A SYSTEM DYNAMICS MODEL FOR STUDYING THE STRUCTURE OF NETWORK MARKETING ORGANIZATIONS

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Abstract

The importance of social structures in analyzing a diffusion of ideas and innovations is widely acknowledged. This paper presents the first step in the construction of a system dynamics model to study social diffusion phenomena using network marketing as the specific structure through which products and services are spread. Network marketing organizations are direct-selling channels that recruit new distributors and form a particular type of social network which is shaped through time and based on preferential attachment. The paper presents a way to generate the topology of such a network so as to have the basis for analyzing the diffusion of products and services through such channels; a variant is introduced to an existing model developed with systems dynamics for generating scale-free networks with preferential attachment. We found that the resulting model generates an adequate network topology for analyzing essential characteristics of the way network marketing organizations are formed. This structure is the base for exploring the diffusion of products through such a business model; exploration of these processes constitutes the next step for this project.

Keywords: *network marketing, scale-free networks, preferential attachment, diffusion*

1. INTRODUCTION

Currently, one prominent issue of research is the relevance and effect of network topology in social diffusion processes in terms of efficiency, reachability and speed. In particular, we are interested in studying the diffusion of attributes through the structure of network marketing organization (NMOs), a type of direct-selling business model.

McCutchan and Campos-Náñez (2007) used systems dynamics (SD) to evaluate the effect of network topology in social diffusion processes; in particular proposing a model for generating scale-free (SF) networks using preferential attachment. In this paper we present a variant of this previous model and apply it to the study of network marketing (NM) structures which can be characterized as SF networks. This is the first step in our project for studying diffusion processes through NM channels.

The paper is organized as follows. The next section presents the business model known as NM. The third section introduces the structure of NMOs as a specific type of social network; it presents an overview of social networks theory and the characterization of NM topology as a type of SF network with preferential attachment. The fourth section describes a way of handling network topology with a SD model using the ideas of McCutchan and Campos-Náñez (2007); it also introduces and assesses a model for generating SF networks with NM characteristics. The fifth section presents the next steps to follow in order to study diffusion processes through the generated network using our findings from this first model. The last section summarizes the main points.

2. NETWORK MARKETING

2.1. Business Model: Network Marketing

Direct selling (DS) is a retail level distribution and commercialization channel for products and services. It is based on direct contact between people. In this model, independent distributors make profits from buying and selling when they provide a product or service to a customer (World Federation of Direct Selling Associations – WFDSA, 2006). NMOs are DS channels that also recruit new distributors into a growing network over time. These independent distributors have access to a compensation plan for inviting other people to join the network, to buy and to distribute products or services. This compensation plan is based on the business group volume, which includes each distributor's volume and the volume of the other distributors that have been recruited (Bhattacharya & Nehta, 2000) —see below. Traditionally, DS has represented a strong force in product distribution. Nowadays, 70% of DS revenues are generated by NMOs (Coughlan & Grayson, 1998).

2.2. Characteristics of Network Marketing Organization

According to Coughlan and Grayson (1998), a network marketing organization (NMO) is defined as a company whose income depends only or principally on DS and whose direct distributors are rewarded for: *(1)* buying products or services, *(2)* selling products or services and *(3)* finding other distributors to buy and sell products or services, too.

The main characteristics of NMOs are (Coughlan & Grayson, 1998):
(a) Human resources: they do not manage a large employee sales force.
(b) Savings: they do not invest in advertising and supply chains.
(c) Remuneration: distributors do not earn a basic salary but a net commission in proportion to their retail markup.
(d) Assistance: they offer products and business training to their distributors.
(e) Rewards: they have a compensation plan depending on each distributor sales volume.

This model has been adopted by different companies around the world. They include: Amway, Excel, Forever Living Products, Herbalife, Mary Kay, Nokken, Nu Skin Enterprises, Primerica and Shaklee (King & Robinson, 2000).

Compensation Plans
Basically, NMOs compensate their independent distributors in three different ways (Coughlan & Grayson, 1998):

(1) Distribution: distributors purchase the product or service at wholesale prices and provide it to a final customer at retail price. Markup profits are approximately between 30% and 50%. They can also purchase it for themselves.

(2) Personal volume (*PV*): distributors receive a commission for their PV, i.e. the value of every product they buy or sell during a period of time established by the NMO. At a higher volume the commission increases.

(3) Group volume (*GV*): distributors are rewarded with a *net commission* for their group volume. This volume includes the PV plus the value of the products or services bought or sold by people that belong to the distributors' networks during the same period (see below). People who are recruited by the distributors into their networks are called *downline distributors*. The distributors' GV is therefore the sum of the distributors' PV plus the PV of each downline in the distributors' network.
Figure 1 shows an example of a distributor network. Distributor 1 has recruited directly Downline 1, 2 and 3; additionally Downline 2 has recruited directly Downline 4 and 5. Note that in the network distributors have their own PV and GV which determine their total net commission at the end of the period. In the example, Distributor 1 completed $200 PV, and including his downline distributors' volume he reaches $700 GV. Likewise, Downline 2 completed $200 PV and $300 GV, including his downline distributors' volume. Note that Downlines 1, 3 and 4 only reach $100 GV each, which corresponds to their $100 PV because there are no downline distributors in their networks yet. Downline 5 has $0 GV and therefore $0 PV, which means that he did not buy or sell any product or service during the period.

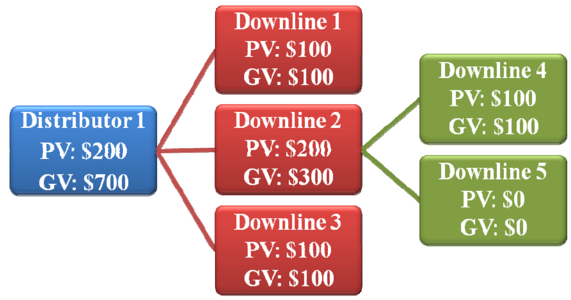

Figure 1. *Example of a distributor's network.*
Adapted from: Coughlan and Grayson (1998, p.403).

The *net commission* of each distributor is the difference between their own commission rate and the commission rates of their direct downline distributors (Coughlan & Grayson, 1998). The next example illustrates this point.

Table 1 shows a NMO's commission system. During a fixed period of time, if the distributors reach between $100 and $299 GV, they earn a commission rate of 2% on their GV; between $300 and $499 they earn a commission rate of 5% on their GV, and so on.

COMMISSION SYSTEM	
GV	RATE
$100 - $299	2%
$300 - $499	5%
$500 - $799	8%
≥ $800	12%

Table 1. *Example of a NMO's commission system.*
Adapted from: Coughlan and Grayson (1998, p.403).

According to the table, in the example of Figure 1, Distributor 1 rate commission is 8% on his $700 GV, this is $56; but this commission includes Downlines 1, 2 and 3 commissions, which for Downlines 1 and 3

are 2% on their $100 GV, which is $2 each and for Downline 2 it is 5% on his $300 GV, which is $15. Distributor 1's net commission is therefore $37 ($56 - $2 - $2 - $15 = $37).
Similarly, Downline 2's rate of commission includes Downlines 4 and 5 commissions, which for Downline 4 is 2% of his $100 GV, which is $2, and for Downline 5 is in this case $0, because his GV is $0 or less than $100. Downline 2's net commission is therefore $13 ($15 - $2 - $0 = $13). Table 2 summarizes the net commissions for each one of the distributors.

GV		$700
GROUP RATE COMMISSION		8%
Total Group Commission		$56
	Rate Commission	Net Commission
Distributor 1	8%	$37
Downline 1	2%	$2
Downline 2	5%	$13
Downline 3	2%	$2
Downline 4	2%	$2
Downline 5	0%	$0
Total Group Commission		$56

Table 2. ***Net commissions for a NMO's distributors.***

Note that these commissions correspond only to the third component of the NMOs' compensation plan. Distributors also receive a commission for their PV, second component, and the markup profit for the distribution, first component (Coughlan & Grayson, 1998).

2.3. A Growing Channel of Distributors for the Diffusion of Products and Services

NMOs show a dynamic structure. The distribution channel is formed continuously as new distributors enter the network. Usually, when a new distributor joins a NMO, he buys a personal code, also called a personal franchise or business kit. Some NMOs sponsor new distributors for free by just filling in an application form. In both cases, this NMO's membership allows – but not forces, the new distributor to *(1)* buy NMO's products or services, *(2)* sell those products or services to their customers – they are not within the network, and *(3)* recruit other distributors to buy and sell the products or services, too; it therefore permits the distributor to access the NMO's compensation plan.

Initially, one person joins the network as an independent distributor of the NMO. Then the distributor begins to buy, consume and sell the products or services that the NMO offers; he also invites other people to join the network. This process continues repeatedly with each distributor within the NMO's network. Figure 2 shows an example of the evolution of a distributor's network over time.

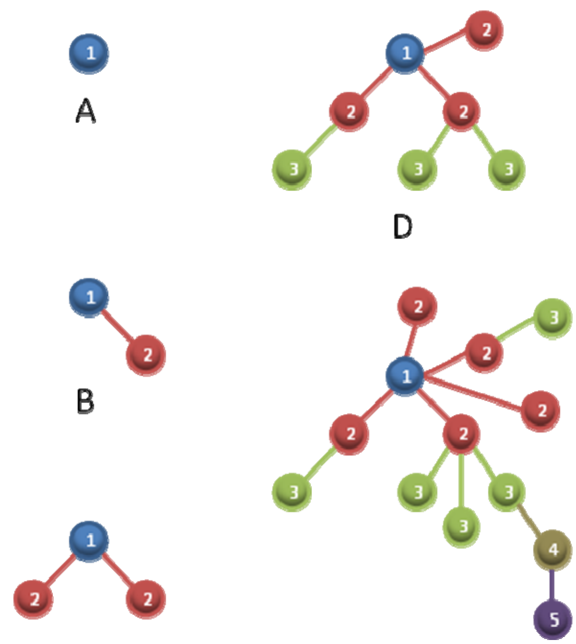

Figure 2. ***Distributor's network over time.***

Stage A illustrates the initial state. When the distributor recruits his first downline, the new distributor enters the network forming one connection with his sponsor, also called *upline* (Stage B). Similarly, another distributor becomes the second downline in the next time-step (Stage C). As the new distributors begin to invite other people, the network will grow, increasing the number of total distributors (Stage D). After several time-steps, the network will exhibit a structure similar to the one shown in Stage E.

Structure Growth and Diffusion

For our study, it is necessary to distinguish two important processes within the NM business model: the network growing process and the diffusion of products or services.

In the ***network growing process*** three relevant issues must be emphasized. First, note that when a new distributor is recruited, only one connection is generated to his/her upline. This feature follows NM characteristics because it is not possible for a person to be recruited by two or more distributors; this would be incompatible with NMO's compensation plans.

Second, the fact that a new distributor joins a particular NMO does not mean that he or she is interested in doing the three activities allowed by the membership; some of them will join only to purchase the products or services for themselves; others will join just to sell them to their customers and to access the markup profit, but not necessarily to recruit new people. This issue is important because only those who are interested in the third activity (recruitment of further distributors) will allow the network to grow.

Third, as a consequence of the last issue, as the network continues to expand, it is possible to identify a few distributors who grow by being successful in their own businesses; these are the distributors who have recruited a larger quantity of direct downline distributors. This means that in the long run, new distributors will not join all existing distributors in the same way, time, speed or quantity. They will be added with a higher probability to those who are being successful. This makes sense because they are the only ones that are spreading the business model structure or inviting others to join the network; they also have more experience of the products or services, the business model, the NMO and its compensation plan, and therefore they are succeeding and reaching higher commissions with their businesses. Over time, those distributors will become stronger and more important within the network.

Concerning the ***diffusion of products or services*****,** one relevant issue must be emphasized. The NMO membership allows but does not force the new distributors to buy the NMO's products or services. Thus, when the network is established, this does not imply that the distributors are spreading those products or services. In other words, if a NMO has a network of a huge quantity of distributors, e.g. 1000, but none of them are consuming or buying its products or services, there will not be a diffusion process and, in consequence, neither the NMO nor the distributors will earn any income.

In this article we are interested in the *network growing process*. We will show that the three issues emphasized within this process determine the network topology of NM and NMOs which allows us to study, later on, the diffusion of products or services.

2.4. Significance of Understanding Growth and Diffusion of NMOs

The NM business model has been recognized as a rapidly-growing industry during the last decade – across the world, sales reached more than US$80.4 billion in 1997, US$85.7 billion in 2002, US$89 billion in 2003 and US$109 billion in 2006, with a sales force of 61.45 million people (WFDSA, 2006). In spite of this increasing acknowledgment and the huge quantity of independent distributors with a dramatic lifestyle change, many people have failed to succeed (Martinez, 2007).

Many factors contribute to the failure of NMOs' distributors. Some of them are related to income expectations, budget investment in the business, different products benefits, and the most relevant characteristic for the study we are presenting here: the marketing plan and the network growth (Martinez, 2007). The problem with this last factor is that in order to be successful and reach high incomes, the NMOs' distributors have to go through a lot of people to find the ones really interested in building the business (Martinez, 2007). One of the authors has been involved with a NMO and has also been interviewed with successful distributors within the NM business model. From the information received through these sources, it has been estimated that each distributor has to recruit approximately five to seven downline distributors to find one agent really interested in building the business. Moreover, in order to recruit two new downline distributors, he or she has to invite and share the NM business model idea with approximately twenty or twenty-five people. Though this is not empirically validated, these figures reflect the efficiency of people already connected with NMOs who achieve significant results in their NM businesses.

As the distributors recruit more downline distributors, the chance of being successful therefore increases. But the target population is not infinite, so unlimited recruiting programs may collapse in the long run (Taylor, 2004). It is considered that 99% of NMOs' distributors lose profits because the costs associated with building the business exceed the returns. A few distributors are therefore getting higher incomes at the expense of many downlines recruited by them who buy the NMO's products or services and receive the lowest incomes (Taylor, 2002).

These statistics may not be encouraging for NMO distributors. Distributors entering a NMO network should consider that building their networks requires a large amount of effort in order to find agents really interested in the NM model, who in turn should succeed in finding further agents, too. This explains why distributors might leave the NMOs' network: because of the time required to achieve significant results and also because of the dependence on the efficiency of other downlines.

Because of these facts, the NM model has been challenged in terms of its capacity to spread a product or service through a population (Taylor, 2002, 2004; Martinez, 2007). Our interest in using system dynamics is to study the possibilities and characteristics of this diffusion process through that specific type of network.

3. NETWORK MARKETING AS A SOCIAL NETWORK

We have presented the NM business model, the main characteristics of NMOs and the source of our motivation for studying social diffusion phenomena using NM as the particular structure through which products or services are spread. Now we can study the structure of these channels using social network theory.

3.1. Network Theory

Networks
A network is a set of items connected by links. Those items are called vertices or nodes and their interconnections are called edges or arcs (Newman, 2003). An example is shown in Figure 3.

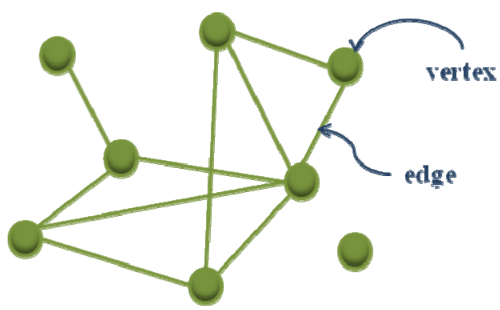

Figure 3. ***Example of a network.***
Adapted from: Newman (2003, p.2).

The number of edges connecting from a particular vertex to other vertices within the network is called its *degree*. Vertices with a high number of edges are called *hubs* (Barabási & Oltvai, 2004).

Properties of Networks
Networks have different properties. Some of them are of particular interest for this study:

(1) Small-world effect: it is presented in networks with short path lengths which imply that, on average, a node is a few steps away from any other node; a second characteristic is high clustering values, which means mutual connectivity within nodes. This property is important in particular dynamic spread processes, e.g. epidemics, innovations (Watts, 2003).

(2) Degree distribution $P(k)$: it refers to the distribution of the degree of the nodes within the network (Newman, 2003). It gives the probability that a selected node has exactly k number of edges (Barabási & Oltvai, 2004). $P(k)$ can be represented as a histogram with the number of nodes of degree 1, 2, 3 and so on. This property is important for distinguishing between different classes of networks. e.g. a peaked degree distribution means no highly connected nodes whereas a power-law degree distribution indicates few hubs connecting with many less connected nodes (Barabási & Oltvai, 2004).

Network Dynamics
As a further issue, we should consider two types of network dynamics. Watts (2003) defines them:

*(1) Dynamics **of** the network*: it refers to the processes of making and breaking network links; this definition denotes the very development of the network structure itself. The dynamic analysis focuses on the processes by which the network is formed.

*(2) Dynamics **on** the network*: it refers to the activities of actors (nodes) that are linked within the network, for example, spreading a rumor. The dynamic analysis focuses on the influence between the individuals (nodes) in the network.

It should therefore be noted that this article focuses on the dynamics *of* social networks following the rules of NMOs. In contrast, the next step for this project will focus on the dynamics *on* NMOs' networks.

Social Networks

According to Newman (2003) there are four types of networks: social networks, information networks, technological networks and biological networks. We are interested in the first kind of network.

A social network is a set of people interacting within a social system. They are flexible and dynamic because of the frequency with which connections are formed and dissolved. These networks have four components (Davern, 1997):
(1) Structural: it refers to both the form in which the nodes and arcs are organized and the strength of the arcs between the nodes.
(2) Resources: it concerns the characteristics of the actors or nodes; for example, knowledge, abilities, gender, a specific attribute, etc.
(3) Normative: it refers to rules that determine the behavior of the actors.
(4) Dynamics: it emphasizes the generation of links and the continuous change in the network.

Following the previous distinction made between dynamics *of* the network and dynamics *on* the network, we underline that these two dynamic aspects are closely interrelated in social network analysis. On the one hand, there is a relationship between the way the links are formed among actors and the roles they have in their social environment. On the other hand, the network can be assumed to be a conduit to spread information (or, for instance, products) and influence others. This condition depends on the position of the actor within the network and on who can access, reach across or influence the links between the actors nearby (Watts, 2003).

Models of Networks

In addition, Barabási and Oltvai (2004) suggest three principal models of networks that we should consider further:

(1) Random networks: known also as Erdos-Rényi models. Given a quantity of vertices or nodes, each pair of nodes is connected with probability *p*, which generates a graph with randomly-placed edges. The node degrees follow a Poisson distribution. In Figure 4 most of the nodes have the same amount of links; it also shows high clustering and short path lengths, reflecting the small-world effect (Barabási & Oltvai, 2004).

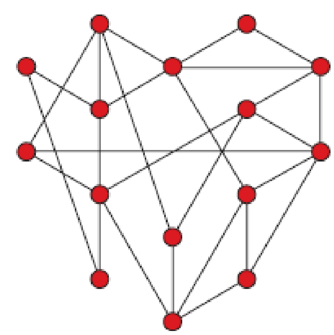

Figure 4. *Random network.*
Source: Barabási and Oltvai, (2004, p.105).

(2) Scale-free networks: it has a power-law degree distribution, which has the form:

$$P(k) \sim k^{-\gamma}$$

where k is the node degree and γ is the degree exponent with range $2 < \gamma < 3$. Two generic aspects characterize this model. The first one is that the 'network continuously expands by the addition of new vertices that are connected to the vertices already in the system' (Barabási, Albert & Jeong, 2000, p.73). The second is that the network exhibits *preferential attachment*, so there is a 'higher probability to be linked to a vertex that already has a large number of connections' (Barabási et al., 2000, p.73). Figure 5 shows that a few nodes, square ones also called *hubs*, are well connected in comparison with the other nodes. For smaller values of γ the importance of the hubs increases and vice versa; for $\gamma = 2$ a network with one large hub connected with a large fraction of all nodes emerges; in contrast, for $\gamma > 3$ the network behaves like a random one. This model is characterized by the ultra-small-world property: its path lengths are shorter than those presented in random networks (Barabási & Oltvai, 2004).

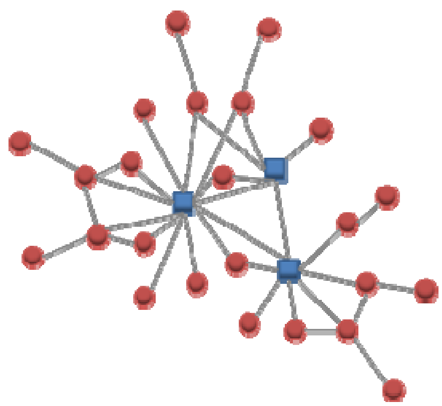

Figure 5. *Scale-free network.*
Adapted from: Barabási and Oltvai (2004, p.105).

(3) Hierarchical networks: their construction starts with a small cluster or module with a given quantity of nodes all linked. Then replicas of this module are generated, all of them connected to the initial one from the external nodes. The resulting module is replicated again, all of them connected to the initial small cluster from the peripheral nodes. Figure 6 shows an example with an initial small cluster of four nodes. This model exhibits co-existence of modularity, the tendency of nodes to form clusters or groups, local high clustering and SF topology. In addition, it implies communication between high cluster areas maintained by a few hubs (Barabási & Oltvai, 2004).

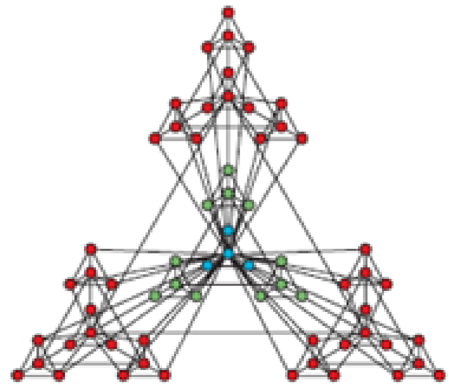

Figure 6. *Hierarchical network.*
Source: Barabási and Oltvai, (2004, p.105).

A last remark regarding models and properties of networks follows. In the past, some authors defined how the 'small world effect' constituted a specific model of network (e.g. Watts and Strogatz cited in Kim, Jun, Kim & Choi, 2006). Various studies, however, have established that the 'small-world effect' is a common feature of *all* networks; this effect has been subsequently shown in several systems (Barabási & Oltvai, 2004). Affiliation networks of movie actors, the power transmission grid of the western United States, neural networks, World Wide Web and random networks are all good examples (Watts, 2004; Barabási & Oltvai, 2004).

We have explored various properties and networks models. Now we can outline the NM business model and NMOs within this theory of networks.

3.2. Network Marketing as a Scale-Free Network

We introduce NM as a business model that can be represented as a SF network. The definition of NM and the characteristics of NMOs as a whole, with the description of social networks and the explanations of the models of networks explored previously, mean that NM can be characterized as a social network and in particular can be modeled as a SF network based on preferential attachment.

Related Work

We have found two precedents for modeling NMOs within the theory of networks.

First, Kim and colleagues (2006) characterized NMOs with a 'small-world network' model based on the two characteristics of the small-world effect property we mentioned above: short path lengths and high clustering values. As regards this previous work, we will make two comments. First, as we mentioned in section 3.1, it has been established that the small-world effect is more a property than a model of networks and it is a common feature in most complex networks; moreover, we are interested in the dynamics of how the network is formed and how the diffusion process is carried out across it – different from the analysis of Kim and colleagues (2006) who focus on path length as the key structural network property. Second, we suggest modeling NMOs as SF networks with preferential attachment because, as Barabási and Oltvai (2004) underline, the degree distribution property is more relevant than the path length in terms of characterizing network classes. Finally, since in NM one downline can be recruited only by one existing distributor, there are no high clustering values within a NMO's network, which is one of the two prime characteristics of the small-world effect. This means that we are proposing to model a SF network with only one of the two characteristics of the small-world property; short path lengths. We will see that despite this, our resulting network will fit with the theoretical power-law degree distribution form which we take as the defining attribute for SF networks.

The other precedent is work developed by Legara, Monterola, Litong-Palima and Saloma (2006); they use agent-based modeling to characterize the specific structure of a *Multilevel Marketing* business model, equivalent to NM, called *binary network structure*, as a SF network with preferential attachment. Within this specific compensation plan, each distributor can only recruit at most two direct downline distributors. In contrast, in our project the compensation plan should not restrict the number of direct downline distributors belonging to a particular node. Moreover, we are interested in using SD (instead of agent-based modeling) both for developing an application of the previous work of McCutchan and Campos-Náñez (2007) and connecting this application with the previous body of literature on SD diffusion models.

Network marketing organization as scale-free networks

As we previously showed, NMOs are channels for spreading products or services to customers. They do not invest huge quantities of money in advertisement and supply chains. On the contrary, they offer the opportunity to their customers to play this role by means of direct contact. The result is a continuously growing network of people interacting and allowing the spread of their products or services through a whole population. This social network of distributors is flexible and reflects the four components: *structural* – upline and downline distributor's interactions, *resources* – distributor's abilities and knowledge, *normative* – NM rules established by NMOs, and *dynamic* – continuous recruitment of new distributors.

Hierarchical networks do not fit the requirements for modeling NMOs. Note that since new distributors connect directly only to one sponsor, it cannot be possible to identify clustered modules to be replicated, let alone a co-existence of modularity.

In addition, NMOs cannot be modeled as random networks either. This model assumes that there is a fixed quantity of nodes to be connected and NMOs do not establish a desired quantity of distributors in their networks; on the contrary, they allow a continuous addition of new distributors over time. Also, NMO's networks do not follow a Poisson degree distribution; new distributors enter the network based on the connections already established and not as independent processes.

A NMO can be better modeled as a SF network with preferential attachment because its characteristics fit with the two generic aspects of this model. First, the distributors' networks continuously expand over time by the addition of new distributors to those already in the system. Second, new distributors connect with a higher probability to those who have more connections or direct downline distributors in their networks. This means that they are spreading the business model idea; have more experience with the products, services, the business model, the NMO and its compensation plan; and finally are succeeding and reaching higher commissions. These special characteristics constitute preferential attachment.

4. MODEL CONSTRUCTION

The previous description of NMOs as social networks that can be particularly modeled as SF networks with preferential attachment is the first result with which we can now proceed to generate the corresponding structure.

4.1. The Model

For the construction of the model we used the one developed by McCutchan and Campos-Náñez (2007) for generating a SF network structure with SD techniques; they used the software *iThink*. We made a variant of that model to introduce the NM structure characteristics.

Modeling Scale-Free Networks

McCutchan and Campos-Náñez (2007) used arrays to build a stock and flow model for SF networks through preferential attachment. These arrays represent the degree distribution of the network; this is the number of nodes with a given degree. There are ten arrays in the model, each one representing the number of nodes with degree 1 to 10. The first array represents the number of nodes with degree 1, the second array the number of nodes with degree 2 and so on; the last one, the tenth array, includes all the nodes with degrees equal to and higher than ten. These arrays or histograms provide the necessary information, quantity of nodes within the network and their number of edges, to represent the network structure.

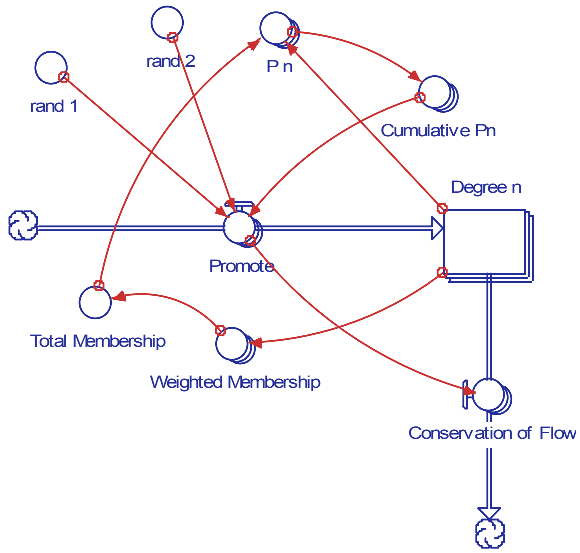

Figure 7. ***Scale-free network model.***
Source: McCutchan and Campos-Náñez (2007).

The model is shown in Figure 7. The degree number is indexed to the elements *Degree n, Conservation of Flow, Promote, Cumulative Pn, Pn* and *Weighted Membership*. The dynamic of the model can be described as follows. *(1)* One node enters the network each time-step. *(2)* The node will be connected with higher probability to those with a higher number of links or higher degree, i.e. the preferential attachment mechanism; this probability is computed using the elements *Total membership, Weighted Membership, Pn* and *Cumulative Pn*. *(3)* Based on the probability, the variables *rand 1* and *rand 2* are used to determine the two connections that the entering node will establish to the existing ones – for each variable one connection is made. *(4)* The two nodes connected to the entering one are promoted from their current degree to the next one; for each of the two links or connections the valves *Promote* and *Conservation of Flow* increment and decrease by one respectively the corresponding value of the array in *Degree n*. In the next time-step a new node enters the network and the process repeats itself. This is the method by which the SF network is formed over time.

Modeling Network Marketing Structure

The model proposed by McCutchan and Campos-Náñez assumes that each new node entering the network establishes two connections to the existing ones. Previously we mentioned that in the NM business model, when a new distributor is recruited, only one connection is generated to his/her upline. We introduce this variation in order to generate a network with the same SF network topology but including this particular characteristic of NM structure. Hence, we eliminated one of the two variables in the model that determine the attachments that the entering node will establish when it joins the network. With this modification, when a new node enters the network it will be connected only to one corresponding upline – in the model *rand 2* is eliminated, the remaining formulas are readjusted and therefore the single connection is determined by *rand 1*. Appendix 1 shows the equations of this model.

Running the Model

Initial conditions are: two nodes mutually connected, both of degree 1. These are the minimum initial conditions for two reasons: *(1)* if there are no nodes in the network, no one can be joined – probability of being linked to a node is zero, and *(2)* then the minimum node degree is 1, to be compatible with the theory of networks and for one node with degree 1 there must be another node in the network which is linked to it – a node cannot be connected to itself.

These initial conditions of the model – first time-step – can be interpreted as the NMO being one of the nodes – square one – and

Distributor 1 being the other node, as shown in Figure 8A. It can also be interpreted excluding the NMO as two initial distributors, Distributor 2 – circle one, being recruited by Distributor 1 – square one, also shown in Figure 8A. For different initial conditions – more nodes connected to the NMO or to one distributor – the model performs in the same fashion. Figure 8B shows an alternative version of the initial network.

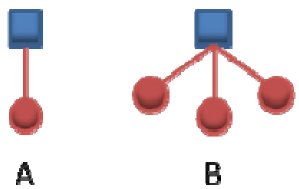

Figure 8. ***Initial network.***

In the model a new node is added to the network at each time-step, which establishes, with the preferential attachment mechanism, one connection to an existing node.

Visualizing the resulting structure

It is important to recognize the particular structure that our model generates. Certainly, visualization techniques exploit human vision and spatial cognition so as to increase understanding of complex processes and to enhance communication of concepts (Card, Mackinlay & Shneiderman, 1999). As we mentioned above, the arrays or histograms provide the quantity of nodes within the network and their number of edges, and this is the degree distribution. This is the necessary information to represent the network configuration and what it looks like. With the information provided by the model, it is possible to construct a diagram of the resulting network. Since we cannot generate the visual representation of the network with *iThink*, we implemented the same dynamics to generate the SF network topology using the software *Mathematica 6*. In this way we obtained the network diagrams that our SD model creates.

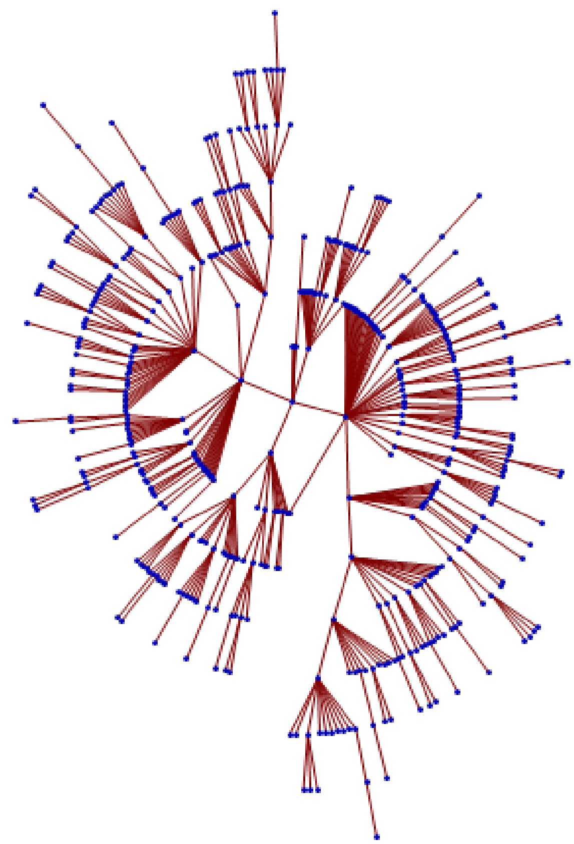

Figure 9. ***Network diagram of 500 nodes.***

The *Mathematica* model constructs a symmetric adjacency matrix of the network; this is a 0-1 matrix which indicates the connections between nodes. This matrix is constructed with the preferential attachment mechanism that was used for the arrays in our SD model; again, each time-step a new node is added and establishes one connection to an existing node. Once the matrix is created, we use a *Mathematica*'s graph function to diagram the network. Figure 9 exhibits a network diagram of 500 nodes obtained with the model. Appendix 2 defines an adjacency matrix which shows the equations of this model and other network diagrams obtained using different number of nodes.

In addition to the diagram, we are interested in observing the growth of the network over time. Figure 10 presents the first six nodes added to the network, including a label for each node. Figure 11 shows the evolution of a network of 153 nodes during four stages – A with three nodes, B with fifty-three nodes, C with 103 nodes and D with 153 nodes. Again, note that the initial condition of the model is a network with two nodes connecting each other.

It is now easier to appreciate the particular characteristic of SF networks generated by our models: the structure grows via preferential attachment showing the consequent few well-connected hubs.

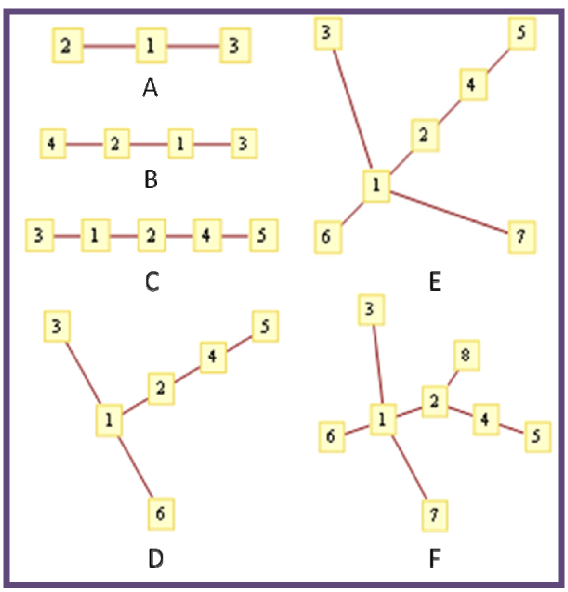

Figure 10. ***First six nodes added to a network.***

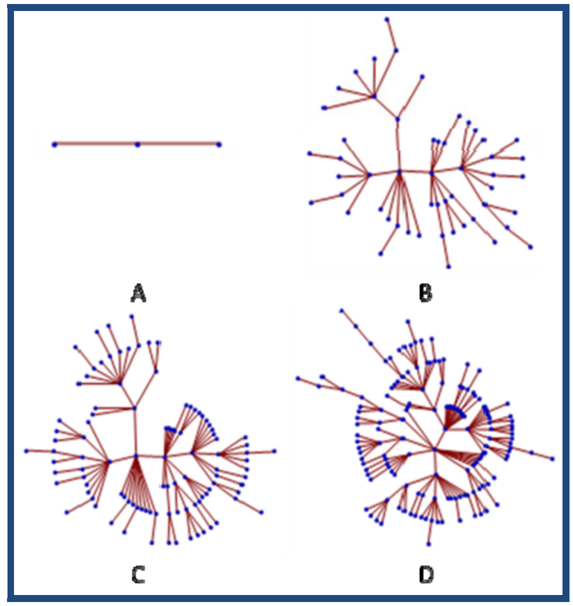

Figure 11. ***Evolution of a network of 153 nodes.***

4.2. Assessment of the Model

We should check that the results from our SD model fit the theory of social networks. In this section we synthesize relevant findings. Appendix 3 gives details.
We have to verify two aspects. On the one hand, the model should generate SF network structures; on the other hand, we should verify that these structures effectively correspond to the logic of NM.

Scale-free Network Structures

First, we verified that the resulting model does in fact generate a SF network. Using least squares regression, we compared the power-law distribution of our model with the one obtained by McCutchan and Campos-Náñez in their original model. We also analyzed the degree distribution using maximum likelihood estimation and testing the goodness-of-fit with the Kolmogorov-Smirnov test.

Least squares regression
According to McCutchan and Campos-Náñez, the analytical solution for SF networks with preferential attachment is:

$$P(k) = 2m^2 k^{-\gamma}$$

where m is the number of connections the new node establishes when it enters the network. McCutchan and Campos-Náñez established the analytic degree distribution for their model as $P(k) = 8k^{-3}$. The analytic solution for our model $(m = 1)$ can therefore be expressed as $P(k) = 2k^{-3}$. McCutchan and Campos-Náñez fit this distribution using least squares regression to obtain $\gamma = 2.71$ and $R^2 = 0.98$. They conclude that the model generates a SF network because their analytical solution was inside the variance that can be found in the real world.

DEGREE DISTRIBUTION			
DEGREE	No. NODES		
	500 TS	1000 TS	5000 TS
1	331	652	3262
2	81	169	838
3	34	69	347
4	18	36	179
5	11	21	106
6	7	14	68
7	4	10	41
8	3	6	31
9	2	4	25
>=10	11	21	103
TOTAL	501	1001	5001

Table 3. ***Average degree distribution across three simulation groups, each one with ten simulations of 500, 1000 and 5000 time-steps.***

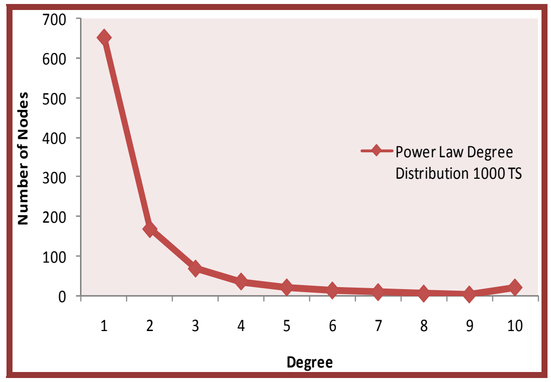

Figure 12. ***Power-Law Degree Distribution for ten simulations of 1000 time steps each.***

We performed three experiments based on this. Each one had ten simulations of 500, 1000 and 5000 time-steps, generating networks of 501, 1001 and 5001 nodes respectively – the first step performs the initial conditions. The average degree distributions across each one of the three simulation groups are shown in Table 3. Figure 12 shows the power-law degree distribution graph for the simulation group of 1000 time-steps. Again, note that degree 10 includes all the nodes with degree equals and higher than ten; this explains the small peak in the tail of the graph. The power-law degree distribution graphs for the simulation groups of 500 and 3000 time-steps are shown in Appendix 3.

The distribution was fitted using a least square regression: we computed the natural logarithm (ln) for the number of nodes of each degree and the slope for the resulting log data for each of the three experiments. The regression results obtained for 1000 time-steps are exhibited in Figure 13. The results obtained for 500 and 5000 time-steps are shown in Appendix 3. The values of γ and R^2 for all the simulation groups are shown in Table 4.

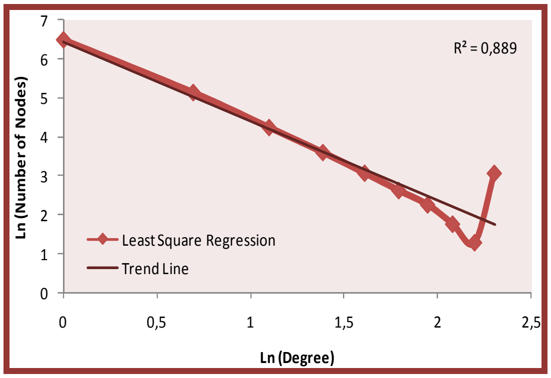

Figure 13. ***Fit Distribution for ten simulations of 1000 time-steps each.***

LEAST SQUARE REGRESSION		
TS	γ	R^2
500	2,052	0,875
1000	2,036	0,889
5000	1,996	0,903

Table 4. ***Values of γ and R^2 obtained for 500, 1000 and 5000 time-steps each.***

Note that the least squares regression do not fit perfectly owing to the peak in degree 10. The values for γ are between two and three except in the case of 5000 time-steps; again, this can be because of the large number of nodes of degree 10 in comparison with the number of nodes for 100 or 1000 time-steps. This is a limitation of the *iThink's* arrays. From this, we estimated the same parameters excluding the number of nodes of degree 10 obtaining an improvement in the fitting as shown in Figure 14 for 1000 time-steps and Table 5 for all three simulation groups. Appendix 3 also shows the fit distribution excluding the number of nodes of degree 10 for 500 and 5000 time-steps.

In sum, the γ values obtained are between two and three and the R^2 values are near to one. Comparing these results with the McCutchan and Campos-Náñez findings, we find no significant difference between the power-law degree distributions of both models.

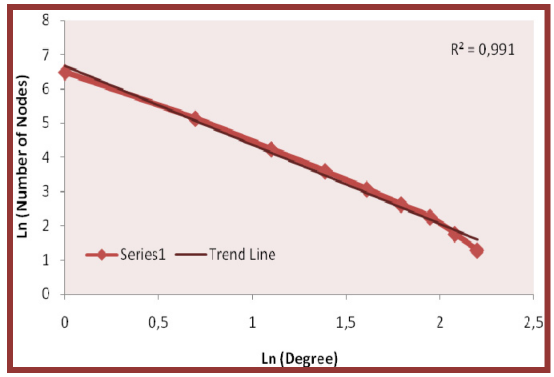

Figure 14. ***Fit Distribution for ten simulations of 1000 time-steps each, excluding the number of nodes of degree 10.***

LEAST SQUARE REGRESSION		
TS	γ	R^2
500	2,358	0,990
1000	2,317	0,991
5000	2,259	0,996

Table 5. ***Values of γ and R^2 obtained for 500, 1000 and 5000 time-steps excluding the number of nodes of degree 10.***

There is one important observation to be made. It should be noted that there must be a minimal number of time-steps necessary to corroborate that the model in fact generates a SF network; that is, the SF network, as such, emerges over time as the network increases its number of nodes. This is relevant in order to assess the reliability of the tests. We used numerous time-steps. We performed a last experiment running the model with fewer time-steps so as to find an approximate minimum number of time-steps that generate a power-law pattern. We found that for 150 time-steps, the average degree distribution fits fairly accurately as a power law, with γ and R^2 values of 2,097 and

0.847 respectively. The corresponding graphs for this result are also shown in Appendix 3.

Maximum likelihood estimation and Kolmogorov-Smirnov test

According to Clauset, Shalizi and Newman (2007), computing power laws empirically can be difficult because of fluctuations that take place in the tail of the degree distribution. They suggest avoiding the use of least squares fitting methods because of the bias introduced to the parameters estimated. Instead, they suggest techniques based on maximum likelihood and the Kolmogorov-Smirnov statistic. In addition, Goldstein, Morris and Yen (2004) demonstrated that the maximum likelihood estimator (MLE) performs better than the linear or logarithmic fit methods, producing more accurate and robust results, and also that it can be assessed with the Kolmogorov-Smirnov test. Goldstein and colleagues constructed a Kolmogorov-Smirnov table for testing power-law distributions derived from the MLE estimation. This table and details of the MLE and the Kolmogorov-Smirnov test are shown in Appendix 3.

Hence we performed a further validation using the MLE and the Kolmogorov-Smirnov test. We analyzed the model constructed in *Mathematica* since it does not have the pragmatic limitation of the SD model regarding the number of arrays — see Appendix 3.

The results for γ and K obtained for simulations of 500, 1000, 3000 and 5000 nodes are shown in Table 6. Figures 15 and 16 show the graphs for the maximum likelihood estimation and Kolmogorov-Smirnov test respectively for the simulation of 3000 nodes. Graphs for the simulations of 500, 1000 and 5000 nodes are shown in Appendix 3

Maximum Likelihood Estimation		
N	γ	K
500	2,302	0.03274
1000	2,284	0,03805
3000	2,293	0,02733
5000	2,295	0,03573

Table 6. ***Values of γ and K obtained with the maximum likelihood estimation.***

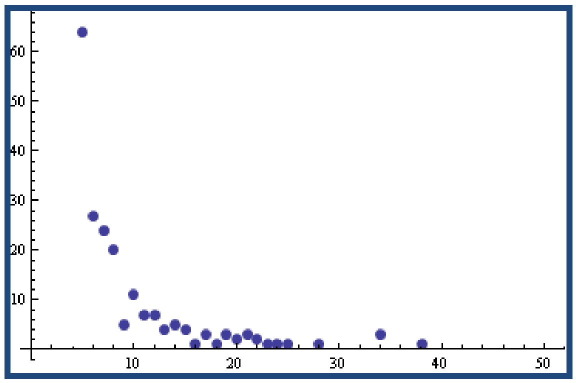

Figure 15. ***Maximum likelihood estimation for simulation of 3000 nodes.***

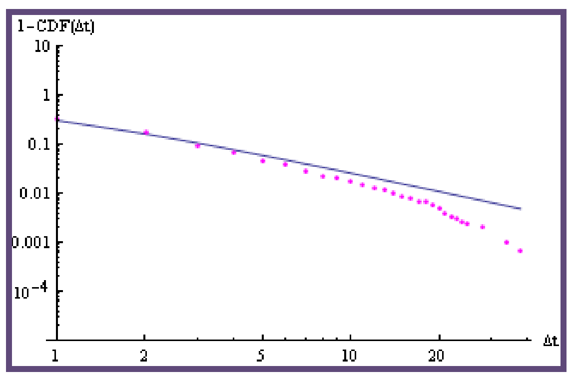

Figure 16. ***Kolmogorov-Smirnov test for simulation of 3000 nodes.***

Our SD model therefore generates SF networks with reliable values for parameter γ. For example, in the case of the 3000 nodes simulation, K=0,02 corresponds to a quantile of 0.999 – see table in Appendix 3;

consequently with an observed significance level greater than 0,1%, there is insufficient evidence to reject the hypothesis that the distribution we obtained is a power law with $\gamma = 2{,}293$.

In sum, taking into account these results, *least squares regression* and *maximum likelihood estimation* with *Kolmogorov-Smirnov test*, together with the Barabási and Oltvai (2004) definition of SF networks, we can safely assert that the networks generated by our model fit SF network properties.

Network Marketing Structures

Second, assuming that the resulting model generates a SF network structure, we verified that this structure does in fact correspond to the logic and the way the NMOs' network is formed over time by comparing the empirical results with the three relevant issues in the network growing process mentioned in Section 2.3.

Time-Step	No. Nodes D1	D2	D3	D4
1	2	0	0	0
2	2	1	0	0
3	2	2	0	0
4	3	1	1	0
5	4	0	2	0
6	4	1	2	0
7	5	0	3	0
8	6	0	2	1

Table 7. ***Degree distribution from degrees 1 to 4 for the first eight time-steps.***

(1) When a new distributor is recruited, only one connection is generated to his/her upline. This was the variant introduced to the McCutchan and Campos-Náñez original model. Table 7 shows the degree distribution of the network from degrees 1 to 4 for the first eight time-steps simulated in the *iThink* model. Note that for each time-step a new node enters the network and only one link is made – step one performs the initial conditions.

(2) Distributors interested in the NMO's commission system for recruiting others will allow the network growing process and in consequence *(3) over time it would be possible to identify a few distributors who have recruited a larger quantity of direct downline distributors*. This can be verified with the graph of the average degree distributions for each group simulation. For instance, Figure 12 shows that the number of nodes decreases as the degree increases. Similarly, Figure 9 shows that there are few nodes well connected – with large numbers of direct downline distributors, and with the larger part of the nodes less connected – and not interested in recruiting others.

In sum, we can confidently use the SF network structures that are generated with our model for modeling the structure and the process of growth of NMOs.

5. FURTHER STEPS

The constructed model generates the structure upon which we can now explore the diffusion of products and services through NMOs. This section summarizes the next steps to take.

Diffusion Processes

The study of diffusion processes started with the analysis of diffusion of innovations within a target market. According to Zabkar and Zuzel (2002), innovation diffusion starts with the launch phase and is influenced by the media and interpersonal interactions. It is the process by which an innovation is spread in a period of time through communication channels within a social system.

Traditional diffusion models, e.g. Susceptible-Infectious (SI) models, (see Sterman (2000)), assume random networks. McCutchan and Campos-Náñez approached the diffusion process introducing the network structure analysis. Using a SF network model and SD, they studied the model's behavior in different social contexts. They explored how infectivity and initial node distributions' parameters influence the diffusion through the network. We want to extend that study introducing a specific network structure: NM. In this paper we have presented the generation of the corresponding topology.

Network Marketing Diffusion Process

In NM structures, the diffusion process is represented by the spread of the NMO's products or services through the network. As we mentioned previously, once the network is established with NM characteristics, the distributors must buy and consume the products or services in order to generate such diffusion phenomena. Owing to the fact that the arrays in the obtained model guarantee a NM structure, now we can use it to proceed with the development of such a diffusion process.

We will connect the obtained model to a SI structure. Here, the element *Degree n* will be the equivalent to the *Potential* or *Susceptible* population; in the NM context these are distributors within the network who are not buying NMO's products or services. A parallel model will be introduced to generate the *Adopter* or *Infected* population representing the number of nodes infected with a given degree; these nodes represent the distributors within the network who are actually buying products or services. When a distributor starts buying the NMO's products or services, the 'infection' flow moves the distributor from the *Potential* to the *Adopter* population.

A central question in diffusion dynamics is related to the probability that a node with a given degree will become infected – in NMOs this means the probability that a distributor starts buying products or services. This probability depends on two parameters.

(1) Probability that a new node will connect to an already infected node (c); this is the probability that the distributor connects to a distributor who is already buying the products.

(2) Probability of infectivity (i); this is the probability that the distributor connected to a distributor who is already buying products starts buying them too, i.e. becomes 'infected'.

Analysis of Network Marketing Diffusion Process

We are interested in studying how the products or services are spread according to the NM structures; this will allow us to evaluate the advantages and restrictions for the effectiveness of the process according not only to the NMO structure but also to the attributes of the products as well.

Taking into account as a whole the NM business model, the NMO's characteristics and compensation plans, the growing channel of distributors' processes and our motivation for studying social diffusion phenomena, linked to the criticism to NMOs, and having a model with these aspects integrated, we are interested in studying the following aspects.

(1) Effectiveness of different NMO compensation plans: including different values for the PV commissions, for the markup profit ranges and for the GV rate commissions within the commission systems. This is relevant because we can inference strategies for achieving better results for both NMOs and distributors.

(2) Influence of the characteristics of the NMOs' products or services: including the attributes, benefits, price, accessibility, sales facilities or difficulties, target market, social stratum, gender, likes and acquisitive capacity.

(3) Influence of the resources of the NMO distributors: these are the distributor's abilities and knowledge. This can be useful for studying how these resources can accelerate or delay the diffusion process.

(4) Understanding of the network structure: how the structure as a whole, along with *c* and *i,* imposes restrictions or benefits for spreading products or services.

(5) Tipping point: estimating how many people a particular distributor needs in order to be successful or to achieve higher income. A further question is how can a NMO increase the speed of the diffusion process?

(6) Extensions of the model: in addition to the SI model, we could evaluate network growing and diffusion processes introducing the SIR model's characteristics (see Sterman (2000)); this variation allows us to model processes in which an infected population recovers. This characteristic makes sense within the NM business model; on the one hand, distributors can change their mind about being in the NMO network; and on the other hand, distributors can also stop buying products or services. The recovery rate might depend on an established average time or on the degree distribution.

Aspects 1, 2 and 3 can be included and combined within the parameter *i*; in this way *i* would be a function with the form:

$$i = f(\textit{compensation plan}, \textit{product's} \\ \textit{characteristics}, \textit{distributors' resources})$$

Also, aspects 3, 4 and 5 can help us to understand the NM's failure factors established by Martinez (2007). Finally, aspect 6 constitutes another extension to the analysis of diffusion processes considering social structures.

In short, we will explore how the network structure influences, allows or obstructs the diffusion process of particular NMO products or services.

6. FINAL REMARKS

This paper introduced NMOs as business models that can be characterized as SF networks. Based on this assessment the article is built on the previous work of McCutchan and Campos-Náñez in order to generate the topology underlying NMOs. The study of this business model using network theory is rather new. Previous academic work has emphasized small-world effects in these networks, but we are more interested in underlining the way these particular networks are formed through time and the manner in which such particular structures allow and restrict the diffusion of goods and services. We highlight the flexibility of representation provided by McCutchan and Campos-Náñez; the work we are presenting here represents a straightforward application of their model which allows us now to proceed to construct an application for studying different aspects related to diffusion processes through network marketing channels.

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REFERENCES

Barabási, A., Albert, R., & Jeong, H. (2000). Scale-free characteristics of random networks: the topology of the world-wide web. *Physica A*, *281*, 69-77.

Barabási, A., & Oltvai, Z. (2004). Network biology: understanding the cell's functional organization. *Nature Reviews*, *5,* 104-113.

Bhattacharya, P., & Nehta, K. (2000). Socialization in network marketing organizations: is it cult behavior? *Journal of Socio-Economics*, *29,* 361-374.

Card, S., Mackinlay, J., & Shneiderman, B. (1999). *Readings in information visualization*. San Francisco, CA: Morgan Kaufmann Publishers Inc.

Clauset, A., Shalizi, C., & Newman, M. (2007). Power-law distributions in empirical data. *SIAM Review*.

Coughlan, A., & Grayson, K. (1998). Network marketing organizations: compensations plan, retail network growth and profitability. *International Journal of Research in Marketing*, *15*, 401-426.

Davern, M. (1997). Social networks and economic sociology: a proposed research agenda for a more complete social science. *American Journal of Economics and Sociology, 56*, (3), 287-302.

Goldstein, M., Morris, S., & Yen, G. (2004). Problems with fitting to the power-law distribution. *The European Physical Journal B*, *41*, 255-258.

Kim, B., Jun, T., Kim, J., & Choi, M. (2006). Network marketing on a small-world network. *Physica A*, *360*, 493-504.

King, C., & Robinson, J. (2000). Los nuevos profesionales: el surgimiento del network marketing como la próxima profesión de relevancia. (*The new professionals: the rise of network marketing as the next major profession*). Buenos Aires, Argentina: Time and Money Network Editions.

Legara, E., Monterola, C., Litong-Palima, M., & Saloma, C. (2006). Preferential attachment model of multilevel marketing. *Proceedings of the 24th Samahang Pisika ng Pilipinas Congress*, *3*, ISSN 1656-2666.

Martinez, M. (2007). Four network marketing lies that can kill your business. *Home Business Magazine*, *July/August*, 54-57.

McCutchan, M., & Campos-Nañez, E. (2007). The effect of network topology in social diffusion: a system dynamics approach. *The 2007 International Conference of the System Dynamics Society.* Curran Associates, Inc. *Sessions by Thread* updated: September 5, 2007, 1-19.

Newman, M. (2003). The structure and function of complex networks. *SIAM Review*, *45*, 167-256.

Sterman, J. (2000). Business dynamics: systems thinking and modeling for a complex world. USA: The McGraw-Hill Companies, Inc.

Taylor, J. (2002). Five causal and defining characteristics of product-based pyramid schemes or recruiting MLM's. *Economic Crime Summit Conference 2002*. Revised March 2006.

Taylor, J. (2004). *Who profits from multi-level marketing (MLM)*? Consumer Awareness Institute.

Watts, D. (2003). *Six Degrees: the science of a connected age*. New York London, USA: W.W. Norton & Company Ltd.

Watts, D. (2004). The 'new' science of networks. *Annual Review of Sociology*, *30*, 243-270.

World Federation of Direct Selling Associations (WFDSA). (2006). Global retail sales and global sales force. *Publications: statistical information*. URL: http://www.wfdsa.org/. Retrieved on March 2008.

Zabkar, V., & Zuzel, B. (2002). Bass new product diffusion model: estimation and findings. *Developments in statistics, Metodoloski Zvezki*, *17*, 209-219.

APPENDIX 1

Equations of the Model for Generating Network Marketing Structures

STOCKS:
Degree_n[Degree_1](t) = Degree_n[Degree_1](t - dt) + (Promote[Degree_1] - Conservation_of_Flow[Degree_1]) * dt
INIT Degree_n[Degree_1] = 2
Degree_n[Degree_2](t) = Degree_n[Degree_2](t - dt) + (Promote[Degree_2] - Conservation_of_Flow[Degree_2]) * dt
INIT Degree_n[Degree_2] = 0
Degree_n[Degree_3](t) = Degree_n[Degree_3](t - dt) + (Promote[Degree_3] - Conservation_of_Flow[Degree_3]) * dt
INIT Degree_n[Degree_3] = 0
Degree_n[Degree_4](t) = Degree_n[Degree_4](t - dt) + (Promote[Degree_4] - Conservation_of_Flow[Degree_4]) * dt
INIT Degree_n[Degree_4] = 0
Degree_n[Degree_5](t) = Degree_n[Degree_5](t - dt) + (Promote[Degree_5] - Conservation_of_Flow[Degree_5]) * dt
INIT Degree_n[Degree_5] = 0
Degree_n[Degree_6](t) = Degree_n[Degree_6](t - dt) + (Promote[Degree_6] - Conservation_of_Flow[Degree_6]) * dt
INIT Degree_n[Degree_6] = 0
Degree_n[Degree_7](t) = Degree_n[Degree_7](t - dt) + (Promote[Degree_7] - Conservation_of_Flow[Degree_7]) * dt
INIT Degree_n[Degree_7] = 0
Degree_n[Degree_8](t) = Degree_n[Degree_8](t - dt) + (Promote[Degree_8] - Conservation_of_Flow[Degree_8]) * dt
INIT Degree_n[Degree_8] = 0
Degree_n[Degree_9](t) = Degree_n[Degree_9](t - dt) + (Promote[Degree_9] - Conservation_of_Flow[Degree_9]) * dt
INIT Degree_n[Degree_9] = 0
Degree_n[Degree_10](t) = Degree_n[Degree_10](t - dt) + (Promote[Degree_10] - Conservation_of_Flow[Degree_10]) * dt
INIT Degree_n[Degree_10] = 0

INFLOWS:
Promote[Degree_1] = 1+(0*Cumulative_Pn[Degree_1]*rand_1)
Promote[Degree_2] = IF
rand_1 <Cumulative_Pn[Degree_1]
THEN 1 ELSE 0
Promote[Degree_3] = IF
(rand_1 >Cumulative_Pn[Degree_1]
AND
rand_1 <Cumulative_Pn[Degree_2])
THEN 1 ELSE 0
Promote[Degree_4] = IF
(rand_1 >Cumulative_Pn[Degree_2]
AND
rand_1 <Cumulative_Pn[Degree_3])
THEN 1 ELSE 0
Promote[Degree_5] = IF

(rand_1 >Cumulative_Pn[Degree_3]
AND
rand_1 <Cumulative_Pn[Degree_4])
THEN 1 ELSE 0
Promote[Degree_6] = IF
(rand_1 >Cumulative_Pn[Degree_4]
AND
rand_1 <Cumulative_Pn[Degree_5])
THEN 1 ELSE 0
Promote[Degree_7] = IF
(rand_1 >Cumulative_Pn[Degree_5]
AND
rand_1 <Cumulative_Pn[Degree_6])
THEN 1 ELSE 0
Promote[Degree_8] = IF
(rand_1 >Cumulative_Pn[Degree_6]
AND
rand_1 <Cumulative_Pn[Degree_7])
THEN 1 ELSE 0
Promote[Degree_9] = IF
(rand_1 >Cumulative_Pn[Degree_7]
AND
rand_1 <Cumulative_Pn[Degree_8])
THEN 1 ELSE 0
Promote[Degree_10] = IF
(rand_1 >Cumulative_Pn[Degree_8]
AND
rand_1 <Cumulative_Pn[Degree_9])
THEN 1 ELSE 0

OUTFLOWS:
Conservation_of_Flow[Degree_1] = IF Promote[Degree_2]=1 THEN 1 ELSE 0
Conservation_of_Flow[Degree_2] = Promote[Degree_3]
Conservation_of_Flow[Degree_3] = Promote[Degree_4]
Conservation_of_Flow[Degree_4] = Promote[Degree_5]
Conservation_of_Flow[Degree_5] = Promote[Degree_6]
Conservation_of_Flow[Degree_6] = Promote[Degree_7]
Conservation_of_Flow[Degree_7] = Promote[Degree_8]
Conservation_of_Flow[Degree_8] = Promote[Degree_9]
Conservation_of_Flow[Degree_9] = Promote[Degree_10]
Conservation_of_Flow[Degree_10] = 0*Promote[Degree_10]

CONVERTERS:
Cumulative_Pn[Degree_1] = P_n[Degree_1]
Cumulative_Pn[Degree_2] = P_n[Degree_1]+P_n[Degree_2]
Cumulative_Pn[Degree_3] = P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]
Cumulative_Pn[Degree_4] = P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]
Cumulative_Pn[Degree_5] =
P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]+P_n[Degree_5]
Cumulative_Pn[Degree_6] =
P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]+P_n[Degree_5]+P_n[Degree_6]
Cumulative_Pn[Degree_7] =
P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]+P_n[Degree_5]+P_n[Degree_6]+P_n[Degr
ee_7]

Cumulative_Pn[Degree_8] =
P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]+P_n[Degree_5]+P_n[Degree_6]+P_n[Degree_7]+P_n[Degree_8]
Cumulative_Pn[Degree_9] =
P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]+P_n[Degree_5]+P_n[Degree_6]+P_n[Degree_7]+P_n[Degree_8]+P_n[Degree_9]
Cumulative_Pn[Degree_10] =
P_n[Degree_1]+P_n[Degree_2]+P_n[Degree_3]+P_n[Degree_4]+P_n[Degree_5]+P_n[Degree_6]+P_n[Degree_7]+P_n[Degree_8]+P_n[Degree_9]+P_n[Degree_10]
P_n[Degree_1] = Degree_n[Degree_1]/Total_Membership
P_n[Degree_2] = (2*Degree_n[Degree_2])/Total_Membership
P_n[Degree_3] = (3*Degree_n[Degree_3])/Total_Membership
P_n[Degree_4] = (4*Degree_n[Degree_4])/Total_Membership
P_n[Degree_5] = (5*Degree_n[Degree_5])/Total_Membership
P_n[Degree_6] = (6*Degree_n[Degree_6])/Total_Membership
P_n[Degree_7] = (7*Degree_n[Degree_7])/Total_Membership
P_n[Degree_8] = (8*Degree_n[Degree_8])/Total_Membership
P_n[Degree_9] = (9*Degree_n[Degree_9])/Total_Membership
P_n[Degree_10] = (10*Degree_n[Degree_10])/Total_Membership
rand_1 = RANDOM(0,1)
Total_Membership = ARRAYSUM(Weighted_Membership[*])
Weighted_Membership[Degree_1] = Degree_n[Degree_1]
Weighted_Membership[Degree_2] = Degree_n[Degree_2]*2
Weighted_Membership[Degree_3] = Degree_n[Degree_3]*3
Weighted_Membership[Degree_4] = Degree_n[Degree_4]*4
Weighted_Membership[Degree_5] = Degree_n[Degree_5]*5
Weighted_Membership[Degree_6] = Degree_n[Degree_6]*6
Weighted_Membership[Degree_7] = Degree_n[Degree_7]*7
Weighted_Membership[Degree_8] = Degree_n[Degree_8]*8
Weighted_Membership[Degree_9] = Degree_n[Degree_9]*9
Weighted_Membership[Degree_10] = Degree_n[Degree_10]*10

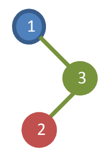
APPENDIX 2

Visualizing the Resulting Networks

Adjacency matrix

An adjacency matrix is a 0-1 matrix which indicates the connections between nodes For a graph with n nodes, the matrix will be of size $n \times n$. It is symmetric because the edges are undirected –if node 1 is connected with node 5, then node 5 is connected with node 1, and its diagonal is filled with zeros which means that a node can not be connected with itself. An example is shown below. The matrix in A represents an undirected network with three nodes, where node 3 is connected with both nodes 1 and 2 as shown in B.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$



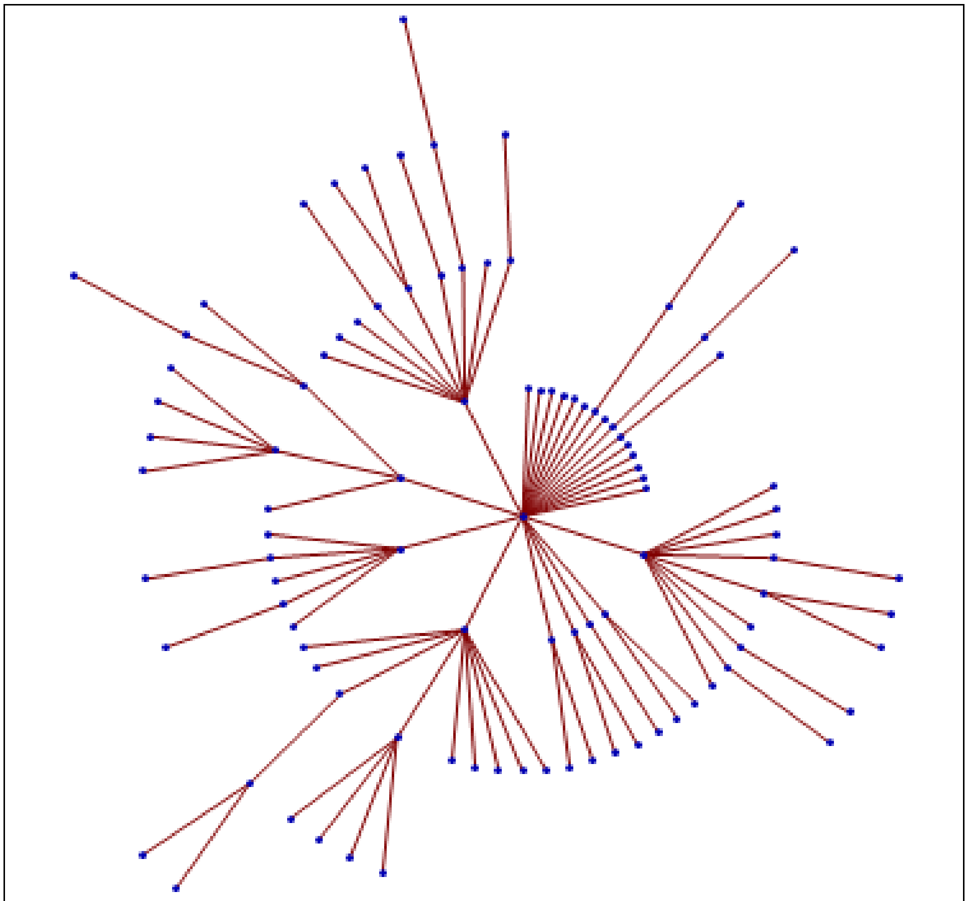
A B

Equations of the model

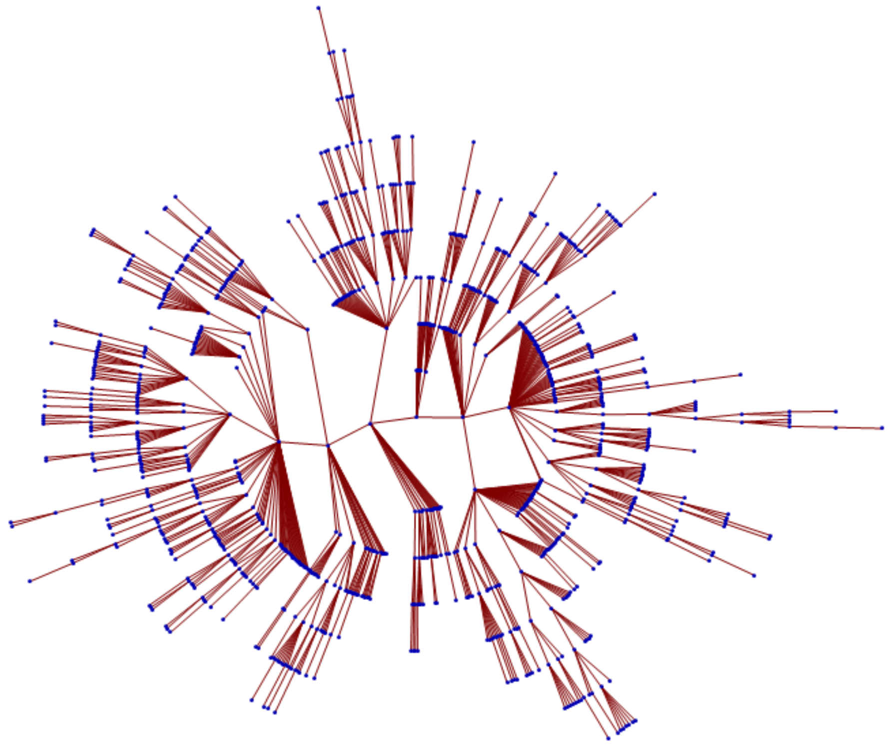
```
<< Combinatorica`
Nnodos = 1000;
min = 0;
max = 0;
pos = 0;
mat = Table[0, {Nnodos}, {Nnodos}];
mat[[1, 2]] = 1;
mat[[2, 1]] = 1;
Do[
  razon = Map[Total, mat] / (iter - 2);
  maxRandom = Total[razon];
  r = Random[Real, {0, maxRandom}];

  min = 0;
  max = razon[[1]];

  pos = 1;
  Do[
   If[min < r ≤ max,
    pos = i;
   ];
```

```
  min = max;
  max = max + razon[[i + 1]];

  , {i, 1, iter - 1}
 ];

 mat[[iter, pos]] = 1;
 mat[[pos, iter]] = 1;

 If[Mod[iter, 50] == 0, Print[iter]];

 , {iter, 3, Nnodos}
];
```

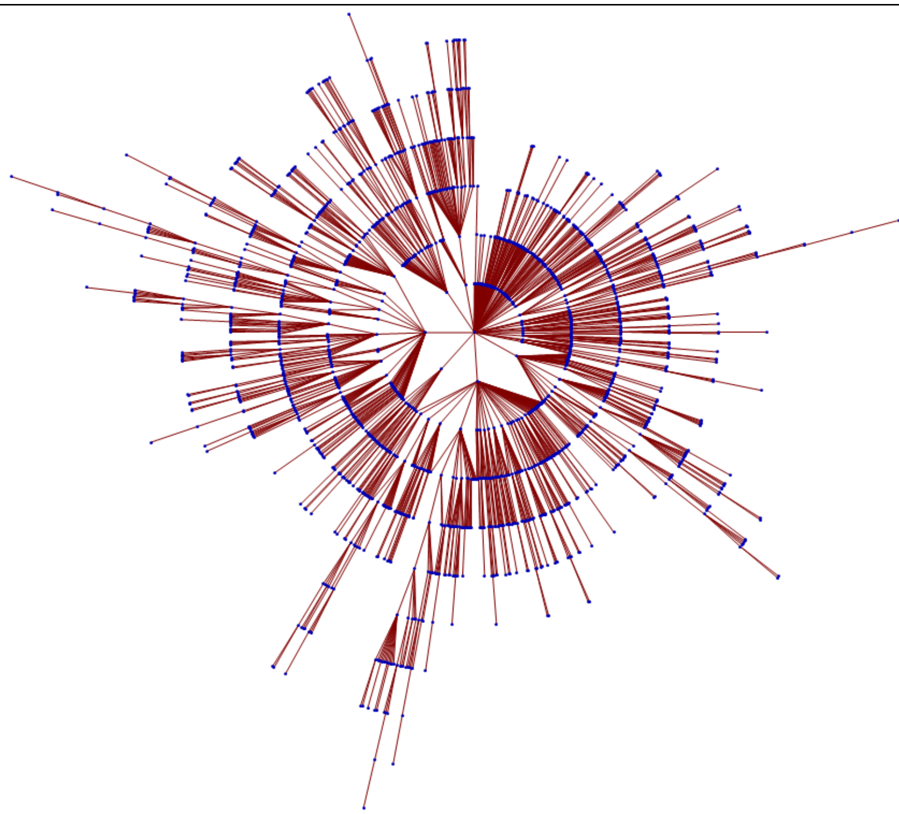
Resulting networks for different number of nodes



Network diagram with n = 100 and γ =**2.3108**



Network diagram with n = 1000 and γ **=2.28444**



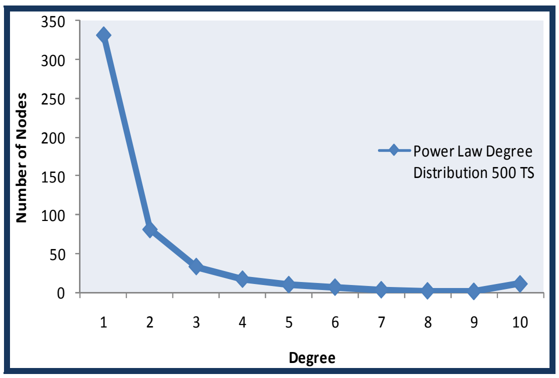
Network diagram with n = 3000 and γ **=2.29316**

APPENDIX 3

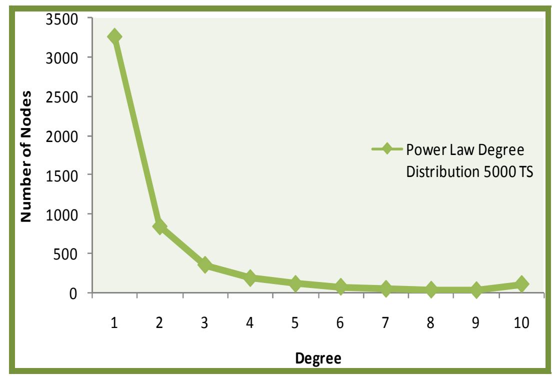
Assessment of the Model for Generating Network Marketing Structures

I Least squares regression

Power law degree distribution graphs

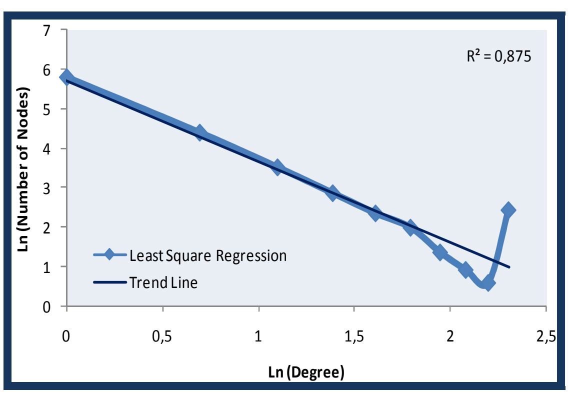


Power law degree distribution for ten simulations of 500 time steps each

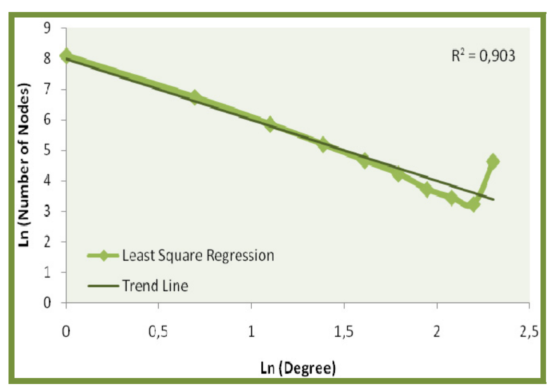


Power law degree distribution for ten simulations of 5000 time steps each

Fit distribution graphs

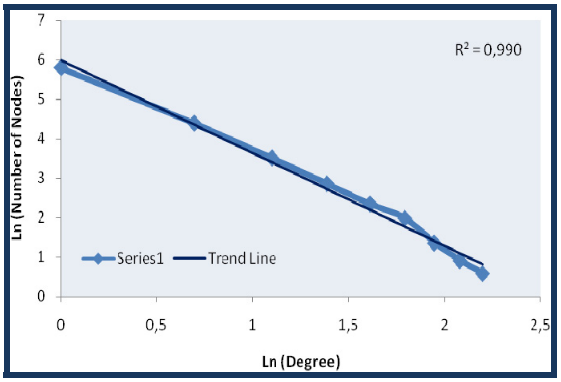
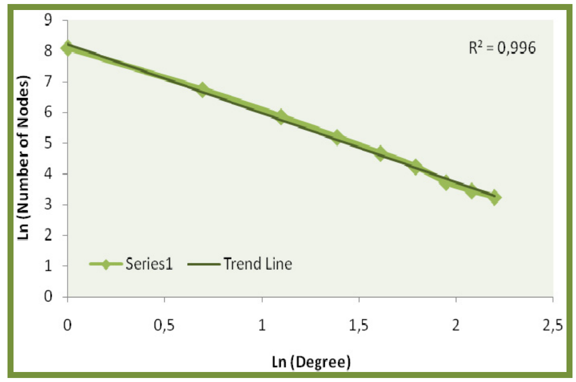


Fit distribution for ten simulations of 500 time steps each

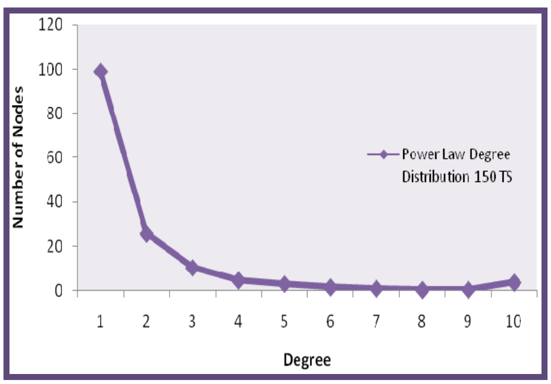
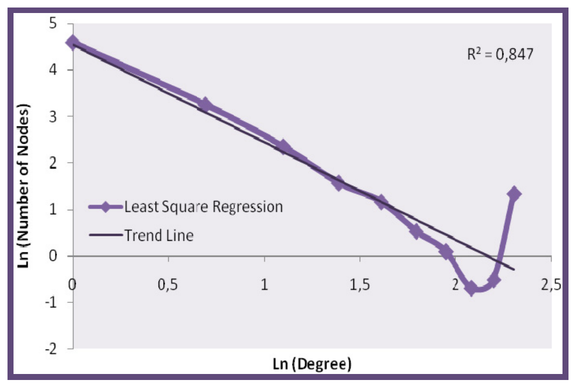


Fit Distribution for ten simulations of 5000 time steps each

Fit distribution graphs excluding the number of nodes of degree 10

	
Fit Distribution for ten simulations of 500 time steps each, excluding the number of nodes of degree 10	***Fit Distribution for ten simulations of 5000 time steps each, excluding the number of nodes of degree 10***

Approximate minimum number of time steps that generate a power-law pattern

	
Average *Degree Distribution for ten simulations of 150 time steps each*	***Fit Distribution for ten simulations of 150 time steps each***

II Maximum likelihood estimation and Kolmogorov-Smirnov test

The maximum likelihood estimator (MLE) and the Kolmogorov-Smirnov test

The MLE estimation maximizes the logarithm of the likelihood function given by (Goldstein, Morris &Yen, 2004):

$$\ell(\gamma \mid x) = \prod_{i=1}^{N} \frac{x_i^{-\gamma}}{\zeta(\gamma)}$$

where $\zeta(\gamma)$ is the Riemann Zeta function. It can be obtained calculating (Goldstein, Morris &Yen, 2004):

$$\frac{1}{N}\sum_{i=1}^{N} \log(x_i)$$

where N is the number of samples or nodes for our model and x_i is the degree of node i. With the value of this function the parameter of a power-law function can be estimated by using *Mathematica*. The Kolmogorov-Smirnov statistic can be used in order to test the goodness of the fit (Goldstein, Morris &Yen, 2004):

$$K = \sup_x \mid F^*(x) - S(x) \mid$$

Where $F^*(x)$ is the hypothesized cumulative distribution function, this is a power-law degree distribution with the γ obtained above and $S(x)$ is the empirical distribution function. The resulting K is finally compared with the table that Goldstein, Morris and Yen constructed. This table indicates the quantile values and the corresponding K values for different numbers of N. The quantile indicates the observed significance level with which there is insufficient evidence to reject the hypothesis that the distribution obtained is a power-law (Goldstein, Morris &Yen, 2004).

Kolmogorov-Smirnov table for testing power-law distributions derived from the MLE

	Quantile			
# samples	0.9	0.95	0.99	0.999
10	0.1765	0.2103	0.2835	0.3874
20	0.1257	0.1486	0.2003	0.2696
30	0.1048	0.1239	0.1627	0.2127
40	0.0920	0.1075	0.1439	0.1857
50	0.0826	0.0979	0.1281	0.1719
100	0.0580	0.0692	0.0922	0.1164
500	0.0258	0.0307	0.0412	0.0550
1000	0.0186	0.0216	0.0283	0.0358
2000	0.0129	0.0151	0.0197	0.0246
3000	0.0102	0.0118	0.0155	0.0202
4000	0.0087	0.0101	0.0131	0.0172
5000	0.0073	0.0086	0.0113	0.0147
10000	0.0059	0.0069	0.0089	0.0117
50000	0.0025	0.0034	0.0061	0.0077

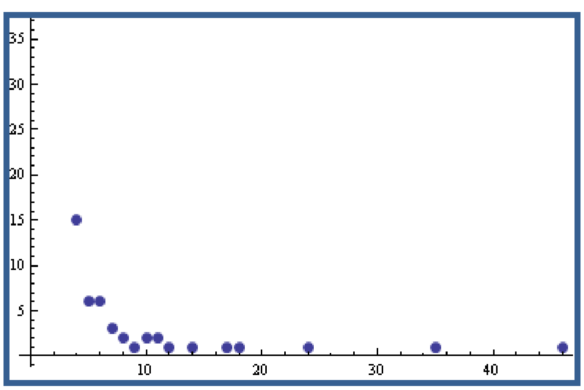
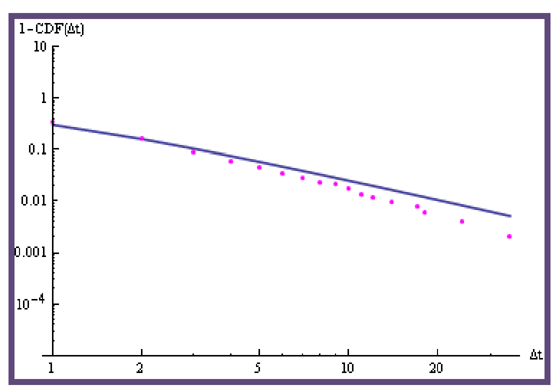
Source: Goldstein, Morris and Yen, (2004, p. 257).

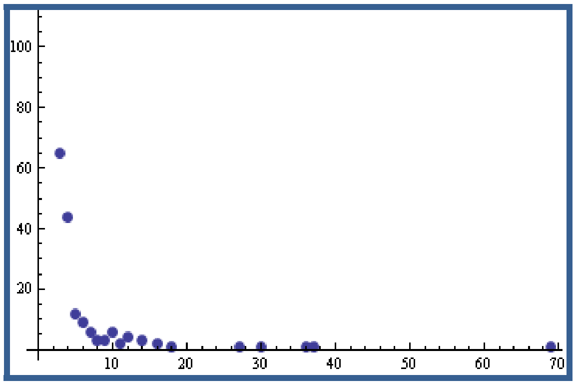
Equations of the model for the maximum likelihood estimation and the Kolmogorov-Smirnov test

```
DispersionPkFastAll[g_Graph] := Module[
   {i, grados, Pk},
   grados = Degrees[g];
   Pk = Table[{i, Count[grados, i]}, {i, 0, V[g]}];
   Pk = Select[Pk, #[[2]] ≠ 0 &];
   Return[Pk];
  ];

MLEKSPowerLawTestFast[l_List] := Module[
   {σ, γ, β, n, z, x, rank, l1 = {}, l2 = {}, i, j, temp, d, f, aa1, aa2, s},

   n = Total[l]⟦2⟧;
   s = Table[l⟦i, 2⟧ Log[l⟦i, 1⟧] + 0.`, {i, 1, Length[l]}];
   σ = Total[s] / n;
   γ = z /. FindRoot[N[Zeta′[z] / Zeta[z]] == -σ, {z, 2.`}];
   β = 1 / Zeta[γ];
   f[z_] := Sum[β / x^γ, {x, 1, z}];

   l2 = {{l⟦1, 1⟧, (n - l⟦1, 2⟧) 1.` / n}};

   Do[
    l2 = Append[l2, {l⟦i, 1⟧, l2⟦i - 1, 2⟧ - l⟦i, 2⟧ 1.` / n}];
    , {i, 2, Length[l]}
   ];

   l2 = Drop[l2, -1];
   d = Table[Abs[(1 - f[l2⟦i, 1⟧]) - l2⟦i, 2⟧], {i, 1, Length[l2]}];
   aa1 = Table[{l2⟦i, 1⟧, 1 - f[l2⟦i, 1⟧]}, {i, 1, Length[l2]}];
   aa2 = Table[{l2⟦i, 1⟧, l2⟦i, 2⟧}, {i, 1, Length[l2]}];

   Print[
    ListLogLogPlot[aa2,
     ImageSize → 800,
     PlotRange → {All, All},
     PlotStyle → {RGBColor[1, 0, 1], PointSize[0.01`]},
     AxesLabel → {"Δt", "1-CDF(Δt)"}]
   ];
```

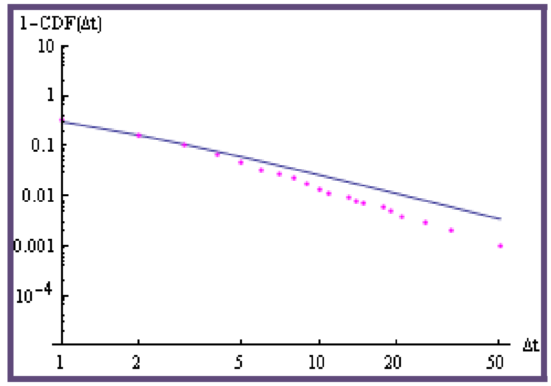
```
    Print[Show[
      ListLogLogPlot[Table[1 - f[x], {x, 1, l2⟦Length[l2], 1⟧}],
       Joined → True,
       ImageSize → 800,
       PlotRange → {All, {0.00001`, 10}},
       PlotStyle → Thickness[0.005`],
       AxesLabel → {"Δt", "1-CDF(Δt)"}],
      ListLogLogPlot[aa2,
       ImageSize → 800,
       PlotRange → {All, All},
       PlotStyle → {RGBColor[1, 0, 1], PointSize[0.01`]},
       AxesLabel → {"Δt", "1-CDF(Δt)"}]
     ]
    ];

    Return[{γ, β, Max[d]}];
  ];

g = FromAdjacencyMatrix[mat];

DPk = DispersionPkFastAll[g];

ListPlot[DPk, PlotStyle → PointSize[0.02]]

ListLogLogPlot[DPk, PlotStyle → PointSize[0.02]]

MLEKSPowerLawTestFast[DPk]
```

Graph results of the maximum likelihood estimation and the Kolmogorov-Smirnov test

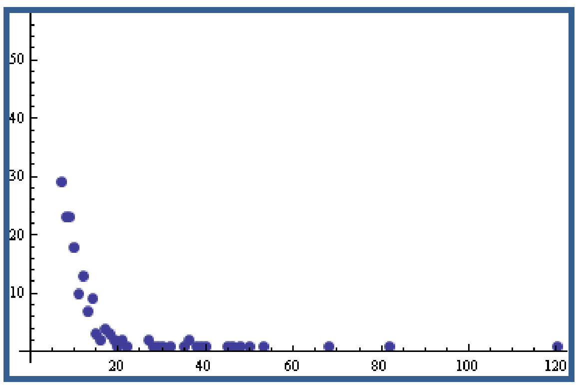
<table>
<tr>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>Maximum likelihood estimation for simulation of 500 nodes</td>
<td>Kolmogorov-Smirnov test for simulation of 500 nodes.</td>
</tr>
</table>



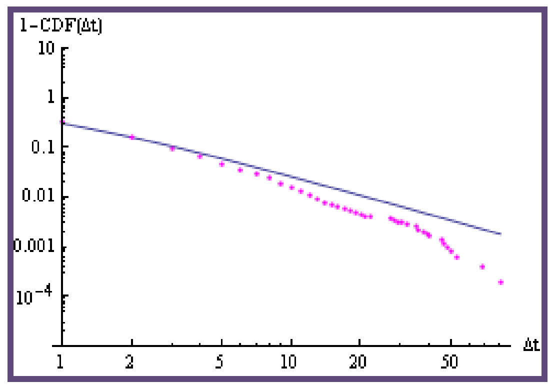
Maximum likelihood estimation for simulation of 1000 nodes



Kolmogorov-Smirnov test for simulation of 1000 nodes.



Maximum likelihood estimation for simulation of 5000 nodes



Kolmogorov-Smirnov test for simulation of 5000 nodes.