The Current Debt Crisis – A System Dynamics Analysis

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Abstract

The near meltdown of the world financial system led in almost all OECD countries to a sharp economic downswing. Even though there are signs for a recovery the political leaders have to cope with another problem: the steep increase in national debt. The increase is due to the automatic stabilizers (decline in tax revenues increase in transfers) but also to discretionary spending in order to stimulate the economy. Public, politicians and media talk of a debt crisis because they have doubts that an upswing will lead to a symmetric decrease in national debt. This paper analyzes the dynamics of government debt and demonstrates that economic shocks may have, in fact, long lasting effects with respect to the debt process.

Keywords

National debt, economic crisis, system dynamics
1. Introduction

The near meltdown of the world financial system which was triggered by the collapse of Lehman Bros. on September 14, 2008 hit a world economy already in the downswing. Unanimously, almost all economist advocated massive government interventions not only in order to save the monetary sectors but also to stimulate the real sectors of the world economy. In contrast to the political reactions following the stock market crash of 1929 and the start of the great depression (first world economy crisis) the policy change reacting to the 2008 financial crisis and to the threat to the real economy was globally coordinated and not an austerity policy. Rather, central banks and governments in all major economies turned to a policy of massive monetary and fiscal expansion. At the same time, due to the worldwide recession tax revenues decreased sharply. Both, the decrease in taxes and the increase in government spending led to a drastic increase in budget deficits and, as a consequence, in national debt. So, after the financial crisis and the economic crisis all major economies now suffer from a “debt crisis”. Accordingly, the question how to handle the enormous budget deficits and the sharp increase in national debt is at the top of the political agenda in many countries. In Europe mainly the so-called PIGS-states (Portugal, Italy, Greece, and Spain) are in the line of fire. As members of the Euro zone all of them have to obey the requirements of the Maastricht treaty and the stability and growth pact. Greece has especially become under fire because the former Greek government has cheated with respect to the deficit and debt statistics: in 2009, the budget deficit was more the 12 percent, while the stability and growth pact requires the deficit to be less than 3 percent. Therefore, the EU deficit procedure imposed tough restrictions on Greece which in turn led to riots. And Greece is just one example: the situation in Portugal, Italy, and Spain (as well as in Ireland, the United Kingdom and other states) is not much better.

All this has renewed the scientific interest in causes and consequences of national debt. This paper contributes to this strand of literature by analyzing the problem of national debt from a system dynamics perspective.

It is structured as follows. The next section starts with a brief review of the economic theories that are offered in the economic literature to explain the phenomenon of an ever-growing national debt and the consequences public debt has. The following section provides some empirical background in looking at the development of budget deficits and national debt in some major economies. Subsequently, a basic SD model is introduced in order to demonstrate the simple dynamics that govern the thinking dominating the public and the political discussion. The model, thereafter, is extended to illuminate under which conditions an ever increasing national debt is sustainable. Finally, we draw some conclusions and point to further directions of research.
2. National Debt, Economics, and System Dynamics

In economics, three aspects of national debt play a central role:

- What are the effects of national debt on growth and welfare?
- If it is true that governments tend to increase national debt in the long run why do they do so?
- Under which conditions is national debt sustainable?

The last of these three questions is of special interest today because the economic crisis (and the fiscal policy response) have rapidly increased national debt in many countries. As this paper aims at shedding some light on the last question we just pick up the first two questions briefly in order to provide some background.

The traditional analysis of the effects of government debt done in a Keynesian framework yields positive or negative effects of budget deficits in term of growth and welfare. For a closed economy, the argument usually runs as follows. If we have full employment and the government lowers taxes financed by a deficit then the higher disposable income leads households to consume more. The higher disposable income also increases household savings. But this increase in private savings is not large enough to compensate the decrease in public savings (equivalent to the deficit). Therefore, national savings decrease which in turn implies a decrease in investment. The lower investment leads to a lower rate of economic growth. Hence, in the long run there is less consumption possible. This in turn is interpreted as a welfare loss. (For the open economy the argument is somewhat different but yields a similar result.)

The traditional view of the deficit was questioned in a seminal paper of Barro (1974). The argument developed in that paper became known as the “Ricardian View” (or “Ricardo-Barro View”) of national debt. It says, in short, that it does not matter whether the government finances its expenditures by taxes or by deficits. The logic behind this statement is that rational households recognize that the government has to pay the borrowed money back. In order to be able to do so the government will raise taxes in the future. The discounted value of the future tax rise is equivalent to the tax cut today. Because the rational households know this they will not change their behavior due to a tax cut financed by a deficit.

Today, only few economists endorse the Ricardian View completely. The general view is that budget deficits are welfare increasing when used for tax smoothing – a hypothesis also put forward by Barro (1979). Were deficits only created in order to smooth taxes, certainly, the national debt would not be subject – of frequently heated – political discussions. The problem is not a cyclical up and down in the national debt due to tax smoothing but the fear of an unsustainable long-run increase in debt. This leads to the second question stated above.

We will briefly point to three possible answers that can be found in the economic literature. The first possible answer refers to the political business cycle and was
introduced into the literature by Nordhaus (1975). The main line of the argument is as follows. The politicians try to manipulate the economy for electoral gain. The incumbents try to bring inflation down by causing a recession and strive towards stimulating the economy such that employment is high as the next election is approaching. They behave asymmetrically over the cycle which leads to a long-run increase in national debt. The hypothesis is intensely disputed. A recent source that sees some empirical support for it in the Euro Area is Mink and de Haan (2006).

A second possible answer is the thesis that weak governments tend to have higher deficits. In a seminal study Roubini and Sachs (1989) offered empirical evidence that in a sample of OECD countries coalition governments had higher deficits than one-party governments. But this thesis is intensely disputed as well. In a more recent study de Haan, Sturm and Beekhuis (1999) do not find evidence that the type of government (coalition or one-party) has a significant effect. However, they argue that the number of political parties in government has an influence on the debt growth of the central government.

A third hypothesis put forward by Woo (2003) is that social polarization plays a role in explaining differences in cross-country variation in fiscal outcomes. He finds that social polarization has strong effects on deficits in the presence of poor institutions.

The question under which conditions national debt is sustainable is usually treated as a question of growth arithmetic. Employing either difference or differential equations the steady state conditions for a growth equilibrium are derived. Usually, these calculations are not done in intermediate textbooks (a notable exception is Blanchard 2006).

But an explicit dynamic analysis that traces the time paths of the relevant variables is, in general, not a subject of the textbook literature. This is also true for the leading advanced textbooks. Romer (2005) just touches the problem of government debt in the second chapter of *Advanced Macroeconomics*. Blanchard and Fischer (1989) build dynamic models in the context of (optimal) fiscal policy but limit themselves to the derivation of optimality conditions and do no simulation studies.

The number of studies that tackle the problem of national debt from a system dynamics viewpoint is quite small. The papers most closely related to this paper are Burns and Janamanchi (2007), Ansah (2006), Radianti (2004), and Arenas (2003).

Burns and Janamanchi (2007) analyze the debt problem basically from a demographic viewpoint. One of their main conclusions is that policy should undertake measures to improve growth conditions especially by increasing the labor force participation rate. Their concluding remark that “We cannot pass this debt on to our children and grandchildren.” (Burns and Janamanchi 2007: 13) is problematic with respect to the general effects of government debt, but presumably correct in the context of the US

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1 See also Tufte (1978).
The US government is financing the national deficit by borrowing abroad.

The paper of Ansah (2006) asks quite a different question and uses a different approach to answer it. Ansah starts with the observation that the IMF and the World Bank require indebted countries to implement structural adjustment programs that have unseen and unwanted side effects. To identify these side effects he uses a qualitative approach in employing causal loop diagrams. But he does not develop a quantitative simulation model. The paper concludes that the implemented programs were only partly successful due to their neglect of the societal framework and stakeholders policy responses.

In a study presented at the 2004 International Conference of The System Dynamics Society Radianti (2004) analyzes the consequences of external debt financing with respect to public finance and the sustainability of fiscal policy for the case of Indonesia. He concludes from his simulation exercises that the usual advices of international institutions like tax increase, cut of public expenditures, and restructuring of state-owned enterprises do only yield temporary relief. In the long run, better institutions seem to be crucial for sustainability of fiscal policy.

Arenas (2003) examines the sustainability of national debt considering the situation in Colombia. The motivation of his work was the Argentina crisis of 2001 and the lack of a system dynamics approach to the problem in the literature. In his paper, he makes a distinction between internal and external (foreign) debt. This is, certainly, appropriate modeling for countries like Colombia. Another unique feature of his model, appropriate for the economies under consideration, is explicitly to allow for military expenditures. Despite this special characteristics of his work, among the cited system dynamics studies of national debt his model comes closest to the work presented here.

Arenas uses as his starting point ideas that have been proposed by Posada and Arango (2001) and puts these ideas in a stock-flow framework. We use a similar starting point by modeling the dynamics of simple debt “mechanics” first. But there are some differences that should be mentioned.

His “starting point model” is build in real terms, thereafter he switches to nominal variables. The model developed in section 3 of this paper is in nominal terms right from the beginning. We do that for three reasons: Firstly, the number discussed in politics and in the media is nominal debt (the world-famous debt clock installed on Sixth Avenue, displays the constantly updated United States gross national debt in nominal terms). Secondly, in order to calculate the usual sustainability measures (for example, the debt-to-GDP ratio) it is not necessary to convert the numbers to real terms because the units cancel anyway. Thirdly, empirical data in nominal terms is easier to get and easier to interpret than data in “constant prices”. (This is especially true since the bureaus of statistics switched to chain indexes world-wide.)

As we are not interested in the difference between “internal” and “external” debt we do not make a distinction between a domestic and a foreign sector. Our paper has two
main purposes: Firstly, to our best knowledge, the traditional economic literature misses by and large an explicit exposition of the dynamics of a problem that is dynamic by its very nature. We show how the basics of the debt dynamics can be captured in a simple model. Secondly, we are interested how exogenous shocks to the economy influence the dynamic properties of the debt process. As this is only the first part of a larger project we limit the scope of our presentation to debt “mechanics”. Before we turn to the presentation and discussion of our model we will briefly provide some empirical background in the next section.

2. Empirical Background

The following Figures show the development of national debt and the development of the ratio of national debt to Gross Domestic Product (GDP) for the United States, Japan, and three European countries: Germany, the United Kingdom, and Ireland. The data are from the OECD Economic Outlook No. 86. Note that the time series the OECD provides differ in length. The longest time series available is for the United States. Figures 1 – 5 show the development of public debt in national currency, i.e. in nominal values. Due to the differences in the size of the economies a cross-country comparison of the numbers is not meaningful. The important message of the diagrams is, firstly, that in all countries national debt has increased more or less steadily. Secondly, we see that there are few time periods with a slow increase (or even a decline). In general, these are periods of above average economic prosperity. Thirdly, it is evident that there are also time periods that are characterized by a fast increase of national debt. This is especially true for the most recent years.

As we will later discuss in somewhat more detail the development of the value of public debt does not tell much about its sustainability. Sustainability is a rather ambiguous concept – not only in the context of the environment but also in the context of public finance. Despite of this, there is a general agreement to evaluate the sustainability of national debt by looking at certain ratios. The ratio of debt service to tax revenue is one measure and the ratio of debt to GDP is another. (Debt service is defined as the sum of interest payments and repayment of principal.) Here, we look at the latter. Figure 6 shows the time development of the debt-GDP ratio for five countries. Figure 7 reproduces this ratio for the U.S. for a longer time period. The diagram for the United States as well as a close inspection of Figure 6 reveal that there are extended time periods which are characterized by a decline of the ratio. But it is also evident that economic shocks often lead to a sustained increase in this measure.
Fig. 1  Public debt, values, United States, national currency. Source: OECD (2010).

Fig. 2 Public debt, values, Japan, national currency. Source: OECD (2010).

Fig. 3 Public debt, values, Germany, national currency. Source: OECD (2010).
Fig. 4 Public debt, values, United Kingdom, national currency. Source: OECD (2010).

Fig. 5 Public debt, values, Ireland, national currency. Source: OECD (2010).

Fig. 6 General government gross financial liabilities, as a percentage of GDP
3. A System Dynamics Approach to the Dynamics of National Debt

In this section, we follow the general advice of model building and introduce a basic system dynamics model that allows to analyze the “mechanical” growth of national debt. These “mechanics” mainly rest upon the fact that the government has to pay interest on the debt and that these interest payments further increase national debt \textit{ceteris paribus}. The structure of this basic model is visualized by Figure 8.

![Fig. 8 Stock flow diagram of the introductory model (model 1)]
The model is based on a single positive feedback loop. We have made the usual distinction between primary deficit and interest on national debt: the total national deficit consists of the primary deficit and the interest on national debt.

The primary deficit is defined as the difference between all public expenditures that are not an immediate result of the national debt and tax income of the government. It is not explained by the model, but is taken as an exogenous variable.

In neoclassical economics this is a standard assumption, justified on the base that the size of the primary deficit is agreed upon in the political process which is typically outside the boundary of (neoclassical) economic models. From a system dynamics viewpoint this is certainly a problematic approach as system dynamics modeling, in general, tends to avoid employing exogenous variables easily – unless there is any good justification for doing so. Right here, we justify the exogeneity assumption by emphasizing the purpose of the model: to analyze the “mechanical” dynamics of national debt in a very simple setting.

The second component of the total deficit consists of the interest payments on national debt. The size of these payments depends on two factors: the size of the national debt and the interest rate. Again, only for sake of simplicity we assume the interest rate to be constant.

The values of parameters and other model constants employed in the basic run of the model are shown in Table 1.²

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value national debt</td>
<td>1644</td>
</tr>
<tr>
<td>Primary deficit</td>
<td>10</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.05</td>
</tr>
</tbody>
</table>

As stated above, the model employs just one (positive) feedback loop linking national debt, interest on national debt and the total deficit. Starting with a certain initial value of national debt, the government has to pay interest on this debt according to the current interest rate. The interest payment together with the primary deficit constitutes the total value, the number by which the national debt increases in each period.

As there is only one positive feedback loop in the model, it exhibits exponential growth with respect to national debt, total deficit, and interest on national debt. Figure 9 shows the time paths of the model variables resulting from a simulation run that employs the values of the model constants given in Table 1.

² As we do not intend to explain the dynamics of national debt in a specific country, using arbitrary parameter values would not create any problem. But to ensure realistic relative magnitudes we used actual numbers of a major industrialized country where possible.
In the basic run we have assumed that there is a permanent positive primary deficit. This, certainly, is an oversimplification because it does not allow for political measures of the government to counteract a national debt that is judged as being too high.

In economics, it is common to differentiate between a structural component of national debt and a cyclical component. The structural component of the primary deficit is positive if the taxes collected from the tax payers fall short of public transfers and government purchases. The structural component of the primary deficit is negative if taxes exceed government expenditures.

Note that we only speak of a structural deficit or surplus if the imbalance is not due to the business cycle. If taxes fall short of expenditures due to a recession we call this a cyclical deficit. While almost all economists agree that a cyclical deficit should be accepted there is much more disagreement about the economic consequences of structural deficits. (In practice, it is a very problematic econometric task to estimate what part of the deficit should be counted as cyclical deficit and which part as structural deficit.)

The distinction between structural and cyclical component of the primary deficit can be easily implemented in an expanded version of our basic model. While we continue to assume the structural component to be a constant, we now model the cyclical component as being driven by a sine shaped process. This, again, is a simplification that
does not lead to a cyclical behavior resembling actual business cycles. We only make use of it because it is the simplest way to add bounded fluctuations to our basic model.\(^3\)

Even this simplistic approach allows us to answer questions that are often debated in the public and in politics: With a zero structural deficit and bounded symmetric fluctuations of the cyclical deficit will the national debt be constant in the long run? If we want to bring down national debt is it enough to have a positive structural deficit provided the cyclical deficit is bounded and symmetric? How long does it take? In addition, if used in class these exercises promote the understanding of the structure and the dynamics of an important aspect of the economic system.

The structure of the system is visualized by the stock flow diagram shown in Figure 10.

![Stock flow diagram with breakdown of the primary deficit (model 2)](image)

**Fig. 10 Stock flow diagram with breakdown of the primary deficit (model 2)**

The only difference compared to the stock flow diagram of Figure 8 is the breakdown of the primary deficit. To answer the questions stated above and to improve our intuition of the debt dynamics we look at three model runs employing different parameter values. The values of the constants underlying the different runs are summarized in Table 2, and the results of the runs are visualized in Figure 11.

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\(^3\) In modern neoclassical macroeconomics usually a random walk is employed to reflect actual cycles. Typically, real business cycle models rely on a random walk with drift which would not allow for the decomposition of trend growth and cyclical deviation we used in our model (e.g. Plosser 1989).
Table 2 Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
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</thead>
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<tr>
<td>Initial value national debt</td>
<td>1644</td>
<td>1644</td>
<td>1644</td>
<td>1644</td>
</tr>
<tr>
<td>Structural deficit</td>
<td>10</td>
<td>-10</td>
<td>-82.5</td>
<td>-83</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

![Fig. 11 Time paths of national debt](image)

The left panel of Figure 4 shows the time paths for all four simulation runs. As national debt in run 1 and run 2 increases much faster than in runs 3 and 4 the evolution of national debt in the latter cases is nearly not recognizable. Limiting the exposition only to the runs 3 and 4 allows appropriate rescaling of the vertical axis. Now the visual inspection of the right panel of Figure 4 reveals, firstly, the fluctuations triggered by the sine movement of the cyclical component and, secondly, the fact of a still, albeit slowly, increasing national debt in run 3. Only in run 4 with a structural surplus of 83 billion national debt decreases over the time under consideration.

The increase of national debt in run 1 is no surprise. National debt starts at a high amount and is further increased by a permanent structural deficit. This and the interest payments yield a fast increase in national debt. The fact that national debt also grows steadily in run 2 despite the yearly structural surplus only comes as a surprise if the positive feedback dynamics of the interest loop are overlooked. The fluctuations of national debt exist but are not visible graphically due to the scaling that is required to show the overall growth process of national debt. The right panel shows an increase of national debt over time for a structural surplus of 82.5 billion. At a first glance, that might be confusing because the first year’s interest payment is only 82.2 billion. The structural surplus of 82.5 should overcompensate this and ensure that national debt decreases in the first round. In the second round the interest payments should be lower than 82.2 billion. Given the assumed constant structural surplus national debt should decrease further, and so on. The pitfall here is the fact that we have assumed a positive and increasing cyclical deficit for the first years by employing a sine-shaped cyclical process. This results in a (positive) deficit by the third time step which then never again reverts to a surplus. Only if the structural surplus is so high that it overcompensates the cyclical deficit during the first steps, the positive feedback process of the interest loop will ensure a decrease of national debt over time.
This observation of our simple model has an important implication with respect to the current debt crisis: even if a country has a structural budget surplus a large (short-term) cyclical deficit might act as a trigger to move the system across a kind of tipping point such that the positive interest feedback loop starts to dominate system behavior. 4

One important point we have mentioned already in section 2 is the fact that a decreasing or constant national debt is not necessary for the “sustainability” of government finance. The term sustainability here refers to the question whether the government can “afford” the debt burden. A debt burden is affordable if a nation can cope with it indefinitely without going to insolvency. Whether a certain size of national debt is sustainable or not is usually judged by referring to several quotas. The most prominent quotas are the deficit-GDP relation and the national debt-GDP relation. (Another important, alas less prominent quota is the ratio of interest payments and GDP.) The national debt is said to be sustainable when the debt-GDP relation converges to a finite number. (In the real world sustainability requires that the number is not too large.)

A meaningful integration of this sustainability criterion into our system dynamics model requires us to expand the model by explicitly incorporating GDP. To keep the model simple we do not consider a random walk (may it have a unit root or not) but assume that GDP growth is governed by a constant and exogenously given fractional growth rate. The structure of this expanded model is visualized by the stock flow diagram shown in Figure 12.

4 Note that the system dynamics terminology reserves the term tipping point usually for a shift in loop dominance (see Sterman 2000: 306). Strictly speaking, this is not the case here because the model encompasses just but one loop.
The modeling of the deficit and of national debt remains the same as in model 2. The expanded model (model 3) now encompasses the dynamics of GDP. We treat the GDP growth rate as having dimension [currency/time]. Hence, the associated stock variable has the dimension [currency] – that is why the stock variable is interpreted as perceived GDP. To analyze sustainability we have included the two indicators usually employed: the debt-GDP ratio and the deficit-GDP ratio.

We start the simulation exercises by doing a base run. The values of the constants of this base run are given in Table 3.

Table 3 Parameter values base run

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Initial value national debt</td>
<td>1644</td>
</tr>
<tr>
<td>Initial value perceived GDP</td>
<td>2495.8</td>
</tr>
<tr>
<td>Taxes</td>
<td>1091</td>
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<tr>
<td>Government expenditures</td>
<td>1292</td>
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<tr>
<td>Structural GDP growth</td>
<td>0.05</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 13 shows some simulation results. In addition to the base run the diagrams show a scenario which assumes that taxes are only 900.
Fig. 13 Simulation results

The panels in the upper half of Figure 13 show the time paths of the total deficit and of national debt, respectively. The budget deficit, as well as the national debt, grows exponentially. The only difference in having lower taxes (and, therefore, a higher primary deficit) is that the growth of deficit and debt is somewhat faster. Considering the structure of the model, this should not be surprising: the positive interest feedback loop dominates the system behavior. In the long run, as long as the primary deficit remains constant, its size has no first order meaning - within certain limits.

The fact that national debt is ever increasing does not mean that it is not sustainable. The panels in the lower half of Figure 13 show the deficit-GDP ratio and the debt-GDP ratio. In this simulation exercise, the deficit-GDP ratio remains constant albeit fluctuating. In the case of lower taxes (and higher primary deficit) the ratio is higher than in the base run, but constant as well (neglecting the oscillations). The debt-GDP ratio increases in both cases, as the right-hand panel in the lower half of Figure 13 shows, but approaches a steady state in the long run. In the sense of a “mechanical” consideration of the debt dynamics the assumed fiscal situation is sustainable even if we look at the simulation run that assumes a higher primary deficit.

The central aspect here is the point that the dynamics of the model are governed by two positive feedback loops: the interest loop and the GDP loop. Whether the model shows sustainable behavior or not depends on the relative size of the interest rate and the growth rate which can be demonstrated easily by employing appropriate values of these parameters in another simulation. Table 4 allows for comparison of the constant differences between the different runs. The results of these runs are shown in Figure 14.
Table 4 Constant differences between runs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Run</th>
<th>Interest Rate Low</th>
<th>Interest Rate High</th>
<th>Structural GDP growth low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>0.05</td>
<td>0.04</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Structural GDP growth</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Fig. 14 Simulation results

The diagrams in the upper panels show that the budget deficit and the national debt exhibit exponential growth for all four runs. In these diagrams the time path of the base run is not visible because it is hidden behind the run "structural GDP growth low". Both runs assume an interest rate of 5 percent, and for the evolution of deficit and debt only the interest rate matters. Both runs can be distinguished in the diagrams shown in the lower panels because for the ratios also the growth of GDP is relevant. The diagrams reveal that parameter constellations characterized by a positive difference between the interest rate and the average growth rate are not sustainable in the “mechanical” sense of a constant debt-GDP ratio: if the interest rate is greater than the growth rate, then the debt-GDP ratio increases forever. The time path of the run “interest rate low” reveals an increase in the debt-GDP ratio at the beginning. Subsequently, the debt-GDP ratio declines. The observed pattern is due to the (constant) primary deficit that is important during the first years. This importance gradually fades away as the feedback processes start to dominate the picture: no matter how large a primary deficit is – as long as it is constant it is dominated eventually by the positive feedback dynamics.
The assumption of the primary deficit to be constant in nominal terms for the whole time period under consideration and independently of the exponential growth of GDP and national debt is a significant limitation even in a “mechanical” model. Therefore, in the next step we consider a version of the debt dynamics model that takes care of this issue. Figure 15 shows a stock flow representation of this extended version.

Fig. 15 Debt dynamics in an extended model

The most important difference to the previous version lies in the inclusion of growth of government expenditures, of taxes, and, hence, of the primary deficit. With respect to the structural component we assume that all the variables grow at the rate of the structural component of GDP. In a simplified growth arithmetic exposition, this ensures that government expenditures and taxes remain in a constant proportion to GDP during the growth process of the economy. With respect to the cyclical component we assume a government expenditures elasticity of -1 and a tax elasticity of 1, respectively. This means that a cyclical increase in GDP growth by one percentage point is believed to decrease government expenditures by one percentage point and to increase taxes by one percentage point. Certainly, one can question the numerical values we have chosen, but we think they are quite acceptable as a first approximation. In addition to this extension, we have expanded the model by including a temporary shock component. This allows to analyze singular events like the economic shock triggered by the near melt-down of the financial system.

We start our analysis by comparing a base run to one run which assumes a balanced budget and another run which hypothesizes that there is a zero cyclical component of GDP growth. The parameter values for these scenarios are given in Table 5.
The differences in the parameter values compared to the base run are emphasized by bold numbers in Table 5. The results of the runs are summarized in Figure 16.

The base run (run 0) does not yield sustainability: the debt-GDP relation grows forever. This is due to the fact that there is an ever-growing primary deficit that adds to the positive feedback of the interest loop. In run 1 (balanced budget) we assume that the taxes have been increased in the initial period such that the budget is balanced. Contrary to the intuition, this balanced budget scenario does not reveal a constant debt-GDP ratio. Rather, this ratio is ever-declining. The reason for this observation is the fact that the scenario under consideration implies a cyclical upswing in the first periods. The upswing leads to an increase in tax revenues and to a decrease in government expenditures. Both developments lower the primary deficit beneath the level it would otherwise have been. The reduction in the primary deficit is big enough to trigger an evolution that leads to a decline in the total deficit that is even followed by a surplus after some periods. The correctness of this interpretation is supported by run 2 (balanced budget, no cycles). Here, we assume that the cyclical component (in our model: the parameter \textit{level of cycle}) is equal to zero. In this case the model yields a constant deficit-GDP ratio as well as a constant debt-GDP ratio, as we would have expected. This result urges us to pay attention not only to the structural component of the budget deficit, but also to the cyclical developments because they may shift the behavior of the system.
Fig. 16 Simulation results runs 0 – 2

So far, we have modeled the cyclical component in a very simple way by employing a sine process. In order to better understand the consequences of shocks we now allow for singular shocks, shocks that decrease or increase the growth of the economy for a certain amount of time in the way the financial crisis did.

We will compare four runs. The first two runs (run 0 and run 2) are familiar already: the base run and the run that assumes a tax increase such that the budget is balanced. Each of the following runs assumes a negative shock which lowers GDP growth by 5 percentage points – this size of shock was seen in a number of countries as a consequence of the financial crisis. With respect to the other parameters run 7 retains the numbers of the base. Run 8 differs in assuming a balanced budget at start time. Table 6 summarizes the assumptions.

Table 6 Parameter values of runs 0, 1, 7, and 8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run 0 (base Run)</th>
<th>Run 1 (balanced budget)</th>
<th>Run 7 (singular shock)</th>
<th>Run 8 (balanced budget, singular shock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value national debt</td>
<td>1644</td>
<td>1644</td>
<td>1644</td>
<td>1644</td>
</tr>
<tr>
<td>Initial value GDP</td>
<td>2495.8</td>
<td>2495.8</td>
<td>2495.8</td>
<td>2495.8</td>
</tr>
<tr>
<td>Initial value gvt expenditures</td>
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<tr>
<td>Initial value taxes</td>
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<td>1292</td>
<td>1091</td>
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</tr>
<tr>
<td>Structural GDP growth</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Magnitude of shock</td>
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<td>0</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>Level of cycle</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Figure 17 visualizes some of the simulation results. The upper left-hand panel shows the evolution of the total deficit, and the upper right-hand panel shows the evolution of national debt. The panels in the lower half show the deficit-GDP ratio (left-hand) and the debt-GDP ratio (right-hand).

As we know already, the base run reveals a non-sustainable scenario. The occurrence of the negative singular shock in period 10 worsens the situation: in the wake of the shock the national debt and the debt-GDP ratio increase even faster. In the previous discussion of run 1 we have learned that a balanced budget together with an economic upswing may result in a sustainable development in the sense of a declining debt-GDP ratio. As Figure 17 shows the positive effects of the upswing are overcompensated by the singular negative shock: just like the initial cyclical upswing has tipped the system towards a sustainable development the negative singular shock reverts this evolution. Even in the presence of a balanced primary deficit and in spite of a favorable economic start the negative shock can lead to an unsustainable development. In order to better visualize what happens during and after the shock, Figure 18 shows the time paths for the period 0 to 20.

Now, it is easier to see that runs 0 and 7 on the one hand and runs 1 and 8 on the other hand are identical for the time until the singular shock hits the economy. The shock increases the deficit by such an amount that even in the case of a balanced budget the system does not revert to a sustainable path after the shock is over. Before turning to an
economic discussion of our model and its results we shall analyze one final question within the given framework: how sensitive is the system with respect to economic shocks?

This question is important because the financial crisis showed that economies might be hurt by shocks more severe than believed up to now. How likely is it that an economy in good conditions (nominal growth above the interest rate, balanced budget) might be tipped from a sustainable path to an unsustainable one? Within our “mechanical” framework the results of a sensitivity analysis shown in Figure 19 may shed some light on this question.

Fig. 18 Simulation results: runs 0,1,7, and 8 for time 0 – 20

Fig. 19 Sensitivity analysis
In Figure 19, run 0 is the base run. The relevant parameter values of this base run are given in Table 6. Run 6 characterizes “good” economic conditions, i.e. the rate of GDP growth is 6 percent, the interest rate is 4 percent, and the budget is in balance. Vensim, the program employed to do the simulations, calculates 50 runs for singular shocks randomly drawn of the [-0.06, 0.02] interval of real numbers. The results are summarized by the fan chart of Figure 12. We do not give a probability interpretation to the results in the sense of a certain likelihood for the system to switch to an unsustainable path. Rather, we note that there is a serious possibility that a once robust sustainable development becomes unsustainable due to a negative shock.

4. Conclusion

The model developed in this paper allows a simple but rich exposition of the dynamic “mechanics” of national debt. If used in class, we believe that this type of model will help students to better understand the dynamical processes that are involved in the development of public finance. From an analytical viewpoint, an main finding of our study is that even in a simple “mechanical” setting one can demonstrate that cycles may have permanent effects on the development even if cycles are symmetric. In addition, we have demonstrated that economic shocks can result in “tipping” a so far stable development.

These observations bring us to what we have not done in our paper. By the same token, these are open questions that demand further research. The biggest flaw is the lack of behavioral elements. As we have stated in our discussion of the economic literature, there are many hypotheses under discussion why governments tend to finance expenditures by deficits and not by taxation. All these hypotheses contain dynamic elements that call for dealing with them by employing system dynamics methods.
References


