–21 July.

Yaşarcan H., 2003, Feedback, delays and non-linearities in decision structures, Bogazici University Industrial Engineering Department Ph.D. Thesis Dissertation.

Yaşarcan H., Barlas Y., 2005, Generalized Stock Control Formulation for Stock Management Problems, System Dynamics Review 21(1): 33-68