SD Approaches for Feedback Dynamic Complexity Analysis

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Abstract: Feedback dynamic complexity is an important feature of complex systems. Professor Jia and his SD group began their study of the theory and application of SD feedback dynamic complexity analysis method since 1985, and proposed a series of approaches successively, which have constructed an approach system of SD feedback dynamic complexity analysis of complex systems. Four major functions of SD feedback dynamic complexity analysis were extracted in this paper, which are formulating feedback model for the system problem; constructing feedback model for Successful case; simulation; feedback loops calculation and management policy analysis. Many questions and further study on each of the four functions were respectively proposed. The paper is also a SD research summary of the group for nearly three decades. We believe that there will be many benefits for the system dynamics community in developing the method system of SD feedback dynamic complexity analysis.

Keywords: Feedback dynamic complexity analysis; the rate variable fundamental in-tree model; vertex weighted causal loop diagram; branch-vector determinant arithmetic

1. Introduction

System dynamics is a subject to study systems complexity arising from the feedback interrelationships between systemic variables (including delay). It is an important branch of systems science, the field developed initially from the work of Jay W. Forrester. The research group of the Systems Engineering Institute at Nanchang University leaded by Professor Jia Renan has taken up with the research on the system dynamics methods for feedback dynamic complexity analysis since 1985, and has developed a series of approaches in succession during the past two decades. We have extracted four major functions of SD feedback dynamic complexity analysis, and proposed many questions and further study on each of the four functions respectively. The intention of this paper is to provide a reference for systems scientific community in developing the dynamic feedback analysis method for complex systems.

The four major functions of SD feedback dynamic complexity analysis we presented in this paper were concluded on the extension of the causal loop diagram and the stock and flow diagram of system generated by Jay W. Forrester. We have done series innovative researches on the theory and application of each major function, and have obtained some new specific theoretical approaches and cases of implementation the four major functions.

2. Study on theory and application innovation of the first SD feedback dynamic complexity analysis function.

The first SD feedback dynamic complexity analysis function is formulating feedback model for system problems and proposing Management countermeasures.

In 1980s, Peter M. Serge has created 9 systems archetypes, which are Balancing Process with Delay, Limits to Growth, Shifting the Burden, Eroding Goals, Escalation, Success to the Successful, and Tragedy of the

Commons, Fixes That Fall, Growth and Underinvestment. These systems archetypes have covered most of dynamic complex problems in society and economy, the management problem-solving policy (leverage solution) of each systems archetype is concluded based on the background of the actual problem. Our research group have made a further study on the first SD feedback dynamic complexity analysis function, the research results are presented as follows.

2.1 The key variables vertex weighted causal loop diagram analysis approach

The key variables vertex weighted causal loop diagram is a model between the SD causal loop diagram and the stock and flow diagram. The key variables vertex weighted causal loop diagram analysis approach was advanced on the basis of the following basic concepts.

2.1.1 Notion of the key variables vertex weighted causal loop diagram

Definition1. Let T be a set of time, for any given $t \in T$, $G(t) = (V(t), X(t), F(t))$ is the causal loop diagram of a complex system, where $V(t)$ is the vertex set of the system, and $X(t)$ is the arc set of the system, $\boldsymbol{F(t)}$ is the map: $X(t) \to \{+,-\}$.Start with the key vertex value at a given time, such as t_0, confirm the value of the key vertex, calculate the value of other vertices in set $V(t)$ by equations of the correlated arcs, real meaning or tested results, then put the value of each vertex into the causal loop diagram, the corresponding causal loop diagram with the vertex value is called vertex weighted causal loop diagram at time t_0, it can be denoted as

$$D(t_0) = [V(t_0), X(t_0), F(t_0)].$$

Definition2. the approach is called the vertex weighted causal loop diagram analysis approach, if it analyze the dynamic system problems based on the vertex weighted causal loop diagram model.

2.1.2 Basic steps of the vertex weighted causal loop diagram analysis approach

*Step*1 Qualitatively analyzes and confirms all of the relative variables in the research system.

*Step*2 Construct the causal loop diagram of the system $G(t) = [V(t), X(t), F(t)]$.

*Step*3 Collect preliminary information and data start with the key vertex value at a given time, such as t_0, calculate the value of other vertices $v_i(t) \in V(t)$ by equations of the correlated arcs, real meaning or tested results.

*Step*4 Put the value of each vertex at time t_0 into the causal loop diagram to generate the corresponding vertex weighted causal loop diagram $D(t_0) = [V(t_0), X(t_0), F(t_0)]$.

*Step*5 Change the value of the key vertex generates other vertex weighted causal loop diagrams if needed.

*Step*6 Analyze quantitatively the structure of the vertex weighted causal loop diagram, the dynamic variety of values of vertices changed with the different key vertex value, shows the law of feedback effect of each positive and negative feedback loops of the system. Identify the key leverage points and design intervention policies via the analysis of the variation law.

2.1.3 Further study on the vertex weighted causal loop diagram analysis approach

The vertex weighted causal loop diagram analysis approach is a qualitative and quantitative combination feedback structure analysis approach. We have obtained a good effect in the feedback dynamic complexity analysis practice with this approach. But, there are still some problems in the approach needed to be further improved.

(1). How to identify the value of each vertex effectively in accordance with practicality?

(2). How to formulate comparative vertex weighted causal loop diagram, and identify management measures or modify management measures been put forward according to the change of the vertices value?

2.2 Principles of Management strategy realization approach

Management principles (lever solution) that response to the 9 archetypes were put forward separately by Peter M.Senge. (1) For Balancing Process with Delay, the management principle is: In a sluggish system, aggressiveness produces instability. Either be patient or make the system more responsive; (2) for the limits to growth, the management principle is: Don't push on the reinforcing (growth) process, remove (or weaken) the source of limitation; (3) For Shifting the Burden, the management principle is: Focus on the fundamental solution. If symptomatic solution is imperative (because of delays in fundamental solution), use it to fain time while working on the fundamental solution; (4) For Eroding Goals, the management principle is: Hold the vision; (5) For Escalation, the management principle is: Look for a way for both sides to "win," or to achieve their objectives. In many instances, one side can unilaterally reverse the vicious spiral by taking overtly aggressive "peaceful" (actions that cause the other to feel less threatened; (6) For Success to the Successful, the management principle is: Look for the overarching goal for balanced achievement of both choices. In some cases, break or weaken the coupling between the two, so that they do not compete for the same limited resource (this is desirable in cases where the coupling is inadvertent and creates an unhealthy competition for resources); (7) For Tragedy of the Commons, the management principle is: Manage the "commons," either through educating everyone and creating forms of self-regulation and peer pressure, or through an official regulating mechanism, ideally designed by participants; (8) For Fixes That Fall, the management principle is: Maintain focus on the long term. Disregard short-term "fix," if feasible, or use it only to "buy time" while working on long-term remedy; (9) For Growth and Underinvestment, the management principle is: If there is a genuine potential for growth, build capacity in advance of demand, as a strategy for creating demand. Hold the vision, especially as regards assessing key performance standards and evaluating whether capacity to meet potential demand is adequate.

Jia Renan and Huang Guihong have studied the way to realize these management measures, and have put forward the principle of practical management measures, which is introduction sub-system with their respective responsibilities and benefits, and through the implementation of the duty and interests of the subsystem, to achieve the overall objective of the system.

This principle has a wide range of guidance. People and organizations should be involved in the implementation of management strategies. People and organizations must have their respective responsibilities and interests, and will strive for the interests of their respective responsibilities, this is the motivation. So 0nly through the realization of sub-system and the organization's responsibility and the benefits, can the general duty and interests be achieved and really continue to implement management. Further, this principle has provided a train of thought for the introduction of specific and operable measures. In our study, we applied the principle to eliminate the growth limit to the supply of agricultural products, and have achieved good effect.

However, there are still lots of issues on principles of Management strategy realization approach to be further study.

(1). How to determine the responsibilities and benefits associated with the subsystem?

(2) Who is the main body of the relevant subsystem respectively?

(3) How to take the initiative to achieve the sub-system responsibility for the implementation and benefits of their efforts to achieve the overall objective of the promotion?

2.3 Further study on the first SD feedback dynamic complexity analysis function.

(1). Besides the 9 archetypes generated by Peter M. Serge, How to establish new archetype in allusion to the typical new problems of system?

(2). How to use the combination of 9 archetypes for feedback analysis, access to new management to address this problem for a specific socio-economic system?

(3). How to evaluate some specific socio-economic system with the archetypes.

(4) How to analyze the respective characteristics of positive feedback loop and negative feedback loop through the nine archetypes

3. Study on theory and application innovation of the second SD feedback dynamic complexity analysis function.

The second SD feedback dynamic complexity analysis function is: formulating the quantitative simulation mode for the specific dynamic complex system, and conducting system simulation analysis of development trends, testing the effect of management measures.

The third function is the core function of system dynamics， which has been used widely.

3.1 The rate variable fundamental in-tree modeling approach

Forrest has generated the stock and flow diagram modeling method, and has created table function, delay function, exponential function, product, difference quotient, and quotient, etc. for formulate the system equations.

System dynamics is a vital science; there are lots of questions for further research.

Renan Jia presented SD rate variable fundamental in-tree modeling approach in 1997, since then his group has carried the relative research continuously, developed a series methods to complex system feedback analysis based on its rate variable fundamental in-tree model, We named these series approaches of SD modeling and analysis as the SD analysis approach based on rate variable fundamental in-tree model.

*Definition3***.** A set composed of the level and its corresponding rate variable pairs of the system is known as a level and rate system. Denoted as $\{[L_1(t), R_1(t)], [L_2(t), R_2(t)], \cdots, [L_n(t), R_n(t)]\}$, Where $R_i(t) = R_{i1}(t) - R_{i2}(t), i = 1,2,\cdots,n$, $R_{i1}(t), i = 1,2,\cdots,n$ denotes the inflow variable and $R_{i2}(t), i = 1,2,\cdots,n$ denotes the outflow variable.

Definition4. An in-tree $T(t)$ is called a rate variable in-tree, if its tree root is a rate variable, and the tree tops are level variables or other rate variables. The branch-order-length is the number of level variables contained in a branch. A rate variable in-tree is called a rate variable fundamental in-tree if the branch-order-length of every branch in the in-tree equals one. All of the fundamental in-trees of a system $T_1(t), T_2(t), \cdots, T_n(t)$ construct a rate variable fundamental in-tree model of the system.

Figure1 is a sketch map of a rate variable fundamental in-tree model, where, $A_i B_j, i, j = 1,2,\cdots,n$ is the auxiliary variable chain, $C_i, i = 1,2,\cdots,n$ denotes the policy parameter. The polarity sign “+”and “-” in each causal chain is omitted here.

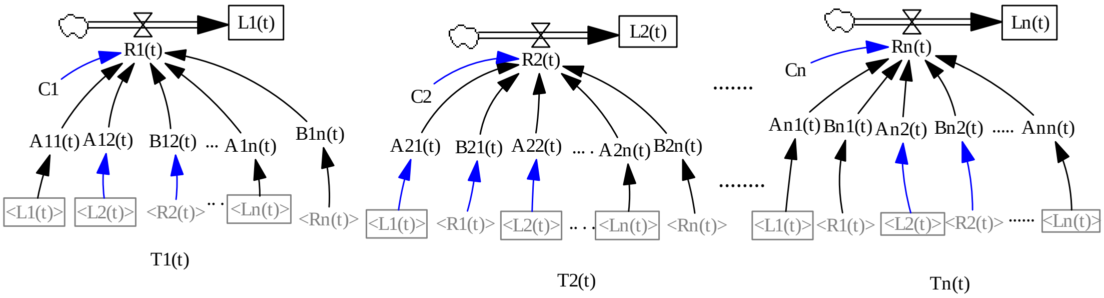

Fig1. Sketch map of a rate variable fundamental in-tree model

A rate variable fundamental in-tree model can be transformed into a network-flow diagram with embedding operation, denoted as $G_n(t)=\overset{\rightarrow}{\underset{i=1}{\overset{n}{U}}} T_i(t)$.The relationship between the rate variable fundamental in-tree model and network-flow digraph (Figure 2) is ‘if and only if’.

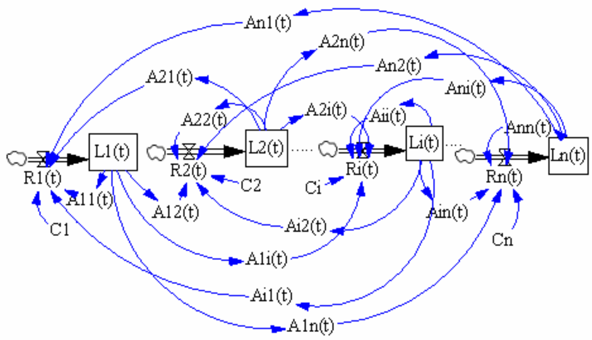

Fig2. Stock and flow diagram model embedded by the rate variable fundamental in-tree model

There are four advantages the rate variable fundamental in-tree modeling approach brings to modelers.

Advantage1. Standardize the SD modeling in branch and in subsystem, improve the concentrated degree and the precision of linear thinking.

Advantage2. Depict the relationship between the various subsystems more clearly.

When modeling, first set up a level and rate system $\{[L_1(t),R_1(t)],[L_2(t),R_2(t)],\cdots,[L_n(t),R_n(t)]\}$, and then formulate the rate variable fundamental in-tree model, transfer the in-tree model into a stock and flow diagram with embedding operation. Since every R$_i$(t) must be a rate of a subsystem, and every in-tree must be a in-tree to depict the change of the rate of the subsystem., the Rate variable fundamental in-tree model may be clearer portrait of the relationship between the various subsystems.

Advantage3. Be in favor of setting up simulation equations.

The rate variable fundamental in-tree model T$_i$(t) depicts clearly the direct relationship between the systemic variable. The qualitative relations can be more clearly set up than in the stock and flow diagram model.

$$\begin{cases}\dfrac{dLEV_1(t)}{dt}=RAT_1(t)\\ LEV_1(t)\big|_{t=t_0}=LEV_1(t_0)\\ \dfrac{dLEV_2(t)}{dt}=RAT_2(t)\\ LEV_2(t)\big|_{t=t_0}=LEV_2(t_0)\\ \cdots\cdots\cdots\cdots\\ \dfrac{dLEV_m(t)}{dt}=RAT_m(t)\\ LEV_m(t)\big|_{t=t_0}=LEV_m(t_0)\end{cases}$$

Where,

$$RAT_i(t) = f_i(LEV_1(t), LEV_2(t), \cdots, LEV_m(t), E_1(t), E_2(t), \cdots, E_n(t), a_1, a_2, \cdots, a_q, c_1, c_2, \cdots, c_r), (i = 1,2,\cdots, m)$$

Here, $E_j(t)$ denotes Exogenous variables, a_j denotes constants, and c_j denotes policy parameters.

Advantage4 Provide an advantageous frame, for system complexity analyze with algebraic method. So it has implemented an ideal combination of the graph theory and linear algebra.

3.2 Further study of the second SD feedback dynamic complexity analysis function

There are lots of *issues* of second function have to be further studied:

(1). How to *formulate* a standardized level and rate system methods?

(2). How to *formulate* high- reliability table function simulation equations of the actual system based on the historical data *and* future development tendency?

(3). How to use the delay function to portray the actual delay problem effectively?

(4). How to *test* the reliability of simulation model?

(5) How to *formulate* the Management policy parameters control equation?

(6) How to set up a more effective combination of the vertex weighted causal loop diagram analysis approach with the formulation of simulation equation?

(7) How to *achieve* the combination of the dominate archetype simulation with the overall simulation more effectively?

4. Study on theory and application innovation of the third SD feedback dynamic complexity analysis function.

The third SD feedback dynamic complexity analysis function is system feedback loops calculation and analysis, Revealing the specific dynamics of complex systems development, structural changes in feedback loops, and proposing management strategies.

Feedback *structure* analysis is an important content of dynamic complexity analysis. We have conducted an in-depth study of this content, and obtained some innovation.

4.1 Feedback Loops Evaluation by vector-determinant arithmetic

The *approach* is presented by Jia Renan & Yang Bo in 1999. It includes feedback loop calculation of the whole flow diagram and the newly added feedback loops calculation after an in-tree entry.

Given a *rate* variable fundamental in-tree model $T_1(t), T_2(t), \cdots T_n(t)$, by transfer the tree branches into a vector, a vector determinant as following can be constructed.

$$\begin{array}{cc}
 & \begin{array}{cccc} L_1(t)/R_1(t) & L_2(t)/R_2(t) & \cdots\cdots & L_n(t)/R_n(t) \end{array} \\
\begin{array}{c} T_1(t) \\ \\ T_2(t) \\ \vdots \\ \\ T_n(t) \end{array} &
\begin{vmatrix}
[R_1(t),\pm, A_{11}(t), L_1(t)] & \begin{array}{l}[R_1(t),\pm, A_{12}(t), L_2(t)] \\ +[R_1(t),\pm, B_{12}(t), R_2(t)]\end{array} & \cdots & \begin{array}{l}[R_1(t),\pm, A_{1n}(t), L_n(t)] \\ +[R_1(t),\pm, B_{1n}(t), R_n(t)]\end{array} \\
\begin{array}{l}[R_2(t),\pm, A_{i1}(t), L_1(t)] \\ +[R_2(t),\pm, B_{i1}(t), R_1(t)]\end{array} & [R_2(t),\pm, A_{22}(t), L_2(t)] & \cdots & \begin{array}{l}[R_2(t),\pm, A_{2n}(t), L_n(t)] \\ +[R_2(t),\pm, B_{2n}(t), R_n(t)\end{array} \\
\cdots & \cdots & \cdots & \cdots \\
\begin{array}{l}[R_n(t),\pm, A_{n1}(t), L_1(t)] \\ +[R_n(t),\pm, B_{n1}(t), R_1(t)]\end{array} & \begin{array}{l}[R_n(t),\pm, A_{n2}(t), L_2(t)] \\ +[R_n(t),\pm, B_{n2}(t), R_2(t)]\end{array} & \cdots & [R_n(t),\pm, A_{nn}(t), L_n(t)]
\end{vmatrix}
\end{array}$$

With this *successful* transformation, we have obtained the Branch-vector determinant feedback loop calculating method

(1) Branch-*vector* determinant feedback loop calculating method application 1: calculating the feedback loops of the whole flow diagram.

Theorem1. Calculate the following diagonal-1 branch-vector determinant, all of the feedback loops of in the in-tree model *from* 2-order to n-order can be obtained.

$$A_n(t)=\begin{vmatrix} 1 & \begin{matrix}[R_1(t),\pm,A_{12}(t),L_2(t)]\\+[R_1(t),\pm,B_{12}(t),R_2(t)]\end{matrix} & \cdots & \begin{matrix}[R_1(t),\pm,A_{1n}(t),L_n(t)]\\+[R_1(t),\pm,B_{1n}(t),R_n(t)]\end{matrix} \\ \begin{matrix}[R_2(t),\pm,A_{21}(t),L_1(t)]\\+[R_2(t),\pm,B_{21}(t),R_1(t)]\end{matrix} & 1 & \cdots & \begin{matrix}[R_2(t),\pm,A_{2n}(t),L_n(t)]\\+[R_2(t),\pm,B_{2n}(t),R_n(t)\end{matrix} \\ \cdots & \cdots & \cdots & \cdots \\ \begin{matrix}[R_n(t),\pm,A_{n1}(t),L_1(t)]\\+[R_n(t),\pm,B_{n1}(t),R_1(t)]\end{matrix} & \begin{matrix}[R_n(t),\pm,A_{n2}(t),L_2(t)]\\+[R_n(t),\pm,B_{n2}(t),R_2(t)]\end{matrix} & \cdots & 1 \end{vmatrix}$$

(2) Branch-*vector* determinant feedback loop calculating method application 2: calculating the newly added feedback loops calculation after an in-tree entry.

Theorem2. Calculate the following diagonal-0 branch-vector determinant, all of the newly added feedback loops from 2-order to n-order of $G_{12\ldots(n-1)}(t)\overrightarrow{\cup}T_n(t)$ can be obtained.

$$A_n(t)=\begin{vmatrix} 1 & \begin{matrix}[R_1(t),\pm,A_{12}(t),L_2(t)]\\+[R_1(t),\pm,B_{12}(t),R_2(t)]\end{matrix} & \cdots & \begin{matrix}[R_1(t),\pm,A_{1n}(t),L_n(t)]\\+[R_1(t),\pm,B_{1n}(t),R_n(t)]\end{matrix} \\ \begin{matrix}[R_2(t),\pm,A_{21}(t),L_1(t)]\\+[R_2(t),\pm,B_{21}(t),R_1(t)]\end{matrix} & 1 & \cdots & \begin{matrix}[R_2(t),\pm,A_{2n}(t),L_n(t)]\\+[R_2(t),\pm,B_{2n}(t),R_n(t)\end{matrix} \\ \cdots & \cdots & \cdots & \cdots \\ \begin{matrix}[R_n(t),\pm,A_{n1}(t),L_1(t)]\\+[R_n(t),\pm,B_{n1}(t),R_1(t)]\end{matrix} & \begin{matrix}[R_n(t),\pm,A_{n2}(t),L_2(t)]\\+[R_n(t),\pm,B_{n2}(t),R_2(t)]\end{matrix} & \cdots & 0 \end{vmatrix}$$

Propose the *management* strategies through the analysis of the feedback loops structural change.

4.2 The branch-vector matrix feedback loop calculating method

The approach is *presented* by Jia Renan and Hu Ling in 2000. It includes feedback loop calculation of the whole flow diagram and the newly added feedback loops calculation after an in-tree entry.

(1) The branch-*vector* matrix feedback loop calculating method application 1: calculating the newly added feedback loops calculation after an in-tree entry.

Set $a_{ij}=[R_i(t),C_{ij}(t),L_j(t)]+[R_i(t),\pm,B_{ij}(t),R_j(t)]$, $i\neq j, i, j=1,2,\cdots,n$

Construct the *diagonal*-0 branch-vector matrix

$$A_{k\times k}=(a_{ij})_{k\times k}=\begin{pmatrix} 0 & a_{12} & \ldots & a_{1k} \\ a_{21} & 0 & \ldots & a_{2k} \\ . & . & & . \\ . & . & \ldots & . \\ . & . & & . \\ a_{k1} & a_{k2} & \ldots & 0 \end{pmatrix}$$

Take row k *from* $A_{k\times k}$ to build row branch vector matrix $X_{1\times k}$ =(a_{k1},a_{k2},…,$a_{k(k-1)}$,0), and build 0-diagonal branch vector matrix $\vec{A}_{k\times k}$

$$\vec{A}_{k\times k}=\begin{pmatrix} 0 & a_{12} & \ldots & a_{1(k-1)} & a_{1k} \\ a_{21} & 0 & \ldots & a_{2(k-1)} & a_{2k} \\ . & . & & . & . \\ . & . & \ldots & . & . \\ . & . & & . & . \\ a_{(k-1)1} & a_{(k-1)2} & \ldots & 0 & a_{(k-1)k} \\ 0 & 0 & \ldots & 0 & 0 \end{pmatrix}$$

Operate one-*power* branch vector matrix multiplication

$$X_{1\times k}\cdot\vec{A}_{k\times k}=(a_{k2}a_{21}+\cdots+a_{k(k-1)}a_{(k-1)1},a_{k1}a_{12}+\cdots+a_{k(k-1)}a_{(k-1)2},\cdots,a_{k1}a_{1k}+\cdots+a_{k(k-1)}a_{(k-1)k})$$

if and only if $a_{ki_1}a_{i_1k}\neq 0(i_1=1,2,\cdots k-1)$, the relevant two-order branch vectors of $a_{ki_1}a_{i_1k}$ build the two-order new feedback loop（i_1=1,2,…,k-1）which is produced by $G_{12\cdots(k-1)}(t)\overrightarrow{\mathrm{U}}\mathrm{T}_k(t)$

Operate two-power branch vector matrix multiplication $(X_{1\times k}\cdot\vec{A}_{k\times k})\cdot\vec{A}_{k\times k}=X_{1\times k}\cdot(\vec{A}_{k\times k})^2$, $(X_{1\times k}\cdot\vec{A}_{k\times k})\cdot\vec{A}_{k\times k}$ is a row matrix, its elements are composed by the sum equation of $a_{ki_1}a_{i_1i_2}a_{i_2i_3}$, if and only if $a_{ki_1}a_{i_1i_2}a_{i_2i_3}\neq 0$, its corresponding branch vector chains form the three-order new added feedback loop which is produced by $G_{12\cdots(k-1)}(t)\overrightarrow{\cup}T_k(t)$.

So on, operate k-1 power branch vector matrix multiplication $X_{1\times k}\cdot(\vec{A}_{k\times k})^{k-2}$, $X_{1\times k}\cdot(\vec{A}_{k\times k})^{k-1}$ is row matrix if and only if $a_{ki_1}\cdot a_{ki_2}\cdots a_{ki_{k-1}}\neq 0$, its corresponding branch vector chains build the k-order new added feedback loop which is produced by $G_{12\cdots(k-1)}(t)\overrightarrow{\cup}T_k(t)$.

In above algorithm if we change the raw matrix $X_{1\times k}=(a_{k1},a_{k2},\cdots,a_{k(k-1)},0)$ into $X_{1\times i}=(a_{i1},a_{i2},\cdots,a_{i(i-1)},0,a_{i(i+1)},\cdots a_{ik}),(1\le i\le k)$, and set raw *i* of $\vec{A}_{k\times k}$ to be 0, other algorithms unchanged, we can get all the new added feedback loop *which* is produced by the embedding operation between a minimum archetype and a in-tree.

Propose the *management strategies* through the analysis of the feedback loops structural change.

(2) The branch-vector matrix *feedback* loop calculating method application 2: calculating the feedback loops of the whole flow diagram.

Jia Xiaojing and Jia *Renan* created diagonal branch-vector matrix feedback loop calculating method. Set $a_{ij}=[R_i(t),C_{ij}(t),L_j(t)]+[R_i(t),\pm,B_{ij}(t),R_j(t)]$, $i\neq j, i,j=1,2,\cdots,n$

Construct the *diagonal*-0 branch-vector matrix

$$A_{k\times k}=(a_{ij})_{k\times k}=\begin{pmatrix} 0 & a_{12} & \cdots & a_{1k} \\ a_{21} & 0 & \cdots & a_{2k} \\ . & . & & . \\ . & . & \cdots & . \\ . & . & & . \\ a_{k1} & a_{k2} & \cdots & 0 \end{pmatrix}$$

Theorem 3 Given 0-diagonal branch-vector matrix $A^{n\times n}=(aij)^{n\times n}$ of a rate variable fundamental in-tree model: $T_1(t),T_2(t),\cdots T_n(t)$, Multiply the corresponding element of up triangle by which of below triangle from the 1st low to n-1 low, then sum them, the branch-vector chain of all elements of

$$F_2(A_{n\times n})=(a_{12}a_{21}+a_{13}a_{31}+\cdots+a_{1n}a_{n1})+(a_{23}a_{32}+a_{24}a_{42}+\cdots+a_{2n}a_{n2})+\cdots+a_{n-1,n}a_{n,n-1}$$

compose the whole two order feedback loops of network flow diagram $G_{12\cdots n}(t)=\overrightarrow{\bigcup_{i=1}^{n}}T_i(t)$, and all two order minimum archetype can be obtained from the two order feedback loops.

The three feedback loop calculating methods stated above have an important role in changes in the structure of the system feedback has an important role in research. Usually, system changes complexly due to the introduction of a new system Management strategy, and the change is mainly expressed in the feedback change. So it is needed to analyze prior to the implementation of the countermeasures. At this point analysis with the above-mentioned additional feedback loops is very effective. In recent years, most related doctor-dissertation guided by JIA Renan, have carried out the calculation and analysis of feedback loop, and have obtained good results.

4.3 The minimum archetype generating set analysis method

The method is presented by Jia Renan, Tu Guoping and Deng Quanzhao in 2004.

Definition 5 In the system structure flow diagraph constructed of all in-tree of a rate variable fundamental in-tree model with embedding operation, a connected sub flow diagram of typical signification including feedback loops which must have rate variable, which is generated by some different in-tree with embedding operation, is called feedback archetype of this system..

Based on Definition 5, some archetype can generate from others instead of being generated from in-tree directly.

Definition 6 In all new archetypes generated from in-tree or archetype with embedding operation, which includes one in-tree factor at least is called the minimum archetype.

Definition 7 The number of flow level variables in feedback loop is the order of feedback loop. The order of the maximum order feedback loop is the order of this archetype.

Definition 8 Given a SD in-tree model. Handle every in-tree as follows: keep generating minimum archetype from the 2 order step by step with embedding operation, until all in-trees is included in the minimum archetype. By this handling, we get the set of minimum archetypes: $A(t) = \{G_{i_1i_2}(t), G_{i_1i_2i_3}(t), \cdots, G_{i_1i_2i_3\cdots,i_p}(t)\}$., and call it the set of minimum archetype of this in-tree model.

Definition 9 Given the set of minimum archetype, from formula

$$G_{xy\cdots z}(t) = \alpha_{i_1,i_2} \cdot G_{i_1i_2}(t) \vec{U} \alpha_{i_1,i_2i_3} \cdot G_{i_1i_2i_3}(t) \vec{U} \alpha_{i_1,i_2\cdots i_p} \cdot G_{i_1i_2\cdots i_p}(t)$$

Here, $\alpha_{i_1,i_2}, \alpha_{i_1,i_2i_3}, \cdots, \alpha_{i_1,i_2i_3\cdots,i_p} \in \{0,1\}$, When giving $\alpha_{i_1,i_2}, \alpha_{i_1,i_2i_3}, \cdots, \alpha_{i_1,i_2i_3\cdots,i_p}$ different value 0 or 1, we can get archetype full of practical meaning.

The set of minimum archetype is similar to the Basic Solution Series in Linear Algebra, with embedding operation; a variety of special significance archetype full of practical meaning can be generated, till the entire network flow diagram. The core structure of the actual system (the set of minimum archetype) can be found, and then we can propose more effective management measures in allusion to the core structure, or by generation change, analyzing the effect of the implementation of management measures.

4.4 Further study of the third SD feedback dynamic complexity analysis function

There are lots of issues of second function have to be further studied:

(1). How to integrate the branch-vector determinant, the branch-vector matrix and the problem-based mode feedback formulated in first SD feedback dynamic complexity analysis function and the successful case-based feedback mode formulated in first SD feedback dynamic complexity analysis function, to conduct feedback analysis?

(2). How to analyze more the new role of new countermeasures effectively through the new added feedback loop analysis?

(3). How to determine the dominant feedback loop through elements classification in the set of minimum archetype?

(4). How to carry quantitative analysis of simulation results with the reference of the feedback loop structure?

5. Study on theory and application innovation of the fourth SD feedback dynamic complexity analysis function

The fourth SD feedback dynamic complexity analysis function is formulating the feedback model for success case, and to sum up experience, guide the development of future.

5.1 Case 1

Peng Yuquan is a farmer pioneer of prosperity in rural areas of China Ministry of Agriculture; he began to engage in scale pig breeding since 1992, and has achieved success. Huan Zhijian and Jia Renan have summed up the experience of Peng Yuquan with the second SD feedback dynamic complexity analysis function, and have generated 5 feedback causal relationship archetypes to depict his successful experience in eliminating the growth limit.

(1). Improve the grade of livelihood to eliminate constraints from stress Survive reducing;

(2). Learn and practice continuously to eliminate constraints from ability insufficiency;

(3). Organize the development of the market to eliminate constraints from market risk;

(4). Enhance risk management and prevention to eliminate constraints from the risk of increasing;

(5). Co-ordinate the relationship between various aspects to remove constraints of environment.

Through the experience summary models listed above, the nature of things is depicted more vividly and specifically.

5.2 Case 2: Cooperation and competition-based mode

Background (the successful experience)

(1).Two organizations seek a win-win policy, learn from each other, and develop in common.

(2).Family business, all of they learn from each other, develops continuously.

(3). Two classmates learn from each other and progress together.

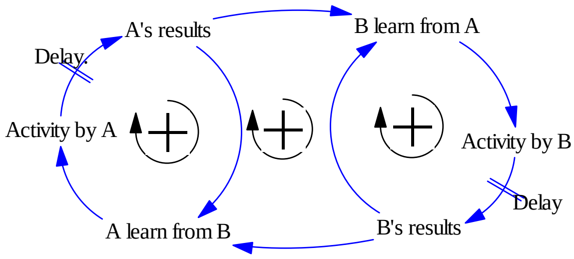

Fig.1 Cooperation and competition-based mode

There are still lots of issues on the second SD feedback dynamic complexity analysis function to be further study.

(1). How to generate a series of common causal loop diagrams for the successful case model?

(2). How to create a set of successful organization theory organization theory expressed by the series of common causal loop diagrams for the successful case model?

(3) How to establish typical matching cases for successful case model?

(4). How to obtain the whole system causal loop diagram with the basis of the successful case model?

(5). How to set up the level and rate system and the rate variable fundamental in-tree model via the analysis of the causal relationship of the feedback model for successful case?

6. SUMMARY

Relative studies on the service sectors are reviewed and analyzed in this paper. From the perspective of supply chain, we used the improved gray relational method to analyze the telecom service supply chain evaluation quantitatively. Case study shows that the improved gray relational evaluation method can help us to select the optimal service supply chain, and is useful for guiding telecom operators to obtain higher returns.

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