The Impact of Aggregation Assumptions and Social Network Structure on Diffusion Dynamics

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Abstract

Diffusion problems in general, and innovation diffusion problems in specific, are one of the most frequently revisited issues in system dynamics domain. Although the models used for analyzing specific diffusion problems differ in details, in most cases a set of assumptions is recognized to be common. In this study, we aim to conduct a set of experiments in order to question the validity and potential impact of fundamental assumptions regarding the aggregation and social network structure. First, a generic model focuses on the impact of information dynamics that accompany the diffusion process of an innovation is introduced. The experiments conducted on the aggregate and individual-level versions of the model reveal that the behavior of the system converges to the aggregate model assuming perfect mixing as the network gets denser. Secondly, the change in diffusion levels as a consequence of changing network densities was monotonic. However, direction of change was different for different groups of scenarios tested. In other words, in some cases diffusion level increases as the network gets denser, while in some other cases the opposite is observed.

1. Introduction

Diffusion problems in general, and innovation diffusion problems in specific, are one of the most frequently revisited issues in system dynamics domain. Although the models used for analyzing specific diffusion problems differ in details, in most cases a set of assumptions is recognized to be common. These simplifying assumptions are generally related to the to the aggregation and/or to the structure of interactions among the individuals in the system, i.e. social network.

In most cases, these assumptions are made relying on the previous innovation diffusion modeling studies, and judged to be acceptable intuitively. However, little effort seems to be put on questioning the appropriateness on these assumptions. Questioning the validity of these assumptions, as well as evaluating the potential impact they may have on the model output is crucial in judging the validity of the results obtained by these models. Hence, analytical as well as experimental exploration on the impact of these assumptions is needed. In this study, we aim to conduct such a questioning process using a diffusion model developed by the authors as a basis, and provide some experimental results regarding the validity of these assumptions, as well as the way these assumption may influence the diffusion behavior.

In the following section, we briefly introduce a diffusion model developed to study the impact of information-related mechanisms in conditioning the diffusion dynamics to be observed. The section covers the description of the model, as well as some interesting outcomes from the model. Following that, we will be questioning the degree to which simplest aggregate diffusion models represent the fully-connected populations, which we implicitly assume that they represent. In the two sections following that, we report our experiments about the impact of social network structure on aggregate diffusion dynamics, as well as adoption chances at the individual level. The article concludes with a brief discussion and summary of the findings.

2. Potential of Information Dynamics of Conditioning Diffusion Dynamics[1]

The impact of the prior adopters on the diffusion dynamics constitutes one of the main components in most of the diffusion models. In the widely known Bass model [1], which is the basis for most of the diffusion models in marketing domains, this impact is represented by the *internal influence concept* [2, 3]. There can be various underlying reasons for such an influence that operate at the level of individual adopters. From a social influence point of view [4], it can be attributed to the imitation tendency of the potential adopters that drive their adoption decisions. Or, from a more economics-driven point of view, it can be attributed to the information flowing from adopters to the potential adopters related to the innovation. In the work we will be summarizing below, we focus on internal influence due to information diffusion.

As Rogers discusses in his seminal work on diffusion of innovations [5], awareness regarding an innovation (i.e. a type of information about the innovation) has its own dynamics within the potential adopter population, which significantly conditions/shapes the diffusion dynamics to be observed following the awareness. Using a simple abstract diffusion model (i.e. a model that does not represent a specific empirical case), we explored interaction between the diffusion of information about an innovation, and the diffusion of the innovation itself.

In the model developed for this purpose two groups are represented; adopters (*Ad*) and potential adopters (*PAd*). The social system represented by these two groups is assumed to be perfectly mixed. In other words, individuals are assumed to interact with everybody in the system (i.e. everybody talks to everybody). The individuals in the system are assumed to be boundedly rational [6], in the sense that they decide in a rational manner using the information available to them, which is imperfect. In other words, what is known/perceived by the actors is not necessarily precise, and may differ from the actual information about the innovation. To simplify the model, without losing generality, the only information represented in the model is about a single attribute, which can be also assumed to be the overall utility delivered by the innovation. The level of the attribute is represented with a quantified index value, and higher values are preferrable. Hence, when we refer to information in the remaining parts of the document, we refer to the information regarding the level of this attribute.

The perceived information about the innovation and the actual properties of the innovation are decoupled in the model. In other words, what is known to the potential adopter (Info_{PAd}) and adopter (Info_{Ad}) groups need not be perfect and identical to the actual properties of the innovation (Info_{Act}).

The adoption flow is defined to be dependent on the perceived information by the potential adopters (Info_{PAd}), and a threshold representing their minimal acceptable level for adoption (*Thold*). The model incorporates two dynamic mechanisms that directly influence the state of information about the innovation among the adopters and

[1] Some parts of this section are reproduced from Yücel, G. and C. van Daalen, *Exploring the interdependencies among mechanisms underlying diffusion dynamics*, in *PICMET Conference*. 2009 (forthcoming): Portland, USA.

potential adopters; *word-of-mouth* and *learning-by-experience*[2]. The *word-of-mouth* mechanism imitates the flow of information from the adopter group to the potential adopter group, and is represented by a first order information smoothing structure. The *learning-by-experience* mechanism is about the learning process that adopters go through after the adoption decision. It is assumed that adopters may possess imperfect information about the innovation even at the point of adoption. Hence, they also go through a learning process during which they improve the precision of their information via experience with the innovation as a user. This learning process is also represented as a first order information delay in the model.

The causal-loop diagram that summarizes the relationships and feedback mechanisms in the model are given in Figure 1, and detailed specifications of the model are provided as an appendix.

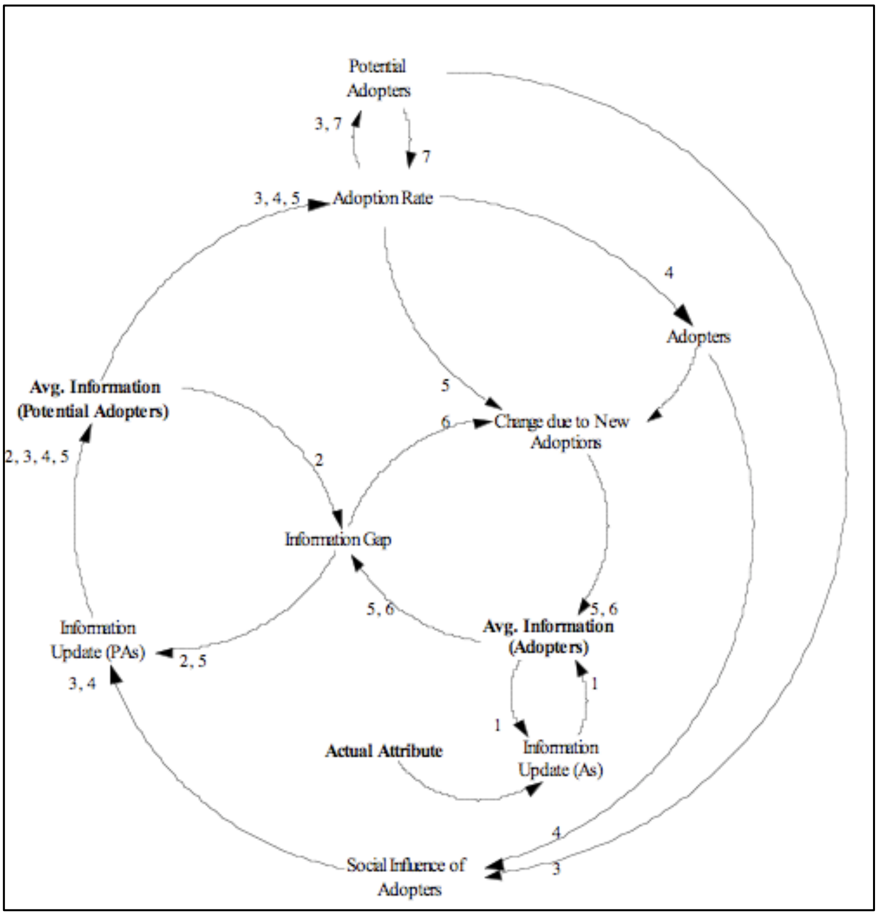

Figure 1. Major relationships in the model

Some important feedback loops we will refer to in the text are introduced below;

<u>*Loop 1 (L1):*</u> Learning-by-experience loop for the adopter group. Works in the direction of reducing the gap between the actual attribute of the artifact and the adopters' information about it.

[2] Although the *learning-by-experience* name resembles *learning-by-doing*, the mechanism discussed here is totally different from the learning-by-doing or learning-curves concepts in technological development and innovation diffusion literature.

Loop 2 (L2): Word-of-mouth loop for the potential adopter group. Works in the direction of reducing the gap between the potential adopters' information about the artifact, and the adopters' information about it.

Loop 3 (L3): Weakening-of-potential adopters loop. Supports the L2 mechanism by altering the Adopter/Potential Adopter ratio. The fewer potential adopters, the higher the ratio of adopters, the more the influence of adopters on potential adopters.

Loop 4 (L4): Empowerment-of-adopters loop. Supports the L2 mechanism by altering the Adopter/Potential Adopter ratio. The more adopters, the higher the ratio of adopters, the more the influence of adopters on potential adopters.

Loop 5 (L5): Dilution of information in the adopter pool. Changes the averages of the adopter group towards the averages of the potential adopters group due to the inflow of newcomers joining the adopter group. Once information on the PA side is favoring adoption, this loop supports the adoption process.

Loop 6 (L6): Market saturation loop. The adoption rate is dependent on the number of existing potential adopters. As potential adopters decrease in number due to former adoptions, the rate of adoption also goes down.

Since the main motivation is to explore the impact of learning mechanisms on diffusion dynamics in general, four key parameters are selected to generate different cases regarding these mechanisms;

- Acceptance threshold (*THold*),
- Actual attribute of the innovation ($Info_{Act}$),
- Information perceived by potential adopters ($Info_{PAd}$), and
- Information perceived by adopters ($Info_{Ad}$).

One of the challenges in working with a generic model is the impact of initial parameter values on the observed dynamics, which hinders the ability to draw general conclusions. In order to minimize this shortcoming, first 24 different scenarios in terms of initial values of the selected four parameters are identified. Although it is possible to initialize these parameters in infinite different ways, there can be only 24 different ordinal scenarios (i.e. ordering of the four parameters of interest). For example, the cases where Thold>$Info_{Act}$>$Info_{Ad}$>$Infor_{Act}$ are all a member of one scenario among these 24 scenarios. After identifying these scenarios, for each scenario numerous instances (i.e. 500) are created randomly and tested. This allowed us to draw, at least, general conclusions related to each scenario, for example when a dominant dynamic behavior is observed almost independent of the initial values in those numerous simulations. Additionally, we were even able to cluster some these scenarios together since we observed that the dominant diffusion dynamic is identical in all of these scenarios, almost independent of the initial values. The results of this extensive experimentation and observations are discussed in detail elsewhere [7]. Under different scenarios, the model generates some interesting diffusion cases, along with the regular S-shaped diffusion to the whole market. In this article, we will only mention some of the interesting dynamics observed, which also constitute the base cases we will be using during the following sections of the paper.

a. Self-deceiving Crowds

We labeled a group of diffusion cases as *self-deceiving crowds* cases due to the way information diffusion mechanisms interact during the diffusion process. Although the underlying explanation of the observed diffusion dynamics is quite similar in these cases, there is considerable variety in the diffusion levels obtained at the end of the simulations. The first scenario in which we can observe this situation is Scenario 7[3] (i.e. $Info_{PAd}$>Thold>$Info_{Ad}$>$Info_{Act}$). We will use this particular scenario in order to elaborate more on what is going on during this type of diffusion.

As can be seen, in this scenario the potential adopters have misleading information about the artifact, and according to what they know their expectation from the innovation is higher than the innovation can deliver. Additionally, in their initial situation the willingness to adopt is non-zero (i.e. $Info_{PAd}$>Thold). The adopter group also has imprecise information about the artifact, but at least this group is aware of the fact that the innovation does not meet expectations (i.e. Thold >$Info_{Ad}$). As seen in Figure 2, the extent of the diffusion may vary significantly; so how come a product that is not able to deliver minimal expectations of users can achieve high diffusion levels? The dynamics of adoption to be observed in this case will be the outcome of interplay between the learning processes and the pace of adoption. While the learning processes takes the state of information of the adopters (*L1*) and potential adopters (*L2*) closer to the real information, new individuals join the adopter group with their deceived information over-valuing the utility of the innovation (i.e. more new adopters who think the artifact is good enough to be adopted) (*L5*).

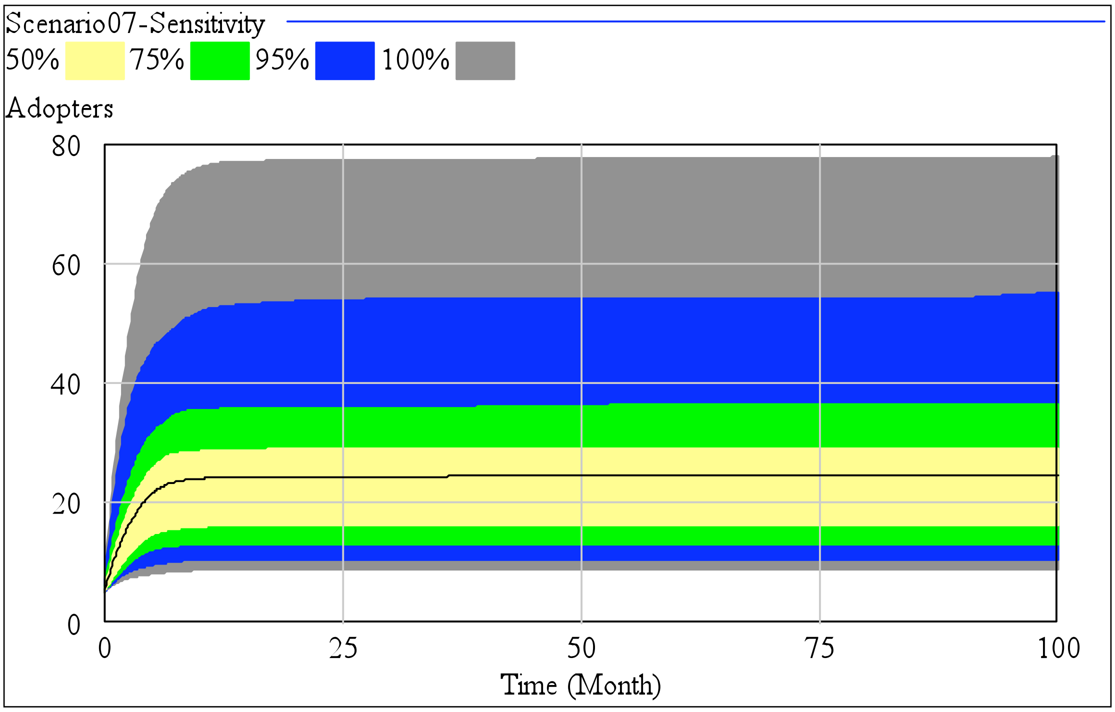

Figure 2. Sensitivity runs for Scenario 07 (behavior envelopes)

Assume that the adoption rate is fast, and the time it takes for adopters to learn about the actual performance of the innovation is long. In such a case, the adopter group will

[3] Not all instances of scenario 7 yield this type of situation. Only in a subset of all instances we observed this type of diffusion.

be dominated by the new adopters having the misleading information about the innovation. Since the time to learn for the adopters is long, the imprecision in the information won't be corrected soon, and the adopter group will be broadcasting a positive, but deceiving, message to the potential adopters about the innovation[4]. Basically, it is the potential adopters who recently made an adoption decision who deceive the rest of the potential adopters regarding the innovation. Hence, the label is "self-deceiving crowds". Although older adopters will have more precise information about the innovation, due to the large number of new adopters with misleading information their effectiveness will decline in influencing the potential adopter. This can be seen as the dilution of experience-driven precise information due to massive adoption movements. Consequently, depending on the relative strength of the learning mechanism (L1 and L2) compared to the adoption-related mechanism (L5), significant differences in the overall diffusion levels are observed.

b. Misguiding Front-runners

As in the previous case, this is also a case where an inferior artifact (i.e. an artifact having an attribute level below the acceptance threshold of the potential adopters) may reach significant diffusion levels. The difference is that this time the adoption process is mainly driven by the initial adopters who have an artificially high expectation about the innovation. As in the former case, it is all about the relative speed of learning and adoption mechanisms.

We will demonstrate this case on scenario 9 (i.e. $Info_{Ad}>Thold>Info_{PAd}>Info_{Act}$), as an example. In this case, the actual attribute of the artifact is below the acceptance threshold of the potential adopters. Additionally, the information that potential adopters have about the innovation is also below the threshold. Thus, if the potential adopter group is isolated and left alone no adoption should be expected. However, the individuals initially defined as adopters (e.g. front-runners) are assumed to have an artificially high positive perception about the innovation. An intuitive expectation would be that adopters would broadcast a deceptive positive message about the innovation during the first phase. However, this will gradually turn into a negative message as they start learning about the innovation as a result of their accumulated experience.

We expect an initial wave of adoptions due to this wrong information from adopters, and termination of the adoption process after a while. However, we can see that the scale of adoption may show significant variations in different cases by conducting a sensitivity analysis. The summary of 500 runs is provided in Figure 3.

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[4] There are some key implicit assumptions that are important in this case. The first of these is the perfect mixing assumption. We assume that every individual is in contact with every other member in the population. Secondly, we assume that potential adopters weight the information coming from different individuals as equal in terms of trustworthiness; thus they do not weight information from experienced adopters more than information from new adopters.

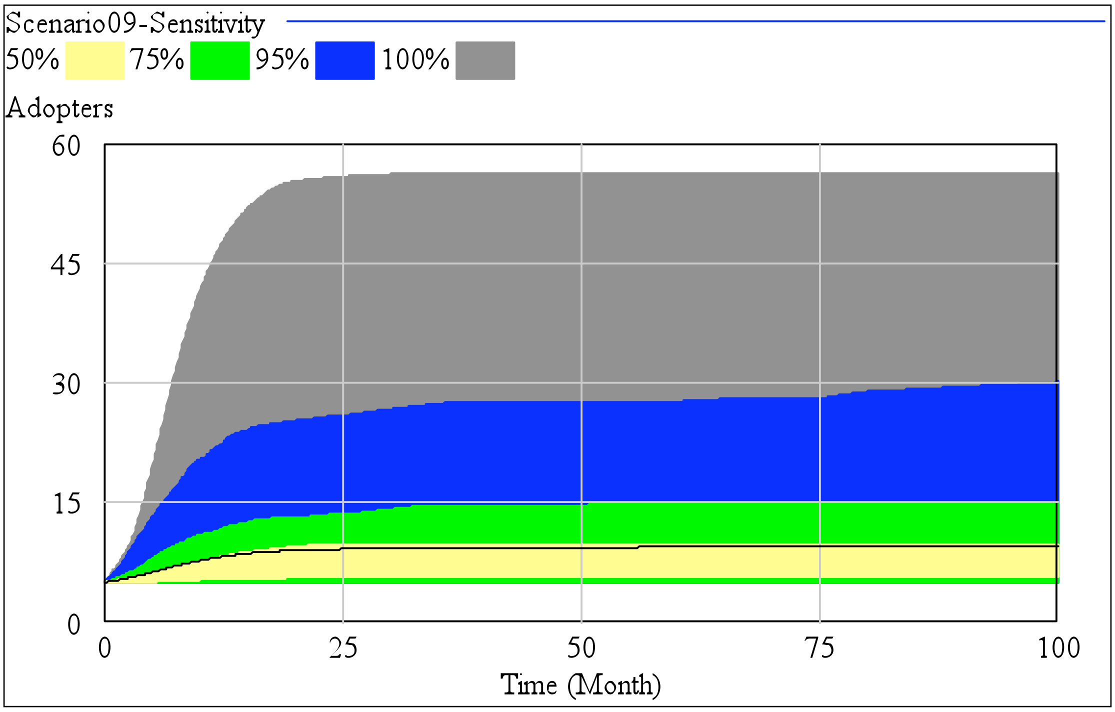

Figure 3. Sensitivity runs for Scenario 09 (behavior envelopes)

The variation in the outcome is a result of the interplay of a couple of mechanisms, as in the former case. In this case, the competing mechanisms are the two learning mechanisms; *learning-by-experience (L1)* and *word-of-mouth (L2)*. So it is all about whether deceptive information of the adopters diffuses to the potential adopters before adopters achieve more precise information via experience, or not.

The following instance demonstrates an extreme case of diffusion in such a scenario. In this case, the information diffusion from adopters to the potential adopters takes place much faster than the *learning-by-experience* process. This may be a case where extensive experience is required to reveal the actual attribute level of the artifact. The diffusion dynamics obtained in this case are presented in Figure 4. Since information diffusion is so fast, the information of the potential adopters quickly converge to the information level of the adopters (see Figure 5). After that point, until the perceptions of both groups converge to the actual level (and hence drop below the threshold level) due to the learning processes, adoption continues. In this case, this happens only after almost a full adoption is realized.

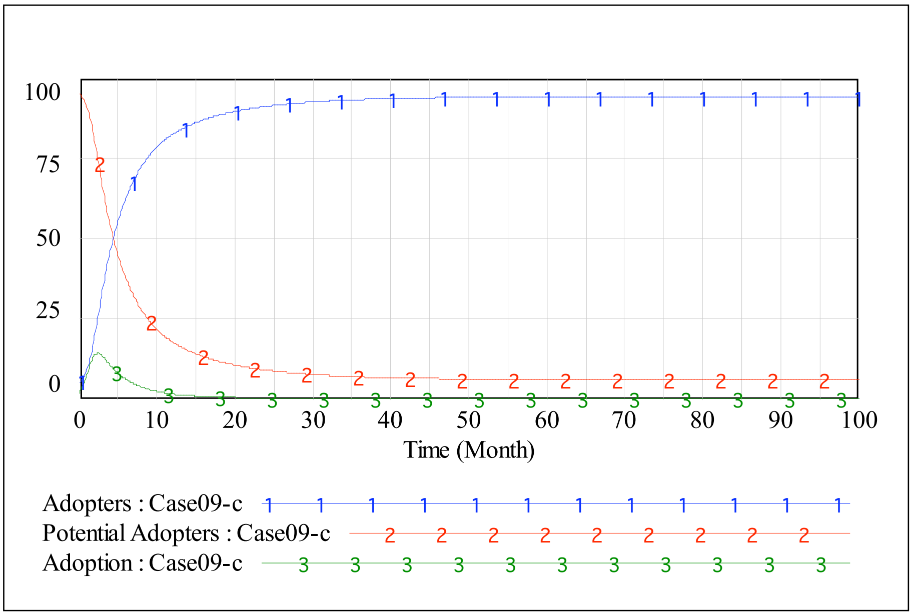

Figure 4. Full-adoption case in scenario 9

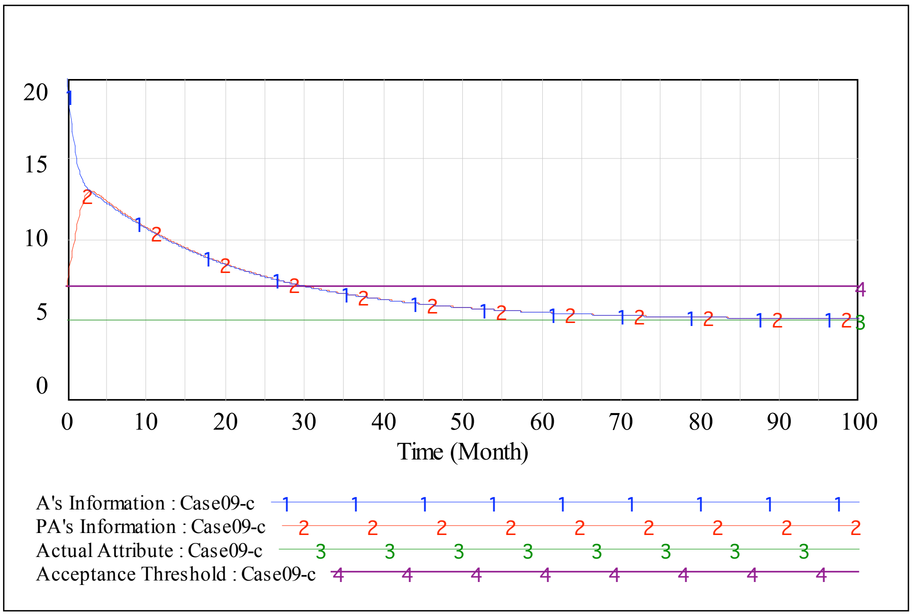

Figure 5. Information dynamics in full-adoption case in scenario 9

The model and experiments introduced briefly in this section, which are presented in detail elsewhere [7], demonstrates the potential of information flow dynamics within the adopter population in influencing the diffusion dynamics to a great extent. Since the information dynamics play such a remarkable role in innovation dynamics, it seems crucial to question related structural assumptions and simplifications made in such models. The following sections will focus on this matter, and explore the impact of assumptions especially made for aggregate representation of the system regarding the social communication network and/or actor heterogeneity on the dynamics observed with diffusion models.

3. Impact of Major Aggregation Assumptions

One of the fundamental assumptions, implicitly or explicitly made in aggregate diffusion models similar to the one presented in the previous section is the *perfect mixing assumption.* According to this assumption a potential adopter is assumed to be in contact with everybody in the system, hence learns from all adopters in the same time. Another major assumption is about the adoption fraction variable. It is assumed that a system, which is composed of individuals having probability p of adoption can be represented by initializing the adoption fraction parameter equal to p in the aggregate representation. Shortly, such a model (i.e. M) is assumed to represent a system perfectly mixed and composed of individuals who have a probability p of adopting the innovation (i.e. S). Equivalence of M to S may be demonstrated analytically, but as an intermediate step of this study we tested this claimed equivalence empirically.

In order to do so, an individual-based disaggregated model of the same system is constructed. Although the model includes the identical mechanisms as the former one, their implementation differs slightly since they have to be defined for the individual level.

a. *Word-of-Mouth*: Each potential adopter checks the average of the adopters' information about the innovation. Based on this average, the potential adopter updates its own perception. The information delay is identical with the aggregate version.
b. *Learning-by-Experience*: Identical to the way it is implemented in the aggregate version. Each adopted individual updates its perception about the artifact based on the gap between its perception and the actual information.
c. *Adoption decision:* This seems to be most significant difference between the two models. In the aggregate model, there was a variable representing the fraction of potential adopter group that will adopt the artifact. The same variable is used in the individual-based model. Its value is calculated for each individual actor. Then it is used as the likelihood/probability of artifact adoption. For each individual a random number between 0 and 1 is generated, and if this number is less than or equal to the probability of adoption of a certain individual, that individual adopts the innovation.

In order to make a comparison between the dynamics of the individual-based and the aggregate model, we used the scenarios discussed previously, namely scenario 7 and 9. According to the former experiments based on the aggregate model, scenario 7 is a case where *self-deceiving crowds* behavior is observed, and scenario 9 is the one in which *misguided crowds* behavior is observed.

Since the cases only specify the ordering of the important parameters, but not their actual values, it is possible to create multiple instances of parameter values that are consistent with the case characteristics. Hence, we have created 50 different instances for each of these cases. These 50 cases are simulated on each model and the results are compared in order to check if our claim of equivalence of these two representations (i.e. aggregate and individual-based) is valid. What is different from the aggregate model is that the individual-based model has a probabilistic nature due to the way adoption decision mechanism at the individual level is implemented. Due to this, it will be misleading to compare an individual run of the individual-based model with the aggregate model. Depending on the random number series generated during a run, it is possible to obtain slightly different output from the individual-based model. In order to minimize the impact of this probabilistic process, for each instance individual-based model is run for 50 replications, using a different random number generator in each of these replications. Then we averaged out the adoption percentage at the steady state of the simulations. This average is compared with the adoption percentage obtained from the aggregate model for that case.

The parameter values used in the 50 instances of case 7 are given in the Appendices. The following table summarizes the comparison of output from these two models. The first three columns are about the individual-based model. It gives the average of the number of adopters at the end of the run in 50 replications of the same instance. The standard deviation of this indicator in the 50 replications is also provided in the second column. The number of adopters obtained by the aggregates model, which is deterministic in nature, is given in the fourth column. The final column gives the ratio of the difference between the average number of adopters obtained with two models, divided by the standard deviation calculated for the individual-based model output. As can be seen, most of the deviation is in the range of approximately 2 standard deviations, which allows us to conclude that statistically the output of the two models are not different[5].

	Individual-based Model			Aggregate Model	Difference/StdDev
	Avg. No of Adopter	Std Dev	StdDev/Avg	No. of Adopters	
1	183.50	8.68	0.05	178.75	-0.55
2	286.20	9.09	0.03	281.00	-0.57
3	369.74	11.37	0.03	364.41	-0.47
4	161.82	8.65	0.05	156.86	-0.57
5	291.98	13.30	0.05	267.55	-1.84
6	185.14	9.56	0.05	180.95	-0.44
7	319.14	13.82	0.04	306.48	-0.92
8	195.92	7.03	0.04	187.03	-1.26
9	203.82	8.65	0.04	191.95	-1.37
10	405.14	15.07	0.04	394.50	-0.71
11	240.78	10.66	0.04	231.83	-0.84
12	254.28	10.74	0.04	244.04	-0.95
13	242.90	9.44	0.04	238.88	-0.43
14	178.98	9.36	0.05	169.21	-1.04
15	164.46	6.80	0.04	165.88	0.21
16	255.92	12.82	0.05	241.77	-1.10
17	340.06	11.34	0.03	330.75	-0.82
18	266.38	12.27	0.05	243.63	-1.85
19	186.46	9.44	0.05	177.05	-1.00
20	727.76	16.24	0.02	740.87	0.81
21	285.60	11.87	0.04	279.22	-0.54
22	382.16	11.16	0.03	374.86	-0.65
23	336.02	8.06	0.02	316.61	-2.41
24	271.14	9.83	0.04	259.81	-1.15
25	261.04	9.31	0.04	254.29	-0.72

[5] A systematic bias is recognized in the deviations Although the deviation between the output of the models is not significant, in almost of the instances, the number of adopters in the aggregate model is less than the number of adopters in the individual-based model. The cause of the bias is under inspection, and the manuscript will be revised in the light of findings in the future.

26	273.54	11.51	0.04	253.02	-1.78
27	256.08	9.89	0.04	246.25	-0.99
28	258.90	11.85	0.05	242.31	-1.40
29	318.98	12.07	0.04	303.62	-1.27
30	263.82	9.54	0.04	246.56	-1.81
31	212.60	9.92	0.05	209.25	-0.34
32	332.50	13.74	0.04	325.15	-0.53
33	248.86	11.53	0.05	226.58	-1.93
34	310.54	10.00	0.03	304.91	-0.56
35	328.12	10.71	0.03	321.72	-0.60
36	318.18	12.67	0.04	314.63	-0.28
37	304.50	12.74	0.04	283.23	-1.67
38	193.38	7.08	0.04	179.58	-1.95
39	233.10	10.17	0.04	216.09	-1.67
40	374.04	11.08	0.03	364.53	-0.86
41	178.30	7.20	0.04	169.87	-1.17
42	331.28	11.07	0.03	310.02	-1.92
43	302.40	13.41	0.04	293.65	-0.65
44	317.70	12.33	0.04	295.37	-1.81
45	246.36	9.92	0.04	231.59	-1.49
46	320.42	12.15	0.04	302.13	-1.51
47	269.34	12.09	0.04	257.60	-0.97
48	187.10	8.55	0.05	175.30	-1.38
49	283.80	12.83	0.05	275.26	-0.67
50	220.90	10.70	0.05	205.74	-1.42

The final number of adopters obtained by these two models in different 50 instances are plotted in the following figure.

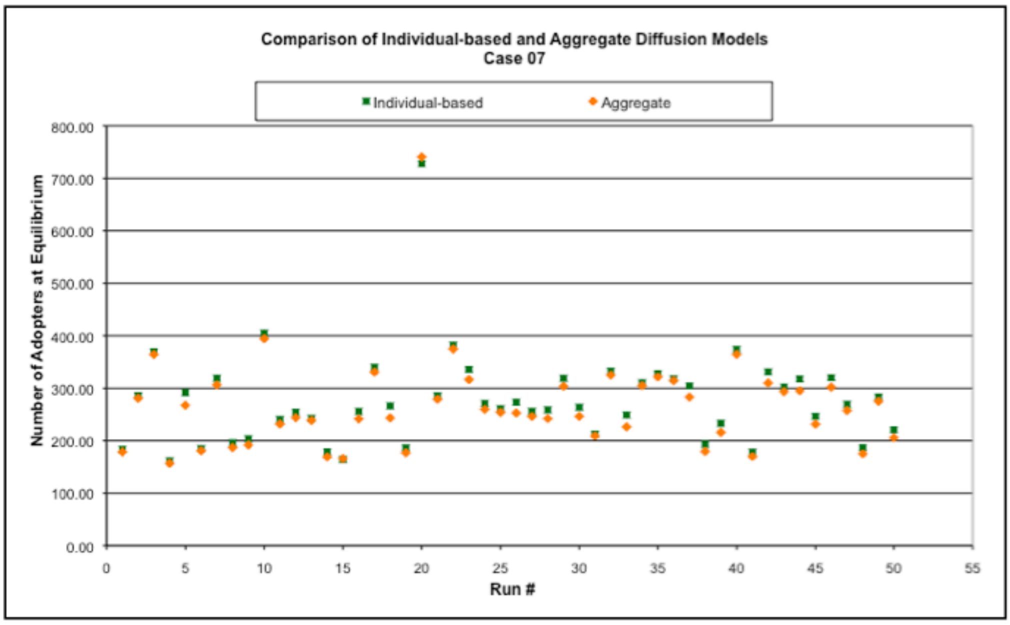

Figure 6, Comparison of results obtained by individual-based and aggregate models (Scenario 7)

As mentioned earlier, the same procedure is repeated for case 9. The tables and graphs summarizing the comparison are given below. As in the previous case, the deviation is evaluated to be insignificant to conclude against equivalence of these two models.

	Individual-based Model			Aggregate Model	Difference/StdDev
	Avg. No of Adopter	Std Dev	StdDev/Avg	No. of Adopters	
1	149.22	6.51	0.04	128.90	-3.12
2	100.00	0.00	0.00	100.00	-
3	176.84	8.41	0.05	148.25	-3.40
4	209.76	10.06	0.05	175.62	-3.39
5	875.74	9.95	0.01	981.19	10.60
6	248.56	10.92	0.04	221.87	-2.44
7	144.86	6.76	0.05	115.37	-4.36
8	100.00	0.00	0.00	100.00	-
9	233.54	9.26	0.04	202.00	-3.40
10	100.00	0.00	0.00	100.00	-
11	268.88	12.13	0.05	248.59	-1.67
12	100.00	0.00	0.00	100.00	-
13	100.00	0.00	0.00	100.00	-
14	225.68	9.01	0.04	203.81	-2.43
15	100.00	0.00	0.00	100.00	-
16	226.74	11.04	0.05	207.09	-1.78
17	233.36	10.26	0.04	215.15	-1.78
18	126.36	5.18	0.04	100.64	-4.97
19	253.68	10.08	0.04	234.13	-1.94
20	100.00	0.00	0.00	100.00	-
21	154.94	6.57	0.04	128.68	-4.00
22	100.00	0.00	0.00	100.00	-
23	165.26	5.47	0.03	127.78	-6.86
24	100.00	0.00	0.00	100.00	-
25	100.00	0.00	0.00	100.00	-
26	100.00	0.00	0.00	100.00	-
27	759.16	12.08	0.02	699.47	-4.94
28	100.00	0.00	0.00	100.00	-
29	285.32	10.13	0.04	238.53	-4.62
30	178.80	8.15	0.05	146.12	-4.01
31	100.00	0.00	0.00	100.00	-
32	100.00	0.00	0.00	100.00	-
33	134.86	5.57	0.04	100.11	-6.24
34	100.00	0.00	0.00	100.00	-
35	219.32	11.25	0.05	185.12	-3.04
36	127.04	5.68	0.04	100.00	-4.76
37	203.68	10.64	0.05	166.97	-3.45
38	100.00	0.00	0.00	100.00	-
39	884.16	9.82	0.01	953.81	7.09
40	302.30	11.45	0.04	274.03	-2.47
41	100.00	0.00	0.00	100.00	-
42	100.00	0.00	0.00	100.00	-
43	236.02	7.95	0.03	213.85	-2.79
44	218.90	9.13	0.04	182.82	-3.95
45	126.64	5.19	0.04	99.82	-5.17
46	287.42	8.83	0.03	247.47	-4.53
47	157.20	6.30	0.04	126.13	-4.94
48	248.22	11.41	0.05	230.94	-1.51
49	100.00	0.00	0.00	100.00	-
50	100.00	0.00	0.00	100.00	-

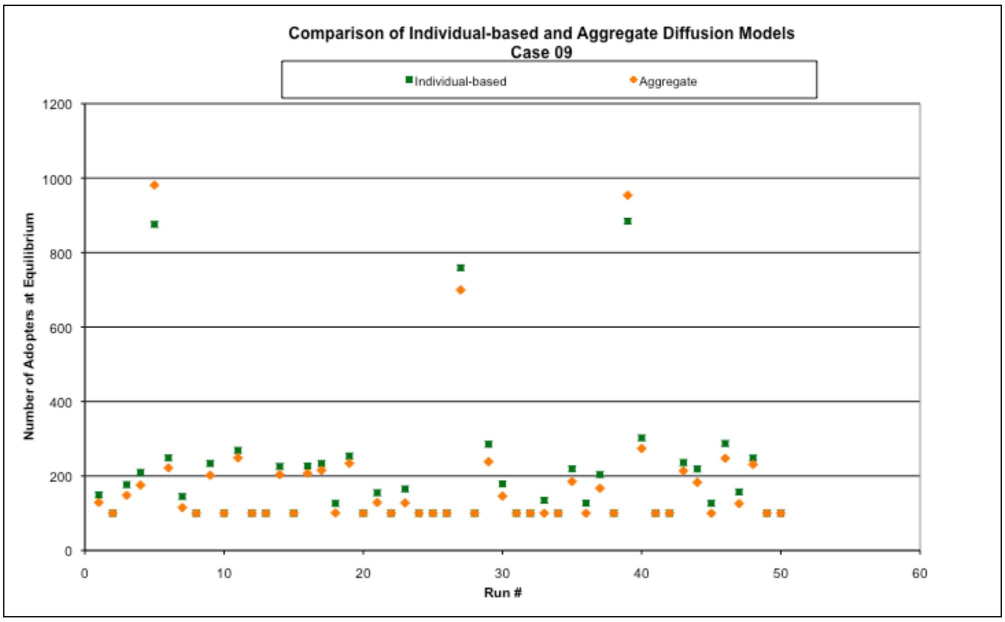

Figure 7. Comparison of results obtained by individual-based and aggregate models (Scenario 9)

Consequently, as a result of 100 simulations with different initial parameter values, we observed that empirical findings also support the claim of equivalence of the individual-based and aggregate models. In other words, the two-stock adoption model with an aggregate *adoption fraction* variable imitates the behavior of a *perfectly mixed* system with individuals having a certain probability of adopting the innovation. These experiments provide some extra empirical support about whether one of the most common stock-flow representations used in diffusion problems, as our aggregate model (see Figure 8), really imitates the behavior of a perfectly mixed system, as it is implicitly assumed.

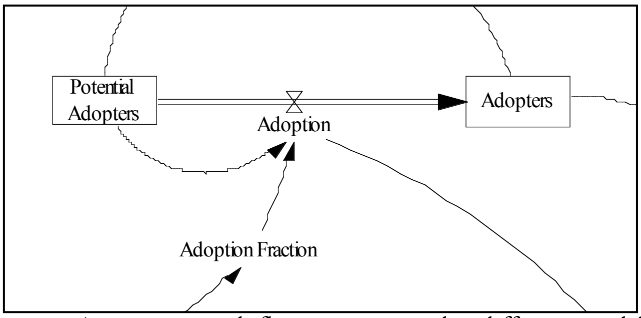

Figure 8. A common stock-flow structure used in diffusion models

As previously mentioned, we aim to explore the potential impact of fundamental assumptions used in this type of models on the diffusion dynamics. Extending the individual-based model discussed in this section, we are able to formulate the same diffusion problem in a social system where contact between individuals is constrained by the social network structure. Using that extended version, we will explore the impact of network structure on aggregate diffusion dynamics, and what we are loosing by making the perfect mixing assumption about a social system.

4. Impact of Network Structure on Aggregate Dynamics

In this part of the study, we studied the impact of network structure on the diffusion dynamics, and the difference caused by this structure compared to the case of perfect mixing, represented by aggregate models. Rahmandad and Sterman [8] conducted an extensive study on this same issue, where they explore the impact of different network structures as *fully-connected*, *random*, *small-world* and *scale-free*, and whether these altered dynamics can be represented by the aggregate model by some recalibration. As an addition to their extensive discussion, we explore the impact of network density (i.e. number of links in the network, or average number of connections per individual) on the aggregate dynamics, and also whether network models' behavior converge to the aggregate models' beyond certain network density levels.

For that purpose, a set of individual-based models with differing network structures are constructed with NetLogo©. The difference between these models is the number of average links per individual (i.e. the number of social neighbors with which an individual communicates directly). With this set of models, we relaxed the perfect mixing assumption. This implies that for a potential adopter, the set of adopters acting as a source of *word-of-mouth* are only the ones directly connected to this potential adopter. In other words, the *word-of-mouth* mechanism works only in the local social neighborhood of the individuals, not on the global scale. An example of the way the social network structure is represented in these individual-based models is given in Figure 9. While white circles represent potential adopters, red ones represent already adopted individuals. The lines connecting the circles indicate the social neighbors of an individual with whom it may communicate.

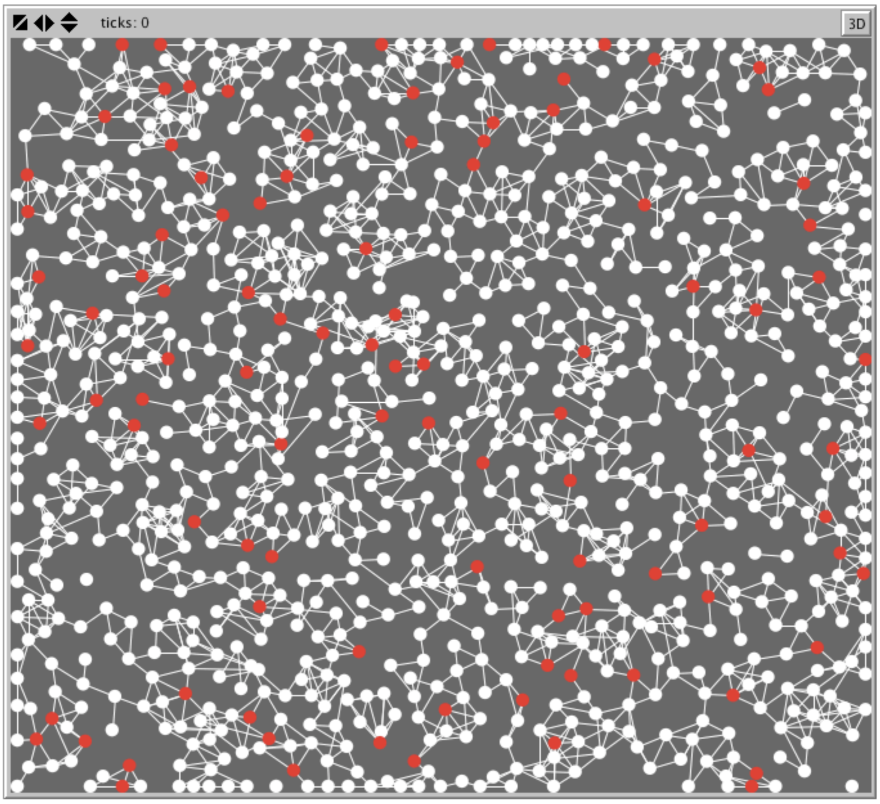

Figure 9. Social network visualization in the NetLogo model (*white:* potential adopter, *red:* adopter)

The experimental procedure we followed can be summarized by the following main points;

- 10 different individual-based models with different network densities (i.e. average link per actor ranging from 1 to 10) are created
- 50 instances already created for the scenario 9 in the previous analysis simulated on these 10 different individual-based models
- For each instance, 50 replications with different random number sequences are performed (i.e. 50 replications for 50 instances, 2500 simulations in total).
- The levels of adoption at the steady state over 50 replications are averaged to find an average adoption level for a particular instance.
- For each instance, the average levels of adoption obtained with different models are compared to see if there is a systematic relationship between the adoption levels and the network density.

The outcome of these experiments is summarized in Figure 10. In the figure each line represents one of the 50 instances of scenario 9. Each instance is simulated on 10 network structures with differing average connection per individual levels. The diffusion levels at the steady state for each network intensity is recorded, and the line for a particular instance is drawn by joining these points[6]. In short, the lines represent the change in the average final adoption levels over different network densities.

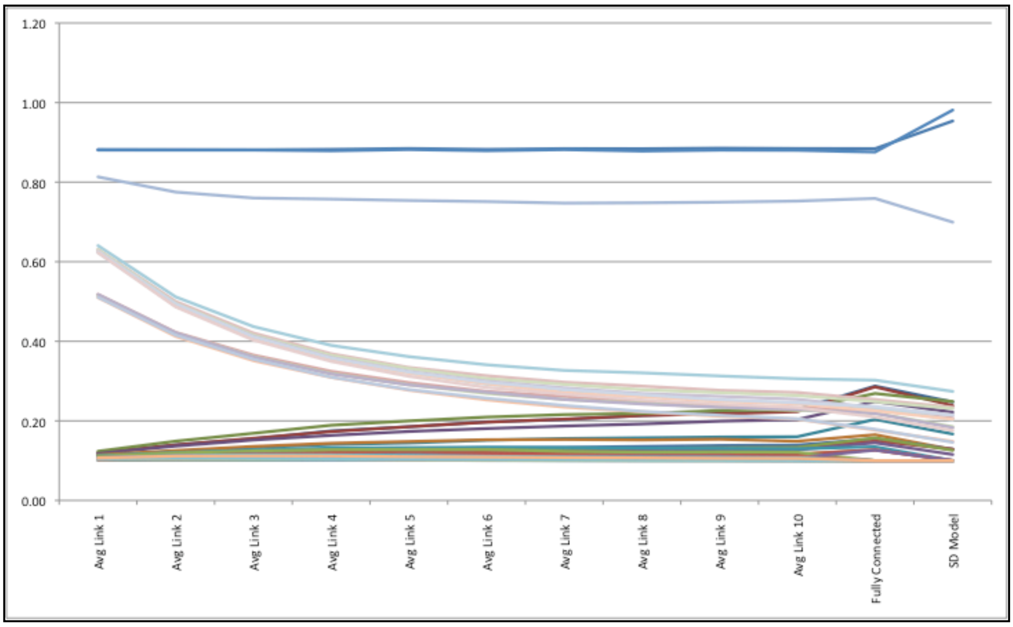

Figure 10. Adoption levels (in % of total market) in steady state behavior of models with differing network densities

Prior to the experiments, we had an intuitive expectations regarding outcome. Independent of the model instance (i.e. initial parameter values used), the behavior of the individual-based model is expected to converge to the behavior of the aggregate

[6] Using continuous lines for each instance may initiate a misleading impression. Those curves are constructed by joining 10 individual points. In other words, the adoption level vs. network density observations we have are discrete in nature rather than continuous. However, representing the results in a discrete way by individual points without connecting them made the information on the graph very hard to follow. We chose this representation for the sake of clarity.

model as network density increases. Results confirmed our expectation in general, though surprising us about another expectation to be discussed later.

In almost all of the instances, we can observe a convergent trend in the adoption levels as network intensity increases (independent on whether adoption level increases or decreases as network density increases). Based on the model used in this experimentation, in can be concluded that the models that have network densities above 10 will have almost identical dynamics to each other, and also to the aggregate model representing the perfect-mixing case. From a practical point of view, the increased network density has significant influence on the performance of the simulation in terms of computational time. The observation made above provides a well-grounded assumption valuable in that sense. Based on this observation, it may be claimed that in cases where social network is highly dense (e.g. above 10 connections per individual), a model with density of 10 connections per individual, or even a simple aggregate model can be used with a reasonable level of precision, and the extra computational burden caused by network representation can be reduced or eliminated. However, this claim has to be tested on other models in which the main dynamics is about the diffusion of information/material among connected members of the social network prior to generalization[7].

Another expectation of the authors was that the relation between the network density and the adoption levels would be a unidirectional one (i.e. adoption level changes in the same direction as a function of network density in all instances). However, experiment results showed otherwise. Two distinct clusters of instances can be seen in Figure 10. In the first group, the adoption level increases as the social network becomes denser. For the second group, the opposite holds. This is interesting in the sense that the relationship between the network density and the adoption levels is not a straightforward and unidirectional one. Hence, further investigation is needed to understand the underlying factors of this difference.

In order to understand what is going on, it is crucial to comprehend the implication of increasing network density (i.e. increasing average node degree) on the mechanisms influencing the adoption process (e.g. *learning-by-experience*, *word-of-mouth*, etc.). Consider an individual in the network. Increasing the number of social connections of this node (i.e. increasing the degree of the node) changes the probability of having at least one adopter in the social network of this selected individual. The probability for a selected individual increases as its node degree increases.

For example, consider a node with degree n

Pr(*Having at least 1 adopted neighbor*) = 1 – Pr(*Having all neighbors potential adopters*)

which is equal to;

Pr(*Having at least 1 adopted neighbor*) = $1 - r^n$

---

7 It is worthwhile to mention an important issue visible on the figures. An abnormality is observed during the experiments, which is about the difference between the fully-connected model and the other network models. In theory, we expect the adoption levels to converge to the fully-connected level as network density increases. Although there seems to be a convergence, the convergence level is slightly lower than the level obtained with the fully-connected model. This point needs further investigation.

where r is the initial ratio of potential adopters in the whole network (<1 by definition). Hence as n (i.e. degree of the node) increases, probability of having at least 1 adopted neighbor increases.

As discussed earlier about the output of the aggregate model, the mechanism responsible for the dynamics observed in scenario 9 is the competition among *word-of-mouth* and *learning-by-experience*. Their relative strength determines the final adoption levels to be observed. Based on the simple probabilistic reasoning given above, increasing the average number of connections reduces the probability of having individuals without any adopters in their neighborhood. For these individuals, the *word-of-mouth* mechanism is not active due to lack of adopters connected to them. Following this reasoning, Hence, if we can increase the chance of having at least one adopted neighbor for each individual in the network, it implies that word-of-mouth mechanism will be active for more nodes, and this means that at the aggregate level the mechanism will be *stronger* or *more active*. This is one of the key points in understating the surprising outcome.

For the second important point, we have to go over how word-of-mouth mechanism works in this model. Contrary to the way word-of-mouth effect is formulated in most of the diffusion models (i.e. assumed to be favoring diffusion), the word-of-mouth mechanism has a dynamic character in this model; it may slow down an adoption process, as well as speeding it up. When the information of the adopters ($Info_{Ad}$) is higher than the information known by potential adopters ($Info_{PAd}$), *word-of-mouth* supports the adoption process by yielding an increase in the information of the potential adopters. In cases where adopters' information is lower than the potential adopters', then it works in the opposite way. In the scenario 9, we observe both of these characteristics of the word-of-mouth mechanism in a single run; it first reinforces adoption by promoting the innovation, and later counteracts it. By definition of the scenario, initially the information of the adopters is higher than the potential adopters. Hence, first they broadcast a positive message, supporting the adoption (i.e. *positive word-of-mouth*). Also by definition, adopters' information is higher than the actual utility of the artifact, so by experience they learn about this and their information gets lower continuously throughout the simulation, and converges to the actual level. At a certain point, the information known by both groups become equal. After that point on, adopters start broadcasting a *negative word-of-mouth*, slowing down the adoption. Their information is always lower than the potential adopters' and due to this they pull down the average information level of the potential adopters.

Combining these two key points, it can be said that having a stronger word-of-mouth mechanism may be both good and bad in terms of adoption levels. It is good in the sense that it speeds up the adoption more during the *positive word-of-mouth* period, and bad in the sense that it slows down adoption during the *negative word-of-mouth* period. At the end, the relative durations of these two phases is crucial regarding the final adoption levels. In other words, for a given case, if the impact of changing the intensity of the *word-of-mouth* mechanism is greater on the *positive word-of-mouth phase*, then we may expect an increase in the final adoption levels. However, if the impact is greater on the *negative word-of-mouth phase*, the same alteration may yield a decrease in the adoption levels.

The explanation will become clear by visually presenting it. We used the aggregate model for the sake of simplicity, without losing generality. Figure 11 and Figure 12 correspond to two specific instances of scenario 9, namely number 4 and 46. Based on our previous observation, the adoption level decreases as network density increases in instance 4. In instance 46, the opposite holds. Since the information possessed by potential adopters is the main driver of adoption behavior, we analyze the change in this information as a consequence of changing the intensity of the word-of-mouth effect. For each, case we conducted 5 runs, by changing the intensity of the word-of-mouth (i.e. changing the delay constant)[8]. In these figures, the *positive word-of-mouth* phase corresponds to the period until the peak point of the information curves, and the rest is the negative phase.

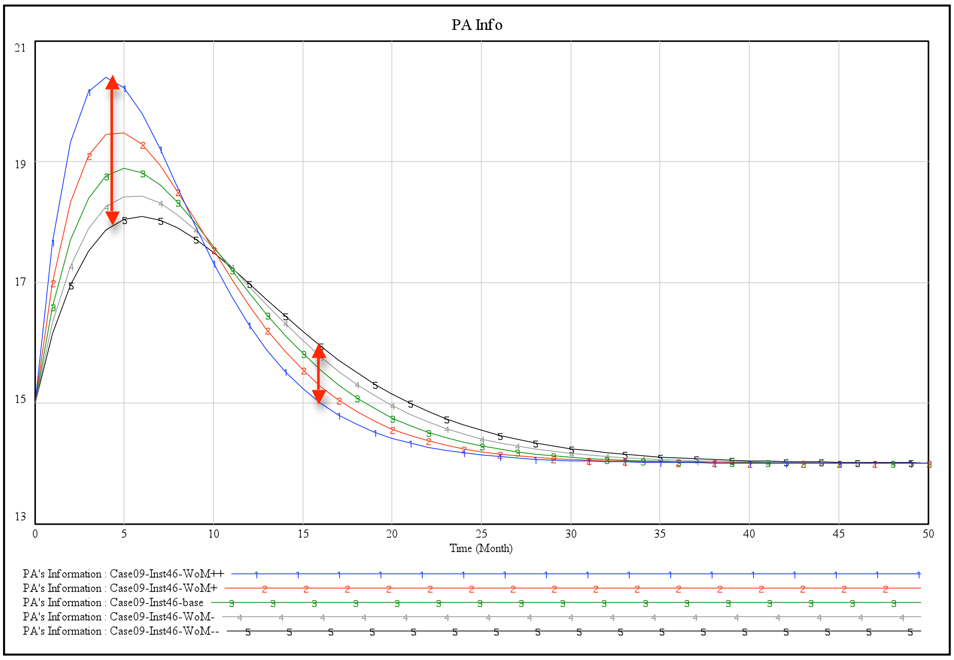

Figure 11. Information dynamics for potential adopters in scenario 9, instance 46

[8] In the figures, (+) sign is used to indicate a case where word-of-mouth mechanism is more intense. (++) indicates stronger. Opposite applies to cases where (-) sign is used.

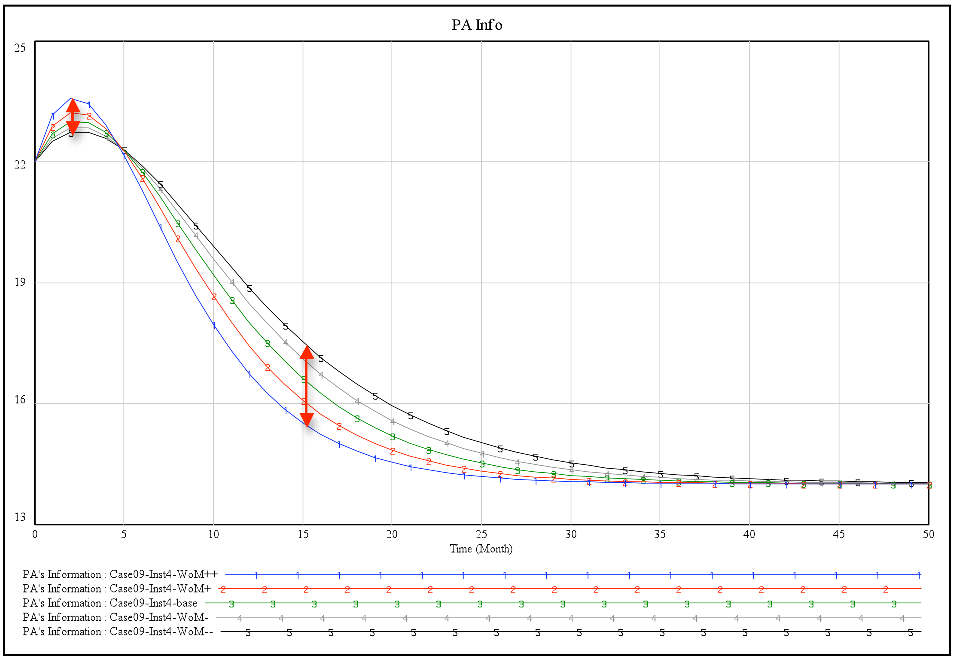

Figure 12. Information dynamics for potential adopters in scenario 9, instance 4

The impact of network intensity during the first phase can be traced through the change in the peak information levels achieved. The higher you can take the perceived information of the potential adopters, the more additional adoption you may expect (e.g. compare WoM++ and WoM-- runs). On the other hand, during the negative phase, the impact is dependent on the pace with which information levels go down; if it goes below threshold level sooner, it means the adoption process stops sooner.

In the light of this explanation, let us compare the two cases. As can be seen, when the word-of-mouth process gets faster (or more intense), the change in the positive phase of case 46 is significant compared to case 4. The opposite can be said for the negative phase. As a result, for case 46 we can say that marginal impact of increasing network intensity on positive word-of-mouth phase is stronger than the impact on negative phase. Hence, as network intensity increases, we observe increasing adoption rates in case 46. The opposite reasoning holds for case 4.

This observation reveals one important conclusion; the density of the network may work in both ways in terms of supporting a diffusion process; it may reinforce or counteract. Based on this, it is seen that overall impact of the network structure on the diffusion dynamics, among other factors, will depend on the initial situation in terms of the gap between what is known by the potential adopters and adopters. Hence it is not possible to make a case-independent generalization about this impact. An extension of this conclusion is related to using aggregate levels. It will be misleading to conclude that explicitly representing network structure always causes a change in the results obtained in the same direction. The change may be in both directions depending on the specific instance being represented by the models.

5. Impact of Network Structure on Individual-level Dynamics

In the previous section, we investigated the impact of social network structure on the diffusion dynamics at the aggregate level. Having the individual-level models, it is also possible to investigate a similar issue from individuals' point of view. What kind of insight can we expect from such an investigation?

During the previous sections of this paper, we conducted some experiments based on a scenario, which results in a poor innovation being adopted by a significant number of potential adopter by misperceived expectations. To be more specific, scenario 9 is a setting in which if adoption takes place, then that is a misguided adoption since the actual utility of the artifact is less than the acceptance threshold of the actors in the system. Departing from this point, we also explored the impact of the network connectivity of an actor (i.e. how many neighbors it has) on the probability of adoption of that actor. In other words, if we assume that adopting this innovation is a wrong choice made due to poor information, we aim to study the vulnerability of individuals differing based on the number of social connections they have in the social system. This is the insight we aim to develop based on this investigation.

In order to study this we conducted the following experiment;

- Grouped the nodes in terms of number of links they have in the network.
- Monitored the percentage of nodes from each group who adopted the artifact during the simulation (the nodes that are initially defined as adopted are excluded)
- For each of the 50 instances of scenario 9, plotted the adoption fractions observed in each node group (e.g. adoption fraction for nodes having 5 links)
- The lines draw by connecting these points give the change in adoption fraction as a function of number of links for an individual, in a particular case
- These plots are produced with all of the 10 models used in the previous analysis (i.e. models with differing densities)

The results reveal a clustering very similar to the one we have observed in the previous section; in some instances individuals with a higher number of neighbors were more vulnerable to misguided adoption, whereas in some other cases the opposite was true. The instances in each cluster were also the same as the clusters in the former analysis. Hence, it indicates that a similar reasoning for also holds for the relationship of number of links (i.e. node degree) and adoption vulnerability. As an example, observations related to two cases of scenario 9 are plotted in Figure 13 and Figure 14. Figure 13 is obtained by using the model, which has 4 links per individual, on average. The average number of links in the model used in generating the data in the Figure 14 is 5. As can be see, in case 4 individuals have a higher probability of adopting the poor innovation if the number of their social links is less. On the other hand, the opposite holds in case 46. This can be observed in both plots.

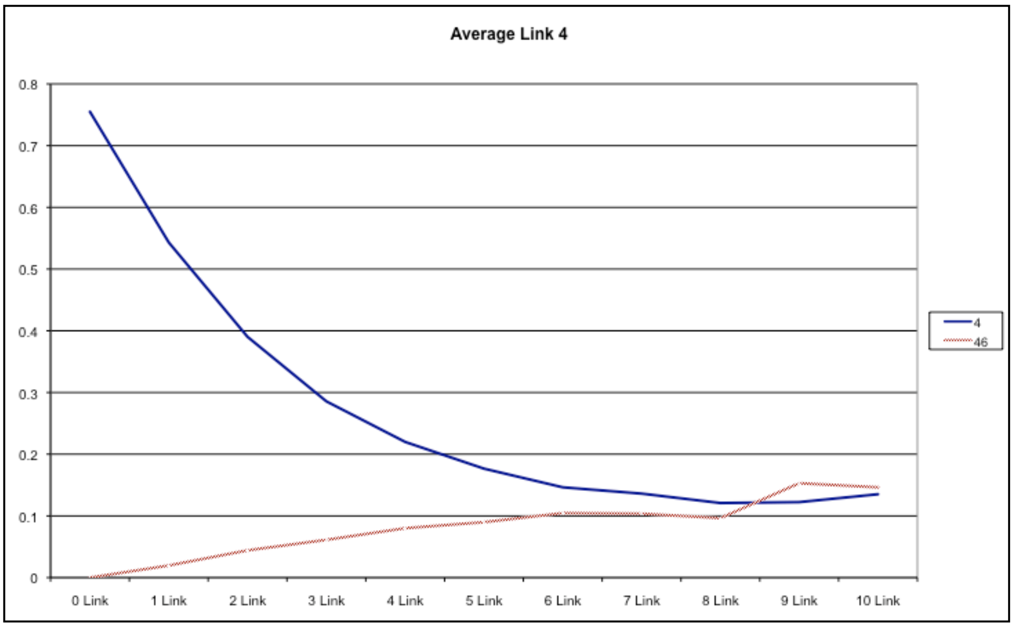

Figure 13. Adoption fractions for individuals with differing number of links (Average no. of links 4)

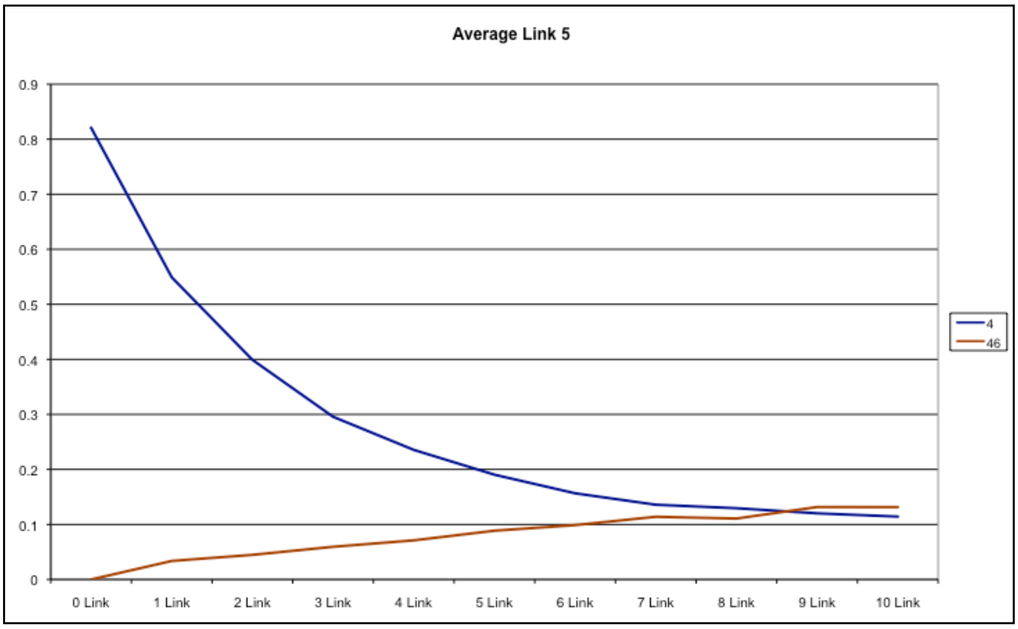

Figure 14. Adoption fractions for individuals with differing number of links (Average no. of links 5)

In order to explain the existence of two clusters of cases in which the adoption fraction behaves different as a function of connectivity of an individual, it is sufficient to go over the discussion given in the previous section. The only difference is the fact that the former discussion was at the aggregate system level (i.e. stronger *word-of-mouth* at the aggregate level). However, in this case the discussion should be interpreted for each individual in the network. The dual direction of impact of number of links on adoption vulnerability is again due to the two key points discussed before.

6. Discussion and Conclusions

The starting point of this study is about the impact of information dynamics that accompany the diffusion process of an innovation. Using a simple generic model, we have conducted an extensive experimental study in order to comprehend the dynamic interactions of a number of mechanisms related to how information changes on adopters' and potential adopters' sides (i.e. *learning-by-experience* and *word-of-mouth*). Although the basic scenarios with this aggregate model are described elsewhere [7], in this paper we provided a couple of interesting diffusion scenarios observed during those experiments (i.e. *self-deceiving crowds* and *misguiding front-runners*), which explain how a poor innovation may diffuse to a significant portion of the total market due to imperfect information.

The main motivation of this study is more focused on representational assumptions made during diffusion studies as the one summarized in the beginning of in this paper. As a first step, we questioned if one of the most commonly used stock-flow representations (see Figure 8) in diffusion models represents what we believe it represents, i.e. perfectly mixed population, in which the probability of individuals adopting the innovation is equal to the *adoption fraction*. Building an individual-based version of the same system, we conducted an empirical comparison, and concluded that the behavior of the aggregate model demonstrates a very precise fit to the behavior of the individual-based model, in which every body communicates with everybody (i.e. perfectly-mixed).

Later we explored the impact of social network structure, or more specifically the density of the network, on the aggregate diffusion dynamics. We can talk about two interesting findings in this phase of the experiments. The first of these is about the convergent behavior of the systems as their network density increases. Especially, the diffusion dynamics of systems having 10 or more links per individual on average differed only insignificantly. In addition, they converge to the dynamics obtained by the aggregate model assuming perfect-mixing. Such an observation provides some empirical grounding for relying on the simplifying assumption of perfect-mixing in representing systems with dense network structures.

The second interesting point was regarding the impact of network density on diffusion dynamics. As expected, the change in diffusion levels as a consequence of changing network densities was monotonic. However, direction of change was different for different groups of scenarios tested. In other words, in some cases diffusion levels increase as the network gets denser, while in some other cases the opposite is observed. A further investigation revealed the cause of this duality; without repeating the detailed discussion given previously, in cases where word-of-mouth changes character, i.e. initially acts in favor of diffusion, and later against it, the relative change in positive and negative word-of-mouth phases of the diffusion process yields such a diversity. A general conclusion of this observation is that it is not possible to make generalization like *"as network gets dense, the diffusion levels decrease"*, since such a conclusion can be made specific to a case having particular conditions. We conducted another round of experiments in order to observe the changes in the individual level adoption tendencies as a function of number of neighbors an individual has. These observations also revealed a similar duality in the response of individuals' adoption chances to changing number of links.

One future step following the observation given above is to study the profile of cases in which increased network density yields increased diffusion, or vice versa. Although we found a set of cases, in which we observe the same behavior, it is difficult to find the common characteristics that are important in this observed phenomenon. For example, it can be the gap between the information levels of potential adopters and adopters, as well as the gap between the information of the potential adopter to their acceptance threshold. Determination of characteristics that determine the direction of the impact of increased network intensity may help in transferring this insight into real cases, and see if there is real empirical data to support such an experimental observation.

Appendix A. Specifications of the Aggregate Model

The differential equation set used in the model and the initial values of the parameters that are not altered during experimentation are given below;

$$\frac{dPAd}{dt} = -Adoption$$

$$\frac{dAd}{dt} = Adoption$$

$$\frac{dAttr_{PAd}}{dt} = \frac{InfoGap_{PAd}}{LearnDelay_{PAd}} \times WoM\_Intensity(\frac{Ad}{PAd + Ad})$$

$$\frac{dAttr_{Ad}}{dt} = \frac{InfoGap_{Ad}}{LearnDelay_{Ad}} - \frac{InfoGap_{PAd} \times Adoption}{Ad + Adoption}$$

$$Adoption = PAd \times AdopFrac(\frac{Attr_{PAd}}{THold})$$

$$AdopFrac = Normal\_AdopFrac \times f(\frac{Attr_{PAd}}{THold})$$

$$InfoGap_{PAd} = Attr_{Ad} - Attr_{PAd}$$

$$InfoGap_{Ad} = Attr_{Act} - Attr_{Ad}$$

$$WoM\_Intensity(x) = x$$

$$LearnDelay_{Ad} = 5$$

$$LearnDelay_{PAd} = 5$$

$$Normal\_AdopFrac = 0.015$$

$f(.)$ is specified using the special interface of the modeling software used (i.e. Vensim). The specification of the function is presented below on the interface of the software, on which range and input-output values of the function can be seen along its visual representation.

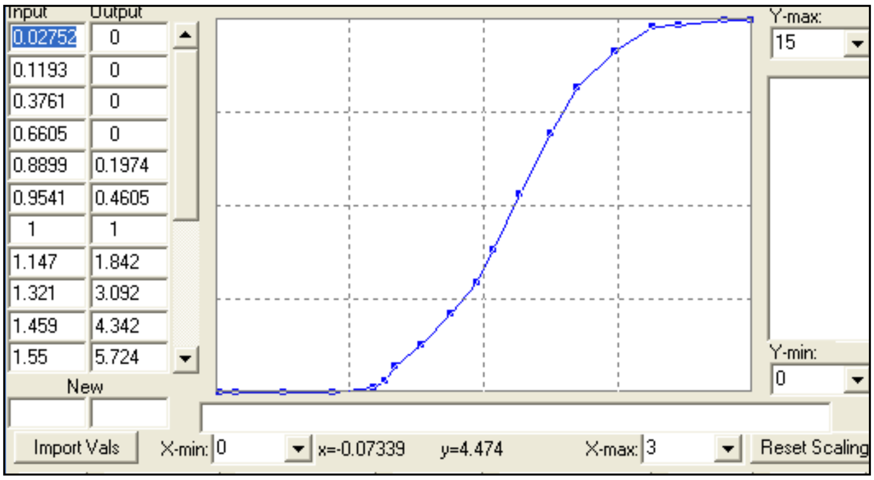

Figure 15. Graphical function used for adoption fraction formulation

The stock-flow representation of the given differential equation system on the simulation software is also given below.

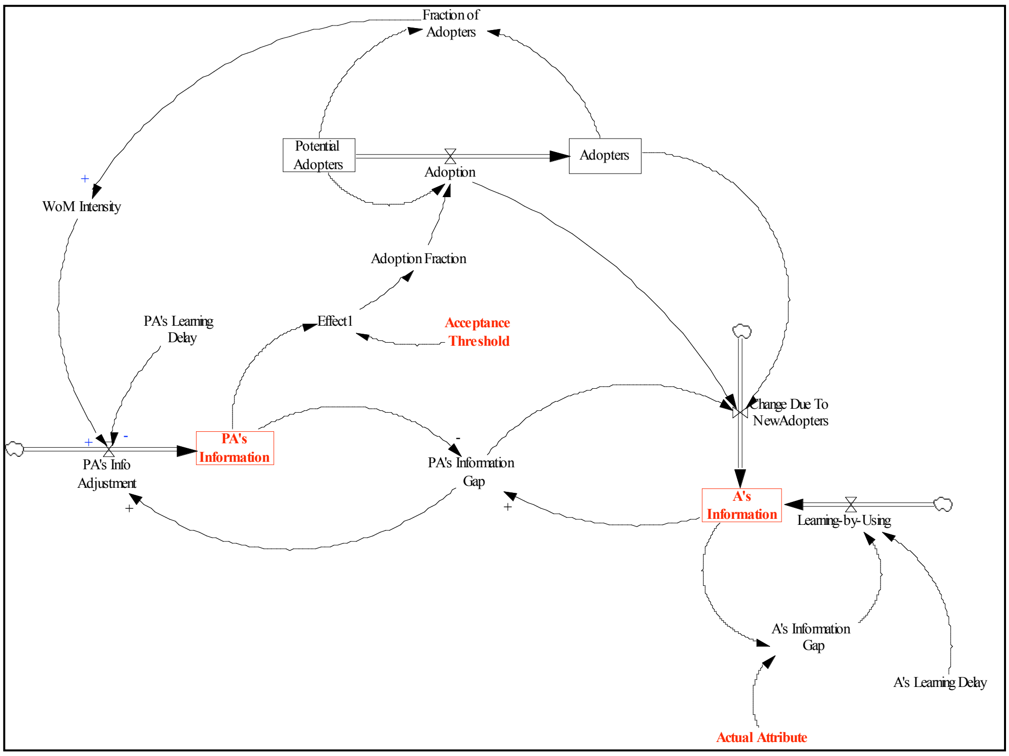

Figure 16. Stock-flow diagram of the model

Appendix B. Specifications of the Individual-based Network Model

The code of the NetLogo model is given below;

```
globals
[
 percUtilAvgAll
 percUtilTotAll
 countAll
 percUtilAvgAdop
 percUtilTotAdop
```

```
 countAdop
 percUtilAvgPotAdop
 percUtilTotPotAdop
 countPotAdop
 willingnessAvg
]
turtles-own
[
 adopted?     ;; if true, the turtle is already adopted the innovation
 threshold    ;; acceptance threshold of the turtle
 percUtil     ;; perceived level of the innovation utility
 percUtilNew   ;; new information about the utility of the innovation
 percUtilNeighAggr ;; an intermediate variable needed to calculate new info
 LbEDelay      ;; learning delay of the turtles. For now equal for WoM and LbU
 WoMDelay
 willingness
]

to setup
 clear-all
 random-seed Seed
 setup-nodes
 setup-network
 ask n-of initial-adopted-population turtles
  [ adopt-ini]
 ask links
  [ set color white]
 update-stat
 update-plot
end


to setup-nodes
 set-default-shape turtles "circle"
 crt number-of-nodes
  [
   setxy (random-xcor * 0.9) (random-ycor * 0.9)
   set adopted? FALSE
   set color white
   set threshold threshold-avg
   set percUtil percUtil-potential-avg
   set LbEDelay DelayLbE
   set WoMDelay DelayWom
   set willingness 0
  ]
end

to setup-network
 let num-links (average-node-degree * number-of-nodes) / 2
 while [count links < num-links ]
 [
  ask one-of turtles
  [
   let choice (min-one-of (other turtles with [not link-neighbor? myself])
         [distance myself])
   if choice != nobody [ create-link-with choice ]
  ]
 ]
```

```
  ; make the network look a little prettier
  repeat 10
  [
    layout-spring turtles links 0.3 (world-width / (sqrt number-of-nodes)) 1
  ]
end

to adopt-ini
  set adopted? TRUE
  set color red
  set percUtil percUtil-adopted-avg
end

to update-plot
  set-current-plot "Network Status"
  set-current-plot-pen "potential"
  plot (count turtles with [not adopted?]) / (count turtles) * 100
  set-current-plot-pen "adopted"
  plot (count turtles with [adopted?]) / (count turtles) * 100

  set-current-plot "Averages Status"
  set-current-plot-pen "potential"
  plot percUtilAvgPotAdop
  set-current-plot-pen "adopted"
  plot percUtilAvgAdop

  set-current-plot-pen "willingness"
  plot (willingnessAvg * 100)

end

to update-stat
    set percUtilAvgAll 0
    set percUtilAvgAdop 0
    set percUtilAvgPotAdop 0
    set percUtilTotAll 0
    set percUtilTotPotAdop 0

    if (count turtles with [adopted?]) > 0
    [set percUtilTotAdop (sum [percUtil] of turtles with [adopted?])
    set countAdop (count turtles with [adopted?])
    set percUtilAvgAdop (percUtilTotAdop / countAdop)
    ]

    if (count turtles with [not adopted?]) > 0
    [set percUtilTotPotAdop (sum [percUtil] of turtles with [not adopted?])
    set countPotAdop (count turtles with [not adopted?])
    set percUtilAvgPotAdop (percUtilTotPotAdop / countPotAdop)
    ]

    set percUtilTotAll (sum [percUtil] of turtles)
    set countAll (count turtles)
    set percUtilAvgAll (percUtilTotAll / countAll)

    ;set fracAdop (countAdop / countAll)

    let willingnessTot (sum [willingness] of turtles with [ not adopted?])
    set willingnessAvg (willingnessTot / countPotAdop)
```

```
end

to go
 ask turtles [ update-info ]
 ask turtles [ update-status ]
 update-stat
 update-plot
 tick
 if (ticks > RunLength) [stop]
end

to update-info
 ifelse (not adopted?)
 [
  let fracAdop 1
  let countNeighAll (count turtles with [(link-neighbor? myself)])
  let countNeigh (count turtles with [(link-neighbor? myself) and (adopted?)])
  if (countNeighAll > 0)
  [ set fracAdop (countNeigh / countNeighAll) ]
  if (countNeigh > 0)
  [
   let percUtilTotNeigh ( sum [percUtil] of turtles with [(link-neighbor? myself) and (adopted?)])
   let percUtilAvgNeigh (percUtilTotNeigh / countNeigh)
   let percGap (percUtilAvgNeigh - percUtil)
   let delay (WoMDelay / (5 * fracAdop))
   let percUpd ( percGap / delay)
   set percUtil (percUtil + percUpd)
  ]
 ]

 [
  let percGap ( util - percUtil)
  let delay  LbEDelay
  let percUpd (percGap / delay)
  set percUtil (percUtil + percUpd)
 ]
end


to update-status
 if (not adopted?)
 [
  let x (percUtil / threshold)
  let adopProb (adopProbFunc x )
  set willingness adopProb
  let dice ((random 100) / 100)
  if (dice < adopProb) [ adopt ]
 ]
end


to-report adopProbFunc [ x ]
 ifelse x > 1.25
  [ report (2 * adopProbNorm) ]
  [ ifelse x < 0.75
   [ report 0 ]
   [
    let y (53.862 * x ^ 3 - 161.59 * x ^ 2 + 162.31 * x - 53.6)
```

```
      report (y * adopProbNorm)
    ]
  ]
end

to adopt
  set adopted? TRUE
  set color red
end
```

Appendix C. Parameter values in the instances of Scenario 7 and 9.

Table 1. Parameter values for the 50 instances of Scenario 7

#	$Info_{Act}$	$Info_{PAd}$	$Info_{Ad}$	Thold		#	$Info_{Act}$	$Info_{PAd}$	$Info_{Ad}$	Thold
1	17	39	24	39		**26**	17	65	32	49
2	1	30	15	19		**27**	3	45	19	32
3	8	25	9	14		**28**	5	36	18	27
4	7	24	10	24		**29**	15	42	25	30
5	14	27	19	23		**30**	12	30	20	25
6	0	25	12	22		**31**	6	39	13	31
7	18	58	29	39		**32**	17	56	35	37
8	16	43	29	42		**33**	18	47	32	41
9	15	45	27	42		**34**	0	33	19	19
10	9	28	13	15		**35**	2	11	3	6
11	11	44	17	34		**36**	7	37	16	22
12	6	43	11	30		**37**	19	61	38	45
13	10	46	20	35		**38**	4	27	23	27
14	10	26	10	25		**39**	11	26	23	25
15	1	19	9	18		**40**	6	24	6	12
16	19	31	25	30		**41**	3	32	11	29
17	2	33	11	16		**42**	18	44	32	33
18	19	40	33	36		**43**	15	53	15	34
19	2	35	14	31		**44**	7	18	9	13
20	14	19	18	18		**45**	13	25	22	24
21	9	41	9	26		**46**	18	52	36	38
22	13	42	24	24		**47**	8	41	26	31
23	13	32	14	22		**48**	2	34	12	30
24	11	49	20	35		**49**	4	48	11	29
25	6	47	24	34		**50**	10	19	15	19

Table 2. Parameter values for the 50 instances of Scenario 9

#	$Info_{Act}$	$Info_{PAd}$	$Info_{Ad}$	Thold		#	$Info_{Act}$	$Info_{PAd}$	$Info_{Ad}$	Thold
1	6	15	23	20		**26**	17	26	46	43
2	13	25	56	43		**27**	10	13	27	13
3	6	25	35	31		**28**	9	18	32	29
4	14	22	29	27		**29**	19	19	41	28
5	18	18	21	19		**30**	12	21	29	27
6	7	11	34	17		**31**	14	19	45	37
7	16	28	40	38		**32**	5	10	36	28
8	14	29	44	43		**33**	3	8	28	16
9	13	22	35	27		**34**	1	20	53	36
10	3	7	30	24		**35**	18	31	49	39
11	3	3	19	7		**36**	11	30	65	48
12	2	5	24	17		**37**	2	12	33	18

13	2	16	34	28		**38**	4	14	43	33
14	7	18	29	21		**39**	17	19	28	20
15	7	7	28	24		**40**	15	19	28	22
16	8	22	47	28		**41**	19	20	42	37
17	18	34	50	39		**42**	10	15	38	29
18	1	19	27	26		**43**	0	5	12	6
19	17	27	36	30		**44**	12	19	29	24
20	3	20	41	37		**45**	10	25	48	38
21	18	33	56	46		**46**	14	15	31	21
22	4	17	26	26		**47**	4	13	23	18
23	18	19	54	35		**48**	17	28	46	33
24	4	13	31	27		**49**	13	27	53	43
25	9	12	48	29		**50**	2	7	14	12

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