On the underlying structure of system dynamics models

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ABSTRACT

The underlying structure of system dynamics models is that of a proportional feedback controller. We propose a broader framework for system dynamics models, where systems are modeled using a combined feedback-feedforward structure. While the traditional structure for system dynamics models only uses proportional feedback of error for control, the proposed structure for information feedback employs the use of proportional, integral and derivative (PID) error. Hence, existing system dynamics models only use a small subset of the proposed structure for modeling systems. We argue that the proposed structure provides a more flexible framework for modeling and designing systems.

KEYWORDS

Feedback, Feedforward, Structure, System Dynamics

INTRODUCTION

Forrester (1961) argues that all "flows" within a system are integrated by information feedback networks. He developed the initial framework for system dynamics modeling to capture the impact of these information networks on system behavior. He describes the system dynamics modeling framework as "one of building models of companies and industries to determine how information and policy create the character of an organization". He also mentions that most mathematical models found in management and economics literature are stable steady state models and that the practical utility of such models in dealing with economic systems has not been significant. One of the strengths of the system dynamics modeling technique is that it is not limited to modeling steady state conditions.

The basic principles of system dynamics are concisely summarized by Wolstenhome (1989,1990). System dynamics is often considered to occupy a position between that of operations research and systems thinking. Keys (1988) concluded that the exact position of system dynamics remains unresolved. However, scientists from both domains can relate to it. Forrester (1994) examines the methodologies followed by operations research, systems thinking and system dynamics practitioners to determine how these approaches overlap and what their unique contributions are to system analysis.

Even though system dynamics uses the formalisms of differential equations to simulate system behavior, diagramming tools are used to communicate the assumptions about the structure of the model. Most system dynamics models are represented using "causal loop diagrams" (CLDs) and "stock and flow diagrams" (SFDs). Sterman (2000) provides an excellent reference for using CLDs and SFDs to build system dynamics models.

Since the interaction of complex causal relationships cannot be fully explored through mental simulation, computers are invariably used to simulate system dynamics models (Sterman 1994). Computer simulation allows system dynamics practitioners to rigorously study the impact of these causal relationships and determine how they lead to counter-intuitive behavior (Forrester 1970).

STRUCTURE OF SYSTEM DYNAMICS MODELS

It is the structure of the system dynamics model, which is the source of the modes of behavior that the model demonstrates. These modes are caused by the interaction of different feedback loops, each of which may involve non-linearities, delays, accumulation, and draining processes. System dynamics modeling aims to explain behavior by providing a causal theory, and then using that theory as the basis for designing policy interventions into the system structure (Lane 2008). The purpose of these policy interventions is to change the behavior and improve the performance of the system.

Schmidt and Taylor (1970) define a system as "a collection of entities, e.g. people or machines, which act and interact together toward the accomplishment of some logical end". The "states" of the system can be defined as the collection of variables necessary to define the system at given point in time. The choice of state variables generally depends on the objectives of the study. Systems are generally classified as either discrete or continuous in nature depending on the behavior of the state variables of the system with respect to time. A discrete system is one in which the state variables change instantaneously at separated points in time, while a continuous system is one where the state variables change continuously with respect to time. System dynamics modeling assumes that the state variables of the system are continuous in nature.

Models provide an effective means for understanding complex phenomena, which may not be easily understood by simple observation. A model can provide information at a lower cost quickly, as compared to the actual system. Askin and Standridge (1993) suggest that the primary use for models include the following:

- Optimization – Finding the best values of decision variables.
- Performance prediction – Predicting performance under different conditions.
- Control – Selecting the desired rules to control the system.
- Insight – Gaining a better understanding of the system.
- Justification – Using the results as a tool to support decisions.

The primary motivation behind the development of the system dynamics methodology was to gain insight into the operations of complex dynamic systems, which Forrester (1961) felt were a barrier to learning. Since its inception, the system dynamics modeling technique has been used to study complex business and social systems through the understanding of the different feedback paths in the system.

The structure of system dynamics models stems from servomechanism theory. A servomechanism is a system that uses information feedback to control the performance of the system. This concept of an information feedback system provides the underlying structure system dynamics models. Forrester (1961) mentions, "The first and most important foundation for industrial dynamics is the concept of servomechanisms (or information feedback systems) as evolved during and after World War II." He also adds that "the information feedback system will become a principal basis for an underlying structure to integrate the separate facets of the management process". An information feedback system is said to exist whenever the environment leads to a decision that results in action which affects the environment and thereby influences future decisions (Forrester 1961).The study of information feedback systems reveals how information is used for control. In order to achieve a desired system response, it is necessary to understand how the amount of corrective action and the associated time delays impact the performance of the system. The behavior of an information feedback system is governed by its structure, delays within the system, and the amplification of the system. Hence, the design of an information feedback system must consider these three characteristics if it is to be successful.

PROPOSED FEEDBACK STRUCTURE

The Proportional-Integral-Derivative (PID) structure is a generic feedback control structure that is widely used in industrial control systems (Ogata 2005). The PID control algorithm continuously measures a process variable and compares it with a desired set point. The error between the two quantities is used to adjust the process. The algorithm used to control the process involves three separate adjustments:

- Proportional adjustment: This is the adjustment based on the current error
- Integral adjustment: This is the adjustment based on the sum of recent errors
- Derivative adjustment: This is the adjustment based on the rate of change of error

The weighted sum of these three adjustments is used to control the process. The weights chosen are dependent on the requirements of the process. Some applications of this control algorithm may not require all three modes for control. In such cases, the weights for the modes that are not desired can be set to zero.

The following are combinations of these three modes, which are commonly used in practice:

- Proportional (P) control
- Proportional-Integral (PI) control
- Proportional-Integral-Derivative (PID) control
  (Ogata 2005)

The PID control algorithm continuously monitors a process variable and compares it with a reference point. The reference point can be fixed or variable over time. Any difference between the measured state variable and its reference point represents an error. This error is then used to control a flow control variable, which is sometimes referred to as the "manipulated variable" (MV). It should be noted that there can exist an error term for each state variable in the model. The proportional, integral and derivative component of each error term can then be used to determine the value of the manipulated variable. This provides a large design space for the feedback control system. The choice of which error values and their corresponding modes are used to control the system depends on the desired response characteristics for the system. If the error associated with each state variable is used for control, it can lead to an increase in system responsiveness. However, such a control scheme can also increase the amplification of the system (Chaudhari 2008).

When using the PID feedback structure, the manipulated variable is calculated as the sum of the proportional, derivative and integral adjustment, as shown below in Equation 1.

$$MV(t) = P_{adj} + I_{adj} + D_{adj} \qquad \textbf{(1)}$$

The proportional adjustment, P_{adj} makes an adjustment that is proportional to the error, E (t). It is defined as a product of the error and a proportional gain constant, K_p. The proportional adjustment is defined as shown in Equation 2

$$P_{adj} = K_p * E(t) \quad \textbf{(2)}$$

A high proportional gain shall lead to a large change in the manipulated variable for a given change in error. While this may make the process responsive to change, it can also make the process unstable if the proportional gain is made very high. The integral adjustment depends on both the magnitude and duration of the error term.

The integral adjustment, I_{adj} is defined as a product of integral of the error and an integral gain constant, K_i. Hence, the integral adjustment is defined as shown in Equation 3.

$$I_{adj} = K_i * \int_0^t E(\tau)d\tau \quad \textbf{(3)}$$

The integral adjustment, when used in addition to the proportional adjustment can enable a system to react in a more responsive manner to disturbances. The impact of the integral adjustment depends on the integral gain constant that is used. However, since the integral adjustment is based on the accumulation of past errors, it can cause the system to overshoot the desired level.

The derivative adjustment, D_{adj} depends on the rate of change of the error term. It is defined as a product of the derivative of the error with respect to time, and a derivative gain constant as shown below in Equation 4.

$$D_{adj} = K_d * \frac{dE(t)}{dt} \quad \textbf{(4)}$$

The impact of the derivative adjustment depends on the choice of the derivative gain constant. This mode of adjustment is often used in conjunction with integral adjustment as it reduces the overshoot that may be caused by the integral adjustment. This mode of adjustment is sensitive to noise and may cause the system to become unstable if a large derivative gain is used.

The choice of gain constants for the PID feedback system reflects the policies of the organization. If the organization has an aggressive policy for correcting discrepancies between the state variables and their ideal values, then the gain constants would be set to large values. Conversely, if the organization has a mild policy for correcting discrepancies between the state variables and their values, then the gain constants would be set to small values. Ultimately, the choice of gain constants is dependent on the desired response characteristics of the system.

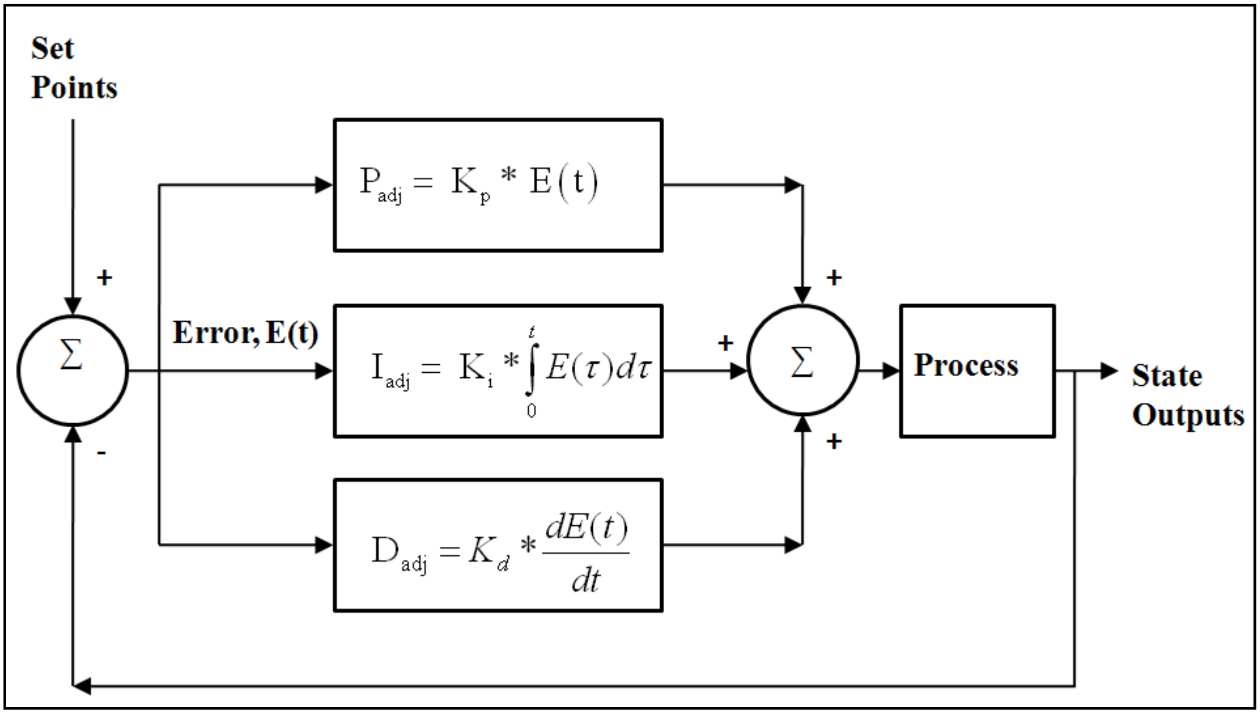

Figure 1: Proportional-Integral-Derivative (PID) feedback structure

COMBINED FEEDBACK-FORWARD STRUCTURE

A feedback system is one that reacts to changes in its environment, usually trying to maintain some desired state. On the other hand, a feedforward system reacts to a measured disturbance in a pre-defined manner. Hence, the disturbance is measured and action is taken before the disturbance affects the system. The difference between the two schemes for control can be discussed in the context of an example. Consider the cruise control mechanism of an automobile, a well-known feedback system. The purpose of a cruise control mechanism is to maintain the car at a steady speed. When the car encounters an uphill slope, the car would slow down. This difference between the actual and desired speed would generate an error signal, which would cause the throttle to open further, and hence bring the car back to the desired speed. If the cruise control mechanism had been using a feedforward mechanism for control, it would have used a sensor to detect the uphill slope and opened the throttle in anticipation of the decrease in speed. Hence, the car would not lose any speed before a correction was made. In the context of this example, it should be noted that there are several other factors such as temperature, wind, altitude etc., which can impact the speed of the car. Since the relationship of these variables with the speed of the car cannot be modeled accurately, it would not be possible for a cruise control mechanism to operate solely with feedforward control. Feedback and feedforward control structures are not mutually exclusive. One could adopt a combined feedback feedforward control structure. Feedforward control would lead to a quick response, while feedback control would correct any error that arose due to the pre-set response.

A standalone feedforward controller is a good choice if the following conditions can be met:

- Disturbances are known before they impact the system
- Disturbances are measureable
- There are no significant unmeasured disturbances

Heylighen and Joslyn (2001) discuss the advantages and disadvantages of feedback and feedforward control. These are summarized below in Table 1. It can be seen that the two control structures are actually complementary to each other. By using a combined structure, one can increase responsiveness to known disturbances, while being robust to unknown disturbances.

	Feedforward Control	**Feedback Control**
Advantages	1. Compensates for disturbance before it affects the system 2. Does not impact the stability of the control system	1. Zero steady state offset 2. Is appropriate for use with all disturbances. 3. Does not require any additional sensor for each disturbance
Disadvantages	1. Shall require a sensor and model for each disturbance 2. Can't eliminate steady state offset 3. The controlled state variable is not monitored. Hence, no error correction is possible 4. Tends to require more calculation/analysis in design phase	1. Requires the disturbance to impact the system before any response is made 2. Affects stability of control system.

Table 1: Comparison of feedback and feedforward control

Figure 2 shows the proposed structure for the combined feedback-feedforward control. The feedback control effort focuses on ensuring that one or more states remain at specified set points. The process output is continuously monitored to compare the values of the state variables with their desired set points. If there is a discrepancy, the error is multiplied by an appropriate gain value and fed back to the flow control variable for the process. If there is a measurable disturbance, its value is measured and multiplied by an appropriate feedforward gain. This information is transmitted to the flow control variable for the process. Hence, the combination of the feedback and feedforward information flows determines the setting for the flow control variable of the process.

It should be noted that in the above framework, the feedback control structure chosen depends on the nature of the process and the state variables of interest. One could use a PID control structure if appropriate. If not, one could use a P, or PI structure. A combined feedback-feedforward does not restrict this choice in any way.

The formulation of the feedforward control loop also depends on the nature of the disturbance and its impact on the process. Depending on the specifics, the disturbance value can be multiplied by an appropriate gain value to determine its effect on the flow value for the process. The choice of gain depends on the units of the disturbance.

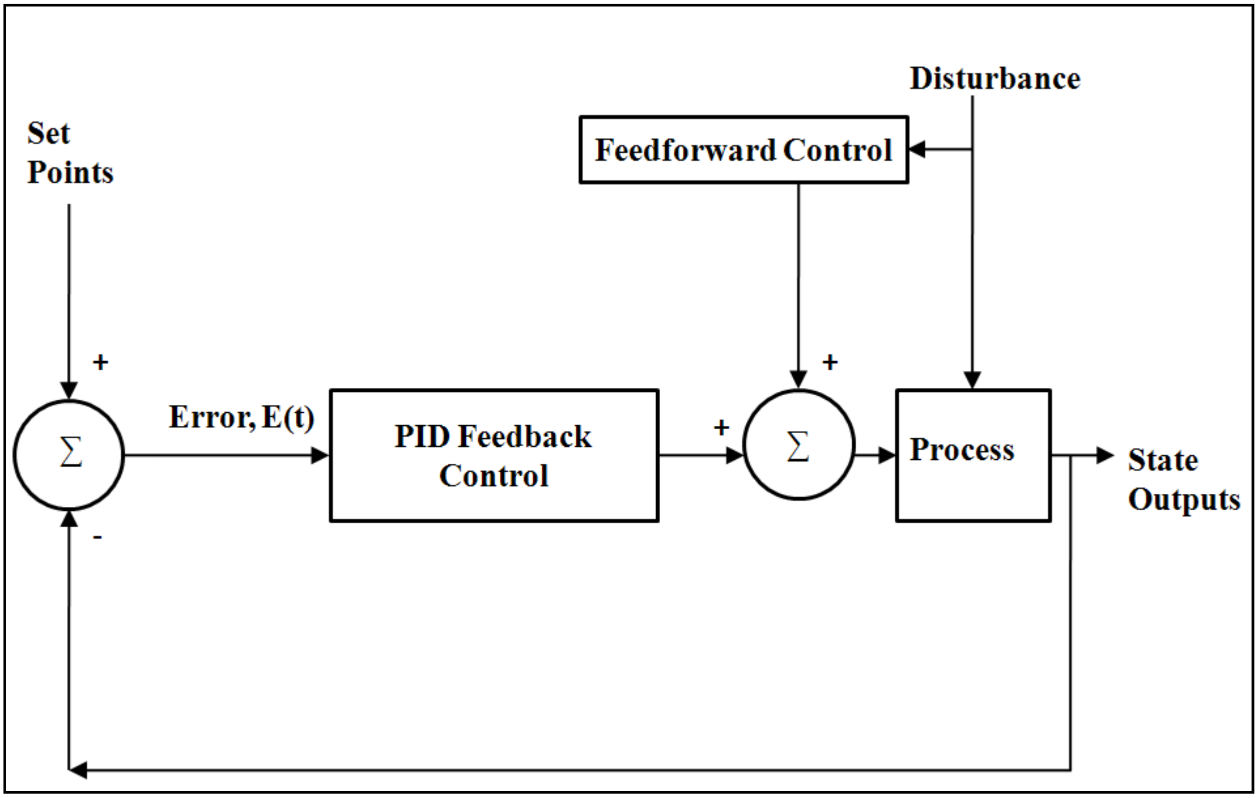

Figure 2: Combined feedback-feedforward control structure

CONCLUSIONS

The primary contribution of this work is to highlight the limitation of the existing structure of system dynamics models. Existing stock and flow control models only use proportional feedback of error to control the process. Hence, existing models only use a subset of the proposed PID structure. We hypothesize that by using integral and derivative feedback of error in addition to the proportional feedback of error, one could better model the dynamics of systems. Such a structure also provides a much larger design space for policy design, as compared to traditional models which only utilize proportional feedback of error.

Traditionally, system dynamics models use a feedback control structure to explain the dynamics of systems. We examine the possibility of using a combined feedback-feedforward structure to model the behavior of systems. The current practice of modeling systems as strictly feedback control systems is based on the premise that systems do not respond to disturbances before they impact the system. This is contrary to the behavior of several business and social systems, which actively respond to disturbances in anticipation of the impact of the disturbance on the system. Further, since feedback and feedforward structures are complementary to each other, a combined structure forms a much broader and robust framework for modeling and designing system behavior.

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