Supposing a Control Law of Capital Accumulation for the Modern Italian Economy

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Abstract

This paper formulates a hypothetic law of capital accumulation (HL) for modern Italian economy mainly owing to three analytical devices.

The first handles the so-called Verdoorn law. HL reconciles a direct relation between growth rates of net output and labour productivity with an inverse relation between growth rates of employment ratio and labour productivity. The second advances a 'Ricardian' view of an inverse relationship between growth of employment and returns. The third transforms constant profit investment share into a secularly declining endogenous variable.

This paper explores analytically and numerically inertia Scenario I and two stabilizing Scenarios II and III of the Italian economic development in XXI century and beyond. In inertia Scenario I, capital accumulation is marked by long swings with a period of about 20 years. Decelerating adjustment of profit investment share to its stationary magnitude depending on profitability would be stabilizing for long swings without altering a non-trivial stationary state. Establishing an inverse relation between profit investment share and capital-output ratio in a control law (CL) not only smoothes long swings but slightly raises stationary profitability in stabilization Scenario II above stationary profitability in Scenario I. Stabilization Scenario III exposes fallacy of the neoclassical golden rule of accumulation.

1. Introduction

Continuing an exploration of the economic law of motion of modern society this paper presents a rather synthetic Goodwinian model. As known Goodwin's model (Goodwin, 1972) has stimulated a huge amount of contributions. Papers that are significantly related to the focus of the present one are: Shah and Desai (1981), van der Ploeg (1985), Manfredi and Fanti (2000), Fanti and Manfredi (2003). Still an encompassing model has not been constructed so far. This paper contributes to filling this gap.

As known, a classical way of extending Goodwin's model is including factors of productivity growth. The extensions previously proposed go after two main lines. The first one, following the famous Kaldor –Verdoorn law, links the productivity gains to the growth rate of production (e.g. Lordon, 1997, Boggio, 2006). The second one assumes a technical change mechanism which relies on mechanization and the diffusion of knowledge embodied in equipment, and is therefore driven by the stock of fixed capital per labourer (e.g. van der Ploeg, 1987). This paper attempts to refine and blend both.

This paper ascertains that the form of dominance of positive returns to scale, typical for the US economy, is not a characteristic of the Italian economy. Therefore the stabilization policies supposed for the modern USA in (Ryzhenkov 2005a, 2005b, 2005c, 2007), would be a debacle for modern Italy.

The progressing modelling of the US economy is helpful in re-formulating a hypothetic law of capital accumulation (HL) for the modern Italian economy mainly owing to three innovative clarifications. The first handles the so-called Verdoorn law that connects empirically the growth rate of labour productivity positively and linearly with growth rate of net output.[1] The second sheds light on a 'Ricardian'

[1] The empirical Verdoorn law is to be connected with a prior Marx idea that the price of reproducible commodity depends on the productivity of labour, and this in turn, – on the scale of production.

view of an inverse relationship between growth of employment and returns during a long swing in Italy because of different levels of efficiency of technologies and labour forces. The third transforms a profit investment share into a secularly declining endogenous variable. This decline serves for stabilizing profitability.

The rest of this paper, besides mandatory references and conclusions, is organised in the following manner.

Chapter 2 re-formulates the hypothetical law (HL) of capital accumulation for the modern Italian economy. Clarifications are mainly achieved by reasonable alterations of the former technical progress and mechanisation functions that enable a generalization of the Verdoorn relation and presentation of the HL deterministic form as a generic model. Assuming a 'Ricardian' effect during a long swing, HL does not imply anti-cyclical behaviour of labour productivity.

Chapter 3 transforms a deterministic form of HL into probabilistic. Its non-observable parameters are identified through application of a simplified version of the extended Kalman filtering (EKF) to macroeconomic data over a basal period 1980–2004. It verifies statistically re-formulated HL that has not been so far refuted for the Italian economy.

A beginning of Chapter 4 explores analytically and numerically inertia Scenario I and its two modifications. Chapter 4 also presents two stabilizing Scenarios II and III of possible Italian economic development in the XXI century and beyond.[2]

The investigation uncovers deeper meaning of 'Ricardian' effects for long swings with periods of about 20 years in inertia Scenario I. An unstable non-trivial stationary state is analysed. A stationary general profit rate is independent of an adjustment speed of profit investment share. Exogenous increases in a stationary labour productivity growth rate can raise a stationary employment ratio and relative labour compensation while a stationary profitability remains thereby intact.

The first modification of inertia Scenario I assume constancy of the profit investment share in the projection period that fosters the employment ratio and relative labour compensation but worsens profitability. The second modification of inertia Scenario I reveals that slower adjustment of the profit investment share smoothes long swings over centuries but not over the initial decades of the projection period.[3] This drawback motivates a design of a more efficient stabilization policy in the same Chapter.

Stabilization Scenario II is based on a surmised control law of capital accumulation (CL) for maintaining profitability and diminishing amplitude of long swings. CL establishes an explicit inverse relation between profit investment share and capital-output ratio. Strengthening this relation not only smoothes long swings but raises the stationary profitability. On the other hand, an exogenous increase of a stationary growth rate of labour productivity is negative for the latter. So due to the 'Ricardian' character of the Italian labour force exogenous increments in labour productivity will be detrimental for profitability and will be, likely, ruled out by capital. Conditions for Andronov – Hopf bifurcations are found analytically for CL, including critical magnitudes of a key parameter of a new technical progress function. Simulations reveal that they are supercritical.

A well-known neoclassical golden rule of accumulation has served as a heuristic tool for stabilization Scenario III. Exploration of global behaviour in this scenario is based on simulation runs. They demonstrate that a dramatic long-term decline in profitability is a condition for stable economic growth under this rule that capital cannot accept. Therefore the neoclassical golden rule of accumulation is not practical outside the ideological area.

The given analysis helps to validate HL and CL that could be useful in controlling structural crises. This study extends macroeconomic applications of system dynamics method and deepens a constructive

[2] The huge time horizons of some presented computer simulations, compared with that in a standard econometric work, do not mean that this paper pretends to predict or forecast dynamics in the respective intervals. The broad intervals of time serve mostly for testing properties of HL and of regulations based on this law, first, in the historical region of the model variables, second, outside it (especially in extreme condition tests).

[3] Terms general profit rate and profit rate are synonymous throughout this paper.

critique of the neoclassical conceptions of growth and distribution. This investigation also uncovers and explains profound structural changes that may reinforce viability of the Italian economy.

2. Re-formulating Hypothetical Law (HL) of Capital Accumulation for Italy

On the one hand, there has been a long tradition, going back to Adam Smith (1776) and Marx (1867), that division of labour and technological progress are somehow intrinsically associated with increasing returns. On the other hand, neoclassical economists emphasise importance of constant or decreasing returns. There is still remaining substantial ambiguity in notion of economy of scale (or increasing return as the synonym) in the modern macroeconomic literature (Weizsäcker 1993). This Chapter moderates this ambiguity conceptually. The statistics for Italy over 1980–2004 serve as empirical base for re-formulating HL paying attention to scale factors and not forgetting a need for refining the 'Ricardian' inverse relationship between growth of employment and returns. Besides that, profit investment share becomes a crucial endogenous variable, dependence of its time derivative on a difference between observed and target profit rates may be used for stabilizing profitability.

2.1. The premises

The advanced capital does not include variable capital since labourers advance capitalists. HL abstracts from capital of circulation. Natural capital is not taken into explicit account in this paper. The capitalist economy is not explicitly restricted by natural resources. The other important premises are such:

(1) two social classes (capitalists and labourers); the State enforces the property rights, yet costs of such an enforcement are not treated explicitly;

(2) only two factors of production, labour force and reproducible fixed assets, with groups and subsystems of different qualities;

(3) only one good is produced for consumption, investment and circulation purposes, its price is identically one;

(4) production (supply) equals effective demand;

(5) all labour compensation consumed, profits are partially saved and invested;

(6) steady growth in the labour force that is not fully employed;

(7) a growth rate of unit real labour compensation rises in the neighbourhood of full employment;

(8) a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time;

(9) during a long swing inputs with different techno-economic efficiencies are used;

(10) labour qualification matches technological requirements;

(11) fixed assets and labour forces are essentially complementary to each other and are also substitutes to some degree depending directly on unit labour value and on employment ratio;

(12) profit investment share shrinks for counter-acting a tendency of general profit rate to fall.

The product-money identity and the supply-demand equivalence stated in the third and fourth assumptions do not mean that this model abstracts from the two-fold character of labour embodied in commodities entirely. It mirrors the twofold nature of labour power, the unity and contradiction of its value and use-value. The creative functions of labour market as an instrument for transmitting impulses to economic change are the focal point of this model.

The model does not describe the formation of real income of the unemployed persons.[4] It supposes that a part of labour compensation covers indirectly the needs of the unemployed. The latter do not play an active role in the model economy. Social security contributions and benefits are not explicitly shown.

The model omits Goodwin's assumptions of fixed capital-output ratio and constant profit investment share, but preserves his premise of the supremacy of production over final demand (Goodwin,

[4] This problem is addressed in Fanti and Manfredi (2003).

1972). This assumption abstracts from the relative independence of final demand and changes in a product mix. It is more acceptable for the long-run as for the short-run: although in the shorter run aggregate demand influences output, in the very long run output dominates over demand. Capital adapts the output to the scale of production.[5]

The assumption (6) means that the labour force grows exponentially over time. This assumption may be substituted by an assumption of a logistic growth or by another hypothesis (Ryzhenkov, 2005a). The assumption (5) is rather abstract and can be relaxed assisted by (van der Ploeg, 1984). The assumption (5) corresponds to the immediate aim of capitalist production. Capital produces surplus product and profit as a monetary form of surplus-value.

Two working hypotheses make the assumption (12) operational below (Section 2.2 and sub-section 4.2.1). The first assumes dependence of accumulated share of profit (respectively, of surplus product and of surplus-value) on deviations of profitability from a target magnitude. The second relates the profit investment share linearly and negatively with the capital-output ratio.

The assumptions (2) and (9) do respect a 'Ricardian' view on utilization of the inputs including a 'Ricardian' type of labour force, as was argued to be the case for the Italian economy in the early paper by De Cecco (1972).[6]

The next peculiarity of this model is that it has only implicit delays. An explicit investment delay is set aside.[7]

2.2. An extensive deterministic form of HL

A deterministic model is formulated in continuous time. A dot denotes a time derivative, while a hat indicates a proportional growth rate. This model consists of the following equations:

$$P = K/s; \tag{1}$$

$$L = P/a; \tag{2}$$

$$u = w/a; \tag{3}$$

$$\hat{a} = h_1 + h_2 \widehat{K/L} + m\psi(\hat{v}), \tag{4}$$

$$\psi(\hat{v}) = \hat{v},$$

$$h_1 > 0,\ \ 1 > h_2 > 0,$$

$$m = 0, \text{ if } v = 0,\ m > 0, \text{ if } 0 < v < v_{\min},$$

$$m \leq 0, \text{ if } 1 > v \geq v_{\min} > 0;$$

$$\widehat{K/L} = n_1 + n_2 u + n_3(v - v_c), \tag{5}$$

$$n_1 < 0,\ \ n_2 > 0,\ n_3 < 0,\ \ 1 > v_c > 0;$$

$$v = L/N; \tag{6}$$

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[5] Capital-output ratio is variable while short run aggregate demand influences output in a model of capital accumulation and income distribution presented in Fanti (2001). In my view, this model is logically contradictory: net output produced is systematically lower than net output finally used. That paper does not explain a difference between the latter (net output finally used) and the former (net output produced), which is essencially an economic surplus with an order of quantity of distributive shares themselves. It could be demonstrated by request.

[6] De Cecco (1972) argued, in particularly, that, especially when investments are low (as in Italy in the 1960-s), firms seek to obtain productivity gains by choosing the most efficient component of the labour supply and exclude the other components. This reference has been prompted by a citation in (Fanti, 2001).

[7] This problem is addressed by Fanti and Manfredi (1997).

$$\hat{N} = n \geq 0\,; \tag{7}$$

$$\hat{w} = -g + rv + b\widehat{K/L},\ \ g > 0, r > 0; \tag{8}$$

$$P = Q + \dot{K} = wL + (1-k)M + \dot{K}\,; \tag{9}$$

$$\dot{K} = k[(1-u)P],\ 1 \geq k > 0; \tag{9a}$$

$$\dot{k} = c_2\left(\frac{1-u}{s} - p_w\right), c_2 \geq 0,\ p_w > 0\,. \tag{10}$$

Equation (1) postulates a technical-economic relation between the advanced fixed capital (K), net output (P) and capital-output ratio (s). Equation (2) relates labour productivity (a), net output (P) and labour input, or employment (L). Equation (3) describes the relative labour compensation, or unit value of labour power (u), as a ratio of real labour compensation (w) to labour productivity.

Equation (4) is an extended technical progress function (TPF).[8] It includes: the rate of change of capital intensity, K/L and direct scale effect, $m\hat{v}$. For Italy for 1980–2004 with its 'Ricardian' labour force, a weak hypothesis is that $m < 0$, a stronger one is $m < -1$.

Equation (6) outlines the rate of employment (v) as a result of the buying and selling of labour power. In the equation (8), the rate of change of the real labour compensation rate (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rate of change of capital intensity (K/L) additionally. The capital intensity (K/L) is a proxy for labourers' qualification.

Mechanisation (automation) manifests itself in growing capital intensity. The rate of change of capital intensity (K/L) in the equation (5) is a function of the relative labour compensation (u), difference between the real employment ratio (v) and some base magnitude (v_c) that is, likely, lower than stationary employment ratio (v_a) defined below for the Italian case (Section 3.2). A high relative labour compensation promotes mechanization (automation).

Ricardo and Marx wrote that machinery is in constant competition with labour and can often be introduced when unit value of labour power has reached a certain height. This idea was applied by the Glombowski – Krüger mechanisation function (Glombowski and Krüger, 1984) that is a special case of the equation (5) for $n_3 = 0$. The equation (5) represents its generalization, which follows from the broader assumption (11) that connects the growth rate of capital intensity (technical composition of capital) with employment ratio (v) besides unit value of labour power (u). The negative sign of the latter parameter ($n_3 < 0$) presents a specific aspect of the Italian 'Ricardian' factor inputs.

While our assumption with respect to the dependence of the productivity growth rate on the capital intensity (4) and the mechanization function (5) are based on the tradition of the Kaldorian technical progress function and the Marxian mechanism, respectively, the assumption stating the dependence of the productivity growth rate on the employment growth rate represents a generalization of the Verdoorn relationship between output and labour productivity growth and has an empirical support for Italy.

According to the equation (7), the growth rate of labour force (N) is a non-negative constant (n). A working hypothesis on endogenous labour supply that connects n with capital intensity (K/L) is introduced and supported by US data in (Ryzhenkov 2005a, 2005b, 2005c).

Turn our attention to the equations (9) and (9a). Here the net formation of fixed capital is $\dot{K}$, a sum of net export, final private and public consumption is Q, a total profit in real terms is $M = (1-u)P$. The equation (10) defines (for $c_2 > 0$) a proportional control over the profit investment share, whereby its

[8] The requirement $m = 0$ if $v = 0$ is 'technically' necessary for avoiding possibility of a stationary state with positive output-capital ratio and zero employment ratio. On economic grounds, this variable has only positive magnitudes ($v > 0$) for the capitalist economy (necessarily with the labour market and labour inputs, $L > 0$).

time derivative depends positively and directly on a difference between observed and target (p_w) levels of profitability. An extended Kalman filtering identifies this target level for the basal period.

2.3. An intensive deterministic form and non-trivial stationary state of HL

The deterministic model in an intensive form, based on the equations (1) – (10), consists of the four non-linear ordinary differential equations (10) and (11) – (13):

$$\dot{s} = \{-h_1 + (1-h_2)[n_1 + n_2 u + n_3(v - v_c)] - m\hat{v}\}s, \tag{11}$$

$$\dot{v} = \{k\frac{1-u}{s} - [n_1 + n_2 u + n_3(v - v_c)] - n\}v, \tag{12}$$

$$\dot{u} = \{-g + rv - h_1 + (b - h_2)[n_1 + n_2 u + n_3(v - v_c)] - m\hat{v}\}u. \tag{13}$$

Figure 1 presents a little bit compacted overall causal loop diagram for the basal model. It contains four main stocks (corresponding to the state variables k, s, v and u) and four main flows (corresponding to the time derivatives of these state variables).

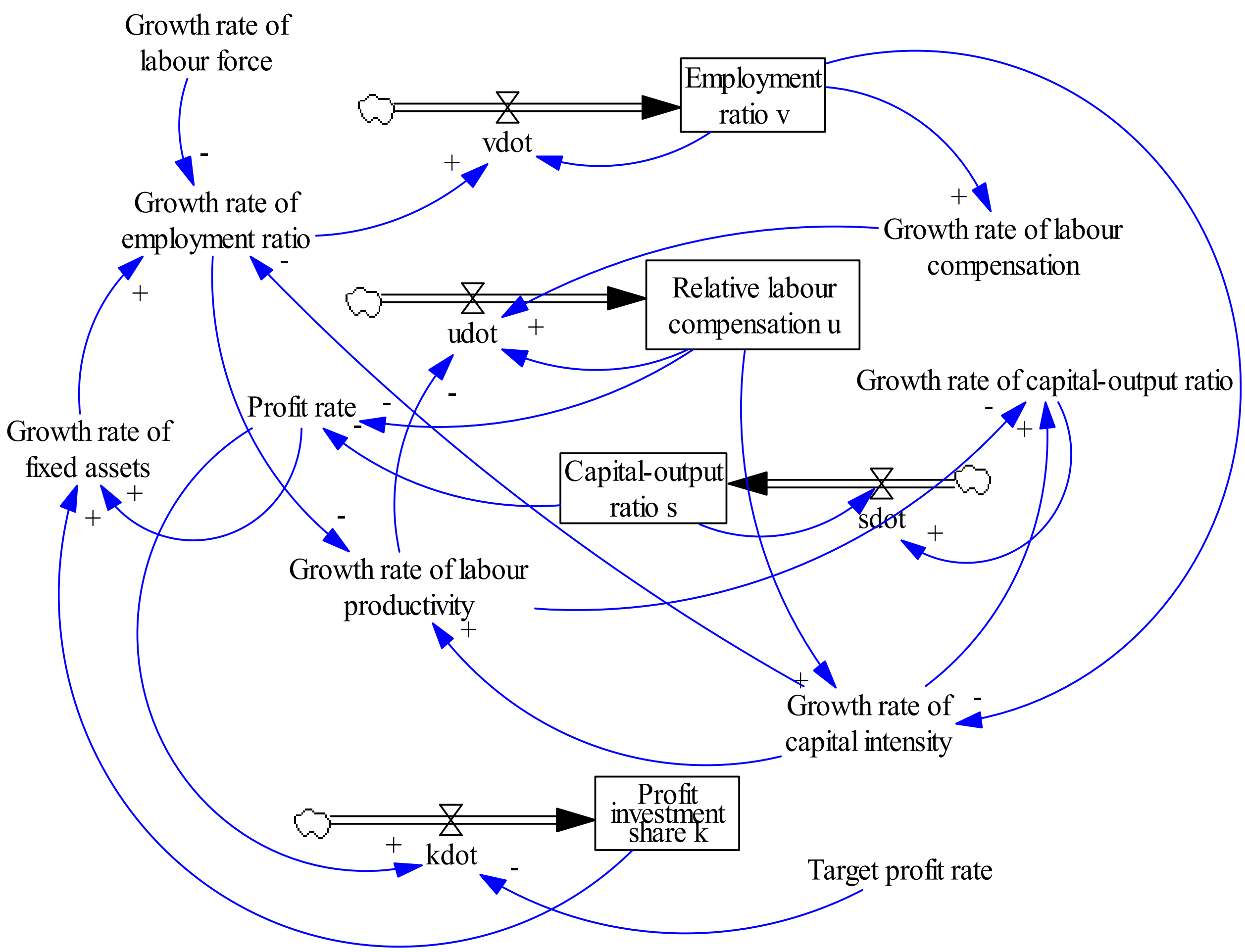

Figure 1. A condensed causal loop diagram of HL

For $c_2 \neq 0$ the system (10)–(13) has a non-trivial stationary state

$E_a = (k_a, s_a, v_a, u_a)$, (14)

where $k_a = \dfrac{d}{p_w}$, $s_a = \dfrac{1-u_a}{p_w}$, $v_a = \dfrac{g+(1-b)(d-n)}{r}$, $u_a = \dfrac{d-n-n_1-n_3(v_a-v_c)}{n_2}$, $d = \dfrac{h_1}{1-h_2} + n$.

A stationary growth rate of fixed capital and net output is: $\hat{K}_a = \hat{P}_a = d$; a stationary growth rate of real labour compensation, labour productivity and capital intensity is $\hat{w}_a = \hat{a}_a = K_a \hat{/} L_a = d - n$. A stationary general profit rate is $\dfrac{1-u_a}{s_a} = p_w$, a stationary growth rate of labour force and employment is $\hat{N}_a = \hat{L}_a = n$. For reasonable parameters' magnitudes the following economic requirements are satisfied: $s_a > 1$, $0 < v_a < 1$, $0 < u_a < 1$, $d > 0$, $d - n > 0$. We assume below that these inequalities are true unless violated for some specific reason(s).

For $c_2 = 0$ and $k \equiv k_0$ the system (10)–(13) is reduced to the system of the equations (11) – (13). The latter has a non-trivial stationary state

$E_g = (s_g, v_g, u_g)$, (14a)

where $s_g = k_0 \dfrac{1-u_g}{d}$, $v_g = \dfrac{g+(1-b)(d-n)}{r} = v_a$, $u_g = \dfrac{d-n-n_1-n_3(v_a-v_c)}{n_2} = u_a$.

A stationary general profit rate is $(1 - u_g)/s_g = d/k_0$. As for the stationary state E_a, the stationary growth rate of fixed capital, net output is: $\hat{K}_g = \hat{P}_g = d$, a stationary growth rate of real labour compensation, labour productivity and capital intensity is $\hat{w}_g = \hat{a}_g = K_g \hat{/} L_g = d - n$, the stationary growth rate of labour force and employment is $\hat{N}_g = \hat{L}_g = n$.

Consider now dependence, if any, of the non-trivial stationary state on the two key parameters m of the TPF (4) and n_3 of the mechanisation function (5) that may be likened to music counterpoints. First, become aware that changes in m produces no influences on this state (14). Second, notice a restricted impact on it of a shift of the key parameter n_3. The stationary employment ratio (v_a), stationary growth rates of labour productivity and of net output, stationary profit rate are independent of this parameter. Still for $v_a > v_c$ that is probable for the Italian economy, the stationary relative labour compensation u_a depends on n_3 linearly and negatively

$$\frac{\partial u_a}{\partial n_3} = -\frac{v_a - v_c}{n_2} < 0, \tag{15}$$

whereas stationary capital-output ratio s_a – linearly and positively:

$$\frac{\partial s_a}{\partial n_3} = \frac{k_a(v_a - v_c)}{dn_2} > 0. \tag{16}$$

An exogenous growth in the stationary growth rate of labour productivity $\hat{a}_a$ benefits the stationary employment ratio v_a:

$$\frac{\partial v_a}{\partial \hat{a}_a} = \frac{1-b}{r} > 0. \tag{17}$$

An exogenous growth in the stationary growth rate of labour productivity $\hat{a}_a$ enhances the stationary relative labour compensation if

$$\frac{\partial u_a}{\partial \hat{a}_a} = \frac{r-(1-b)n_3}{rn_2} > 0. \tag{18}$$

We see that exogenous growth in the stationary growth rate of labour productivity $\hat{a}_a$ raises the stationary relative labour compensation u_a if the following condition is satisfied:

$$n_3 < \frac{r}{1-b}. \tag{19}$$

The condition (19) was probably valid for the Italian economy as shown below (Section 3.2), unlike neoclassic models that have an opposite property. If this condition is true, then a drop in the stationary capital-output ratio follows from increases in the stationary growth rate of labour productivity since

$$\frac{\partial s_a}{\partial \hat{a}_a} = \frac{\partial s_a}{\partial u_a}\frac{\partial u_a}{\partial \hat{a}_a} = -\frac{1}{p_w}\frac{r-(1-b)n_3}{rn_2} < 0. \tag{20}$$

The stationary general profit rate p_w does not depend on the stationary growth rate of labour productivity $\hat{a}_a$.

3. A Historical Fit of HL for the Italian Economy in the Basal Period 1980–2004

3.1. A probabilistic form of HL

For taking into account measurement errors and an impact of factors neglected in the model assumptions, the deterministic model (10) – (13) has been transformed into a stochastic model. This makes implicit allowances for short-term and middle-term economic fluctuations by specification of the random components. The latter model includes state equations and measurement equations for discrete moments of time

$$\mathbf{x}(t) = \mathbf{f}\,[\mathbf{x}(t-1)] + \mathbf{w}(t), \tag{21}$$

$$\mathbf{z}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{v}(t), \tag{22}$$

where $t = 1, 2,\ldots, T$ is an index of data samples, $\mathbf{x}(0)$ – a vector of an initial state of the system, $\mathbf{w}(t)$ – a vector of equations errors (driving noise), $\mathbf{v}(t)$ – a vector of measurement errors. The deterministic part $\mathbf{x}(t) = \mathbf{f}[\mathbf{x}(t-1)]$ corresponds to the system (10) – (13). The symbol $\mathbf{H}$ is for a rectangular matrix. The residuals are not due entirely, or largely, to pure random influences. On the contrary, these residuals contain highly systematic, non-random components.

This paper applies a simplified version of an extended Kalman filtering (EKF), realised in the Vensim software developed by Ventana Systems, Inc. This software has enabled to estimate magnitudes of the unobservable parameters of probabilistic HL by a procedure of maximum likelihood.

3.2. Parameters' magnitudes identified

The main variables have the following units of measurement: a [millions of chained 2000 euros per working hour], u and v [fraction of unit], s [years]. Calculations of $s = K/P$ are done with the nominators and denominators measured in 2000 prices. The employment ratio v is for the civil labour force (without accounting hidden unemployment). Private and governmental produced non-residential fixed assets present fixed capital. Simulation runs have used the observed magnitudes for the initial year (1980) posted in Table 1. Data sources are Economic Report of the President (2007) and Timmer et al. (2005).

An application of the EKF to the Italian macroeconomic data for the basal period 1984–2004 has identified probable, sub-optimal magnitudes of unobservable parameters of the above probabilistic model (Section 3.1): $b = 0$, $c_2 \approx 0.519$, $g \approx 0.233$, $h_1 \approx 0.01$, $h_2 \approx 0.071$, $m = m_i \approx -1.255$, $n_1 \approx -0.104$, $n_2 \approx 0.194$, $n_3 \approx -0.109$, $r \approx 0.263$, $n \approx 0.005$, $d \approx 0.016$.

There are important qualitative differences with the identified parameters for Italy and for the US (cf. Ryzhenkov 2005a) presented in Table 2.

Table 1. Initial, average magnitudes of main variables in the basal period and their stationary magnitudes in three different scenarios of Italian economic development

	Profit investment share (k)	Capital-output ratio (s)	Employment ratio (v)	Relative labour compensation (u)	Profit rate ($(1-u)/s$)
Initial for the year 1980	$k_0 \approx 0.2$	$s_0 \approx 1.837$	$v_0 \approx 0.920$	$u_0 \approx 0.612$	$(1-u_0)/s_0 \approx 0.191$
Average for 1980–2004	$k_{\text{mean}} \approx 0.127^{\spadesuit}$	$s_{\text{mean}} \approx 1.639$	$v_{\text{mean}} \approx 0.917$	$u_{\text{mean}} \approx 0.661$	$[(1-u)/s]_{\text{mean}} \approx 0.2064$
Stationary in inertia Scenario I and in its second modification	$k_a \approx 0.072185$	$s_a \approx 1.861$	$v_a \approx 0.928 > v_c \approx 0.915$	$u_a \approx 0.600$	$p_w \approx 0.2148$
Stationary in the first modification of inertia Scenario I$^{\clubsuit}$	$k = k_0 \approx 0.103 < k_{\text{mean}}$	$s_g \approx 2.660 > s_a > s_b$	$v_g = v_a \approx 0.928 > v_c \approx 0.915$	$u_g = u_a \approx 0.600$	$(1-u_g)/s_g = d/k_0 \approx 0.150 < p_w$
Stationary in stabilization Scenario II	$k_b \approx 0.071962 < k_a < k_0$	$s_b \approx 1.856 < s_a < s_g$	$v_b = v_a = v_g \approx 0.928 > v_c \approx 0.915$	$u_b = u_a = u_g \approx 0.600$	$(1-u_b)/s_b > p_w \approx \approx 0.2154$
Stationary in stabilization Scenario III	$k_r = 1 >> k_0 > k_a > k_b$	$s_r \approx 25.797 >> s_g > s_a > s_b$	$v_r = v_a = v_b = v_g \approx 0.928$	$u_r = u_a = u_b = u_g \approx 0.600$	$(1-u_f)/s_f = d \approx 0.0155 << p_w$

♠ For 1981–2004. ♣ For 2004 and later years.

Table 2. Comparison of HL realizations for Italy and the USA

Condition or equation	Italy, 1980–2004	USA, 1969–2002
technical progress function (4)	$\psi(\hat{v}) = \hat{v}$	$\psi(\hat{v}) = sign(\hat{v})\lvert\hat{v}\rvert^{j}, 0 < j < 1$
	$m < -1$	$m > 0$
mechanisation function (5)	$n_3 < 0$	$n_3 > 0$
labour force equation (7)	$n = const > 0$	$n \geq 0$ is a function of capital intensity
generalised Phillips equation (8)	$b = 0$	$1 > b > h_2 > 0$
profit investment share equation (10)	$\dot{k}$ is a function of profitability	$k = const > 0$
inequality (19)	valid	not valid

3.3. Targeting profitability as stabilization policy instrument

Turn attention again to the intensive deterministic form and non-trivial stationary state of HL. It was found that the structure of HL contains, in particular, eight feedback loops of growth rate of labour productivity (seven negative and one positive) that do not include the time derivative of profit investment share, and six additional feedback loops that include the latter (five negative and one positive).

There are also ten additional feedback loops of employment ratio that do not contain the growth rate of labour productivity, six of them do not include the time derivative of profit investment share (one with a variable sign of the growth rate of employment ratio, four negative and one positive), the other four include the time derivative of profit investment share (two positive and two negative).

Feedback loops containing *k* are of the second, third and fourth order depending on a number of the level variables in them. We see that the time derivative of profit investment share is a significant element of the model structure, although not all feedback loops contain it.

The control over the profit investment share according to the equation (10) is stabilizing as Figures 2–4 demonstrate. Not all feedback loops (negative and positive) with an order higher than two are presented. The negative polarity of the three presented feedback loops is due to the negative magnitude of the parameter *m* from the TPF (4) that is probable in the Italian economy case.

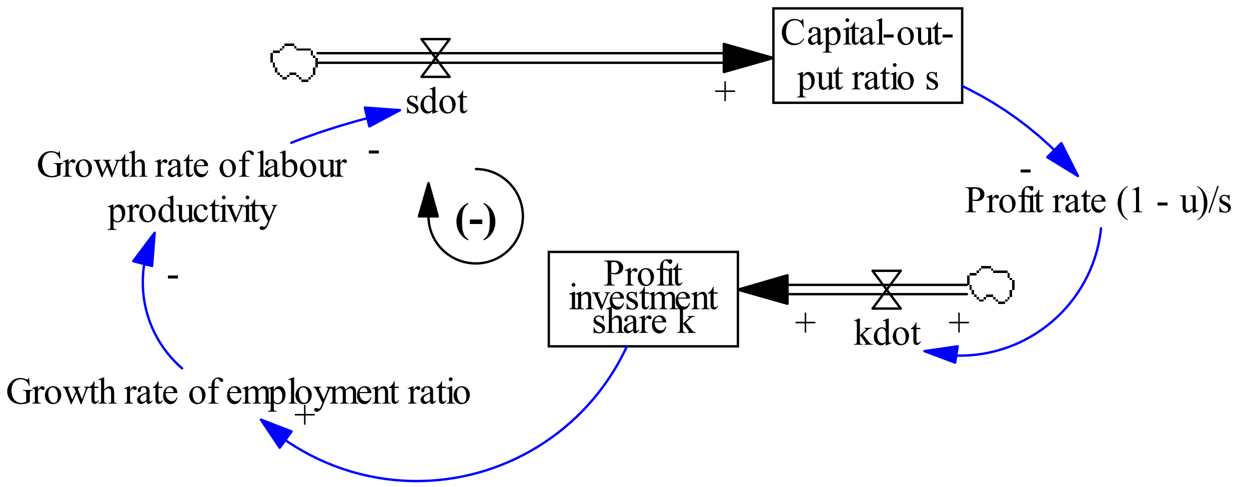

Figure 2. A first negative feedback loop of the second order (for $m < 0$) controlling profitability

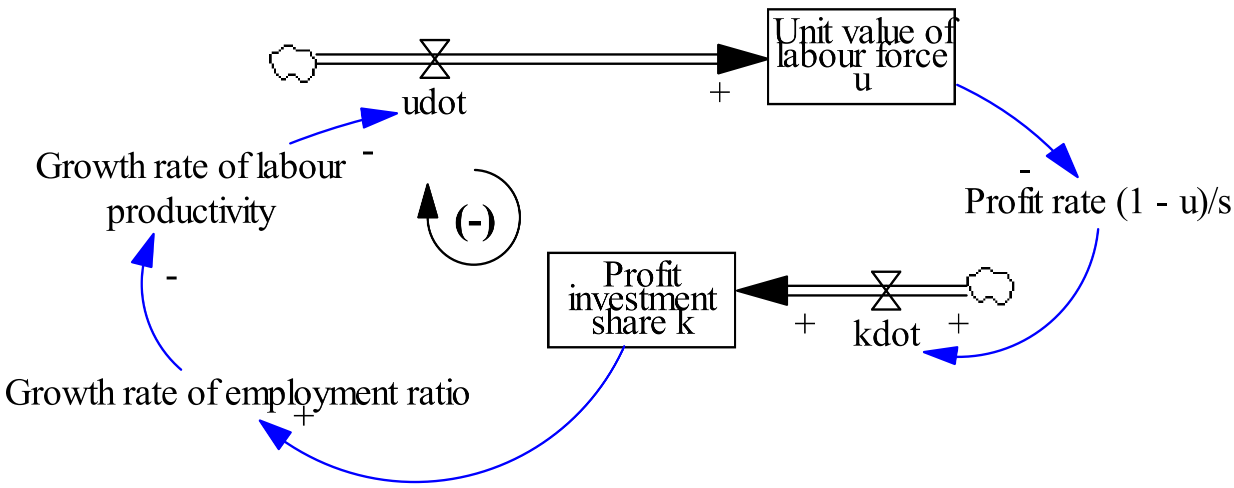

Figure 3. A second negative feedback loop of the second order (for $m < 0$) controlling profitability

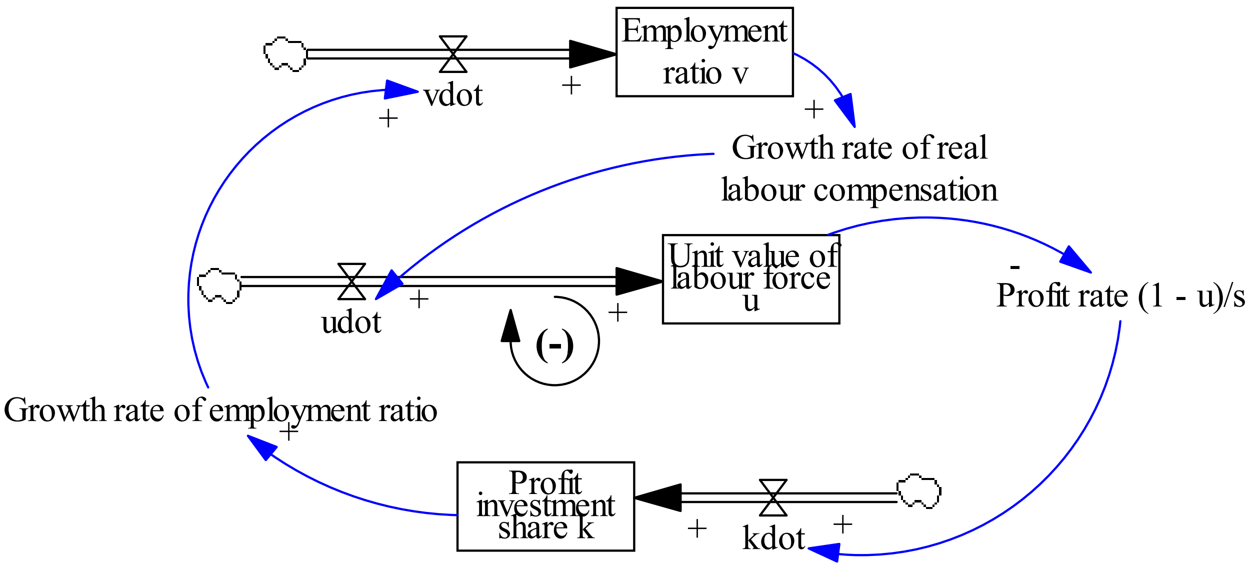

Figure 4. A first negative feedback loop of the third order (for $m < 0$) controlling profitability

A counter-acting destabilizing role of the negative parameter n_3 from the mechanisation function (5) is to be emphasised (Figure 5).

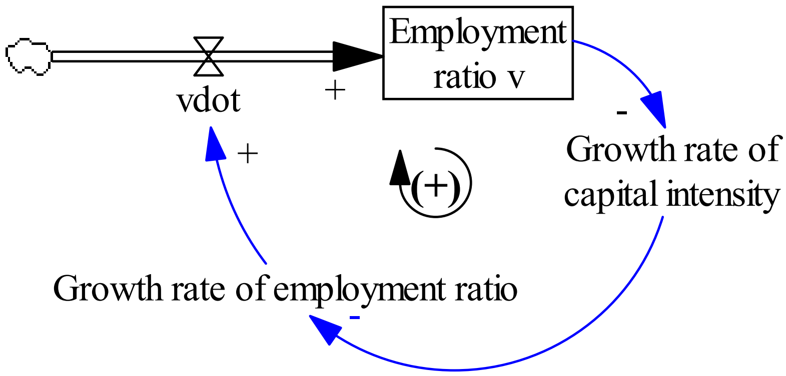

Figure 5. A positive feedback loop (for $n_3 < 0$) including the employment ratio of the first order

There is no a first order feedback loop containing the profit investment share (k). This incompleteness prompts a search for another partial control law for this variable for enhancing profitability.

3.4. Retrospective forecast for Italy over 1980–2004

A retrospective forecast is a well-known (rather weak) behaviour reproduction test. Available statistics allow comparison of imitated and observed magnitudes of main variables in 25 points (1980–2004). For this period, the retrospective forecast fits the observed development of the Italian economy.

The small total errors in variables show the model suitably tracks the major variables. The discrepancy between observable and simulated magnitudes is mostly due to fluctuations with higher frequencies and divergence on a point-by-point basis. On the on hand, our model presents rather well the declining tendencies of relative labour compensation (u) and of profit investment share (k) (Figures 6 and 7); on the other hand, it presents a dominant secular tendency of the profit rate to go up (Figure 8) in the basal period. This probabilistic model also presents an interaction of the short-term and middle-term cycles with long swings in the basal period. Such an interaction is not modelled in the remaining (perspective) period when the applied model abstracts from short-term and middle-term economic fluctuations altogether.[9]

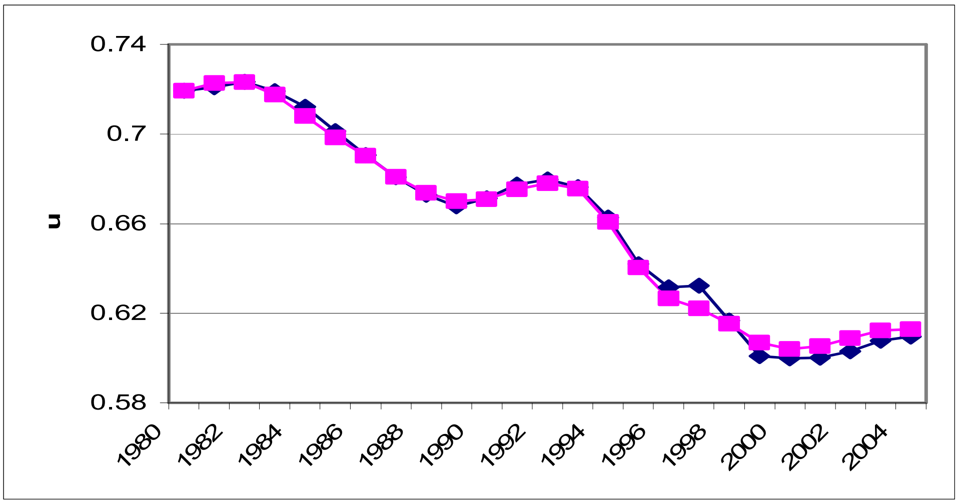

Figure 6. Observed (blue) and simulated (violet) relative labour compensation (u) in Italy, 1980–2004

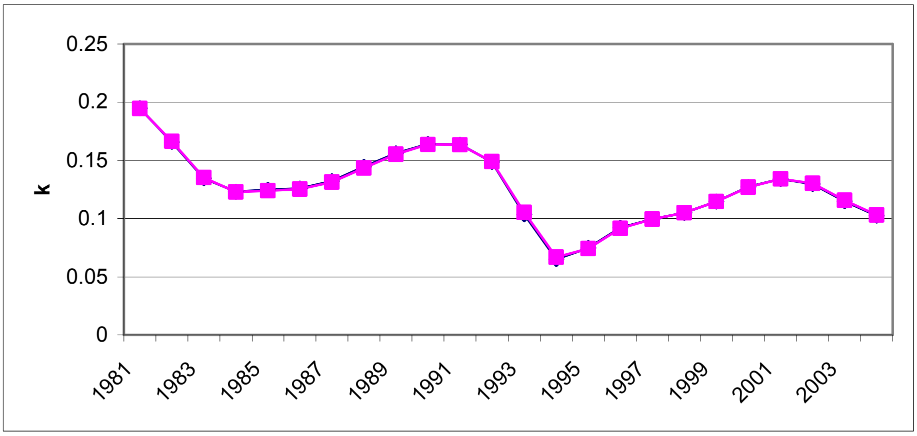

Figure 7. Observed (blue) and simulated (violet) profit investment share (k) in Italy, 1981–2004

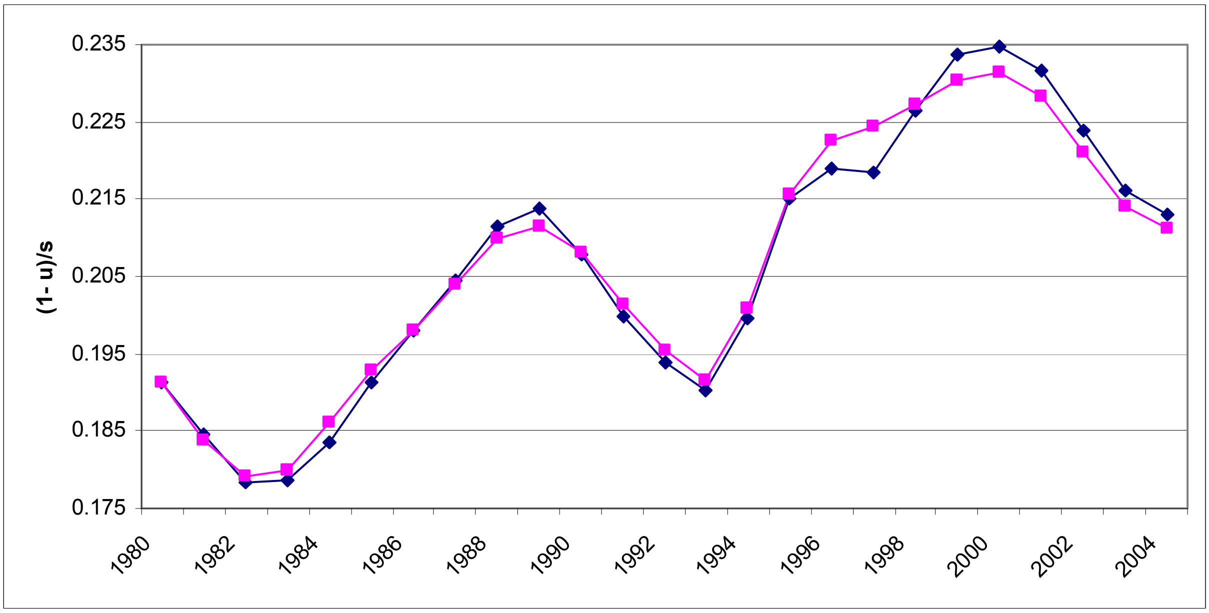

Figure 8. Observed (blue) and simulated (violet) profit rate $(1 – u)/s$ in Italy, 1980–2004

[9] Notice that economic fluctuations, especially short- and middle-term, are smoothed by EKF. This procedure flattens off peaks and troughs even of long swings.

Our calculations use the Theil inequality statistics (Theil, 1966; Sterman, 1984). Table 3 posts results for six main variables.

Mean square error (MSE) is divided into three components. Incomplete co-variation (U^C) accounts for more than 84 per cent of this error in relative terms for all these variables. Unequal variation (U^S) is non-negligible mostly for profit rate (15.4%), relative labour compensation (13.0%) and for the profit investment share (8.6 %).[10] The bias (U^M) is almost non-existent in these cases. The root-mean-square percent error (RMSPE) of these variables is lower than 1.1 per cent.

Table 3. Error Analysis of HL for 1980–2004

Variable	RMSPE (%)	MSE	U^M	U^S	U^C
Labour productivity (*a*)	0.579	2.18E-08	0.005	0.027	0.968
Capital-output ratio (*s*)	0.246	1.61E-05	0.0	0.038	0.962
Employment ratio (*v*)	0.068	3.88E-07	0.004	4.7E-05	0.996
Relative labour compensation (*u*)	0.579	1.31E-05	1.3E-05	0.130	0.870
Profit rate ((1 – *u*)/*s*)	1.038	5.06E-06	1E-04	0.154	0.846
Profit investment share (*k*, 1981-2004)	0.693	4.50E-07	7.2E-05	0.086	0.914

The rather small root-mean-square percent errors (RMSPE), low biases (U^M) and prevailing non-systematic errors of incomplete co-variation (U^C) confirm that this probabilistic model tracks the major variables observed in the basal period agreeably.

4. Prospective scenarios of Italian Economic Development

The simulation runs have generated magnitudes of variables in the subsequent years. These simulations have started at the most probable (still sub-optimal) magnitudes of the four phase and auxiliary variables in the year 2004 ($a_0 \approx 0.0284$ [in 2000 prices, thousands of Euros per working hour], $k_0 \approx 0.103$ [fractional], $s_0 \approx 1.832$ [years], $v_0 \approx 0.919$ [fractional], $u_0 \approx 0.613$ [fractional], $P_0 \approx 1111.6$ [in 2000 prices, millions of Euros per year]).

4.1. Inertia Scenario I and its two modifications

An extrapolation of the retrospective forecast by the deterministic HL with the identified parameters magnitudes given above (Section 3.2) is called inertia Scenario I.

4.1.1. Long swings and profound meaning of 'Ricardian' labour force

Computer simulations reveal that the phase variables (*k*, *s*, *v*, *u*), profit rate, growth rates of labour productivity and real labour compensation as well as some other auxiliary variables fluctuate. The generated oscillations are long-term and anharmonic (Figures 9 and 10).

A period of the completed long swings 1998–2200 in inertia Scenario I is about 19–20 years. Economic growth does not decelerate and fluctuate around the stationary magnitude. The troughs (peaks) of employment ratio have a lead time of about one year against peaks (troughs) of profit rate and about five years against troughs (peaks) of the growth rate of net output. In other words, local maximums of the employment ratio roughly coincide with local minimums of profitability, while the employment ratio leads the GDP growth rate by nearly one phase of a long swing.

[10] In all these calculations profit includes amortisation of fixed capital (without residential stock).

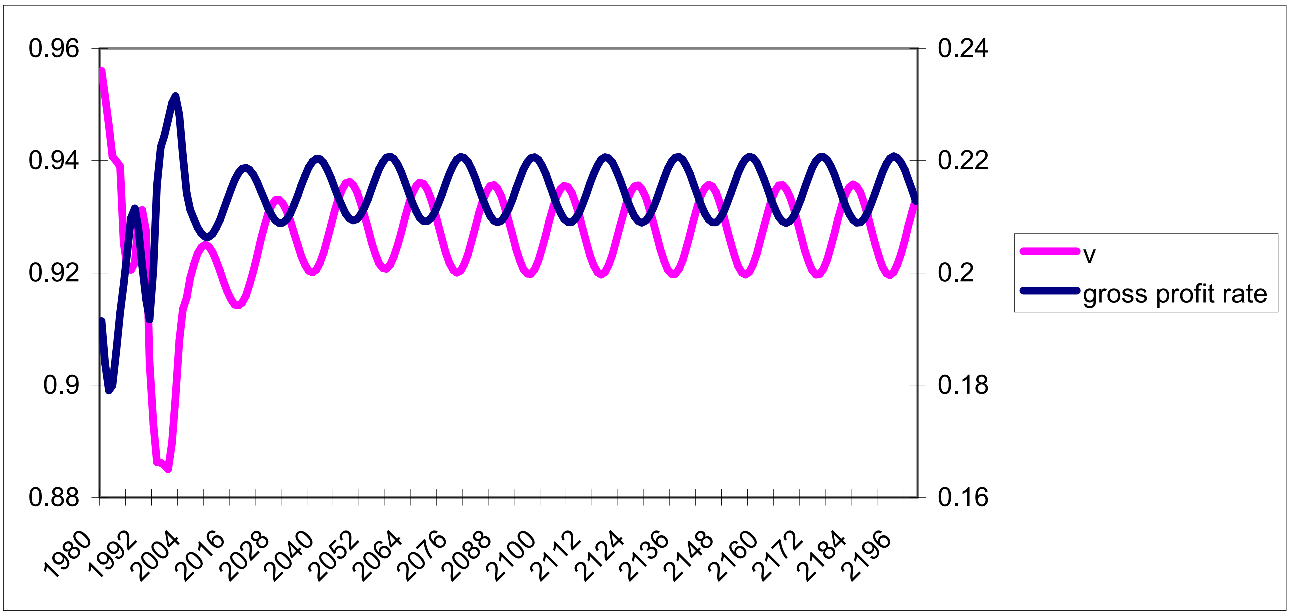

Figure 9. The simulated profit rate (right scale, blue) and employment ratio (left scale, violet) in inertia Scenario for Italy, 1980–2200

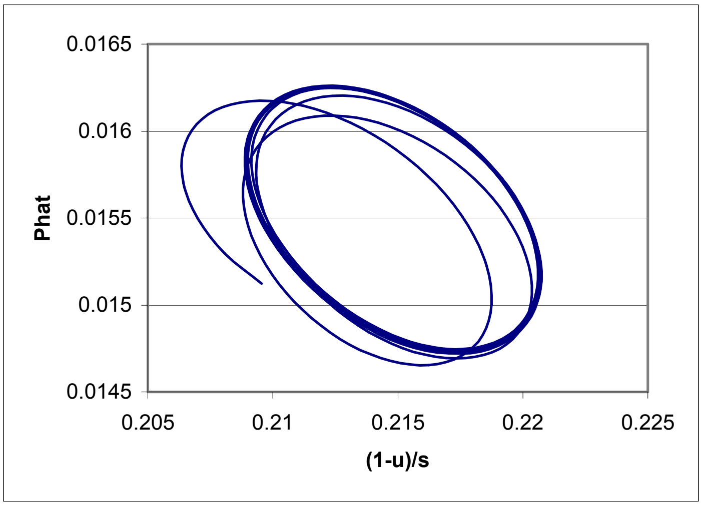

Figure 10. A secular tendency of profit rate $(1-u)/s$ and growth rate of real GDP ($\hat{P}$ = Phat) to rise and their long swings in inertia Scenario I for Italy, 2005–2200 (clockwise shifting to the right and up)

The TPF (4) is the generalization of the Verdoorn relation and Kaldorian technical progress function. Its application as an element of HL enables us to reconcile a direct relation between growth rates of net output and labour productivity with an inverse relation between growth rates of employment ratio and labour productivity (Figures 11 and 12).

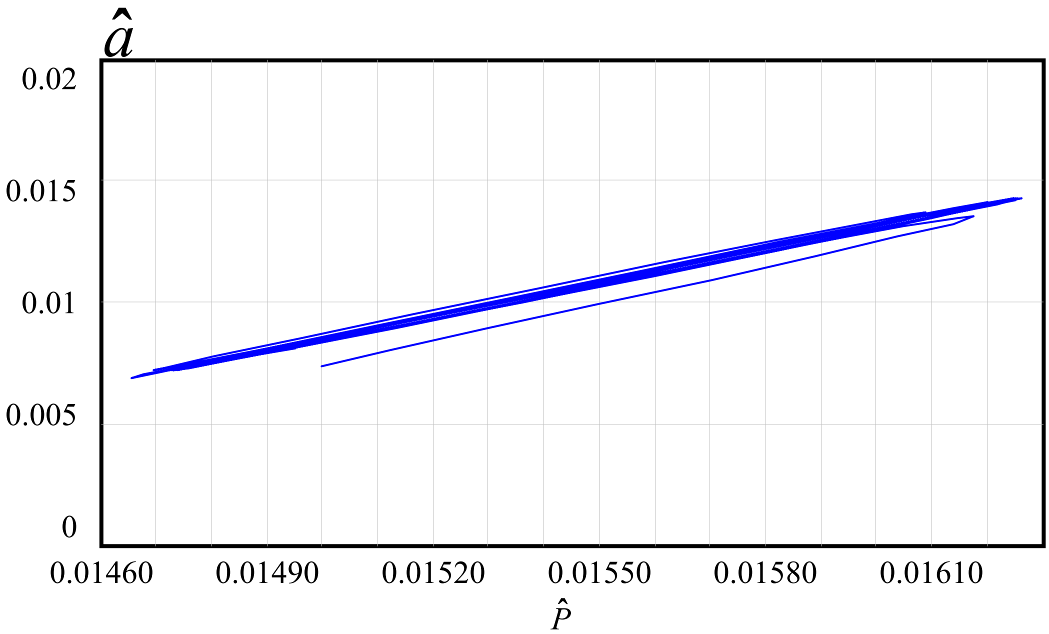

Figure 11. The direct relation of growth rate of real GDP ($\hat{P}$) with growth rate of labour productivity ($\hat{a}$) in inertia Scenario I, 2004–2200

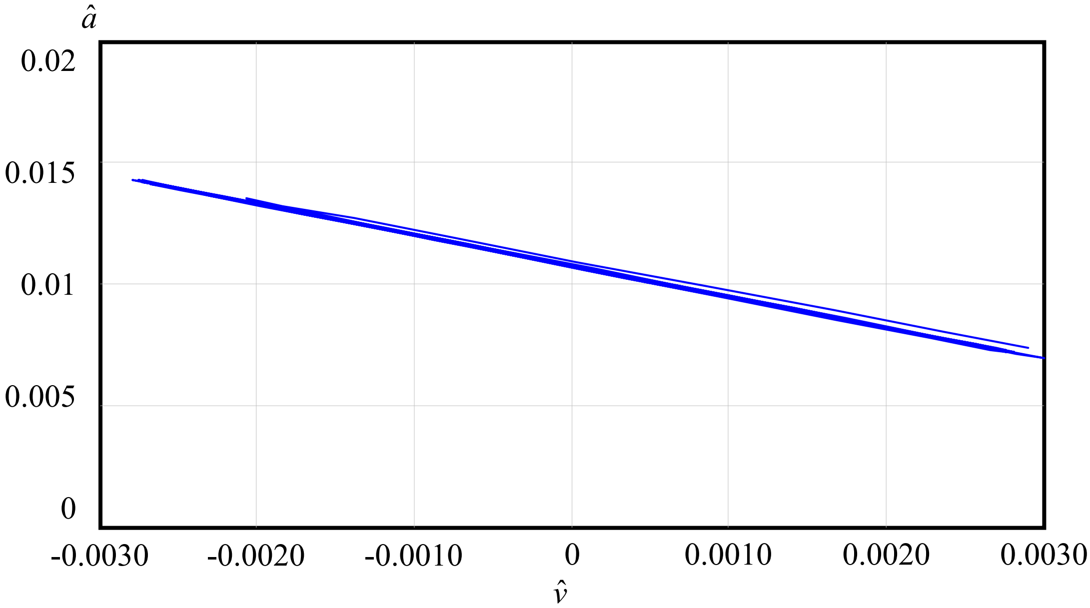

Figure 12. The inverse relation of growth rate of employment ratio ($\hat{v}$) with growth rate of labour productivity ($\hat{a}$) in inertia Scenario I, 2004–2200

Inertia Scenario I generated by the applied theoretical model sheds also light on the peculiarity of the Italian labour force (noticed in Section 1.2) that was called 'Ricardian', namely the inverse relationship between growth of employment and returns (Figures 9, 13 and 14) within a long swing. A period of pseudo limit cycle is about 19.5 years.

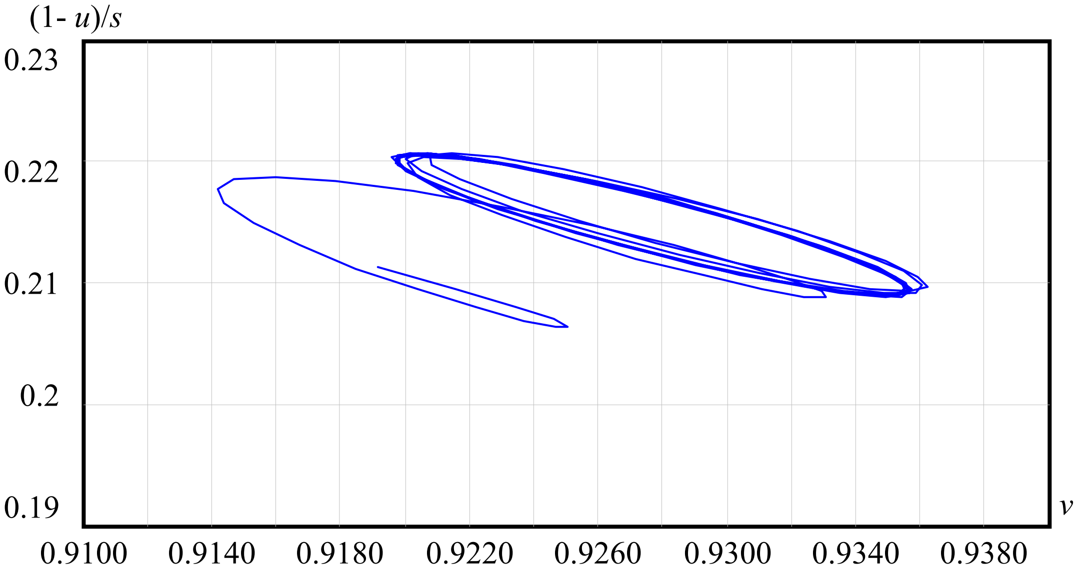

Figure 13. Secular tendency of profit rate $(1 - u)/s$ and employment ratio to rise and their long swings in inertia Scenario I for Italy, 2004–2200 (clockwise shifting to the right and up)

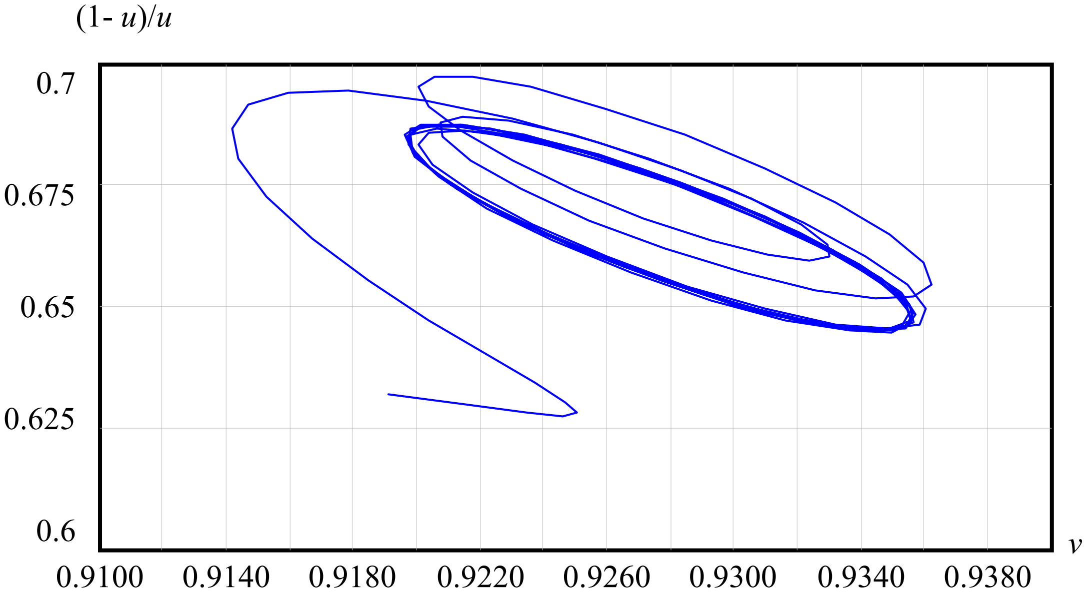

Figure 14. Employment ratio (v) and rate of surplus value $(1- u)/u$ in inertia Scenario I for Italy, 2004–2200 (clockwise)

According to the simulation run, the Italian economy will reach next locally maximal employment ratio ($v \approx 0.925$) in 2008 while the profit rate will continue to fall from the year 2000 until 2009. After that the GDP growth rate will be locally maximal (0.016) in 2012. The employment ratio will hit its next local maximum ($v \approx 0.933$) in 2028. Applying employment ratio as criterion for demarcating phases of long swing, it may be expected that a new crisis will start in the current year 2008.

4.1.2. Instability of the non-trivial stationary state

For the magnitudes of the HL parameters as in inertia Scenario I (in particular, $m \approx -1.255$ and $n_3 \approx -0.1092$) the stationary state $E_a = (k_a, s_a, v_a, u_a)$ (14) is, according to multiple simulation runs, unstable focus (hence slowly diverging fluctuations in the phase space). A limit cycle in the American case (for $n = const$), which is attracting trajectories in the phase space, is generated by a singularity-based bifurcation owing to the properties of the derivatives of the function $\psi(\hat{v}) = sign(\hat{v})|\hat{v}|^j, 0 < j < 1$, that is a key element of the technical progress function.[11] Instead of a singularity-based bifurcation, a parametric shift can generate the Andronov – Hopf bifurcation in the CL realisation for Italy (see sub-section 4.2.2 below).

Timely structural changes could enhance viability and efficiency of the model economy. Pro-growth stabilization can be achieved by different strategies. As m is, likely, negative nowadays in Italy, the stabilization policy, advised for the modern USA in (Ryzhenkov 2005a – 2005c, 2007), would be a debacle for modern Italy. More reasonable pro-growth stabilization policies for the latter country could be based on lowering m or on raising n_3 within some intervals. These policies will not affect the stationary profit rate that is independent of these both parameters.

Stabilization of the long swings could be achieved by other policies as well. For example, steadiness of the profit investment share in the year 2004 and later ($k = k_0 > k_a$) when $c_2 = 0$ in the equation (10) guarantees for the identified magnitudes of the other parameters (Section 3.2) transition to the stationary state (stable focus) $E_g = (s_g, v_g, u_g)$ defined by (14a) yet with a lower profitability (a red curve on Figure 15). A period of the transient fading oscillations is about 33–34 years. The new stationary profit rate is lower in this *first* modification of inertia scenario I (Table 1) than the former (before modification): $(1- u_g)/s_g = 0.150 < p_w$.

The former stationary state (unstable focus) $E_a = (k_a, s_a, v_a, u_a)$ could be transformed into stable focus in the *second* modification of inertia scenario I by lowering the positive magnitude of the parameter c_2 (a green curve on Figure 15) that increases the pseudo period of oscillations (in this particular case by about 1–2 years) compared with that in the unmodified inertia Scenario I (a blue curve on Figure 15). Still the stationary profit rate (p_w) remains the same (unlike stabilization Scenario II in Section 4.2 below).

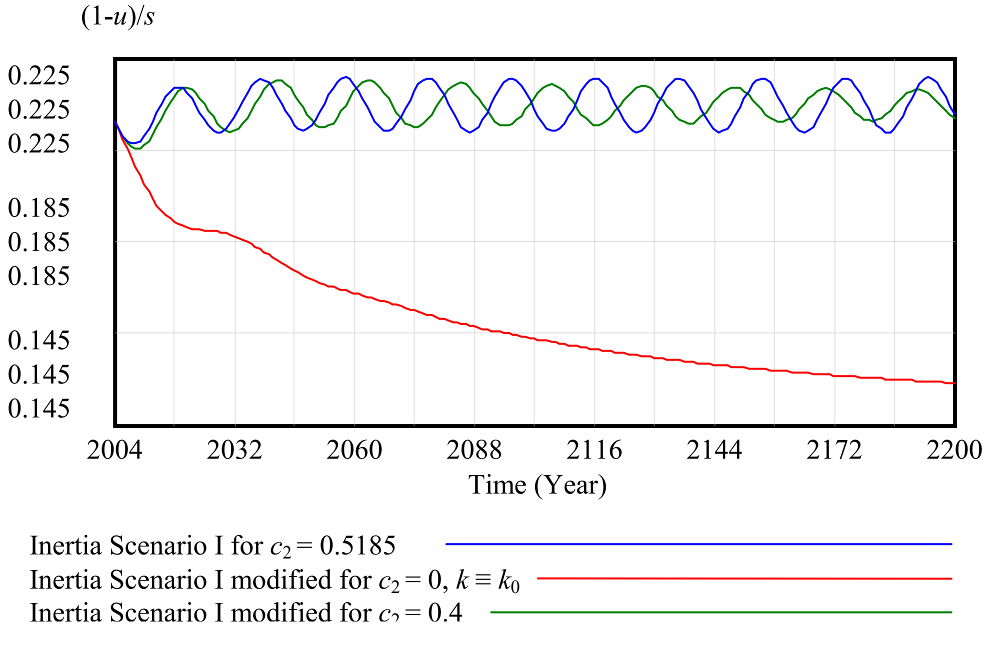

Figure 15. Profit rate in inertia Scenario I and in its two modifications, 2004–2200

[11] The non-linear continuous function $\psi(\hat{v})$ is analytical except at singular points with $\hat{v} = 0$ where its positive first derivative ($\psi'(\hat{v}) = j|\hat{v}|^{j-1} > 0$) becomes infinite. The derivatives of the function $\psi(\hat{v})$ of higher orders go to plus or minus infinity at the vicinity of $\hat{v} = 0$. This substantial singularity explains why the growth rate of labour productivity changes stepwise at local extrema of the employment ratio. Abruptness of economic crises follows from this essential singularity too if a closed loop control over total profit, profitability or total surplus value is not enforced.

The next section supposes and elaborates a stabilization policy that is able not only to eradicate long swings faster than the stabilization policy in the second modification of the inertia scenario I but to raise the stationary profit rate a bit above that in inertia Scenario I. These improvements will be at cost for labour power compared with the stabilization policy targeted at $E_g = (s_g, v_g, u_g)$ in the first modification of the inertia scenario I. These results will be exposed in the next section.

4.2. Stabilization Scenario II

4.2.1. A synthesis of control law (CL) including a new partial law for profit investment share

For enhancing stabilization, it is reasonable to suppose a policy that includes at least one negative feedback loop of the first order containing a variable profit investment share absent in the structure of HL. We will use the probable peculiarity of the Italian economy that the parameter m of TPF (4) is negative and turn the former changeable stock k into an auxiliary variable. Then the following possibility suggests itself (Figure 16).

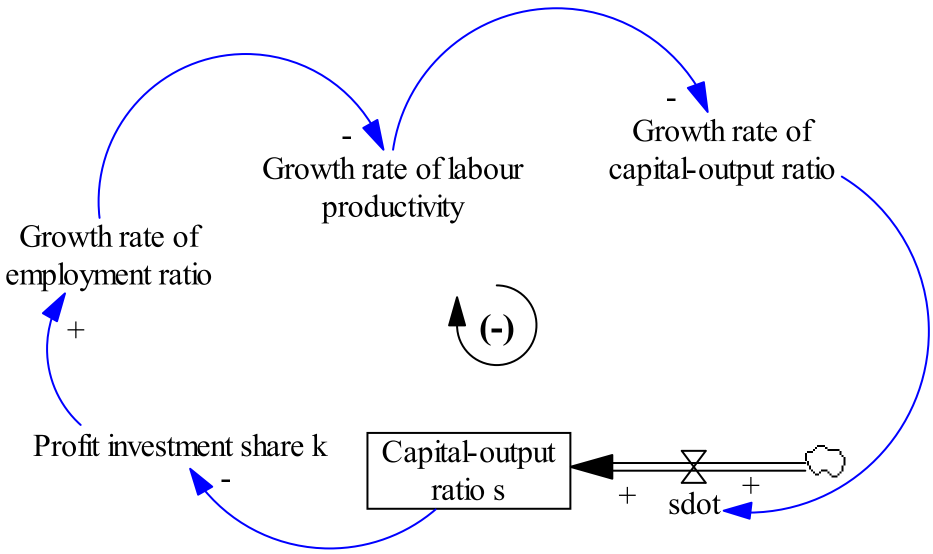

Figure 16. A negative first order feedback loop (for $m < 0$) controlling the profit investment share (k)

This feedback loop is brought about if we suppose that the profit investment share is defined by the following equation (23) that substitutes the equation (10)

$$k = c_0 - c_1 s > 0, \qquad (23)$$

where $1 \geq k > 0$, $c_0 > 0$ and $c_1 > 0$. Besides that, initially $k = k_0$ for the year 2004.

An extensive deterministic form of CL with the variable profit investment share (k) consists of the equations (1) – (9a) and (23); its intensive form is the system of the three ODEs (11) – (13) together with the equation (23) for the new auxiliary variable (k).

Figure 17 presents a little bit compacted overall causal loop diagram for the new model. It contains three main stocks (corresponding to the state variables s, v and u) and three main flows (corresponding to the time derivatives of these state variables).

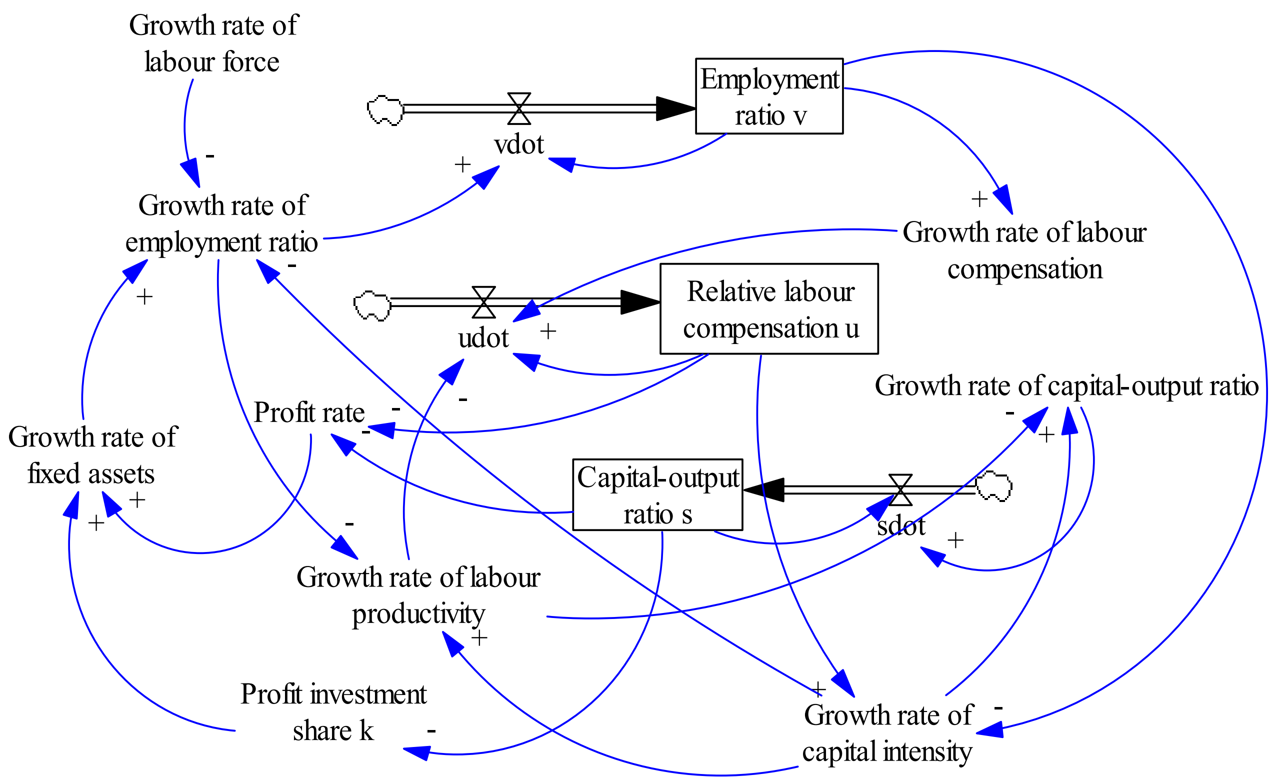

Figure 17. A condensed causal loop diagram of CL

A non-trivial stationary state of the adapted system (11) – (13) is defined as

$E_b = (s_b, v_b, u_b)$, (14b)

where $s_b = c_0\dfrac{1-u_b}{d+c_1(1-u_b)}$, $v_b = (g + (1 - b)(d - n))/r$, $u_b = (d - n - n_1 - n_3(v_b - v_c))/n_2$. Notice that the latter two remain the same: $v_b \equiv v_a$ and $u_b \equiv u_a$.

The new stationary profit investment share is

$$k_b = \frac{c_0 d}{d+c_1(1-u_b)}. \qquad (24)$$

We assume that parameters are such that the requirement $0 < k_b \leq 1$ is always satisfied.

The stationary profit rate exceeds the same stationary growth rate of net output (for $c_0 \neq 1$)

$$\frac{1-u_b}{s_b} = \frac{d+c_1(1-u_b)}{c_0} > d \qquad (25)$$

if $\dfrac{c_1(1-u_b)}{c_0-1} > d.$ (26)

If the stationary profit investment share equals unity ($k_b = 1$), for instance, for $c_0 = 1$ and $c_1 = 0$, then the stationary profit rate coincides with the stationary growth rate of net output

$$\frac{1-u_b}{s_b} = \frac{d+c_1(1-u_b)}{c_0} = d. \qquad (27)$$

The new stationary capital-output ratio is lower than the former if

$$s_b = c_0\frac{1-u_a}{d+c_1(1-u_a)} < s_a = \frac{1-u_a}{p_w}$$

or (as $0 < u_a < 1$)

$$c_0 \frac{1}{d + c_1(1 - u_a)} < \frac{1}{p_w} \ . \qquad (28)$$

The new stationary profitability is also higher than the former if the condition (28) is valid.

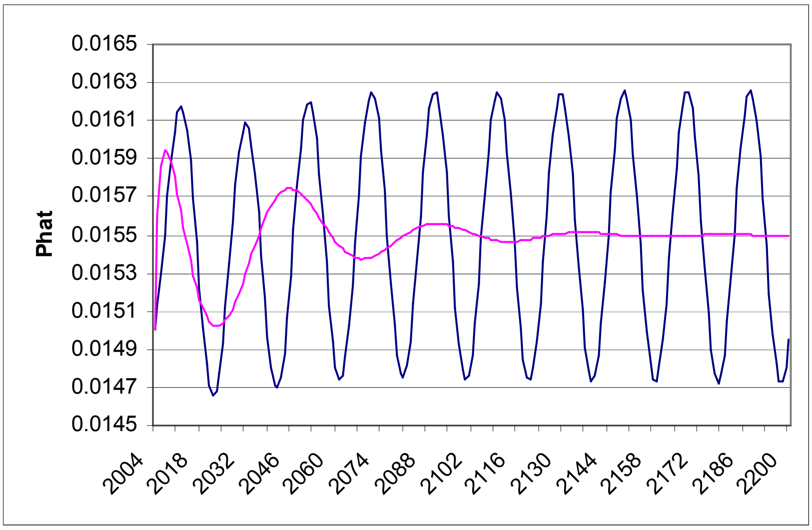

Figure 18. Growth rate of real GDP ($\hat{P}$ = Phat) in the inertia scenario I (blue curve) and in stabilization scenario II (violet curve) for $c_0 \approx 2.485$ and $c_1 \approx 1.3$, 2004–2200

An equivalent requirement is

$$k_b < k_a. \qquad (28a)$$

This condition is satisfied for different magnitudes of the two new parameters. In stabilization Scenario II specific magnitudes are chosen $c_0 \approx 2.485$ and $c_1 \approx 1.3$ for a bit improving of stationary profitability (see Table 1 above in Section 3.2). In result of the new stabilization policy, amplitude of converging oscillations is lower whereas their quasi period (39–45 years) is longer than that (19–20 years) in the inertia scenario I (Figure 18).

The new stationary profitability is higher than the former after exogenous increases in the stationary growth rate of labour productivity if

$$\frac{\partial \dfrac{d + c_1(1 - u_b)}{c_0}}{\partial \hat{a}_b} = \frac{1 - c_1 \dfrac{\partial u_b}{\partial \hat{a}_b}}{c_0} = \frac{1 - c_1 \dfrac{r - (1 - b)n_3}{rn_2}}{c_0} > 0. \qquad (29)$$

For $c_0 > 0$, $1 - c_1 \dfrac{r - (1 - b)n_3}{rn_2} > 0$

if $rn_2 > c_1[r - (1 - b)n_3]$

or

$$c_1 < \bar{c}_1 = \frac{rn_2}{r - (1 - b)n_3} \ . \qquad (30)$$

In particular, $\bar{c}_1 < n_2$ for $n_3 < 0$.

For $c_1 \approx 1.3 > n_2 \approx 0.194 > \bar{c}_1 \approx 0.1372$ the condition (30) is violated therefore an exogenous increase of the stationary growth rate of labour productivity brings about a drop in the stationary profit rate together with rise in the stationary profit investment share and in stationary relative labour compensation.

Reconciliation of the conditions (28) and (30) is not possible for the chosen magnitudes of CL parameters unless at least one of them is changed. The following opportunity suggests itself.

The upper bound $\bar{c}_1 \to \infty$ if $n_3 \to \frac{r}{1-b}$ in the inequality (30). For n_3 sufficiently close to the latter ratio, the right hand side of the inequality (30) can exceed any $c_1 = const > 0$. Therefore for moving up the stationary profitability by raising the stationary growth rate of labour productivity, capital could be interested in shifting the (initially negative) magnitude of the parameter n_3 closer to a positive ratio $\frac{r}{1-b}$. But this may be not accepted by the Italian labourers since increases in this parameter, if all other conditions remain the same, cause decreases in the stationary relative labour compensation, according to the equation (15).

It was found that the structure of CL contains, in particular, eight feedback loops of growth rate of labour productivity (seven negative and one positive) that do not include the time derivative of profit investment share, and four additional feedback loops that include the latter (three negative and one positive). There are also eight additional feedback loops of employment ratio that do not contain the growth rate of labour productivity, six of them do not include the time derivative of profit investment share (one with a variable sign of the growth rate of employment ratio, four negative and one positive), the other two (negative) include the time derivative of profit investment share (k). We see that the time derivative of profit investment share is a significant element of the CL structure.

Sufficiently high absolute magnitudes of $m < 0$ secure local stability of the stationary state (14b) if the other conditions remain the same. The destabilizing role of $n_3 < 0$ is not completely eradicated under this policy as Figure 19 shows.

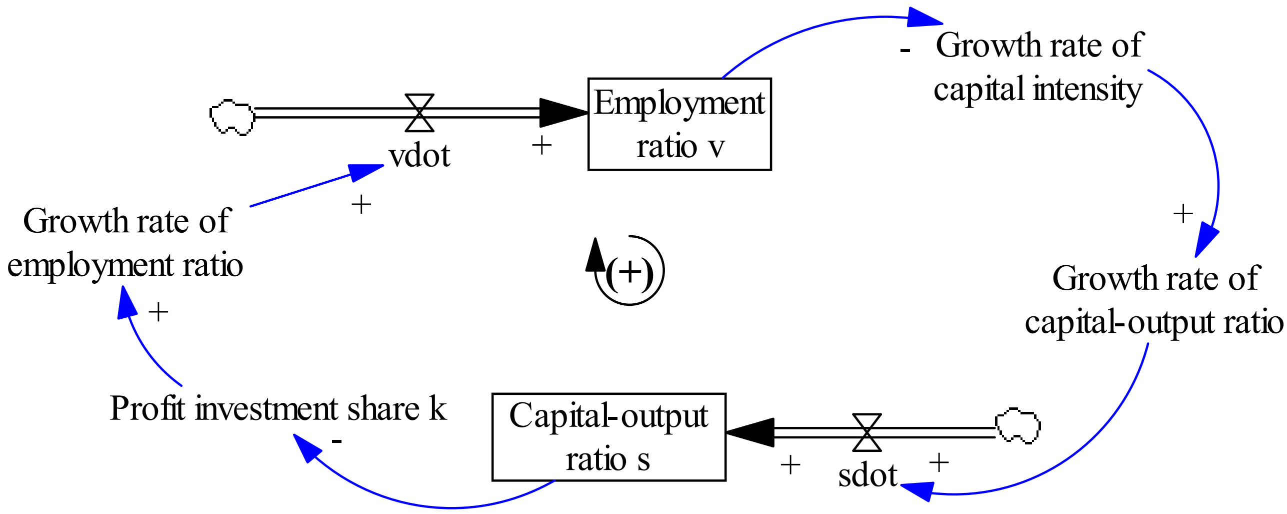

Figure 19. A positive feedback loop of the second order (for $n_3 < 0$)

Table 4 compares some important characteristics of our scenarios for the four initial decades. The first modification of Inertia scenario I is mostly advantageous for labourers (both mean relative labour compensation and mean employment ratio are maximal whereas their variation is minimal). The stabilization scenario II is mostly advantageous for capitalists (the highest mean profitability and the minimal variation of this variable). It is the second best for labourers.

Advantages of four scenarios are conditioned to some degree by length of a projection period. Still the scenarios mostly favourable for labourers and for capitalists for lengthier periods, such as 2004–2200 remain the same (cf. Table 4 and Table 5).

These comparisons shed light on limitations of evaluations of different scenarios mostly based on properties of stationary states. The statistical characteristics of transients are no less important practically.

Turn attention to the second modification of the inertia scenario I for a moment. Although the investment policy transforms the same unstable focus into stable focus it raises variation and a bit worsens profitability in the first four decades compared with the initial inertia scenario I. Therefore this investment policy is impractical.

Table 4. Summary statistics of the three main social variables in four main scenarios for 2004–2044

	Mean			Normalised standard deviation (variation)		
Scenario	Relative labour compensation (u)	Employment ratio (v)	Profit rate ($(1-u)/s$)	Relative labour compensation (u)	Employment ratio (v)	Profit rate ($(1-u)/s$)
Inertia I	0.599	0.924	0.214	0.014	0.006	0.020
1st modification of Inertia I	0.620	0.928	0.190	0.008	0.004	0.042
2nd modification of Inertia I	0.600	0.923	0.213	0.015	0.006	0.022
Stabilization II	0.599	0.925	0.216	0.013	0.007	0.017

Table 5. Summary statistics of the three main social variables in four main scenarios for 2004–2200

	Mean			Normalised standard deviation (variation)		
Scenario	Relative labour compensation (u)	Employment ratio (v)	Profit rate ($(1-u)/s$)	Relative labour compensation (u)	Employment ratio (v)	Profit rate ($(1-u)/s$)
Inertia I	0.600	0.927	0.215	0.010	0.006	0.020
1st modification of Inertia I	0.610	0.928	0.168	0.011	0.002	0.079
2nd modification of Inertia I	0.600	0.927	0.214	0.009	0.005	0.016
Stabilization II	0.600	0.927	0.216	0.006	0.004	0.009

Particularly, in the two initial centuries a transient to the stationary state E_b (14b) in the stabilization scenario II preferred by capital would be less advantageous for labourers than a transient to the stationary state E_g (14a) in the first modification of inertia scenario I with the constant profit investment share ($k = k_0$) preferred by labourers: whereas profitability would be lower on the average (Figure 20), both the employment ratio (Figure 21) and relative labour compensation (Figure 22) would be a bit higher on the average if the profit investment share remained unchanged.

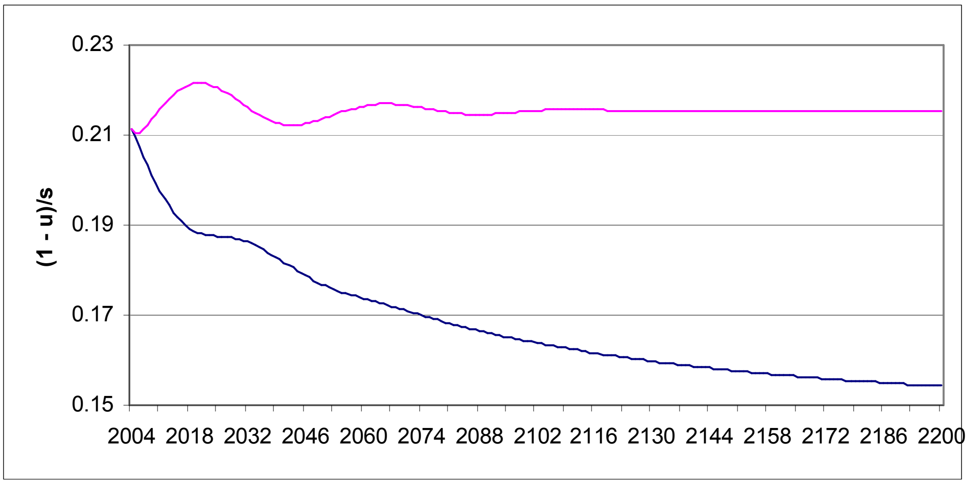

Figure 20. Profit rate ($(1-u)/s$) in the 1st modification of inertia scenario I (blue curve) for $k = k_0 \approx 0.103$ and in stabilization scenario II (violet curve) for $c_0 \approx 2.485$ and $c_1 \approx 1.3$, 2004–2200

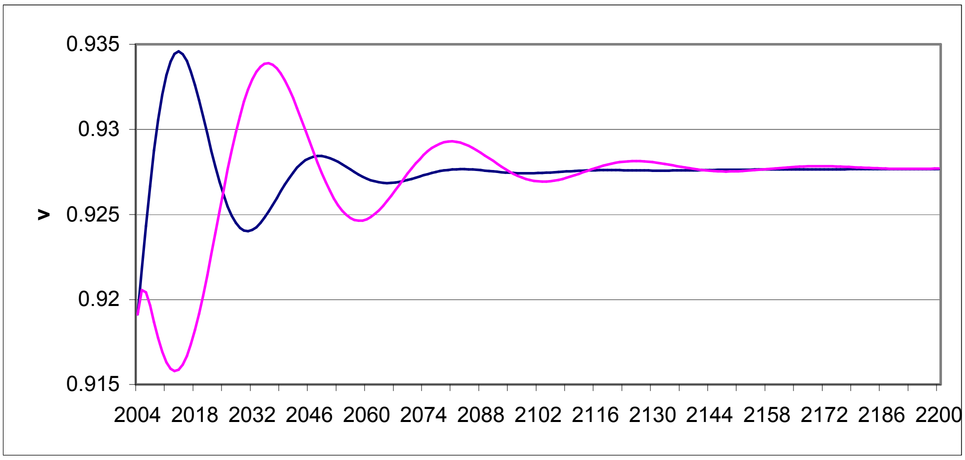

Figure 21. Employment ratio (v) in the 1st modification of inertia scenario I (blue curve) for $k = k_0 \approx 0.103$ and in stabilization scenario II (violet curve) for $c_0 \approx 2.485$ and $c_1 \approx 1.3$, 2004–2200

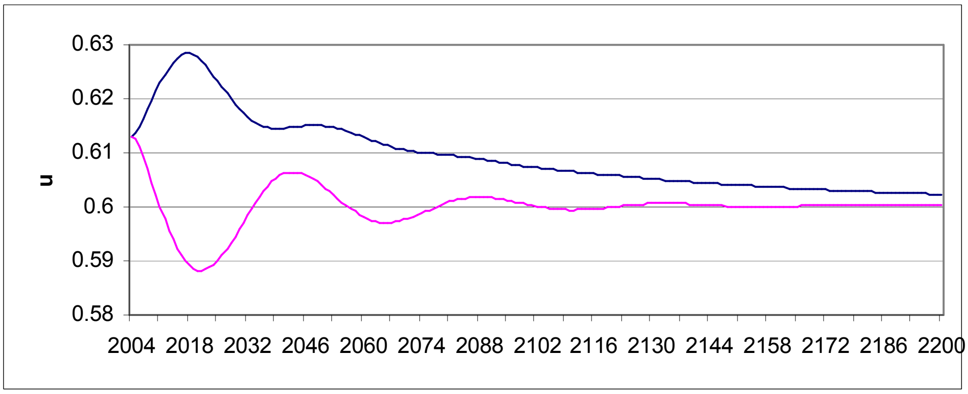

Figure 22. Relative labour compensation (u) in the 1st modification of inertia scenario I (blue curve) for $k = k_0 \approx 0.103$ and in stabilization scenario II (violet curve) for $c_0 \approx 2.485$ and $c_1 \approx 1.3$, 2004–2200

It becomes visible that the labourers, rather paradoxically, are more interested in more vigorous capital investment in the domestic economy than the capitalists. Although total capital (both in real and labour value terms) is higher in the first modification of inertia scenario I than in the stabilization scenario II, the capital-lead society would likely chose the latter scenario because of its higher profitability.

Still the supposed stabilization policy is at risk or not possible for critical magnitude(s) of control parameter(s). The following analysis illustrates this reservation.

4.2.2. An Andronov – Hopf bifurcation based on the key TPF parameter in CL

The Jacoby matrix evaluated at the non-trivial stationary state E_b (14b) of CL is

$$J_b = \begin{pmatrix} m\dfrac{c_0(1-u_b)}{s_b} & n_3(1-h_2+m)s_b & [(1-h_2)n_2+m(-c_1+\dfrac{c_0}{s_b}+n_2)]s_b \\ -\dfrac{c_0(1-u_b)}{{s_b}^2}v_b & -n_3v_b & -(-c_1+\dfrac{c_0}{s_b}+n_2)v_b \\ m\dfrac{c_0(1-u_b)}{{s_b}^2}u_b & [r+(b-h_2)n_3+mn_3]u_b & [(b-h_2)n_2+m(-c_1+\dfrac{c_0}{s_b}+n_2)]u_b \end{pmatrix}. \quad (31)$$

The characteristic polynomial is

$$\lambda^3 + a_2(m)\lambda^2 + a_1(m)\lambda + a_0(m) = 0, \qquad (32)$$

where the coefficients are functions of the control parameter m.

The Routh – Hourwitz necessary and sufficient conditions for the local stability are

$$a_0(m) > 0, \qquad \text{(C-1)}$$

$$a_1(m) > 0, \qquad \text{(C-2)}$$

$$a_1(m)a_2(m) > a_0(m). \qquad \text{(C-3)}$$

For this system $a_0(m) = a_0 = const.$ These requirements confine the region $S \subset R$ such that for $m \in S$ the non-trivial stationary state E_b of the system (11) – (13) is locally asymptotically stable.

For the applied magnitudes of the CL parameters, stationary state E_b is stable since the all three above conditions (C-1) – (C-3)are satisfied ($a_0 \approx 0.0141 > 0$, $a_1 \approx 0.0621 > 0$, $a_2 \approx 0.7535 > 0$, $a_1a_2 - a_0 \approx 0.0326 > 0$).

The Andronov – Hopf bifurcation theory is a tool for establishing the existence of closed orbits. In this study we choose m in the equation (4) of TPF as the main bifurcation (control) parameter.

Consider the stationary state of the system (11) – (13) as dependent on this control parameter m:

$$\dot{x} = 0 = f(x, m). \qquad (33)$$

The determinant of the Jacoby matrix (J_b) for the system (33) evaluated at the stationary state E_b (14b) differs from zero in our case as $a_0 = const > 0$ (independently of m). The implicit function theorem ensures that for every m in a neighbourhood $Br(m_0) \in R$ of the parameter value m_0 there exists a unique stationary state x_b, moreover, changes of m do not affect E_b.

It is assumed the following properties are satisfied:

(a) the components of the function $f(x, m)$, corresponding to the system (33), are analytic (i.e. given by power series);

(b) the Jacoby matrix $J_b(m_0)$ has a pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts;

(c) the derivative $\dfrac{d(\operatorname{Re}\lambda_{2,3}(m))}{dm} > 0$ for $m = m_0$ (it is the transversality condition);

(d) the stationary state E_b is asymptotically stable (for $m < m_0$).

Then, according to the Hopf theorem, there exists some periodic solution bifurcating from $x_b(m_0)$ at $m = m_0$ and the period of fluctuations is about $2\pi/\beta_0$ ($\beta_0 = \lambda_2(m_0)/i$). If a closed orbit is an attractor, it is called a *limit cycle*. The Hopf theorem establishes only the existence of closed orbits in a neighbourhood of x_b at m_0, still it does not clarify the stability of orbits, which may arise on either side of m_0.[12]

The characteristic polynomial for $m = m_0$ is

$$\lambda^3 + a_2(m_0)\lambda^2 + a_1(m_0)\lambda + a_0 = \lambda^2[\lambda + a_2(m_0)] + a_1(m_0)[\lambda + a_2(m_0)]$$
$$= [\lambda + a_2(m_0)][\lambda^2 + a_1(m_0)] = 0. \qquad (34)$$

It has the following roots:

$$\lambda_1 = -a_2(m_0) < 0; \qquad (35)$$

$$\lambda_{2,3} = \pm i\sqrt{a_1(m_0)}. \qquad (36)$$

PROPOSITION 1. The Andronov – Hopf bifurcation does take place in the system (11) – (13) in a local vicinity of E_b at $m = m_0$ defined by the equation (40).

Lemma 1. The quadratic equation based on the above characteristic polynomial (32)

$$a(m) = a_1(m)a_2(m) - a_0 = 0, \qquad (37)$$

where

$$a_1(m) = e - om, \qquad (38)$$

[12] A particular closed orbit as well as frequency and amplitude of fluctuations along this closed orbit may depend on an initial point x_0.

$$a_2(m) = c - hm, \tag{39}$$

always has two real roots m_3 and m_0 such that $m_3 > m_0$:

$$m_{3,0} = \frac{oc + eh \pm \sqrt{(oc-eh)^2 + 4oha_0}}{2oh}. \tag{40}$$

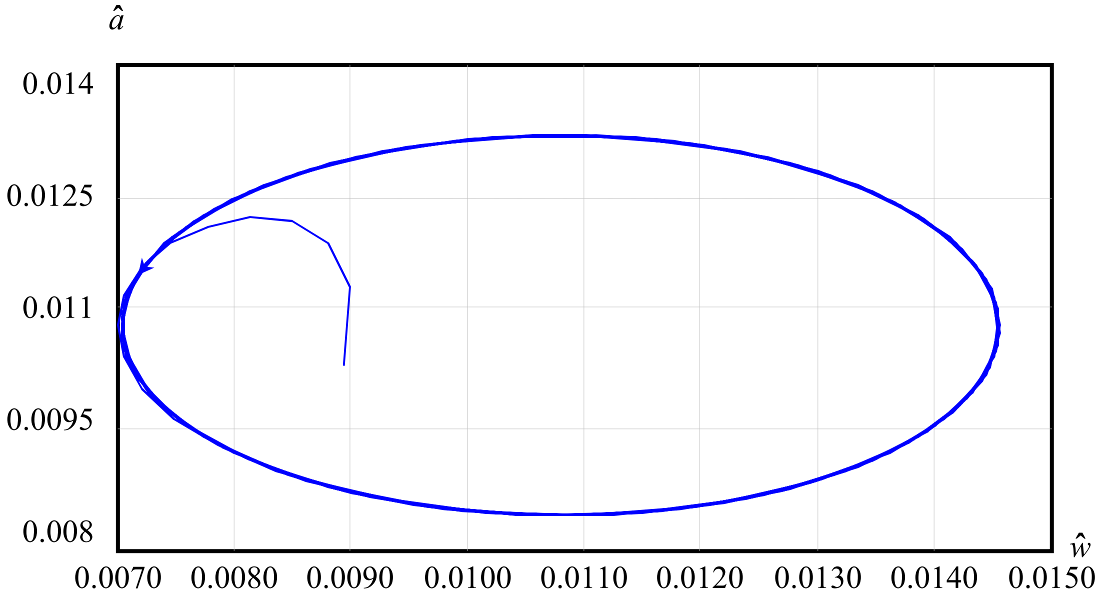

Figure 23. Transition to a closed orbit of growth rates of real wage $(\hat{w})$ and labour productivity $(\hat{a})$ in modified stabilization Scenario II for $c_3 = 1.3$ and $m = m_0$, years 2005–2500. Counter-clockwise

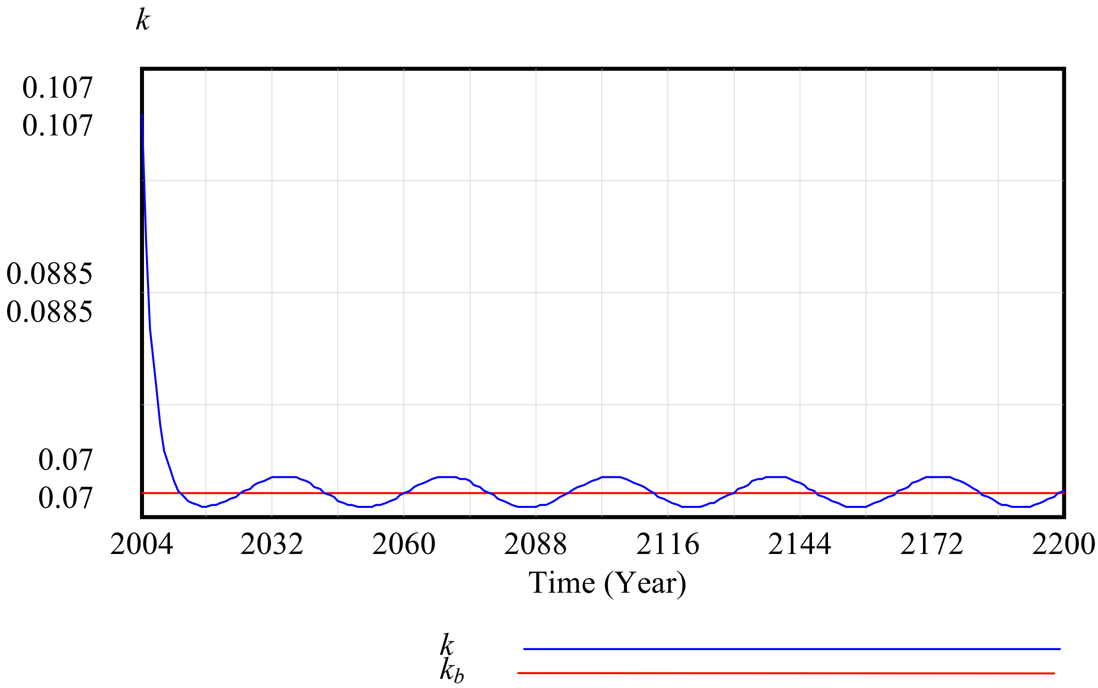

Figure 24. Transition of profit investment share (k) to steady oscillations around its stationary magnitude (k_b) in modified stabilization Scenario II for c_3= 1.3 and $m = m_0$, years 2004–2200

It is true that $-\infty < m < m_0 < m_3 < \infty$; in this particular case $m_0 \approx -0.7803 < m_3 \approx 0.3839$. I have proved that E_b (14b) is locally asymptotically stable for $m < m_0$ and that the Andronov – Hopf bifurcation does take place in the system (11) – (13) at $m = m_0$ (Figures 23–24). According to simulations, a supercritical bifurcation occurs. The period of oscillations near E_b is about $2\pi/\sqrt{a_1(m_0)} \approx 34.81$(years).

For $n_3 \approx -0.1092$ and $m_0 \approx -0.7803$ (critical), there is a rather fast transition (within a period of about 40 years) to a limit cycle vicinity even from the initial phase vector x for the year 2004. Within the initial half a century and beyond amplitude of fluctuations of the employment ratio and of the relative labour compensation is slightly higher whereas amplitude of fluctuations of the capital-output ratio and of profit investment share is substantially lower than that in the inertia scenario I.

4.3. Stabilization Scenario III

We elaborate a new stabilization policy in this section as a reminiscent of the golden rule of accumulation that was formulated by the neoclassical school.[13] An analysis of these new policy possible consequences will illustrate drawbacks of a chronic investment fever.

In our case the investment share in net output equals the capital share in net output if the profit investment share $k = 1 >> k_0 > 0$. Let this profit investment share asymptotically approaches the target profit investment share (that equals one) according to a new partial dynamic rule

$$\dot{k} = c_3(1-k),\ c_3 > 0. \tag{41}$$

An extensive form of the new eclectic model consists of the new equation (41) and the former equations (1)- (9a). The system of four autonomous ODEs (11) – (13) and (41) represents its intensive form.

The stationary state of this system is defined as

$$E_r = (k_r, s_r, v_r, u_r), \tag{14c}$$

where $k_r = 1$, $s_r = \dfrac{1-u_a}{d}$, $v_r = \dfrac{g+(1-b)(d-n)}{r}$, $u_r = \dfrac{d-n-n_1-n_3(v_a-v_c)}{n_2}$, $d = \dfrac{h_1}{1-h_2} + n$.

A stationary growth rate of fixed capital and net output is: $\hat{K}_r = \hat{P}_r = d$; a stationary growth rate of real labour compensation, labour productivity and capital intensity is $\hat{w}_r = \hat{a}_r = K_r \hat{/} L_r = d - n$. A stationary general profit rate is $\dfrac{1-u_r}{s_r} = d$, a stationary growth rate of labour force and employment is $\hat{N}_r = \hat{L}_r = n$. For reasonable parameters' magnitudes the following economic requirements are satisfied: $s_r > 1$, $0 < v_r < 1$, $0 < u_r < 1$, $d > 0$, $d - n > 0$. We assume below that these inequalities are true unless violated for some specific reason(s).

As $\dot{k}$ depends only on k the system (41), (11) – (13) is decomposable and k may be defined explicitly as a function of time

$$k = 1-(1-k_0)e^{-c_3(t-t_0)}. \tag{42}$$

Moreover, as $\dot{k} \to 0$ for $t \to \infty$ this system is locally equivalent to a system of three autonomous ODEs (11) – (13) near the stationary state $E_g = (s_g, v_g, u_g)$ defined by (14a) for $k = 1$. At this stationary state the profit rate equals growth rates of fixed capital and net output (d).

[13] Not going into details, we recall the essence of this rule: when the investment share in net output equals the capital share in net output, consumption per working hour (measured in efficiency units) reaches and sustains maximum in a standard neoclassical growth model (Phelps 1961). This assertion was restricted to exponential growth paths.

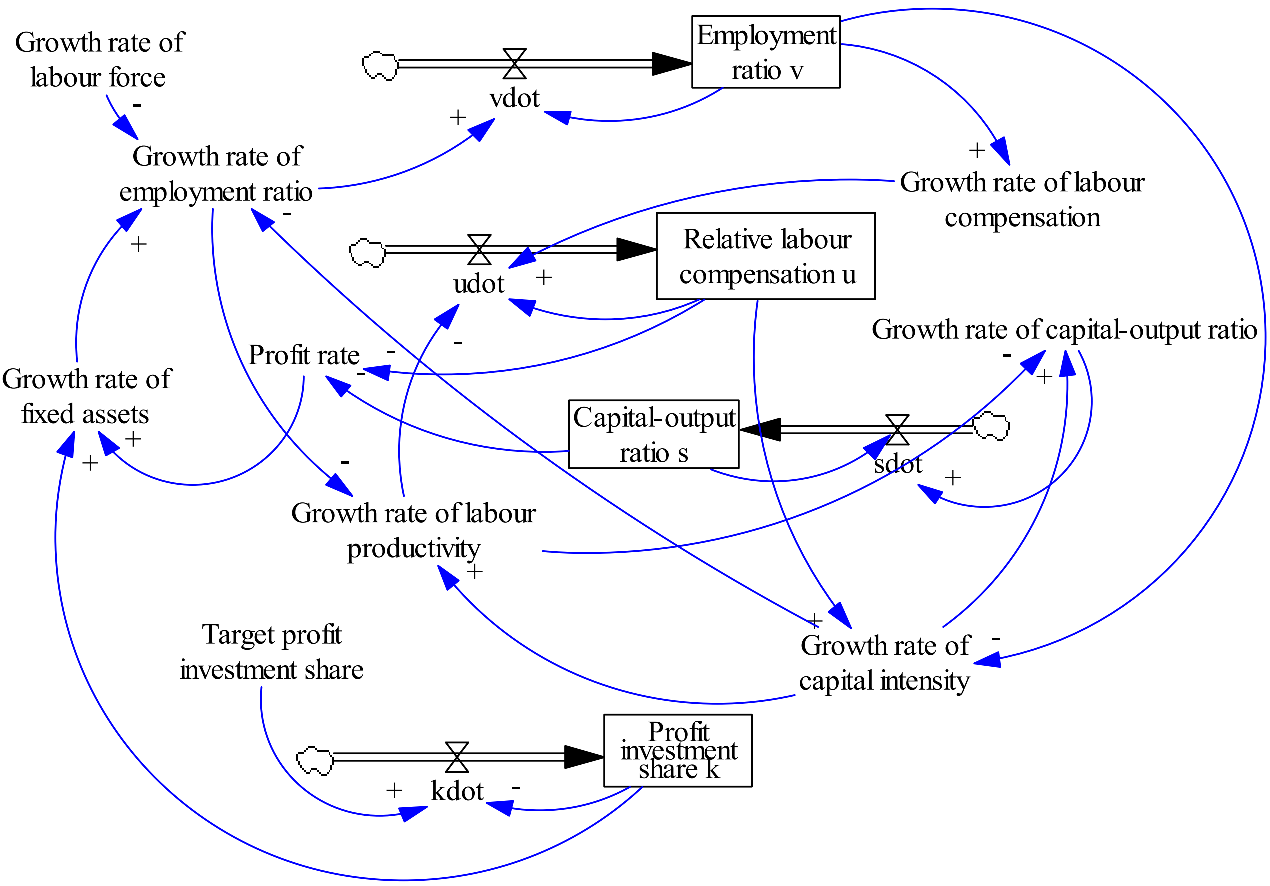

Figure 25. A condensed causal loop diagram of the eclectic model

Figure 25 presents a compacted overall causal loop diagram for this eclectic model. It contains four main stocks (corresponding to the state variables k, s, v and u) and four main flows (corresponding to the time derivatives of these state variables).

The positive adjustment coefficient in the equation (41) must be rather low for allowing solutions to remain within viable bounds. So for avoiding, in particular, $v > 1$, on transient to the stationary state E_g we have used $c_3 = 0.02$ (if $c_3 = 0.025$ $v > 1$ on transient to E_g in some years).

For $k = 1$ and $m \approx -1.254$ (initially from inertia Scenario I) the stationary state $E_g = (s_g, v_g, u_g)$ of this system of three autonomous ODEs is locally stable node or focus (independently of c_3) as $a_0 \approx 0.00041 > 0$, $a_1 \approx 0.035 > 0$, $a_2 \approx 0.1018 > 0$ and $a_1 a_2 - a_0 \approx 0.0031 > 0$. There is a rather smooth asymptotic transition to E_g from the initial point (x_0) for the year 2004 without noticeable fluctuations, unlike the two previous scenarios.

Still this policy is rather contradictory. It brings some advantages for workers. In particular, owing to this new policy, a number of unemployed in the years 2004–2030 is substantially lower on the average than in the other scenarios. The critical shortcomings of this scenario are extremely high stationary capital-output ratio ($s_f \approx 25.8 >> s_0$) and profit rate that is substantially lower than the profit rates in inertia Scenario I and in stabilization Scenario II (respectively, green, blue and red curves on Figure 26, see also Table 1 in Section 3.2). In other words, this scenario exacerbates the drawbacks of the modified inertia scenario with the constant profit investment share mentioned above.

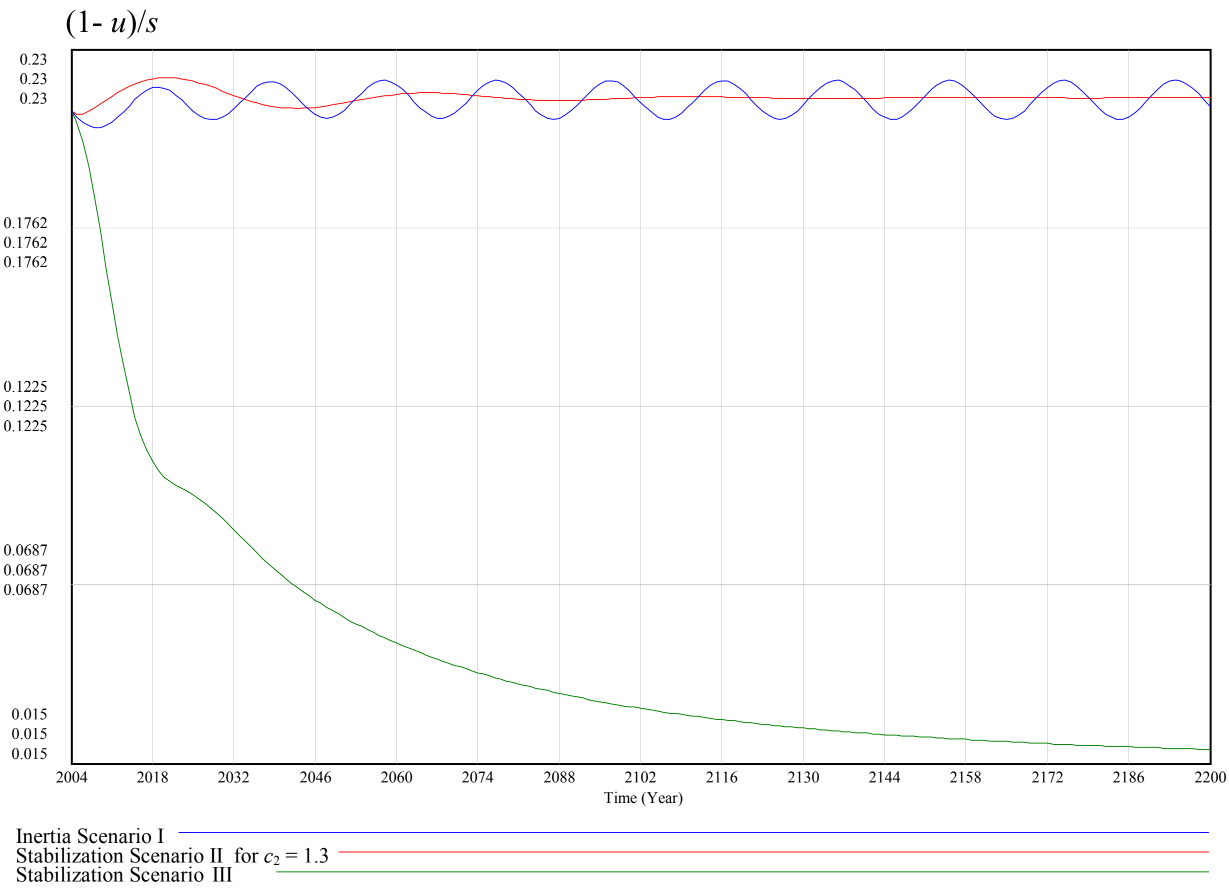

Figure 26. Profit rate $(1 - u)/s$ in the three scenarios, 2004–2200

Besides that, this policy does not guarantee stable economic growth for higher magnitudes of the parameter m of TPF (4). Therefore this stabilization policy associated with the neoclassical golden rule of accumulation would be socially objectionable, mostly for capital.

Conclusion

This paper has modified and refined both deterministic and probabilistic forms of the hypothetical law of capital accumulation (HL) for the modern Italian economy. It applies the new technical progress function (4) as the generalization of the Kaldorian technical progress function and Verdoorn relation. This enables HL to reconcile the direct relation between growth rates of net output and labour productivity with the inverse relation between growth rates of employment ratio and labour productivity.

In this paper, it has been assumed that factor inputs with different productivity are used. This assumption is coherent with the 'Ricardian' view of an inverse relationship between growth of employment and returns because of different levels of efficiency of technologies and labour forces. The suggested contribution to interpretation of the 'Ricardian' type of labour force in Italy is mainly focused on two key parameters ($n_3 < 0$ and $m < -1$) of the mechanization and technical progress functions reflecting the roots of the 'Ricardian' effects within the HL structure.

This paper also advances understanding of the role of the variable profit investment share in capital accumulation. It demonstrates that a secular decline in this ratio serves mitigating the tendency of profit rate to fall.

Magnitudes of non-observable parameters of HL are identified through application of a simplified version of the extended Kalman filtering (EKF) to macroeconomic data for Italy over a basal period 1980–2004. For this period, the retrospective statistical analysis is in a rather good agreement with the development of the Italian economy.

This paper explores analytically and numerically inertia Scenario I, its two modifications and two stabilizing Scenarios II and III of the Italian economic development in XXI century and beyond. After the year 2004, the Italian economy can show converging, diverging or structurally stable fluctuations depending on the intensity of the 'Ricardian' effect and on adjustment of the profit investment share.

In inertia Scenario I with a variable profit investment share, capital accumulation is marked by long swings with a period of about 20 years. Although the stationary state E_a is unstable focus, respective orbits in the state space diverge very slowly.

In the first modification of inertia Scenario I, keeping the profit investment share constant transforms the unstable focus E_a into the stable focus or stable node E_g. Compared with the unmodified inertia Scenario I, this improves labourers' well-being but worsens profitability. It becomes visible that the labourers, rather paradoxically, are more interested in more vigorous capital investment in the domestic economy than the capitalists. On the premise that the society is lead mostly by capital, it will hardly accept stabilization policy of this kind.

In the second modification of inertia Scenario I, decelerating adjustment of profit investment share to its stationary magnitude depending on profitability would be stabilizing for long swings over centuries without altering the non-trivial stationary state E_a. Still this is not practical because of a bit lower profitability and due to higher variation of main variables on the transient to the stationary state during initial decades than in the initial inertia Scenario I.

By establishing the explicit inverse relation between profit investment share and capital-output ratio this paper supposes the control law (CL) of capital accumulation. Stabilization Scenario II based on CL describes a transient to a new stationary state E_b (stable focus). CL smoothes long swings and prolongs their quasi period in this scenario compared with inertia scenario I. The new stationary profitability is a bit higher than stationary profitability in inertia Scenario I. The improvement in profitability (compared with the inertia scenario and its two modifications) is also achieved on the transient to the stationary state E_b.

It is demonstrated that stability of the new stationary state E_b is vulnerable to particular changes of the control parameter (m) from the technical progress function (4). This stationary state loses stability and stable 'local' limit cycles emerges through the Andronov – Hopf supercritical bifurcations as the modified Scenario II demonstrates (at $m = m_0 < 0$).

The fallacy of the stabilization policy that is reminiscent of the neoclassical golden rule of accumulation is demonstrated with the help of the eclectic model in the stabilization Scenario III, which is not practical for too low transient and stationary profitability. This analysis deepens a critique of the neoclassical conceptions of growth and distribution.

Finally, this new macroeconomic application of the system dynamics method explains why finding suitable compromise of material interests of the two main social classes is complicated in modern Italy. A future research will elaborate stabilization policies in the context of *open* economy.

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