

# On Modeling Some Essential Dynamics of the Subprime Mortgage Crisis

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## Abstract

*We develop a Systems Dynamics model for capturing the key interactions involved in the evolution of the subprime mortgage crisis. In particular, we propose an aggregate modeling resolution that involves three main sub-systems, namely, an aggregate banking system, an aggregate housing market and an economic environment. The model exposes the physics of each individual system as well as influences and interactions among the three systems. The model is useful for developing intuition about the evolution of the crisis as well as the lagged timing and magnitude of the effects of various corrective actions, such as an economic stimulus package. We present three scenarios using simulated data. In the first scenario, we establish an equilibrium state that represents a steady state normal condition. In the second scenario, we introduce a step function for the availability of subprime loans and hold it for certain duration. This practice eventually culminates in a credit crisis, where the aggregate bank experiences insolvency. In the third scenario, we study the application of an economic stimulus, which steers the entire system back to a new equilibrium state. We note that the economic stimulus needs to be larger than a certain critical lower threshold in order to enable the system towards reaching a new equilibrium.*

**Keywords:** System Dynamics, Subprime Mortgage Crisis, Bank Assets and Liabilities, Capital, Insolvency

## 1. Introduction

Today, the world economy is in recession and the financial sector is experiencing a severe credit crisis. The origins of the credit crisis can be traced back to the *subprime* mortgage market in the U.S., where *subprime* refers to mortgagees who are unable to qualify for prime mortgage rates due to myriad reasons. These include past payment delinquencies, personal bankruptcies, low credit scores, large existing liabilities, or high loan to value ratios. As such, they represent a high-risk class of loan-borrowers with respect to defaulting on prospective payments.

In the last five years or more, a boom in subprime lending was fueled by, and it in turn propelled, a bull-run in the market for mortgage-backed securities. A Mortgage-backed security (MBS) is simply a merged pool of multiple mortgages that has a recurring stream of annuity payments associated with it over a horizon of 15 to 30 years. The annuity stream originates from the monthly mortgage payments that are purportedly expected from the corresponding loan borrowers. The lending boom sparked a spike in demand for homes, which in turn artificially

inflated home prices and made investments in MBS assets very attractive for the banking sector. Banks consequently invested heavily in MBS assets, many of which had significant exposure to underlying subprime borrowers. They sought to neutralize the default risk to the associated annuity streams by investing in a form of insurance known as Credit Default Swaps (CDS). However, this turned out to be a superficial and temporary transfer of the underlying default risk. This is because of the high counterparty risk that was later realized in the overextended CDS market when subprime borrowers started to default *en masse*, and home foreclosures started to rise. Defaulting-led foreclosures in turn led to depressing home prices, particularly in subprime zip codes. These dynamics implied a significant downward pressure on the value of MBS assets as well as real physical home assets that ended up on the books of the banking sector through write-downs. A faster depreciation of *assets* relative to *liabilities* in turn put a downward pressure on the capital held by various banks.

As a consequence of pressure on capital, banks were concerned about the magnitude of future write-downs and counterparty risk. They have been trying to keep as much cash as possible as a cushion against potential losses. They have been wary of lending to one another and, consequently, have been charging each other much higher interest rates than normal in the inter bank loan markets (Crouhy et. al. (2008)). This has led to a downslide in the availability of commercial credit and business loans that form the life blood of the economy. As a result, a recessionary downturn has materialized on the economy that has led to lower consumption and higher unemployment. This has led to even higher downward pressure on affordability at the level of individual families, thereby increasing the rate of defaults and leading to more foreclosures and further downward asset valuation in the banking sector. The chain-reaction dynamics that we have described above is in effect a vicious, self-fulfilling negative spiral, also referred to a 'deflationary spiral' that is triggered by 'systemic risk' in the financial literature. We are now at a point where the government is embarking on a bold economic stimulus package to revive the economy.

There has been a lot of recent research work aimed at analyzing the financial crisis. Allen and Gale (2006) investigate transferring system risk from banking sector to insurance sector, as witnessed with the proliferation of the CDS market. They conclude that depending on the dynamics of the market, it could be beneficial or adverse. Murphy (2008) concludes that there were serious problems in valuation of CDS instruments, which exploded upon cataclysmic rises in residential mortgage defaults and led to significant worry about counter-party risk, thereby fueling the 'contagion'. Whalen (2008) discusses causes of the subprime crisis in greater detail. Especially, three factors are pointed out, 1) multiple agencies (including government) enhanced the availability of "affordable housing" via the use of "creative financing techniques", 2) federal regulators encouraged practice of over-the-counter derivatives like CDS, and 3) the security exchange commission (SEC) embraced "fair value accounting". Demyanyk and Van Hemert (2008) provide evidence that the rise and fall of the subprime mortgage market follows a classic lending boom-bust scenario, in which unsustainable growth leads to the collapse of the market.

Our take-away message from the above scholarly articles is that modeling and understanding the dynamics between multiple aggregate entities, namely the banking sector, the housing market, and the prevailing economic environment is critical for developing intuition about the evolution of the financial system. It is also important with respect to developing intuition about the lagged timing and magnitude of the effects of various corrective actions such as economic stimulus packages. The chain-reaction dynamics that we have attempted to

elucidate above leads us to believe that a Systems Dynamics model is an appropriate modeling approach. We follow the methodology proposed by Forrester (1960). He proposed to use systems-thinking to capture causal relationships in any target system and described a mental model in the form of stock-flow diagram. A stock is accumulative quantity that increases by inflow and decrease by outflow. Its flow rates are formulated as a consequence of driving forces in the system – physical reasons that lead to flows. The methodology has been successfully used to model various applications, like supply chain, disease propagation, decision rules in project management etc. summarized in Sterman (2000).

In the context of the financial sector that is the target system of our article, there has been some limited work using system dynamics. For instance, Garcia-Ochoa (1996) develops a model to study the dynamics and feedback effects of the borrower-lender relationship under regulation. We propose an aggregate modeling resolution that involves three main sub-systems, namely, an aggregate banking system, an aggregate housing market and an economic environment, using individual system dynamics as well as the interaction among the three systems. A short overview of key physical interactions in each aggregate entity is as follows. Details are presented in later subsections for each entity.

- Flows within the banking system among different assets are driven by the housing market as well as the economic conditions. The dynamics of the valuation of *assets* vis-à-vis *liabilities* is important in the banking system towards assessment of available capital. When capital is driven towards zero, bankruptcy kicks in.
- The purchasing power that drives demand in the housing market and home prices, and results in exchanging flows among homeowners and non-homeowners, is influenced by mortgage loan availability from the banking system.
- The economic model captures business activity as a consequence of liquidity in the banking system and availability of business-credit. When business activity slows down, production and gross output drops, leading to rise in unemployment, and lower household income which will further drive consumption down. It leads to loss of affordability at the level of individual home-owners, leading to home-loan defaults, foreclosures and depressed home prices. The total bank asset value drops relative to liabilities, thereby eroding available capital and lower availability of and business-credit as well as mortgage-loans.

We attempt to capture the physics of the above evolution process and simulate the implied consequences. Finally, we also examine the impact of economic stimulus with respect to steering the gross output back to a new equilibrium through boosting the aggregated demand. The paper is organized as follows. In section 2, we present an overview of bank system, housing market and economic models. Section 3 gives the formulation detail of the system dynamics model. Section 4 demonstrates some simulation results for certain scenarios. Section 5 concludes the paper and discusses further research direction.

## **2. Subprime Mortgage System Model Overview**

Figure 1 shows a schematic of the overall model with essential flows that capture the interactions among the three aggregate sub-systems: Banking sector, Housing market and the Economy.

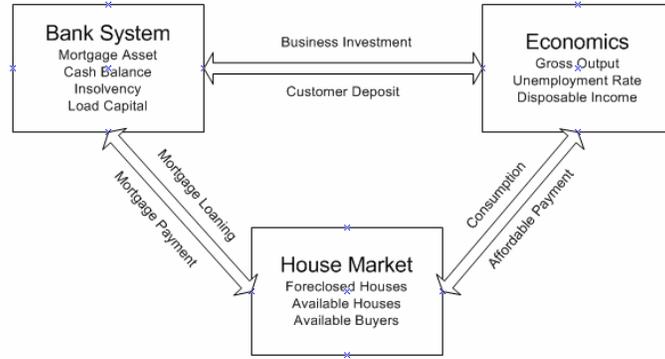


Figure 1: Subprime Mortgage Model

The interaction of three systems will determine the evolution of whole process. We present each of the subsystems in detail next.

### 3. System Dynamics Model and Formulation

#### 3.1. Bank System

The banking sector provides leverage for all economic activities in today's society. It provides cash circulation and loan capital for mortgage loans as well as business investments. The core business model of the banking sector is to collect deposits and then to use it for investments such as loans and other securities. The difference in interest rates between lending and depositing activities make banks profitable and cover the operation cost.

In our model, an aggregated bank is assumed and it includes a portfolio of several classes of assets, and is in a healthy condition if total asset value is greater than its liability. The first asset class included in our model is Cash ( $A_c$ ) that is used to conduct daily operations. It has in- and out- flows in relation with other assets, since it is used to mediate financial transactions. Cash does not create additional value nor does it lose its face value, unless transformed into other assets that can change in value. Since cash interacts with all other asset classes, we present the equation for this asset class at the end of this subsection.

The second asset in the model is short term securities ( $A_s$ ). This class of assets can be easily converted into cash at short notice with relatively small risk. In-and-out flows between Cash and short term security occur whenever the amount of cash is out of baseline value ( $A_{c0}$ ). The equation for tracking the value of this asset class is as follows.

$$\frac{dA_s}{dt} = R_{si} - R_{so} = \frac{1}{T_s} \left[ (A_c - A_{c0})^+ - \min((A_{c0} - A_c)^+, A_s) \right] \quad (1)$$

Note that  $T_s$  is the short term security selling/buying processing time, and out-flow of this asset class is restricted by its availability. The expression  $(\cdot)^+$  denotes the positive part of the value inside its parenthesis. For simplicity, we assume that the value of this asset would not change value. In other words, we assume that this asset class is fully equivalent to Cash.

The third asset class is mortgage-backed securities ( $A_m$ ). It is used to account for the value associated with the underlying mortgage loan repayment stream. The value ( $A_m$ ) of this asset increases when a home-buyer gets a loan from the bank. Its value decreases when home-buyers make their monthly mortgage payments. Its value also decreases when some buyers default on their mortgage loan. In this case, bank would foreclose the corresponding properties and assume ownership of foreclosed physical assets (i.e. houses). Mathematically, we have the following equation for tracking the value of this asset class.

$$\frac{dA_m}{dt} = P_h \cdot R_h - \min(R_a, R_m) - R_f, R_m = \frac{A_m}{T_m} \quad (2)$$

Let us examine Equation (2) term by term. The first term on right corresponds to the rate of increase due to bank lending activities. It equals to the average unit house price ( $P_h$ ) times house purchasing rate ( $R_h$ ). The house purchasing rate ( $R_h$ ) is determined dynamically in the Housing market subsystem, which is presented in the next subsection.

The second term represents an outflow related to mortgage repayment rate  $R_m$ , and at any point in time is equal to the current mortgage backed value  $A_m$  divided by the mortgage cycle time  $T_m$  (i.e. 240 months for 20 years loan). Note that the repayment rate is restricted by a quantity we refer to as, affordable repayment rate ( $R_a$ ). The affordable repayment value is determined in the economic model that will be presented later. The second term simply gets transferred into the asset class, Cash, as shown later in the equation for Cash.

The third term,  $R_f$ , represents the rate of change of this asset due to the foreclosures. Say, the defaulting home-owners have already repaid  $T_b$  periods of mortgage payments before defaulting and foreclosure. Then,  $R_f$  is simply the remaining (unpaid) value of the original loan. The rate at which it is transferred from the mortgage backed asset to the bank-owned house value is written as the following.

$$R_f = (R_m - R_a)^+ \cdot (T_m - T_b) / T_f \quad (3)$$

In Equation (3),  $T_f$  is the foreclosure processing time and is much shorter compared to the mortgage cycle time  $T_m$ .

The fourth asset class considered in our model is bank-owned-houses ( $A_h$ ), resulting from loan defaults and house foreclosures. Since a repayment for  $T_b$  time periods has been made prior to defaulting and foreclosure as captured in Equation (3), the value that gets transferred upon foreclosure (due to rate,  $R_f$ ) from  $A_m$  into  $A_h$  is not the original loan amount. In order to account for the re-paid principal, we simply add a flow from Cash to the bank-owned-house that has rate  $R_p$ .

$$R_p = (R_m - R_a)^+ \cdot (T_b) / T_f \quad (4)$$

Note that the sum of both  $R_f$  and  $R_p$  over  $T_f$  period needs to equal the original loan amount corresponding to the foreclosed houses, i.e.  $(R_f + R_p) \cdot T_f = (R_m - R_a)^+ \cdot T_m$ .

There is another important flow associated with the bank owned house value ( $A_h$ ). This asset can change value as determined by the Housing market conditions. The rate of change of market value is captured using rate,  $R_{chv}$ ,

$$R_{chv} = \frac{1}{T_f} \min(H_b \cdot (P_{orig} - P_h), A_h), \quad P_{orig}(t) = P_h(t - T_b) \quad (5)$$

where  $H_b$  represents the number of houses owned by Bank and is modeled in the housing market model.  $P_{orig}$  is the unit house price at the point in time of the original home-purchase. When the bank sells these foreclosed houses in the market, the corresponding value gets transferred from  $A_h$  to Cash at the following rate.

$$R_{hr} = \min\left(R_{bank} \cdot P_h, \frac{A_h}{T_h}\right), \quad (6)$$

where  $R_{bank}$  is the house-buying rate from bank-owned-houses, and  $T_h$  is the buying processing time in the House Market model. Using all the above relevant flows, we finally have the balance equation for the bank owned house value ( $A_h$ ).

$$\frac{dA_h}{dt} = R_f + R_p - R_{hr} - R_{chv} \quad (7)$$

The fifth asset considered is the non-mortgage backed asset class ( $A_n$ ), which is used to account for the rest of aggregate bank's business. We use this asset to capture business investment that impacts gross production output in the economy model. There are three flows affecting the value of this non-mortgage asset.

When the bank's capital, namely,

$$C_a = (A_t - L)^+ = (A_c + A_s + A_m + A_h + A_n - L)^+, \quad (8)$$

which is equal to total asset value ( $A_t$ ) minus bank liability ( $L$ ), is positive, a portion ( $r_i$ ) of it is used to fund business investment activities at the following rate,

$$R_{bi} = \frac{r_i \cdot \min(C, A_c)}{T_{prod}}, \quad 0 \leq r_i \leq 1, \quad (9)$$

where  $T_{prod}$  is the production time-constant defined in the economy model. The rate is restricted by available cash  $A_c$ . A second flow that captures the rate of increase of  $A_n$  is the business loan-payback rate, namely,

$$R_{bp} = \frac{A_n}{T_{blc}}, \quad (10)$$

where  $T_{blc}$  is the business loan cycle time. The third flow captures a rate of increase that is proportional to the rate of increase of the gross production output,  $R_{cgv}$ . Finally, we arrive at the balance equation, which is expressed as the following,

$$\frac{dA_n}{dt} = R_{bi} - R_{bp} + \alpha \cdot R_{cgv} = \frac{r_i \cdot \min(C, A_n)}{T_{prod}} - \frac{A_n}{T_{blc}} + \alpha \cdot R_{cgv}. \quad (11)$$

After defining all flows that interact with cash, we have the balance equation for  $A_c$  as the following,

$$\frac{dA_c}{dt} = R_{so} - R_{si} + \min(R_a, R_m) - R_h \cdot P_h + R_{hr} - R_p + R_{bp} - R_{bi}. \quad (12)$$

The first two terms are the in-and-out flows with the short term security asset  $A_s$ . The following two terms are the in-and-out flows with the mortgage backed asset  $A_m$ . The fifth and sixth terms are the in-and-out flows with the bank-owned-houses asset  $A_h$ . The seventh and eighth terms are the in-and out flows with the non-mortgage asset  $A_n$ .

Note that in the case of both  $R_{chv}$  and  $R_{cgv}$  being zeros, the total bank asset value is conserved and does not lose or gain during bank operation. But in the case of either  $R_{chv}$  or  $R_{cgv}$  or both being non-zero, then the total asset value could drop and potentially become lower than bank liability. We assume for simplicity that the rate of interest paid to depositors (liability) is the same as the interest rate that is charged to borrowers, since the primary concern of this study is the aggregate bank's viability, and not its profitability. The aggregate bank's insolvency is measured by the following

$$C_i = (L - A_i)^+. \quad (13)$$

In the processing of formulating in-and-out flow, we see that some quantity is determined by state values in the other models. It also holds true that, some state valued in this model would affect some change-of-state rates in the other models. Figure 2 shows the system dynamics model of bank asset flows.

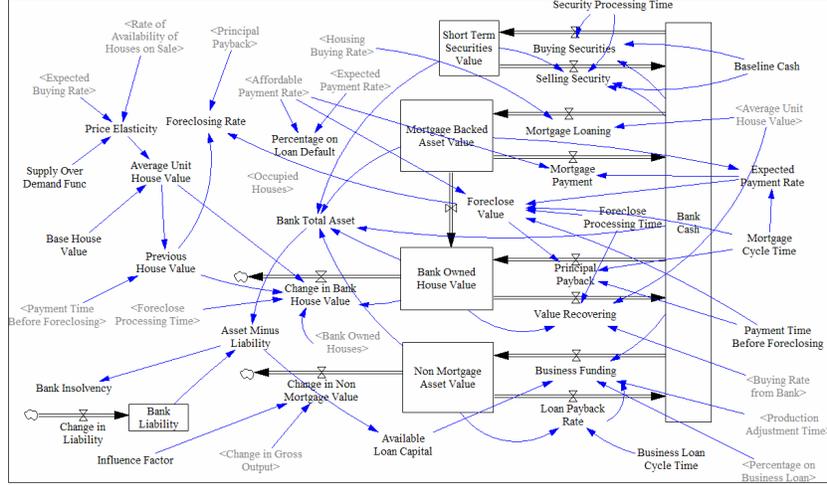


Figure 2: System Dynamics Model of Bank Asset Flow

### 3.2. House Market

We include three types of houses in our model: the currently occupied houses, non-bank owned available houses and bank-owned houses. The banking model provides the available loan capital ( $C_a$ ) for house buyers. Due to co-flow nature of this model with the banking system, the overall formulation needs be consistent. For instance, bank loaning-out rate in banking model is determined by house-purchasing rate in housing market model.

House-purchasing rate is determined by three factors, available buyers – families that do not own a house right now, available houses on sale and available loan capital from the bank. Let  $F_n$  be the number of families that do not own a house. The expected buying rate is written as the following

$$R_{fam} = \frac{F_n \cdot (1 - r_{uem}) \cdot (1 - r_{int})}{T_{fam} \cdot (1 - L_{sub})}. \quad (14)$$

Where  $r_{uem}$  is unemployment rate;  $r_{int}$  is mortgage interest rate since lower interest rate would increase buying rate;  $L_{sub}$  is for subprime loan availability ( $0 \leq L_{sub} < 1$ ) and higher availability would motivate the buyers and increase buying rate;  $T_{fam}$  is the average family lifespan. The rate of availability of houses on sale can be formulated as,

$$R_{av} = \frac{H_{av}}{T_h}, \quad (15)$$

where  $H_{av}$  is total number of available houses on sale in the market, and  $T_h$ , mentioned in bank system model, is the time-constant associated with processing a house purchase.

Note that buying rate is limited by the availability of loans, and such a rate can be formulated as,

$$R_{fin} = \frac{r_m \cdot \min(C_a, A_c)}{P_h \cdot T_h}, \quad (16)$$

where  $r_m$  indicates the portion of the bank's capital available for mortgage lending, restricted by available cash. The rate corresponds to how many houses can be financed, since it is divided by the average unit house price. Hence the effective house purchasing rate can be captured as the minimum of these three rates, namely,

$$R_{buy} = \min(R_{fam}, R_{av}, R_{fin}) \quad (17)$$

Before formulating the stock (level) variables in the housing market model, we present our pricing logic vis-à-vis house prices. Based on usual price elasticity theory, price increases when demand is greater than supply, and decreases when demand is less than supply. We defined a lookup function,  $r_p$  (price attractiveness) as a function of the ratio of supply  $R_{av}$ , over demand  $R_{fam}$ , satisfying  $r_p(1)=1$ , as illustrated in Figure 3.

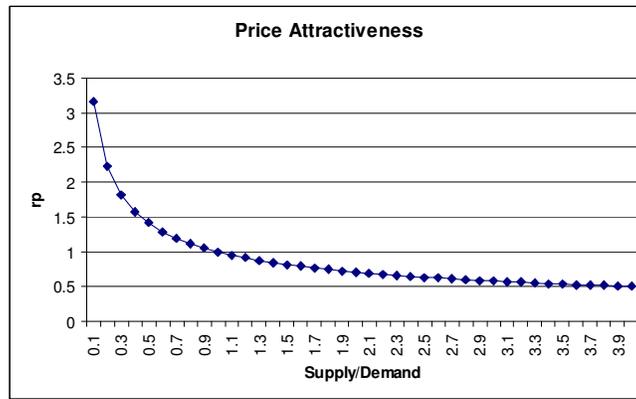


Figure 3. Price Attractiveness

The current unit house price is then given by

$$P_h = P_o \cdot r_p \left( \frac{R_{av}}{R_{fam}} \right), \quad (18)$$

where  $P_o$  is a baseline unit house price that corresponds to the equilibrium (steady) state in which demand equals supply.

The non-bank-owned available houses ( $H_n$ ) exchange with the occupied houses ( $H_o$ ) through selling and purchasing. The increasing rate of the ( $H_n$ ) is written as,

$$R_{sell} = \min \left( \frac{H_o \cdot r_p}{T_{dur}}, \frac{(P_{lim})^+}{P_h} \right). \quad (19)$$

Essentially, the rate is equal to the number of occupied houses  $H_o$  divided by the average house occupying duration  $T_{dur}$ , modified by pricing attractiveness  $r_p$  and limited by loan-availability, i.e., available value  $P_{lim}$  in a unit period divided by unit house price  $P_h$ . Derivation of  $P_{lim}$  comes from the following physical constraint: the total out-flow rate from the occupied houses ( $H_o$ )

times unit house price should be less than or equal to the total out-flow rate from the mortgage backed asset value ( $A_m$ ). This would guarantee consistent flows in the banking model and the housing market model. Otherwise, we could get some unphysical result, for instance,  $H_o$  becoming negative. The equation for  $P_{lim}$  is as follows,

$$P_{lim} = \min(R_a, R_m) - (R_m - R_a)^+ \frac{T_b}{T_f} \quad (20)$$

An outflow from available houses associated with purchasing rate from market

$$R_{mkt} = \min\left(R_{buy} - R_{bank}, \frac{H_n}{T_h}\right), \text{ where } R_{bank} = \min\left(R_{buy}, \frac{H_b}{T_h}\right), \quad (21)$$

i.e. the buying rate  $R_{buy}$  subtracted by the house-buying rate from bank-owned-houses,  $R_{bank}$ , restricted by its rate of availability on sale. Also  $H_n$  can increase through construction captured by the following rate,

$$R_{con} = r_{lag} (R_{fam} - R_{av})^+, \quad (22)$$

where  $r_{lag}$  is a delay factor. The construction rate is responsible for increasing the number of available houses, as a response to increased demand. The resulting balance equation for the non-bank owned available houses is finally given as,

$$\frac{dH_n}{dt} = R_{sell} - R_{mkt} + R_{con}. \quad (23)$$

For bank owned houses  $H_b$ , its inflow rate is implied by the foreclosure rate  $R_f$  from the banking model, and its outflow rate is the house purchasing rate  $R_{buy}$ , restricted by its rate of availability on sale. Hence we have the following balance equation,

$$\frac{dH_b}{dt} = \frac{R_f + R_p}{P_0} - R_{bank} = \frac{R_f + R_p}{P_0} - \min\left(R_{buy}, \frac{H_b}{T_h}\right). \quad (24)$$

The occupied houses ( $H_o$ ) exchange with both bank-owned and non-bank-owned available houses,

$$\frac{dH_o}{dt} = R_{mkt} - R_{sell} - \frac{R_f + R_p}{P_0} + R_{bank} = R_{buy} - \min\left(\frac{H_o \cdot r_p}{T_{dur}}, \frac{P_{lim}}{P_h}\right) - \frac{R_f + R_p}{P_0}. \quad (25)$$

The number of families that do not own a house  $F_n$ , satisfies the following equation,

$$\frac{dF_n}{dt} = \min\left(\frac{H_o \cdot r_p}{T_{dur}}, \frac{P_{lim}}{P_h}\right) + \frac{R_f + R_p}{P_0} - R_{buy}, \quad (26)$$

Note that Equations (25) and (26) imply a closed, conserved system, with respect to the number of families in the housing market.

We will see that, by using subprime loan availability as a trigger, more buyers would be attracted towards purchasing houses. This will push the aggregate mortgage-backed asset value in the banking system higher, thus making the aggregate expected mortgage payment higher than the aggregate affordable payment. As a result, this leads to loan-defaulting and foreclosures, which lead further to cause bank insolvency. It demonstrates its vicious cycle in the whole system. Figure 4 shows the corresponding system dynamics model for house market.

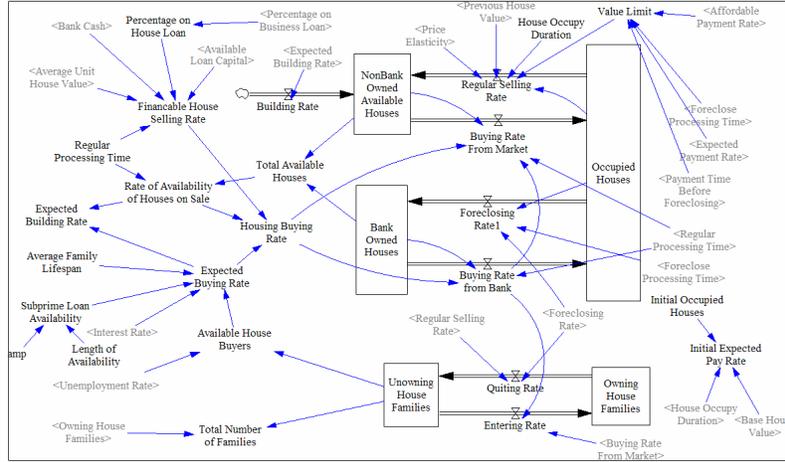


Figure 4: System Dynamics model for House Market

### 3.3. Economy model

The economy model includes gross production output per unit time-period, household income per unit time-period, and unemployment per unit time-period. The aggregated demand ( $D$ ) per unit time-period consists of overall consumption and business investment. The change in gross production output per unit time-period ( $G$ ) will be formulated as the following,

$$\frac{dG}{dt} = \frac{D - G}{T_{prod}}, \quad \frac{dM}{dt} = \frac{G - M}{T_{inc}}, \quad (27)$$

where  $D$  is the aggregated demand per unit time-period,  $M$  the household income per unit time-period and  $T_{prod}$  is the production time-constant. We assume that the household income is distributed among mortgage payment  $\min(R_m, R_a)$ , tax payment and overall consumption. Let  $r_t$  be tax rate and  $r_c$  be marginal propensity to consume, then tax portion is  $M_t = r_t * M$  and the minimum disposable portion is  $M_d = r_c * M$ . The affordable payment rate,  $R_a$ , is equal to,

$$R_a = M - M_t - M_d = (1 - r_t - r_c) \cdot M. \quad (28)$$

The aggregated demand consists of overall consumption  $D_{con}$ , the extent of business investment  $D_{inv}$  and government expenditure  $D_{gov}$ , all on a per unit time-period basis.

$$D = D_{con} + D_{inv} + D_{gov} = (M - M_t - \min(R_a, R_m)) + \alpha_{inv} \cdot R_{bi} + \alpha_{gov} \cdot M_t. \quad (29)$$

Note that in Equation (29), the amount of overall consumption  $D_{con}$  is roughly equal to the leftover amount, after tax and mortgage payments. The amount of business investment  $D_{inv}$  is proportional to business investments obtained from the banking model, and the amount of government expenditure  $D_{gov}$ , is proportional to the collected tax. We chose  $\alpha_{inv}, \alpha_{gov}$  in such a way that, initially  $D = M = G$ . Figure 5 shows the corresponding system dynamics model for economics.

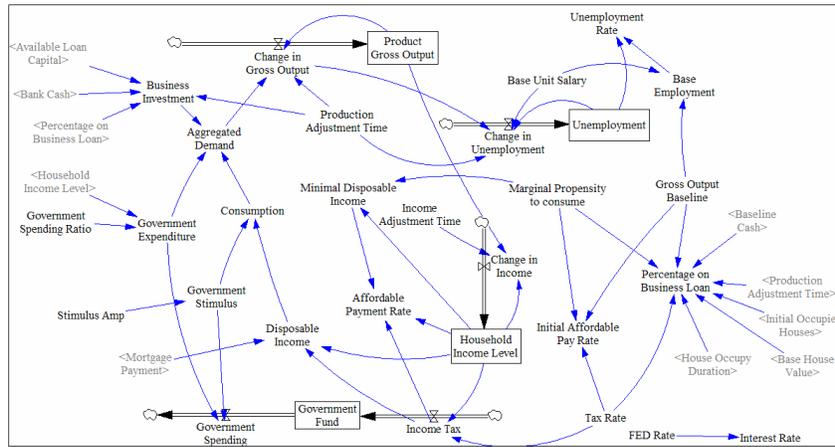


Figure 5: System Dynamics model of economics

## 4. Simulation Scenarios and Results

### 4.1. Steady State

We assign some values for all parameters in our system. Then we demonstrate that under proper initial values, an equilibrium (steady) state can be held, which also validates our model.

- a. Base unit house value (bank/house)  $P_o = 300K$
- b. Mortgage cycle time (bank)  $T_m = 240$  (months)
- c. House Occupy Duration (house)  $T_{dur} = 720$  (months)
- d. Business loan cycle time (bank)  $T_{blc} = 120$  (months)
- e. Production adjustment time (economics)  $T_{prod} = 3$  (months)
- f. House sell process time (house)  $T_h = 3$  (months)
- g. Average family lifespan (house)  $T_{fam} = 480$  (months)
- h. Portion used for business loan (bank)  $r_b = 40\%$
- i. Tax rate (economics)  $r_t = 28\%$
- j. Marginal propensity to consume (economics)  $r_c = 50\%$
- k. Income adjustment time (economics)  $T_{inc} = 3$  (months)

1. Interest Rate (bank)  $r_{int} = 4\%$

Initial values for stocks are chosen to have balanced in-and-out flows in order to satisfy an equilibrium state. Specifically, the banking model has a steady cash flow, the economy model has aggregated demand, gross production output and household income that are in balance, and the housing market model has balanced selling and buying rates.

- a. Occupied house (house)  $H_o(0) = 3,000,000$
- b. Bank owned houses (house)  $H_b(0) = 0$
- c. Non bank owned available houses (house)  $H_n(0) = H_o(0) * T_h/T_{dur} = 12,500$
- d. The number of families that do not own houses (house)  $F_n(0) = 2,170,000$
- e. Bank cash asset (bank)  $A_c(0) = \$10,000,000K$
- f. Short term security (bank)  $A_s(0) = 2 * A_c(0) = \$20,000,000K$
- g. Mortgage backed asset (bank)  $A_m(0) = H_o(0) * P_o * T_m/T_{dur} = 300,000,000K$
- h. Bank owned House Asset (bank)  $A_h(0) = H_b(0) * P_o = 0$
- i. Non-mortgage backed asset (bank)  $A_n(0) = r_b * A_c(0) * T_{ble}/T_{prod} = 160,000,000K$
- j. Bank liability (bank)  $L(0) = A_s(0) + A_m(0) + A_h(0) + A_n(0) = 480,000,000K$
- k. Product gross output (economics)  $G(0) = \$5,700,000K$
- l. Household income level (economics)  $M(0) = G(0) = \$5,700,000K$
- m. Unemployment rate (economics)  $r_{uem}(0) = 4\%$
- n. Subprime loan availability (economics)  $L_{sub} == 0$

For current setting, three rates defined in Section 2.2, will be

$$R_{fam} = \frac{2,170,000 * (1 - 0.04) * (1 - 0.04)}{480 * (1 - 0)} \approx R_{av} = \frac{12,500}{3} = 4,166 < R_{fin} = \frac{0.6 * 10,000,000}{300 * 3} = 6666 .$$

So supply matches demand and financial support is also available. We also have roughly the same rate  $R_{sell} = 3,000,000/720 = 4,166$ . The in-and-out flows are balanced for house market. Now let us look at the in-and-out flows for the mortgage-backed asset class in the banking model. The rates are,

$$P_h \cdot R_h = 300 * 4,166 = R_m = \frac{300,000,000}{240} = 1,250,000 < R_a = 5,700,000 * (1 - 0.28 - 0.5) = 1,254,000 .$$

The total amount lent by the aggregate bank balances with the total mortgage payment that is received. Further, the mortgage payment per unit time-period in the steady state is less than the affordable portion of household income per unit time-period. For economic model, the aggregated demand also matches gross production output ( $D=G$ ). As a result, the household income and unemployment rate will stay the same. We have an equilibrium state and an ideal stable system.

## 4.2. Subprime Mortgage Crisis

We perturb the system with an increase in the availability of subprime mortgage loans in order to study the resulting dynamics in each of the three subsystems

Specifically, we assume that there is subprime loan availability starting at the period 24 and lasting for two years as shown in the left of Figure 6. Based on our formulation, the incentives for home buyers lead to a higher expected buying rate.

$$R_{fam} = \frac{2,170,000 * (1 - 0.04) * (1 - 0.04)}{480 * (1 - 0.2)} = 5,208 > 4166$$

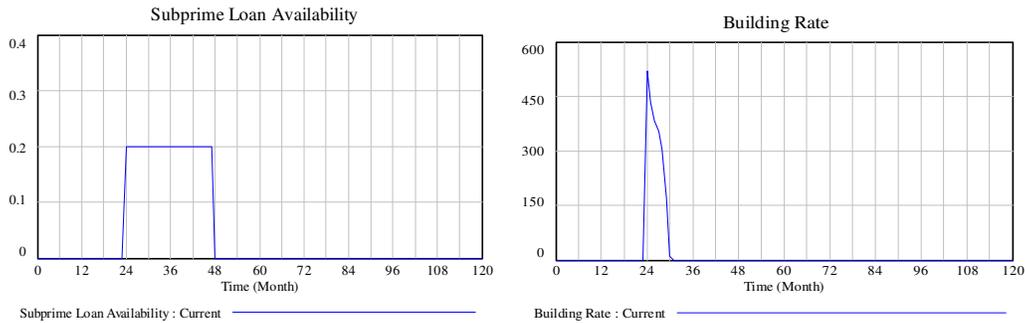


Figure 6: Subprime Load Availability and House Building Rate

Then the expected building rate becomes positive, due to  $R_{fam} > R_{av}$ . In turn, the  $R_{av}$  (the right of Figure 6) becomes higher to support the higher demand. In the beginning, the aggregate bank still has enough capital to loan ( $R_{fin}=6666$ ). This pushes the expected mortgage payment per unit time-period higher, and finally the expected mortgage payment amount reaches the affordable amount, as a part of household income. After that, the loan defaults start and house foreclosing begins.

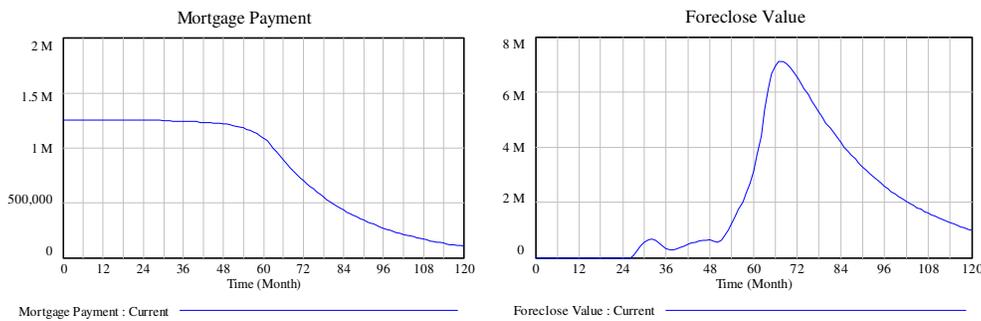


Figure 7: Mortgage Payment Drop and Foreclose Rise

Figure 7 shows the mortgage repayments drop and foreclosure rate rises starting at the period  $t = 27$  and becomes serious at period  $t=67$ . In fact, after houses are foreclosed, the house price begins to drop due to the number of available houses getting larger than the number of purchases that may be financed through the aggregate bank. As a result, the value of bank-owned-houses will drop, and bank total asset value will drop. Our aggregate bank gets into insolvency after period  $t = 62$  as shown in Figure 8.

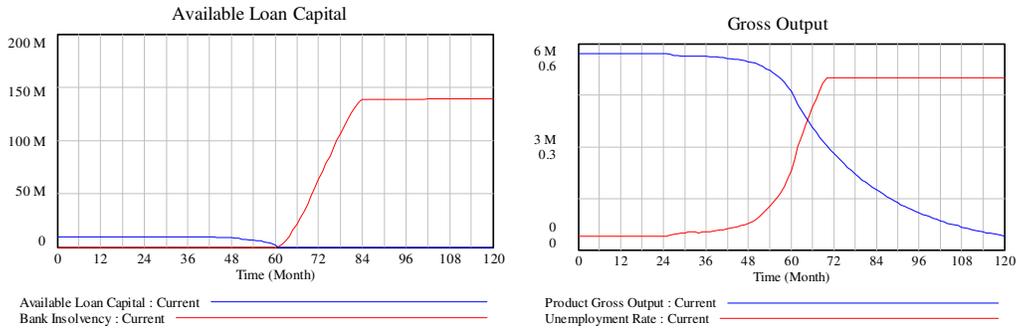


Figure 8: Available Loan Capital and Gross Production Output Drop

At the same time, business investment will drop, which leads to drop in gross production output as well as household income. As a vicious cycle, affordable payment drops further, equivalently, mortgage repayment drops further. Of course, what we demonstrate is the worst scenario without any intervention. The vicious cycle and chain reactions lead the whole economy towards a systemic disaster situation.

### 4.3. Economical Stimulus

We investigate the impact of a corrective action, such as intervention from the government in the form of an economic stimulus. For example, the government issues stimulus money to families to drive consumption higher, or provides financial help for troubled businesses by acquiring an ownership stake, etc.

In our economic model, we introduce a government stimulus, that is added to aggregated demand starting at a period  $t = 42$  and lasting 12 months with magnitude of 863K as shown in the left of Figure 9.

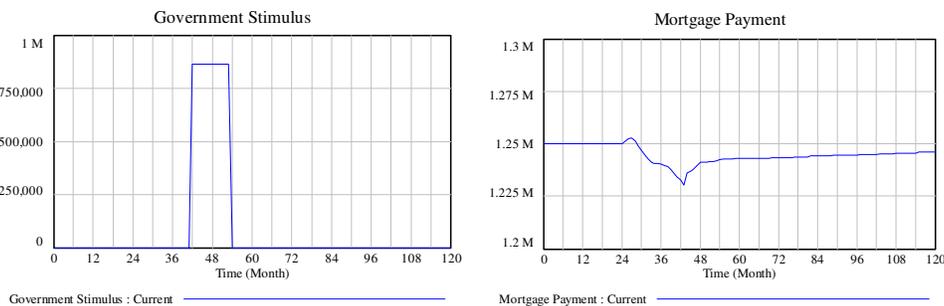


Figure 9: Government Stimulus and Mortgage Repayment

Its effect is shown in the right. The mortgage repayment per unit time-period returns to a stable level. In order to maintain the over-expanded housing market, it is necessary to have a relatively higher gross production output (relative to the original steady state), in order to reach a new equilibrium state. This means that the size of the economic stimulus needs to be larger than a critical lower threshold in order to enable a new equilibrium (steady) state; else the stimulus effort will not achieve its desired goal of turning downward spiral system around. Figure 10 shows the corresponding curve in gross production output. Note that, there will be another negative consequence of government stimulus, i.e. government reaches a new deficit level, also

shown in Figure 10. Addressing the issue of how government can remedy this situation is beyond the scope of this article.

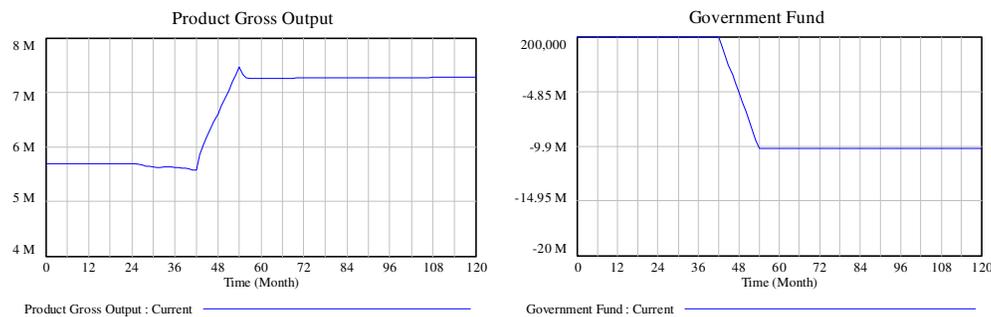


Figure 10: Gross Production Output after Stimulus and Government Financial State

## 5. Conclusion and Discussion

We use system dynamics to model three key systems that are relevant to the subprime mortgage crisis. The causal relationships in each system and across systems are captured and qualified in the model. We expose the physical conservation laws in the evolution of the subprime movement. We validate the model by simulating three scenarios. An equilibrium state exists in a normal condition with a steady cash flow in the aggregate bank, matched supply and demand in the aggregate housing market and balanced gross production output with consumption in the economy.

Mortgage crisis in our model originates with a spike in the subprime loan availability and climaxes with a vicious cycle and chain reactions in multiple causal loops. Economic stimulus through boosting aggregated demand improves the gross output in economics, at the same time, the total asset value of bank returns to a healthy state and an over-expanded housing market is recovered. Of course, our study mainly concentrates to expose physics of the system and to demonstrate qualitative behavior of evolution. Calibration of the simulated result with some observable data is desirable in future work.

Our model is built on an aggregated level and involves multiple time-constants that can be interpreted as long-time average duration values of various processes. It is possible to use an agent based model to simulate multiple heterogeneous and autonomous participants in the system. In such a setting, each individual family may be modeled with independent economical status and different loaning history. Each bank has its own characteristic (commercial or retailer bank) in its portfolio of assets. Each region has its own economic situation and dependence. The underlying fundamental physics could be the same except for including interaction among families and banks themselves.

The model can also be extended to include some negative feedback loops corresponding to mitigation actions and regulations. The robustness of the new equilibrium (steady) state resulting from corrective measures can be studied through systematic eigen-value analysis around the resulting steady state.

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