Church Growth via Enthusiasts and Renewal[1]

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Abstract

A word-of-mouth model is constructed for the growth of a church through enthusiasts, a subset of the church members who are active in recruitment. Two mechanisms for the source of new enthusiasts are proposed: new converts, and existing church members through a process called renewal. The conversion and renewal processes are compared and policies that could enhance the growth of a church are analysed. It was found that renewal is able to lower the epidemiological threshold present in the conversion process making rapid revival growth easier to achieve, even when that conversion process is inadequate. This is in part due to the expanding network of contacts a growing church produces that can enhance the renewal process. Policies to foster church growth centre on encouraging renewal, even at the expense of traditional evangelism. Some re-grouping of churches is suggested in order to attain critical masses of enthusiasts and church members that could tip a church into growth.

1 Introduction

1.1 Church Growth

Churches by their very nature are organisations that desire growth. Whereas many clubs and societies would be happy with a healthy membership, churches have a mission to see outsiders become members, regardless of the church's size. This is particularly true of Christian churches which, unlike most other world faiths, usually see membership of the organisation, and regular attendance, as the defining signs of religious belief. Because of this much effort is expended in seeking to understand how churches grow and what policies can effectively enhance that growth. Such studies of church growth are pursued by two strands: church organisations themselves; and by sociologists of religion.

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Church growth studies within churches were effectively founded by Donald McGavran in the late 1950's who investigated various factors that enhanced and inhibited growth in the mission field (McGavran 1990). Since then a thriving "church growth" movement has been established where the emphasis is on examining factors within the church institution that affect growth (Gibbs & Coffey 2000). This has been honed into major consultancy programs, such as Natural Church Development (Schwarz 1996), and the Purpose Driven Church (Warren 1996). This strand of church growth promotes good organisational practice within the belief systems of those churches.

By contrast sociologists of religion tend to emphasise contextual factors within society as the main forces affecting church growth. One of the earliest proposals, secularisation theory, states that as society becomes more advanced churches will decline, as people have more reason to doubt the simplistic faith that was offered (Finke and Stark 1992; Wallace 1966). This theory is used to explain the decline of church life in Western Europe, where many of the historic denominations face extinction within a generation or so (e.g. Bruce 2001). However with the continuing growth of churches in other western societies, such as the USA, other theories have come to the fore that seek to explain this growth and the partial failure of secularisation theory (Warner 1993; Stark & Iannaccone 1994; Stark 1999). This "new paradigm" in the sociology of religion received much impetus from Kelley (1986), whose thesis was that strict churches are organisationally strong and thus have much better growth potential. This thesis has proved controversial as it implies that church decline is a result of organisational weakness derived from leniency. Thus much work has been produced to either demonstrate or refute the thesis (e.g. Perrin & Mauss 1991; Iannaccone 1994). A consequence of this work has been the attempt to explain church growth in market terms and apply ideas from economics to the demand for religion (Iannaccone 1998; Mangeloja 2007).

Although both the church growth movement and sociologists of religion make extensive use of statistics, neither investigate the growth of the church by examining quantitatively the process by which people become members of the church. Hayward (1999; 2000; 2002) proposed the theory that churches grow by word-of-mouth through the activities of a subset of the church, called enthusiasts, who are alone responsible for its growth, and that only for a limited period of time. Thus it was proposed that church growth resembles the nature of an epidemic or the social diffusion of a fad or fashion. The model successfully reproduced the behaviour seen during religious revival, where the growth of the church is rapid, and has been subsequently extended to model long-term church growth and decline Hayward (2005).

1.2 Social Diffusion

One of the earliest theories of social diffusion is due to Penrose (1952) who suggested that epidemiology was a suitable model for a range of social phenomena from crazes to ideas, because all involved some aspect of diffusion by word-of-mouth. The work was a largely qualitative description based on statistical modelling and does not appear to have been used by subsequent authors[2]. Independently Rapoport (1953) proposed a stochastic model of the spread of information through a population based on ideas from mathematical biology. However it was Coleman (1964) who produced the first simple deterministic models, applying differential equations to the diffusion of medical innovations. This has led to an extensive literature on innovation diffusion (Mahajan, Muller & Bass 1990; Kumar & Kumar

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[2] Penrose's major contribution, described briefly in this work, was the power law voting mechanism used in some political assemblies such as the United Nations.

1992, Wejnert 2002) which has benefited from system dynamics insights (e.g. Maier 1998; Milling & Miczka 2008) .

However the church growth model of Hayward differs from most social diffusion models because the enthusiasts who spread the religion only retain their enthusiasm, or ability to convert others, for a limited period. Thus the mechanism is closer to that of the spread of a disease rather than the standard diffusion models.

Mathematical epidemiology is one of the most developed areas of mathematical modelling (Anderson & May 1987) with many models using the SD paradigm (e.g. Dangerfield, Fang & Young 2001; Bagni, Berchi & Cariello 2002). However, although epidemiology is applied to social modelling at the popular level, often through the concept of tipping points (e.g. Gladwell 2000), computational applications are more limited. One of the earliest models was by Burbeck, Raine, & Stark (1978) who used the Kermack-McKendrick model to analyse the spread of rioting. Subsequent applications of epidemiology to social ideas with word-of-mouth effects include diffusion of street gangs (Crane, Boccara & Higdon 2000); spread of bulimia (Gonzalez et. al. 2003); cigarette smoking (Rowe et. al. 1992); alcohol drinking (Sanchez et. al. 2006); rumours (Kawachi 2008); sexual behaviour (Rodgers & Rowe 1993); political party growth (Jeffs 2009) and scientific ideas (Bettencourt et. al. 2006). In each case the models are largely developed independently of each other; they are deterministic and a variation on the general epidemic model.

1.3 Models of Church Growth

It is perhaps Penrose (1952) who could be credited with the first model of church growth, as he applied his idea of "mental" epidemiology to outbreaks of religious enthusiasm. However as the work was not quantitative it is difficult to verify his conclusions. A more quantitative attempt at church growth modelling, using ideas from gas dynamics, was proposed by Logan & Dye (1984), but the approach was not pursued further. However, more recently, others from the physics community have attempted models of the growth of religion. Ormerod & Roach (2004) used scale free networks to model the spread of heresies in history. Ausloos & Petroni (2007; 2009) have used ideas from statistical physics. A more general model of ideological competition, which includes religion, was constructed by Vitanov, Dimitrova, & Ausloos (2009) using differential equation modelling, following a generalised Verhulst law.

With all these approaches to social diffusion, church growth or otherwise, the method is to take an established model from a different paradigm and apply it to the social situation in question. By contrast the system dynamics approach is to construct a model appropriate to the situation rather than borrow one from another paradigm. There are few examples of church growth tackled this way. Gaynor, Morrow & Georgiou (1991) modelled the sustainability of a religious order, but the growth of the organisation was not linked to the behaviour of individuals. Likewise (Bullock 1999) applied system dynamics to the analysis of parish growth; here the major emphasis was the financial management of the parish, rather than its growth mechanism. Hayward (2000; 2002) outlined how a system dynamics model of the limited enthusiasm principle could work, with (Acuña Moreno et. al. 2001) constructing a micro-world applied to a South American Christian denomination.

The latest version of the limited enthusiasm model (Hayward 2005) was constructed using mathematics and used to classify a range of Christian denominations in terms of revival growth, stability, decline and threatened extinction. The model has subsequently been re-

constructed as a system dynamics model and presented at a range of conferences and meetings[3]. The system dynamics methodology has proved very useful at explaining the model to church practitioners.

1.4 Need for Model Extension – The Renewal Process

Whereas many extensions could be made to the existing limited enthusiasm model of church growth, there is one crucial issue that needs attention. The model assumes that the only source of religious enthusiasts, the ones who are responsible for conversions and thus the growth of the church, are new converts. Although new converts are an excellent source of enthusiasts, because they have many contacts with unbelievers, nevertheless new enthusiasts come from the inactive members of the church as well. This feature of a revival is called renewal as the members of the church are reactivated to become more enthusiastic about their beliefs and subsequently start recruiting again. Some examples should help clarify this process.

The Welsh Revival of 1904-5 saw 100,000 people converted in the space of just over one year, as verified by denominational statistics (Hayward 1999). The revival is generally deemed to have started in one Welsh village on October 31st 1904, and quickly became noticed within a few weeks with many conversions recorded (Evans 1987, Ch. 6). However prior to this date, in other parts of Wales, churches had seen a significant rise of enthusism, as noted by local newspaper reports, although with only slow growth taking place (Evans 1987, Ch. 4; Hayward 2004). Although reasons are not given for the earlier slow growth it is clear that many of the "converts" mentioned in the revival were existing believers making re-commitments, as many of the meetings were almost exclusively among church members (Hayward 2004). Re-commitment is one example of renewal among church members.

In January 1994 an outbreak of religious enthusiasm started in Toronto Canada that was so intense that it caught the attention of the secular media, especially in the United Kingdom who dubbed it the "Toronto Blessing". However for the first six months the movement was largely unknown outside a small number of Christian churches, with the Toronto Airport church, its place of origin, acting as a renewal centre for other Christians from around the world. Following a newspaper article in the summer of 1994 the movement became public, and for the remainder of the decade a number of new enthusiastic churches with the "Toronto" ethos appeared, many of whom saw a significant number of conversions through evangelism (Hayward 2002; Poloma 2003, Ch. 9).

The Vineyard Christian Fellowship is now a worldwide church that had its origins in the Californian Jesus People revival of the 1970s. It took on its current form when leadership of the church was taken over by John Wimber who brought his own congregation into the church in 1982. From that date it grew from a handful of congregations in southern California to a major international Christian denomination by the mid 1990s, largely through church planting and conversion (Jackson 1999; Miller 1997, Ch. 3, 7). However prior to 1982 much of the work in Wimber's church had been among existing Christians from different churches seeking renewal, with significant conversion growth only coming in that congregation after 6 years (Jackson 1999, p. 61-89, table p. 64). It should be noted that surveys still show that a major source of recruitment into the Vineyard church are existing Christians from other

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[3] www.church-growth-modelling.org.uk

denominations looking for a renewed faith, i.e. renewed believers not genuine converts (Perrin & Mauss 1991; Perrin, Kennedy & Miller 1997).

1.5 Purpose of Paper

Thus before a complete model of church growth can be constructed the short-term process of renewal needs to be added to the limited enthusiasm model of church growth, and its effects understood. This paper describes a church growth model that includes the renewal process, and gives an assessment of its relative importance for church growth, compared with evangelism. The model will be limited to the short-term revival growth phenomena, thus births and deaths and recycling will be ignored, a time horizon of at most 10 years.

This paper will describe the model construction, including both the conversion and renewal processes, in particular the model of word-of-mouth contacts appropriate to each. The different modes of the model will be analysed, particularly those where renewal can encourage church growth, and policies suggested as a result. Data from the Welsh Revival of 1904-5 will be used to obtain typical values of the model parameters in a historic setting.

2 Model Development

2.1 Limited Enthusiasm

The central dynamic hypothesis of short-term church growth is that the church grows in membership[4] through the activity of a subset of its members, called the enthusiasts, who alone are responsible for the recruitment and conversion into the church. These enthusiasts make personal contact with unbelievers, those outside the church but susceptible to its influence, either by being directly involved in their conversion, or by taking them to a meeting where the unbelievers are converted at the hands of others. Thus it is proposed that the major mechanism for the short-term spread of the church is through the word-of-mouth activities of these enthusiasts, as noted by a number of authors (Stark & Bainbridge 1985; Olson 1989; Hadaway 1993)**.** Studies from within the church growth community also confirm word-of-mouth as the primary mechanism for recruitment (Greig 1998) with numerous evangelistic methods based around the training of such enthusiasts (Green 1990; Neighbour 1990; Thackery 2000). Enthusiasts would be among the most committed members of the church. Studies show that there is a strong correlation between commitment, expressed in intentional activity, and church growth (Dougherty 2004; Mangeloja 2007).

In addition to this dynamic hypothesis, it is assumed that new converts are a source of enthusiasts, because they have the most extensive network of potential recruits and an initial enthusiasm for the faith as a result of their conversion (Stark & Bainbridge 1985, p. 363; Olson 1989; Hayward 1999; 2005). Converts are frequently noticed because of their missionary zeal and their changed lifestyle, both of which make them effective in spreading the faith to other people (Kelley 1986).

It is further assumed that the enthusiasts only retain their recruitment potential for a limited period of time. This might be because the new believer loses their network of unconverted friends as they become integrated into the life of the church. Alternatively they may retain

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[4] In this paper the membership of a church will be interpreted informally and can include all who are committed to its cause rather than limited to the formal membership system that exists in some churches.

their friends, but their influence declines as their friends adjust to the changed ways of the new convert. Indeed the enthusiasm of the new convert to recruit may also decline as they become established in the church and find evangelistic activity has too high a social cost to be continued (Stark & Bainbridge 1985, p. 363; Hayward 1999; 2005). It was proposed by Kelley (1986) that this latter mechanism, also known as Wesley's law of the decay of pure religion, is a major cause of church decline in the long term (Hayward 1999; 2002).

Thus the central dynamic hypothesis, with its subsidiary assumptions, can be outlined in causal loops familiar in word-of-mouth and epidemic models (Figure 1) (Sterman 2000, Ch. 9). The enthusiasts are the cause of the church's growth (R1), but the dwindling pool of unbelievers opposes the growth by making it harder to achieve (B1). Although the church can continue to grow, the number of enthusiasts will be depleted as individuals cease to be active (B2)[5].
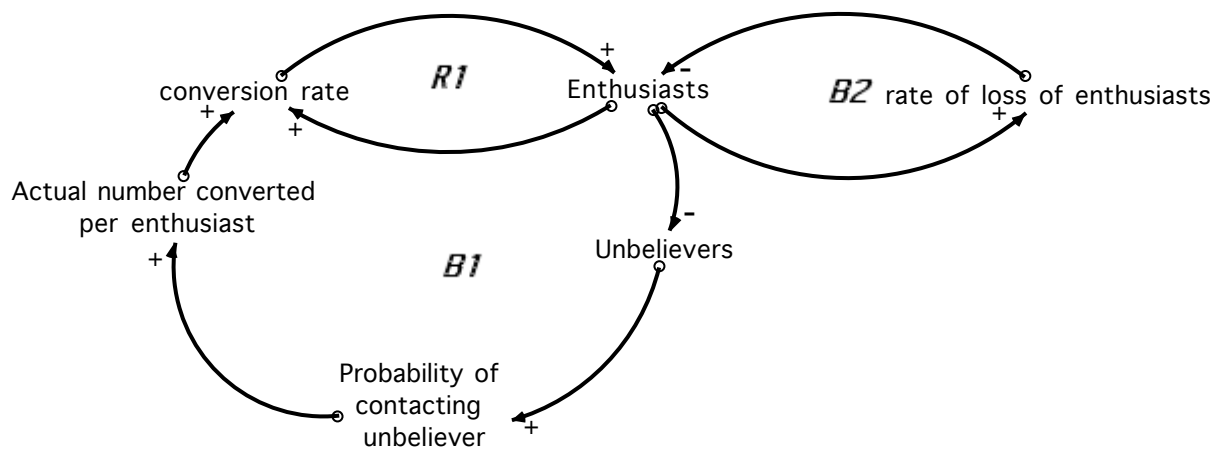

Figure 1: Causal Loop Diagram of Limited Enthusiasm Hypothesis

From the hypothesis in Figure 1 three categories of people are identified which will represent stocks on a main chain: the unbelievers, those without the faith; the enthusiasts, those members of the church responsible for the conversions; and the inactive believers the enthusiasts become after they lose their enthusiasm. Although these inactive believers are members of the church, and may contribute to aspects of its life, they do not directly contribute to its growth through conversion (Figure 2). The enthusiasts are like those infected with a disease, in this case the religious belief of the church. The inactive believers are like the removed, however they are still members of the church; they have the "disease" of religion, but are not contagious.
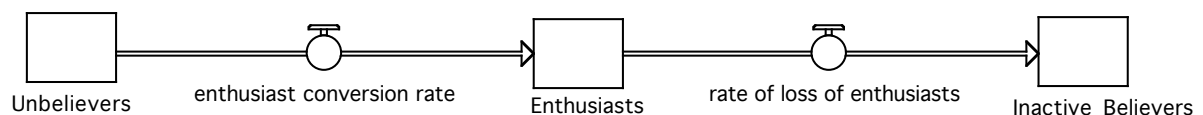

Figure 2: Stocks of the Limited Enthusiasm Model

Additionally it will be assumed that not all new converts will become enthusiasts, but will instead be inactive immediately. Such converts will be happy to be involved in church life

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[5] There are a number of ways that different authors express the causal loops of an epidemic (Lyneis & Lyneis 2007). A three-loop version has been chosen as the most intuitive to the situation being modelled here.

but not in evangelism, either due to shyness, or their conversion being for family reasons rather than one that carries deep independent conviction (Hayward 2002). Such secondary conversions, leading to believers with little enthusiasm for the faith, have been identified as a major dynamic in church life throughout its history (Stark 1996, p. 111-115). Thus a certain fraction of converts will flow directly from unbelievers into the inactive believers governed by similar loops to R1 and B1.

The limited enthusiasm model is implemented by the system dynamics model shown in Figure 3. Only the causal loops discussed above are indicated, however there are similar loops through the flow to inactive believers.
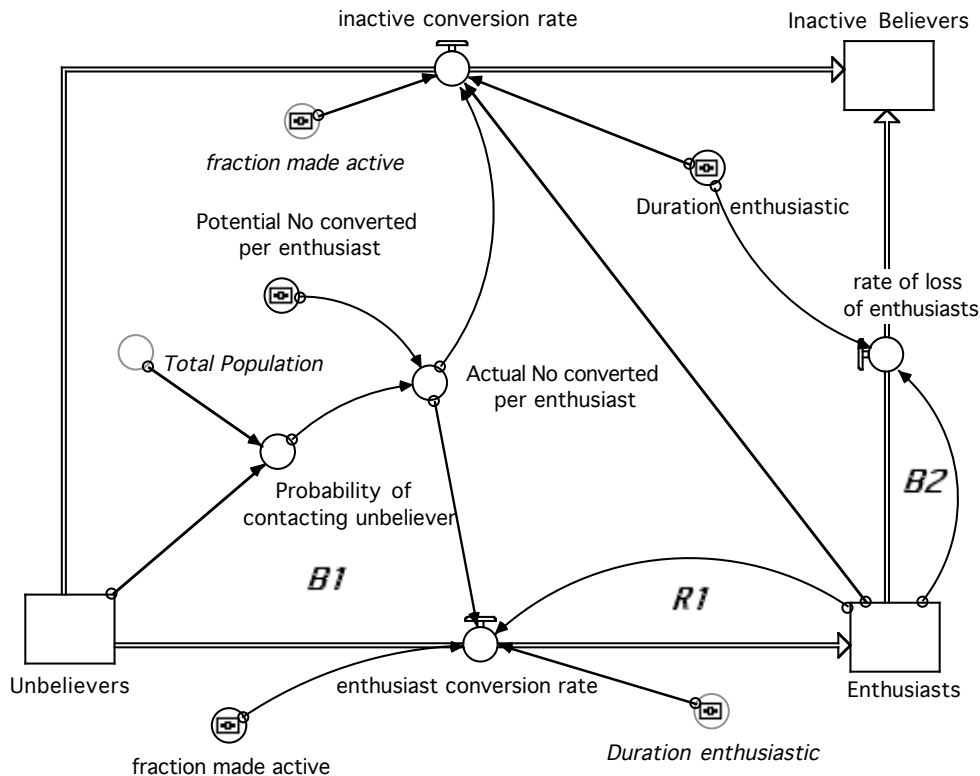
Figure 3: Limited Enthusiasm Model of Church Growth

Homogenous mixing is assumed for enthusiasts among the general population. Although churches can take a significant proportion of a person's time, most are not so demanding that the believers are isolated from the world. Also, although enthusiasts may engage in deliberate acts of recruitment, they nevertheless are still involved in the life of the church. Thus the probability of an enthusiast contacting an unbeliever is the proportion of unbelievers in society:

Probability of contacting unbeliever = Unbelievers/Total Population

The parameter that governs how effective an enthusiast is at making conversions is the *Potential No. converted per enthusiast*, that is how many converts a single enthusiast could make throughout their entire enthusiastic period, if the whole of the population were unbelievers. The actual number of unbelievers an enthusiast converts will be this potential number multiplied by the probability a person contacts an unbeliever. In terms of epidemic models this is the standard incidence model commonly used for sexually transmitted diseases

(STDs) where each infected has a fixed number of contacts independent of the population size (Hethcote 1994). For church growth, as with many word-of-mouth models, this is the best model of contacts as the number of people a person will contact in most communities is limited by the number of friends they can hold down, rather than the population size or density of that community. The one exception would be if the church were based in a small village geographically isolated from other communities.

The number of converts is found by dividing the actual number converted by the *duration enthusiastic*, another parameter. This parameter also governs the flow out of the enthusiasts. A third parameter, *fraction made active*, determines the proportion of converts who become enthusiasts and those who become inactive directly.

With this arrangement it is possible to define the reproduction potential, the number of enthusiasts one enthusiast could make during their enthusiastic period if the whole population were unbelievers:

Reproduction Potential = fraction made active X Potential No converted per enthusiast

This is the same as R_0 the reproductive ratio in epidemiology which defines the strength of an epidemic in a transparent way.
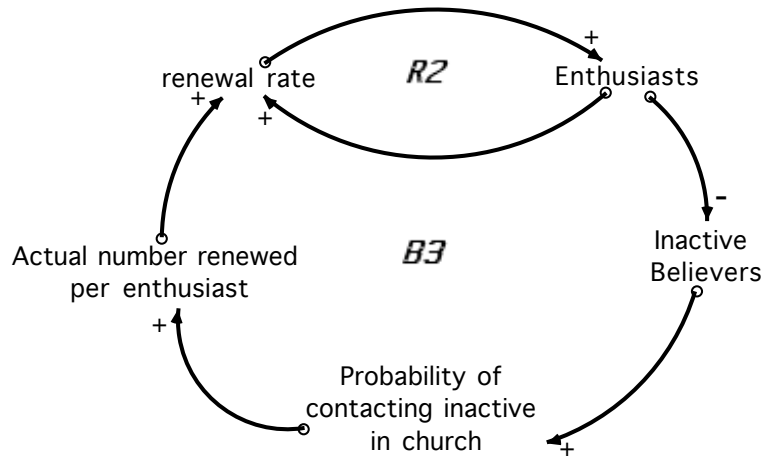
2.2 Renewal Hypothesis

The second dynamic hypothesis is that enthusiasts are also generated through the existing enthusiasts making contact with inactive believers within the church. The most enthusiastic members of the church do not just engage in evangelism but spend a substantial amount of their time interacting with fellow believers encouraging a deeper faith. This is the central activity of many churches and movements and can be illustrated with some examples.

The Vineyard Christian Fellowship specialised in running renewal meetings for other churches, as well as their own members, and they have exerted a huge influence on the renewal of the church (Jackson 1999, Ch. 8; Miller 1997, pp. 102-107). The Toronto Airport church has from its inception run courses to renew the church, a feature it has passed on to other churches (Poloma 2003, Ch. 7, 8). The Alpha course which started from a single church in London in the early nineties, and claims to have had over 34,500 courses operating worldwide, is nevertheless used with existing church members to renew their faith as well as to make new converts (Hayward 2002, Alpha Course).

However, like the conversion process, this method of reproducing enthusiasts will be limited by the supply of inactive believers. As renewal spreads through a church enthusiasts will have a lower probability of contacting an inactive believer. Thus there are two processes: a compounding one where enthusiasts renew more enthusiasts; and an opposing one through the limited pool of inactive believers. These can be expressed as the reinforcing loop R2 and the balancing loop B3 respectively (Figure 4).

Figure 4: Causal Loops of the Renewal Hypothesis

The causal loops of the renewal process have a similar structure to the limited enthusiasm hypothesis of growth, and lead to a similar implementation in the system dynamics model. The additions are given in Figure 5.
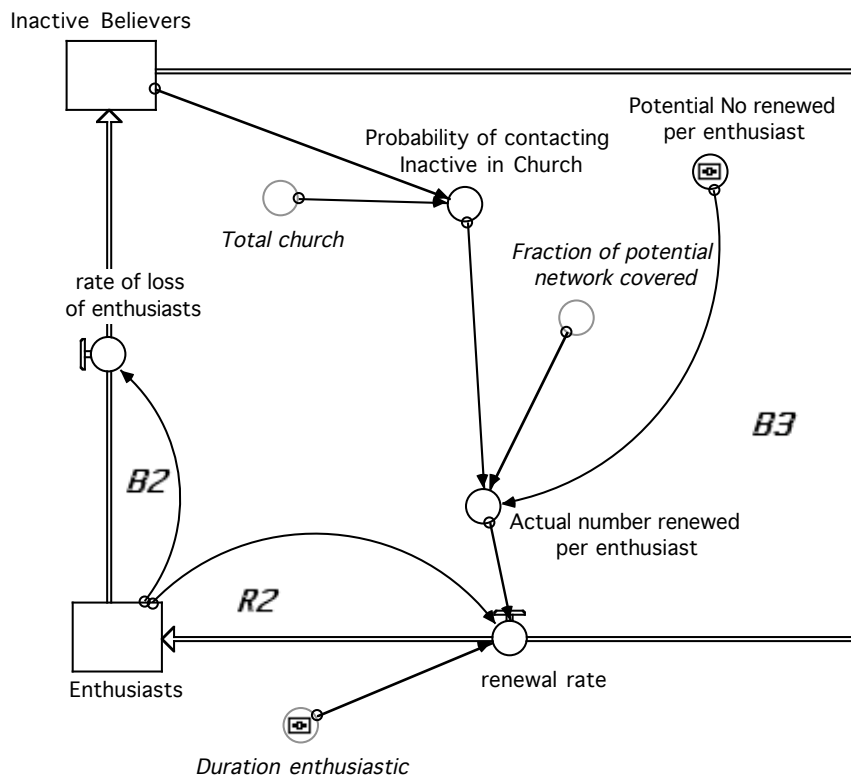
Figure 5: Renewal Model of Production of Enthusiasts

However there are some crucial differences in the details of the model. Renewal is a process that only takes place within the church. Thus the probability that an enthusiast contacts an inactive believer depends only on the fraction of inactive believers in the church. Thus, assuming the homogeneous mixing of believers:

Probability of contacting inactive in church = Inactive Believers/Total church

where the total church is the sum of enthusiasts and inactive believers. Even in the short term the denominator of this fraction is no longer constant and it is anticipated that this will affect the dynamics of the model.

In the early stages of a church's life it should be noted that the total church will be quite small, perhaps less than 50 people, and as such, the limit to an enthusiast's network of church members will not be limited by the largest friendship network a typical person can hold down, but by the size of the church. Thus the model of contacts for the renewal process, particularly for a small church, is closer to the mass action (crowd) model of epidemiology, rather than the standard incidence model. The mass action model is the one normally considered for diseases whose spread is airborne and is dependent on the density and size of the population (Anderson & May 1987; Hethcote 1994). In this case the potential number renewed per enthusiast would be directly proportional to the population size, the size of the church in this case.

Thus as the church grows the friendship network expands leading to more renewals through the increased contacts. This in turn leads to more enthusiasts who make more converts and thus a larger church. This leads to another reinforcing loop R3 (Figure 6).
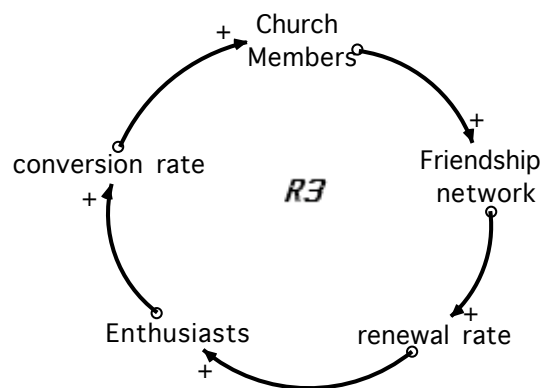

Figure 6: Causal Loop of Expanding Friendship Network in Church[6]

Additionally though a church will grow to a size where the mass action model will become less relevant. Congregations of over 1000 are not untypical, and by this size the network of influence, even among believers, will be more limited by the maximum network size a typical person can hold down. Thus as the church grows the mass action model of contacts becomes closer to the standard incidence model, with the loop R3 having diminishing effect.

In the earliest version of the renewal model of church growth the expanding friendship network was represented by a graphical converter which linked the renewal potential to the size of the church. However calibration proved difficult due to the lack of reliable data on the effect of different size populations on friendship networks. Instead an alternative approach was found by multiplying the potential number renewed per enthusiast, as it would be in the standard incidence model, by the fraction of a person's potential network of contacts that is covered in the church. This depends on the size of the church and corrects for churches with low numbers. A similar number/density correction is used in the model of STDs where the disease can lead to death (Anderson & May 1987, Ch. 11.2). Under these circumstances the

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[6] The links from Enthusiasts to Church Members covers conversions to both enthusiasts and inactive believers.

reduced density of the population changes the contact rate as the population declines through death, one of the few epidemiological circumstances where such effects could be significant.

Following Anderson & May (1987, p. 308)[7] the fraction of the potential network covered can be written:

Fraction of potential network covered
   = Total church/(Population for half density ratio + Total church)

where the parameter, *Population for half density ratio*, indicates the population at which the maximum network of contacts is 50% reached. For a suitable value of *Population for half density ratio* then, when the church is small, the renewal contacts follow the mass action model, whereas when the church gets larger the model becomes the standard incidence one. The parameter controls the speed of that transition. This formula ensures the positive polarity on the church/network link in R3 (Figure 6) is always valid.

An example of the behaviour of the *Fraction of potential network covered* on church size is given in Figure 7. In this example the mass action model is relevant from a church size of zero up to about 30. From 100 onwards the standard incidence model is a reasonable approximation.
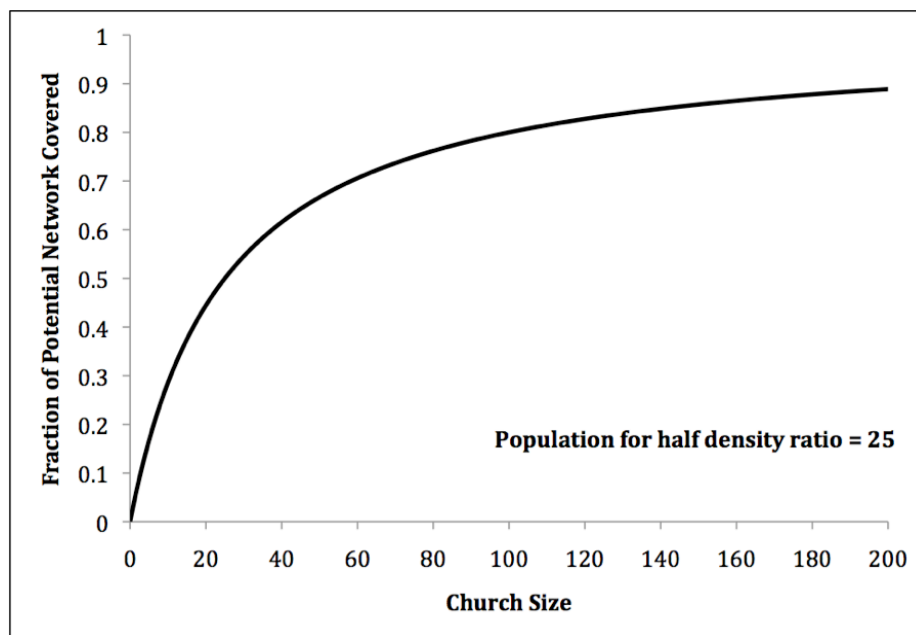
Figure 7: Example of how Fraction of Potential Network Covered depends on Church Size

For completeness the actual number an enthusiast will renew in the church is given by:

Actual number renewed per enthusiast
   = Fraction of potential network covered*Potential No renewed per enthusiast
      *Probability of contacting Inactive in Church

The *Potential No renewed per enthusiast* will also be referred to as the renewal potential to be compared with the reproduction potential defined above.

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[7] Hethcote (1994) also describes a variety of models of such horizontal incidence.

2.3 Scope of the Model

At this point it should be noted that the meaning of "church" in the model needs further clarification. The word "church" can refer to an individual congregation, the sum total of all believers in one denomination in a country, or the total number of church members in a geographic region from a town to a nation, regardless of denomination. The original limited enthusiasm model has been successfully used for all these levels of church because, for most countries, in any region or denomination, congregations tend to be reasonably homogeneously spread. Given that the limited enthusiasm model, in common with epidemic models, can scale with population, then it is possible to interpret population numbers such as enthusiasts and unbelievers proportionally. It is this feature of epidemic models that makes them applicable to most aggregate population groups without modification.

However in the renewal model, with the actual number renewed dependent on the church size, then more care will be needed if it is applied to church numbers of multiple congregations. As outlined above, the model was constructed for a single congregation, and the fraction of a person's potential network that is covered levels off at unity as the *congregation* size grows. If the model is applied to a wider region with many congregations, then a person's potential network size at that regional level of church is wider than their congregation, including people from other congregations. This is due to congregations having shared meetings for a range of spiritual and social activities. When the church grows then not only will the congregations grow but so will the interaction between congregations. Thus a person's potential network of contacts will increase in this wider regional church. Even if congregations do not grow as much as the regional church due to new congregations being planted, the interaction between congregations will grow, thus the network still expands with increasing regional church size. Due to new congregations being planted the *proportional* size of the congregation (measured as a fraction of its community) and the proportional size of the regional church (measured as a fraction of the region) will not be the same. However this will just be reflected in a different value of the *Population for half density ratio* at the regional level compared with the congregational level, even taking into account population scaling.

Thus it can be concluded that the renewal model can be used at different aggregate levels of church, provided a suitable parameter *Population for half density ratio* is taken, and that the rapid growth, or revival, is reasonably homogenously spread through the population of churches. It should be noted that using the density correction from Anderson & May (1987) rather than a graphical convertor makes it possible to calibrate the model for an appropriate level of aggregation.

2.4 Reference Modes

Descriptive and anecdotal evidence for rapid revival growth in the Christian church abounds. Specific numerical data is much harder to come by. Although most Christian denominations return membership figures and there are surveys by external bodies, the data is generally spread over periods wider than is typical in rapid revival. For example the Welsh Revival of 1904-5 had a national impact in Wales and this is clearly shown in the membership figures for the churches, but as the revival lasted at most 18 months, the figures only provide data points for its beginning and end. This proved sufficient to estimate the reproduction potential, but not sufficient to provide a detailed reference mode (Hayward 1999).

However there is newspaper evidence for the number of conversions in the first few months of the revival (Phillips 1906, Ch XLVIII). Although the reliability of the data is less robust

than membership data as it relied on congregations contacting the newspaper, nevertheless, given the historical fact of a revival taking place, and the eventual change in denominational membership figures, the data is remarkably convincing. Thus an attempt will be made to compare this data with the model.

Clearly the model should reproduce the rapid, epidemic-like, spread of enthusiasts, typical of revival. This is to be expected as the model is built on the limited enthusiasm model which also reproduces that mode. However the renewal model should be able to show that the church has a slower period of growth prior to there being rapid growth, due the presence of renewal in the church.

3 Model Behaviour & Results

3.1 Effect of Renewal on Revival Growth

The model was set with parameters typical of revival conditions for a church that initially occupies 10% of the population in which it is embedded (see Hayward (2002; 2005) for a variety of measured parameter values). The total population was normalised to 1. The renewal potential was allowed to increase from zero up to 0.4, showing a clear enhancement to the growth of the church (Figure 8), where "Total church" is the sum total of its members, enthusiasts and inactive believers. Thus a revival is enhanced by enthusiasts in the church seeking the renewal of inactive believers[8].

Parameters
Reproduction Potential **1.3**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0, 0.1, 0,2, 0.3, 0.4**
    Curve 1 to curve 5 respectively
Duration Enthusiastic **1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
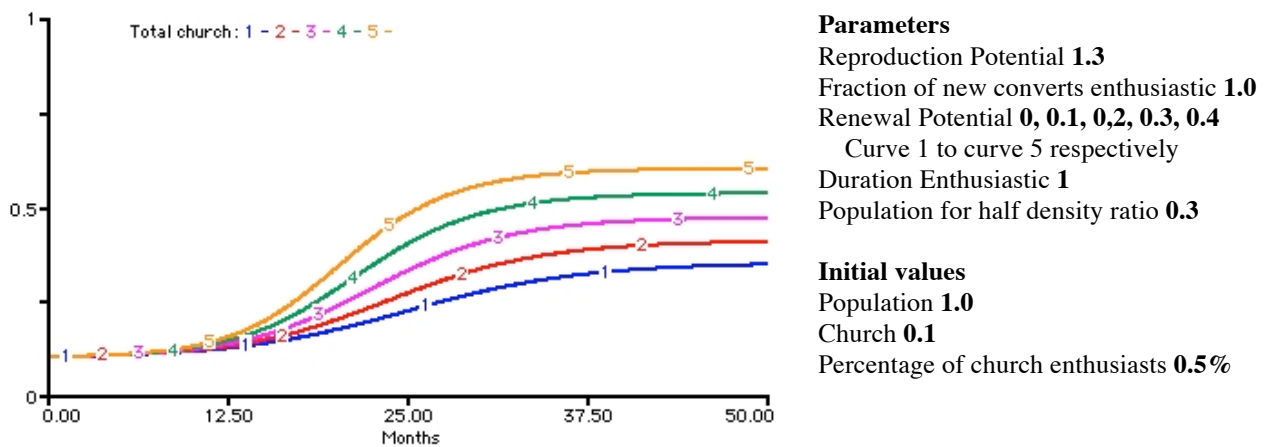Percentage of church enthusiasts **0.5%**

Figure 8: Revival Growth Enhanced by Increasing Renewal

For a church where conversion is the dominant mechanism the result of renewal has a uniform effect on the number converted in the revival. This needs to be noted as later there will be scenarios where the effect of renewal will be highly non-uniform.

The increase in the final church numbers is due to renewal increasing the number of enthusiasts (Figure 9), through the loop R2. The effect on the peak number of enthusiasts is less than uniform, thus higher renewal gives a larger than proportional peak in enthusiasts, and additionally brings the revival growth in marginally earlier. Thus a revival can occur faster when there is renewal in addition to conversions.

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[8] As in standard epidemic and word-of-mouth models loop R1 drives the growth of the enthusiasts. It slows through B1, and stops before all people have been influenced through B2 depleting the number infected, enthusiasts in this case.
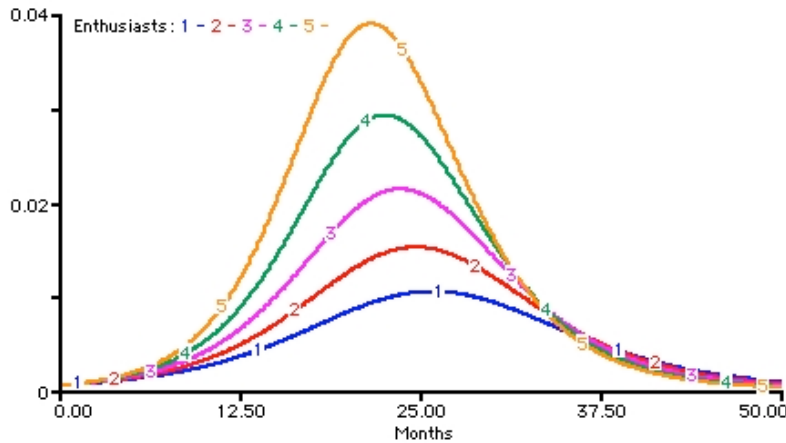
Parameters
Reproduction Potential **1.3**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0, 0.1, 0,2, 0.3, 0.4**
    Curve 1 to curve 5 respectively
Duration Enthusiastic **1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **0.5%**

Figure 9: Number of Enthusiasts Enhanced by Increasing Renewal

It should also be noted that the peak in the total conversion rate[9] occurs before the peak in the renewal rate (Figure 10). This pattern is seen in historic revivals. For example, most of the converts in the 1904 Welsh Revival occurred in the earlier period of the revival whereas the later period was dominated by meetings mainly for church members. Indeed the newspapers at the time stopped recording the number of conversions before the revival was one third of the way through, as the rate of conversion was already falling (Phillips 1906, Ch XLVIII).
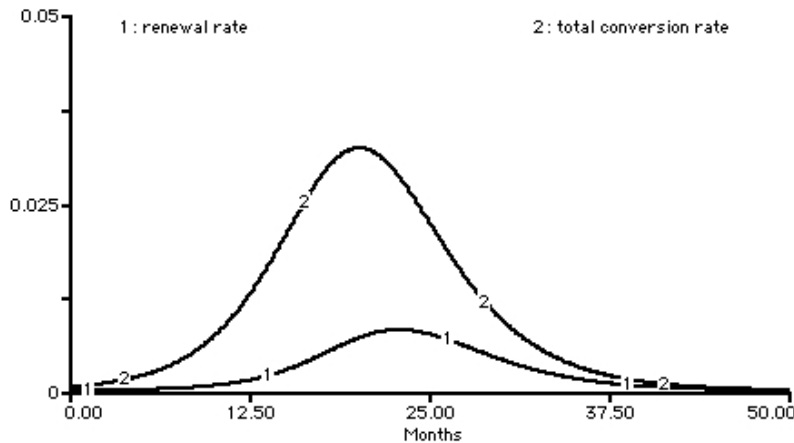

Parameters
Reproduction Potential **1.3**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0.4**
Duration Enthusiastic **1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **0.5%**

Figure 10: Conversion Rate Peaks before the Renewal Rate with Moderate Renewal

3.2 Growth Where Renewal Dominates over Conversion

There is a second scenario where the renewal process dominates over conversion, in particular where conversion is inadequate to achieve revival growth. This case is best understood in terms of the reproduction potential, which measures the number of enthusiasts one enthusiast could make during their enthusiastic period. This is R_0, the reproductive ratio of epidemiology, and is critical in determining revival growth. In the absence of renewal, for revival growth to take place the reproduction potential needs to be at least one, indeed $R_0 > 1/(U/N)$ the reciprocal of the fraction of the population who are unbelievers (Anderson & May 1987; Hayward 1999). This is the <u>critical reproductive value</u> of the epidemic, or revival growth. Thus consider a scenario where the reproduction potential is 0.75, well under

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[9] The sum of the inactive conversion rate and the enthusiast conversion rate.

the critical value and insufficient for growth. Figure 11 shows that if the renewal potential is large enough then revival growth can again be brought about.
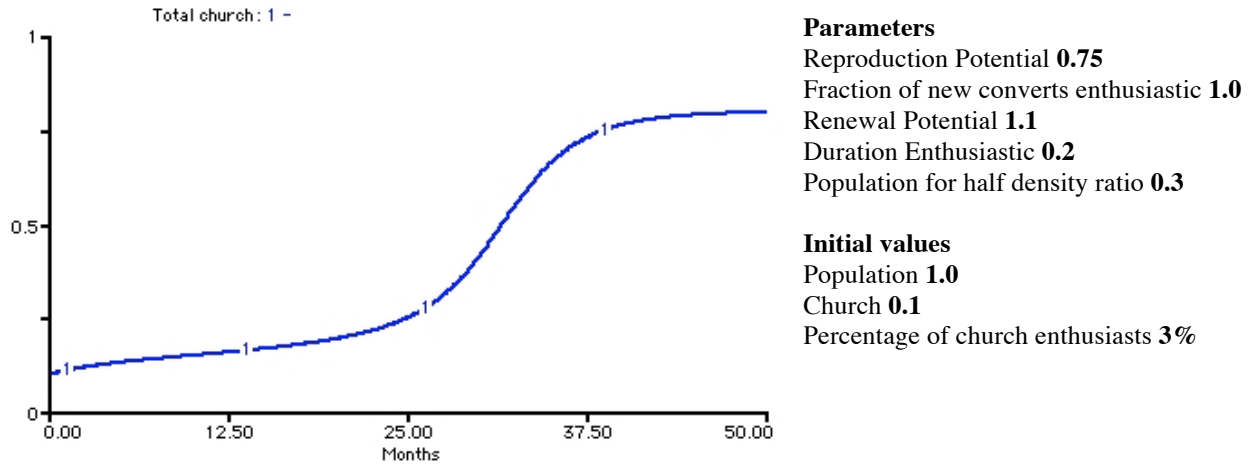

Parameters
Reproduction Potential **0.75**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **1.1**
Duration Enthusiastic **0.2**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **3%**

Figure 11: Church Growth Through Renewal with an Inadequate Reproduction Potential

In this case revival growth is delayed following a much slower period of growth. This is closer to the pattern of the Welsh Revival of 1904 which had seen slow growth with renewal prior to the revival breaking out, as noted earlier (Evans 1987, Ch. 4; Hayward 2004). Similar behaviour was seen in the Nagaland revival in the late 1970s where the rapid growth in the indigenous Baptist churches had been preceded by very slow growth where the church focussed on its own renewal (Orr 2000; Hattaway 2006).

Thus it can be concluded that renewal can induce revival growth in a church where the reproduction potential, that is the church's ability to make enthusiasts, is inadequate to bring that growth on its own. This is achieved because renewing inactive believers enhances the source of enthusiasts, and the fact that the church is growing slowly through the few enthusiasts that are present. It is this growth, although small, that enables the renewal process to become more effective as a larger church allows enthusiasts to access a larger network of inactive believers. Figure 12 shows how the potential network for the enthusiasts within church expands as the church grows, thus allowing the enthusiasts to become more effective at reproducing themselves through renewal, until there are enough to tip the conversion process. The expansion of the friendship network, loop R3, accelerates the diffusion process.

Parameters
Reproduction Potential **0.75**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **1.1**
Duration Enthusiastic **0.2**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
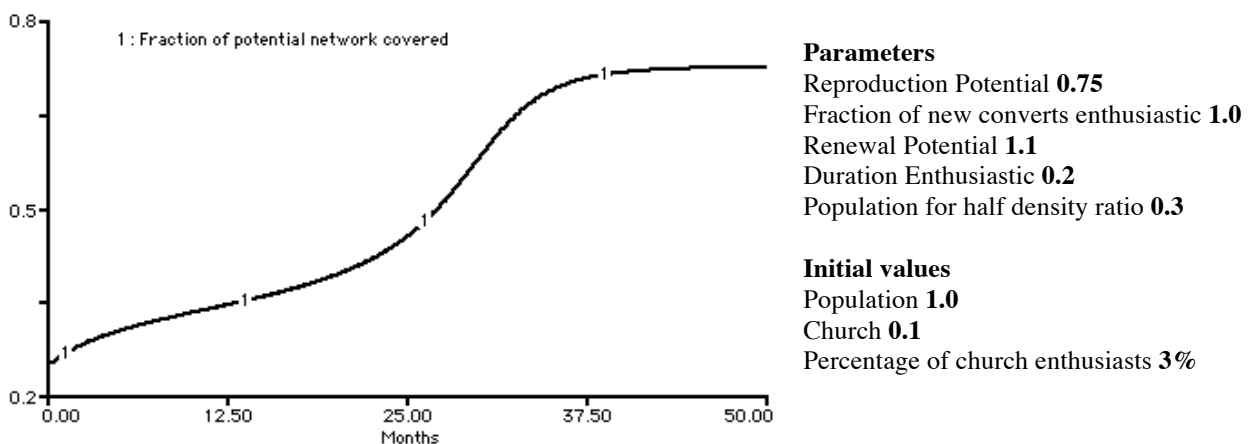Percentage of church enthusiasts **3%**

Figure 12: Expanding Network of Influence of Enthusiasts Within Church Due to Growth

In this case, when the reproduction potential is inadequate, the value of renewal is important. Too low, and there is no effect, but when a critical value is achieved the change in the growth pattern is dramatic, and far from uniform. Figure 13 compares the total church numbers with no renewal and negligible growth, curve 1, with values of the renewal potential near the critical value, curves 2-4. The growth in the church, and the onset of revival, is very sensitive to the renewal potential near its critical value, a non-uniformity due to the nonlinear interaction of the two causal loops R2 and R3.
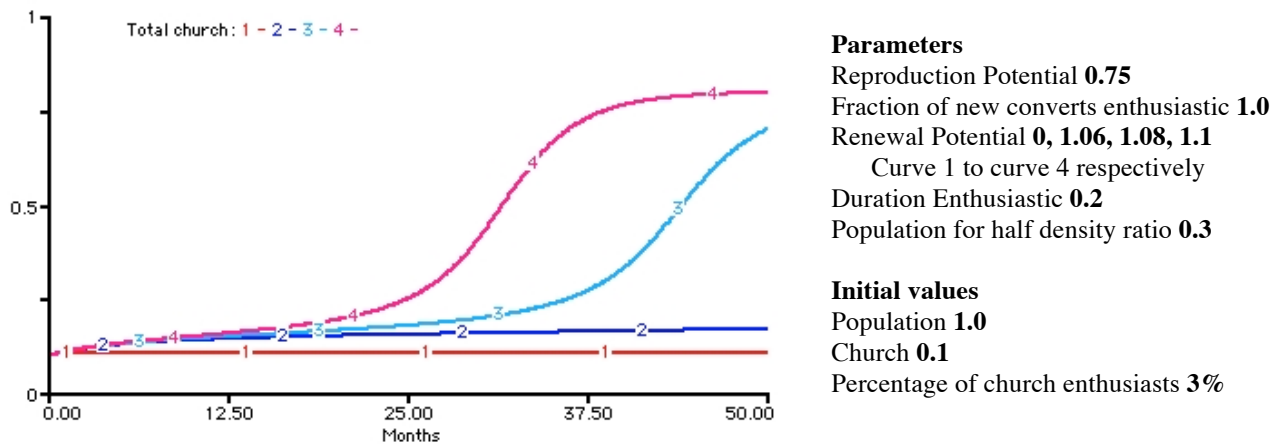

Parameters
Reproduction Potential **0.75**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0, 1.06, 1.08, 1.1**
      Curve 1 to curve 4 respectively
Duration Enthusiastic **0.2**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **3%**

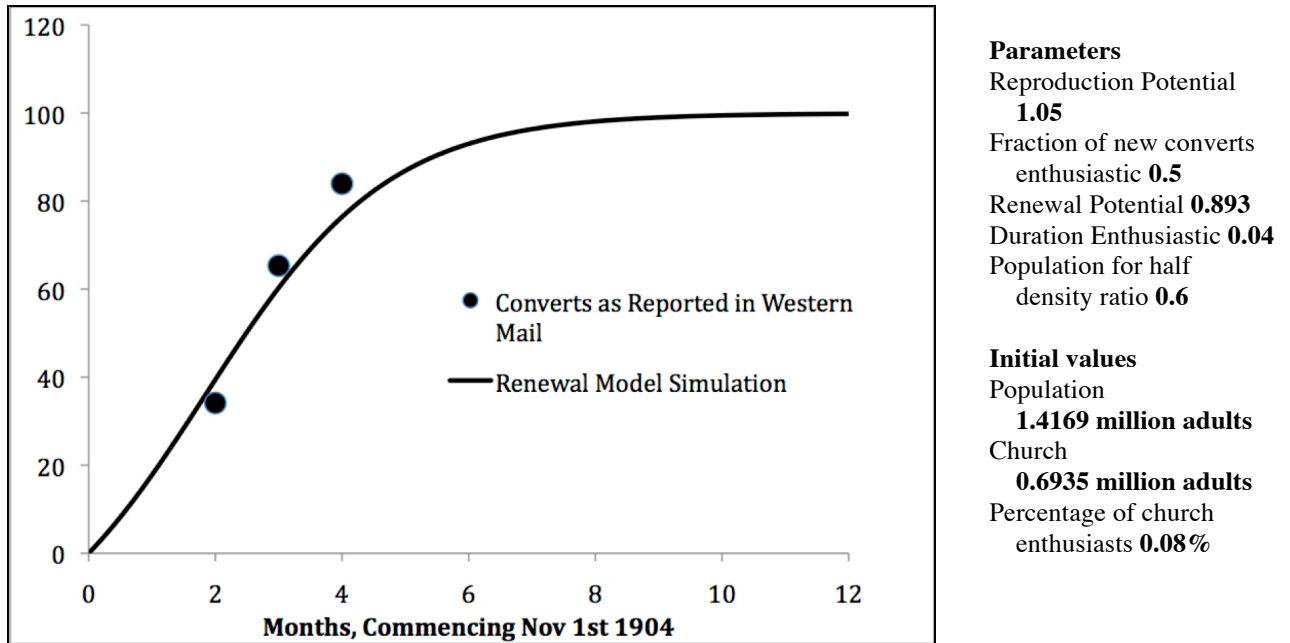**Figure 13: Rapid Growth at Critical Value of Renewal Potential**

3.3 Welsh Revival 1904

The last major revival to take place in Wales UK was in 1904-5. Unlike its previous revivals, and those in other countries, there is some reliable data. As discussed earlier, the public phase of the revival started on October 31st 1904. The population of the country and the membership of the churches are well documented, the latter by denominational returns across most of the churches in Wales (Williams 1985). This source records a rise in church membership by 100,000 people in 12 months during 1905, the same figure claimed by the revival leaders for the number of converts. A previous attempt was made to use this data to estimate parameters in the limited enthusiasm model (Hayward 1999).

Additionally the national Welsh newspaper, The Western Mail, recorded the cumulative number of converts at the end of December, January and February. These appeared in special supplements using data received from churches involved in the revival. The supplements were subsequently published in the first documented history of the revival (Phillips 1906). Although this data is less reliable than the denominational returns it gives a reasonable picture of what took place between the beginning and the end of the revival.

Although it is ambitious to obtain a good fit between the model and conversion data, due to a number of missing factors, the data can be used to get an estimate of parameter values that could be typical of a revival. The best fit is given in Figure 14, where it can be seen that a significant amount of renewal is required to obtain that fit. Given that the church was initially around half the adult population, then a reproduction potential of over 2 would have been required, using $R_0 > 1/(U/N) \approx 2$. Thus, with this additional data, conversion is insufficient to explain the source of enthusiasts in the revival, with $R_0 = 1.05$, and that some renewal of existing believers is required (renewal potential almost 0.9).

Parameters
Reproduction Potential
1.05
Fraction of new converts
enthusiastic **0.5**
Renewal Potential **0.893**
Duration Enthusiastic **0.04**
Population for half
density ratio **0.6**

Initial values
Population
1.4169 million adults
Church
0.6935 million adults
Percentage of church
enthusiasts **0.08%**

Figure 14: Cumulative Conversions in a Simulation of the Welsh Revival 1904-5 compared with Data

That the model is unable to match the steep rise in conversions could be due to a number of factors. Firstly it is possible that the conversion statistics may be over estimated. The newspaper relied on individual churches submitting their figures, and there may have been a degree of over enthusiasm in reporting in order that churches could be seen to be supporting the revival. Secondly it is possible that some existing church members were reported in these conversion statistics, people who in the model would be classed under renewal. The word "convert" was used differently in different churches. Thirdly there is the effect of the national press advertising the revival and the effect that had on people attending meetings. There are no doubt other factors that could be listed.

One feature of the revival was the slow build up of church numbers for about a year prior to the public phase of the revival. Although the model can reproduce this mode of behaviour, as shown in Figure 11, data fitting over this period proved more problematical. This may be partly due to the uncertainty of the length of that slower period, but perhaps also that the pool of available unbelievers was significantly less during that period as most churches and their communities knew little of the emerging revival. So although the revival can be used to estimate typical parameters for the renewal model, the model is still some way from encapsulating the whole story behind the entire progress of the revival.

3.5 Modes of Behaviour

Five different modes of behaviour can be identified in the model.

Mode 1 is where both the reproduction and renewal potentials are too small to bring about growth. The number of enthusiasts always declines, and church growth is negligible. Mode 2 is where renewal is low and the reproduction potential is enough to bring revival growth. This is the epidemic mode where the church fails to convert all the population due to the shrinking susceptible pool (see the phase plot in Figure 15, read right to left).
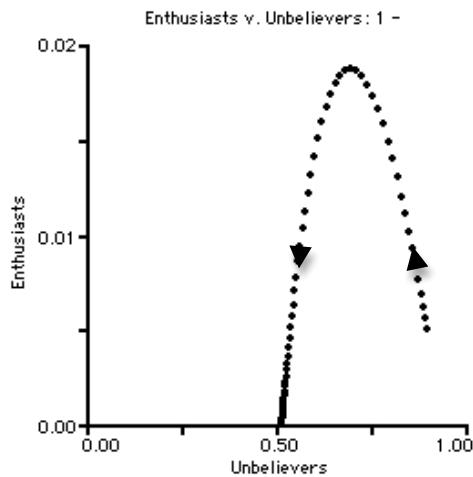
Parameters
Reproduction Potential **1.1**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0.5**
Duration Enthusiastic **0.1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **5%**

Figure 15: Epidemic Mode of Growth Largely Through Conversion (read right to left)

Mode 3 is the case where the reproduction potential is low and the renewal potential is sufficient to bring about revival growth. In this mode the number of enthusiasts declines initially, but not so fast that their numbers cannot recover through renewal and conversion. Again the result is the epidemic pattern of revival growth where the church fails to convert the whole population (Figure 16). In this case it is renewal that has proved critical in bringing about the growth. As will be shown later, the initial values of the church numbers, and the enthusiasts, will be crucial in achieving this mode.
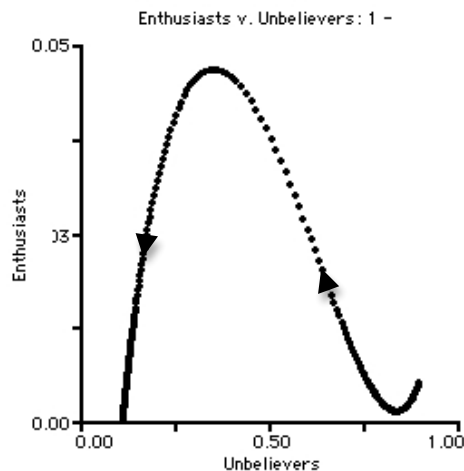

Parameters
Reproduction Potential **0.7**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **1.2**
Duration Enthusiastic **0.1**
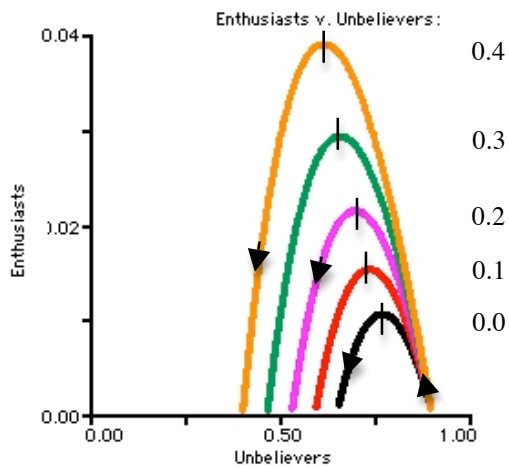Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **5%**

Figure 16: Epidemic Mode with Growth Driven by Renewal

Mode 4 is where the reproduction potential is low and the renewal potential high, and mode 5 is where both are high. In these cases the parameters are so high the whole population is converted. Compared to measured values of the reproduction potential, these values are unrealistically high and probably of no practical importance. They are distinguished from each other by mode 4 starting with the enthusiasts decreasing, like mode 3, whereas mode 5 has them increasing straight away, like mode 2.

3.6 Thresholds

A useful concept for understanding epidemic type growth is that of the threshold. In standard epidemiological modelling (Anderson & May 1987; Hayward 1999) the threshold occurs when the number of infected people changes from increasing to decreasing, marked by the

peak in the graph on the phase plot in Figure 17. If the number of unbelievers is larger than this threshold, revival growth occurs. Increasing the renewal potential effectively lowers the revival threshold, shown by comparing the plots in Figure 17. Thus increasing renewal can tip a declining church into growth by lowering this revival threshold below its reproduction potential.

Parameters
Reproduction Potential **1.3**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0, 0.1, 0,2, 0.3, 0.4**
Duration Enthusiastic **1**
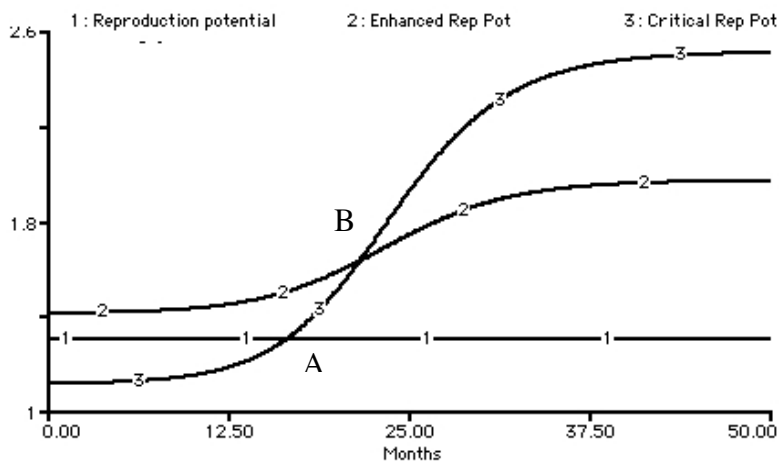Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **0.5%**

Revival Growth Threshold
For different renewal potentials

Figure 17: Revival Growth Threshold Drops With Increasing Renewal

An alternative approach to the threshold is to think of renewal as "enhancing" the reproduction potential and comparing it with the critical reproductive value $1/(U/N)$ noted earlier. The enhanced reproduction potential is equal to the normal reproduction potential plus a factor dependent on renewal, obtained by equating the flows in and out of the enthusiasts.

In the case of the model in section 3.1, where renewal enhanced the existing revival growth, the enhanced renewal potential rises as the enthusiasts' network of believers expands with the initial slow growth of the church (curve 2, Figure 18). Thus the point at which it crosses the critical value, B on curve 3, is later than it would have been if there had been no renewal, A on line 1). It should be noted that the critical reproductive value (Curve 3) always rises as the unbelieving pool shrinks. The critical value is a measure of how much harder it is to achieve growth by the conversion (word-of-mouth) mechanism as the church grows.

Parameters
Reproduction Potential **1.3**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **0.4**
Duration Enthusiastic **1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **0.5%**

Figure 18: Growth Occurs when Enhanced Reproduction Potential Exceeds the Critical Value (B).

In the case where renewal dominates over a low amount of conversion (section 3.2 and mode 3 in section 3.5), the enhanced reproduction potential rises almost in parallel with the critical value (Figure 19). In this case the church is initially in a state where slow church growth can have a significant effect on the potential network of the enthusiasts among inactive believers (Figure 12).
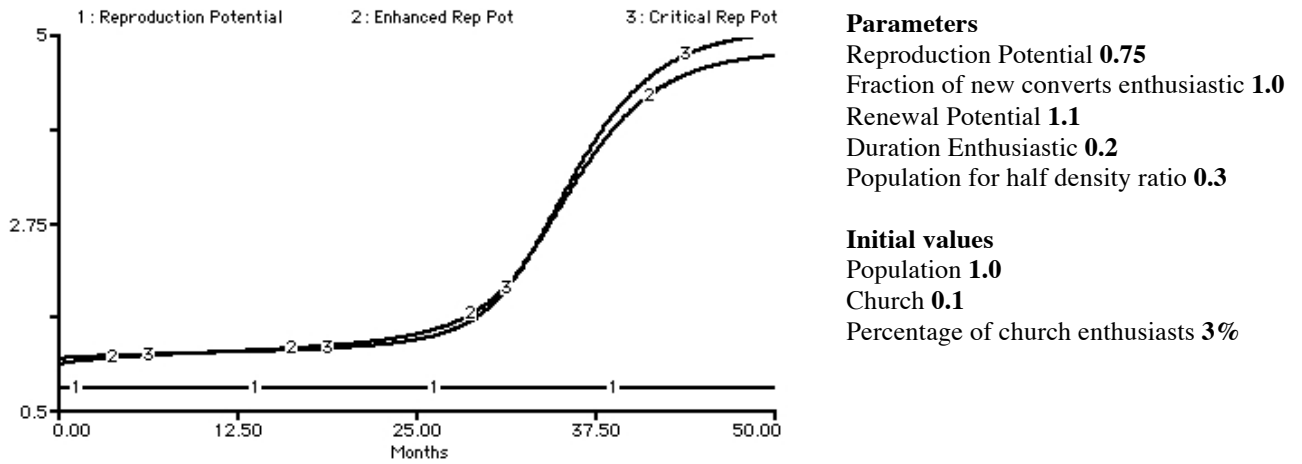

Parameters
Reproduction Potential **0.75**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **1.1**
Duration Enthusiastic **0.2**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.1**
Percentage of church enthusiasts **3%**

Figure 19: Enhanced Reproduction Potential Tracks Critical Value when Renewal is Dominant

3.7 Critical Mass

The rapid growth in mode 3, where renewal dominates over conversion, is also dependent on initial conditions. In particular for revival growth to occur there has to be a sufficient number of enthusiasts in the church. If those numbers are too low revival growth does not occur, but increasing the number to a critical mass of enthusiasts allows revival growth to take off, growth that can be dramatic. This is illustrated in Figure 20. If enthusiasts only compose 1% of the church initially then their numbers collapse with little growth in the church, that is reduction in unbelievers (right hand curve). However increasing this initial value of enthusiasts to 3% sees them recovering after an initial drop and a large increase in the church, that is a decrease in unbelievers. A further increase in initial enthusiasts sees little extra gain for the church. Thus ensuring there are sufficient enthusiasts in a church can be critical for growth to take place.

Parameters
Reproduction Potential **0.65**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **1.1**
Duration Enthusiastic **0.1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.2**
Percentage of church enthusiasts
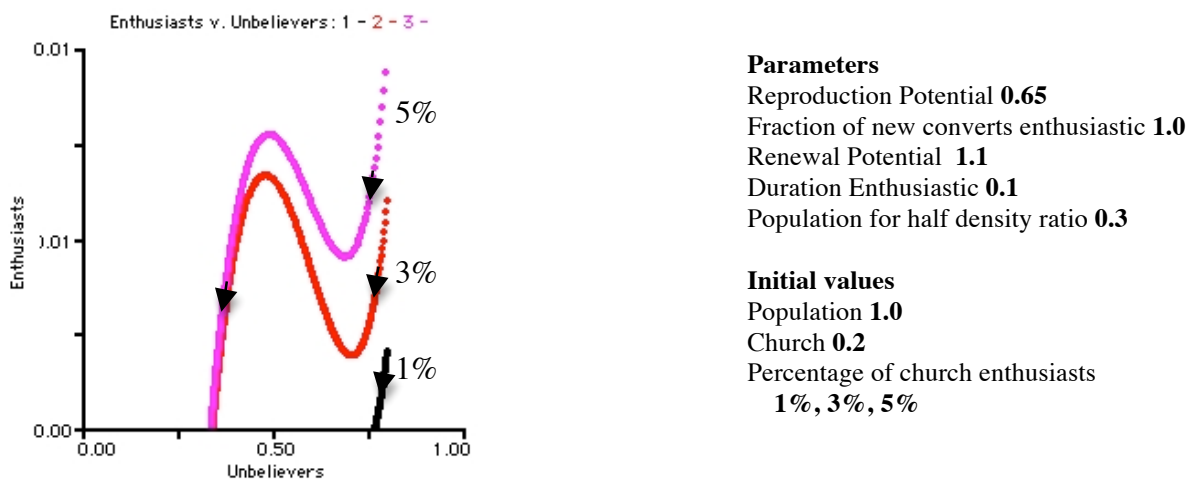   **1%, 3%, 5%**

Figure 20: Revival Growth Depends on Critical Mass of Enthusiasts

Likewise revival growth also requires a critical size of church. Figure 21 shows three churches with the same percentage of enthusiasts, but different sizes. The smallest church, the right hand curve, sees little growth with the enthusiasts failing to reproduce themselves. A small increase in church size not only sees the enthusiasts recover and increase, but the church have revival growth. Thus ensuring a church is a sufficient size can be critical for its growth.
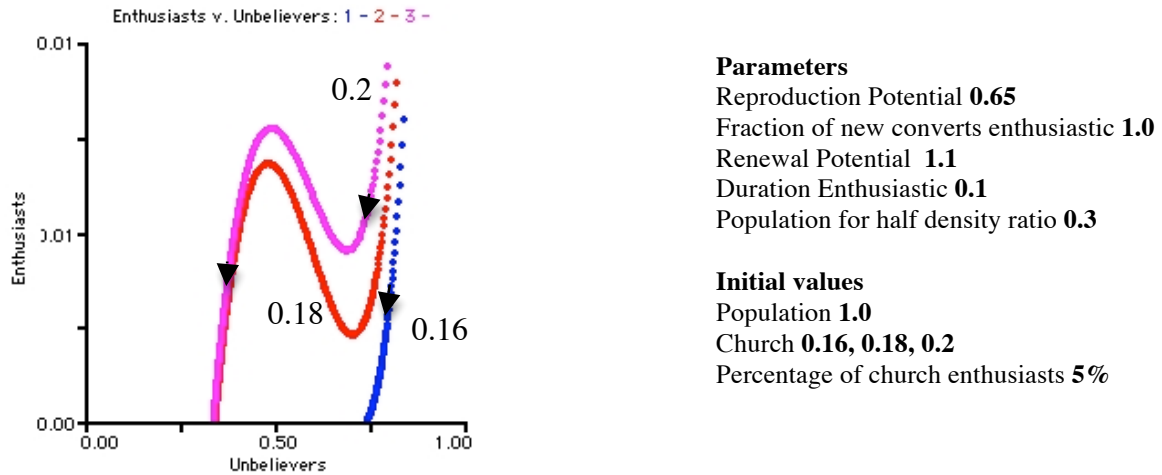

Parameters
Reproduction Potential **0.65**
Fraction of new converts enthusiastic **1.0**
Renewal Potential **1.1**
Duration Enthusiastic **0.1**
Population for half density ratio **0.3**

Initial values
Population **1.0**
Church **0.16, 0.18, 0.2**
Percentage of church enthusiasts **5%**

Figure 21: Revival Growth Depends on Critical Church Size

4 Principles and Policies to Encourage Church Growth

Using the results of section 3 some principles and policies can be established to encourage growth in the church.

Following the limited enthusiasm model, increasing the effectiveness of enthusiasts can make a dramatic effect on the church's growth if its parameters are near the threshold value. Thus churches could be advised to make reproduction of enthusiasts, rather than just seeking converts, a priority. That is, it is the ability of the converts to reproduce themselves that is key, perhaps through the training of new converts and those newly renewed, or perhaps by creating a church environment where that enthusiasm can be enhanced and maintained. A small change here can make a large impact, especially given that it is estimated that some major UK and USA denominations are not far from threshold values (Hayward 2005).

Renewal can enhance revival growth. Church growth is not just about recruitment or evangelism, it is about increasing enthusiasts, the church members who drive the growth of the church. Renewal of existing church members who are inactive can be an important additional source of enthusiasts, and can make growth easier to achieve. It is renewal, not evangelism, that is the key to church growth, as renewal can allow conversions to occur even when the conversion potential is quite low. Given that churches find it easier to mobilise their members for church-based meetings, rather than engage in the more costly evangelism (Kelley 1986), a policy of concentrating on renewal could be an attractive strategy.

Renewal does not just enhance growth; it could help tip a church whose reproduction potential is inadequate into revival growth. Given that there are a number of declining denominations whose reproduction potential is very low (Hayward 2005), this aspect of

renewal could give hope to those churches that they could achieve recovery through a renewal strategy.

The impact of renewal is enhanced if there are more enthusiasts initially. Thus another policy for growth, particularly in a congregation, is to bring enthusiasts together. This could make a difference between a congregation dying out before the enthusiasts have had time to reproduce themselves, and the recovery of the church. This critical mass of enthusiasts could be the difference between revival growth and extinction. Although this could prove a controversial policy, as it might leave some congregations declining faster at the expense of others growing, it would prove an effective strategy for overall growth.

Likewise renewal has more effect when the host church is bigger. Again this could mean either bringing congregations together, or alternatively organising meetings where different congregations mix more, allowing enthusiasts a larger network of inactive believers they could influence. This latter approach has often been the strategy used prior to revival taking place. The Welsh Revival of 1904-5 was influenced by the emerging Keswick Convention in Wales (Evans 1987, Ch. 4); the Toronto Blessing saw numerous inter-congregational renewal centres set up around the world (Poloma 2003, Ch. 7,8); and the Vineyard movement used the renewal meeting strategy to promote its ethos (Jackson 1999, Ch. 8). The model, together with past successful strategies, should encourage churches to pursue similar policies again.

Finally it should be noted that the renewal and reproduction potentials act as limits to the growth of the church. Hayward (2005) showed that dealing with the reproduction potential was more effective than stemming losses as a means of raising this limit to growth. This is due to the nonlinear nature of the word-of-mouth process compared with the linear nature of the losses. Improving renewal should provide a similar effective way of raising the growth limit, as it is also a non-linear process.

5 Conclusion

A model was developed that dealt with the renewal aspect of church growth, where the renewal of existing but inactive believers provides an alternative source of enthusiasts who drive the growth of the church. The model was constructed along similar word-of-mouth lines to the conversion process, but with the inactive believers as the susceptible pool. In addition it was found that the mass action model of contacts was more appropriate for renewal than standard incidence as the network size among church members was limited by church size, at least until the church became a substantial proportion of society.

It was found that renewal was able to enhance the reproduction of enthusiasts, allowing the critical reproduction value to be achieved with insufficient conversion potential for growth. This enabled growth to occur in circumstances where there would have been no revival without renewal. Renewal effectively lowered the epidemiological threshold making revival growth easier to achieve. This was one of five modes of growth in the model and the one that could influence church policies for growth. Those policies centre on encouraging renewal, even at the expense of evangelism, and suggest some regrouping of churches in order to attain critical masses of enthusiasts and church members.

The model omitted factors that affect churches such as organisational constraints and the context of society in which the church is embedded. The model deliberately excluded long-term effects, dealt with elsewhere (Hayward 2005) in order to examine more fully the short-

term generation of enthusiasts which is well documented historically. Also there is, as yet, no dynamic model of the relationship between enthusiasm and the potentials for conversion and renewal. Nevertheless, because word-of-mouth is such a significant fact in church life, it is believed the model as presented gives sufficient insight into short-term growth to encourage policies of renewal to be developed that enhance growth.

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