Modeling Exploration Dynamics and Uncertainty of Natural Gas Discoveries

Yris Olaya
Universidad Nacional de Colombia
yolayam@unal.edu.co

Isaac Dyner
Universidad Nacional de Colombia
Center for Complexity Studies, CeiBa
idyner@unalmed.edu.co

Abstract
Reserves estimations in natural gas markets are fundamental for decision making of private and public agents. When markets are mature, market signals such as demand growth, costs, and price expectations activate exploration and new reserves are continually added to the proven reserves base. We can describe the process of discovering new reserves in a market with a simple dynamic hypothesis in which investment in exploration eventually leads to increase proven reserves while probable reserves decrease. The simple dynamic hypothesis, however, does not seem able to explain the large reserves additions occurring in immature markets. In immature markets it is frequent to discover large natural gas reservoirs independently of the degree of exploration activity. Instead of rejecting our simple dynamic hypothesis, we broaden it by including discoveries as a stochastic component, aiming to capture some of the major uncertainties observed in immature natural gas markets.

Keywords: Energy, natural gas reserves, system dynamics, stochastic processes, modeling.

Introduction

The uncertainty of oil and natural gas exploration is high. Today, exploratory and developing drilling is the only certain way of assessing the quantity of oil and gas that can be economically produced with current technology. Whereas exploratory drilling discovers new fields that add to the proven reserves base, development drilling extends the frontier of known reservoirs also increasing the proven reserves base.

Uncertainty of discoveries comes from two different sources. One is the randomness of resource distribution and the second is the uncertainty of government policies, taxes and industry performance which affect exploration activity. Understanding the relationship between natural gas discoveries, natural gas reserves and market evolution can decrease uncertainty, improving decision-making, particularly in recently formed markets.

The relationship between gas discoveries and market formation has not been directly addressed by economic and engineering research. Economic models tend to focus on optimal expansion paths while their approach to reserves discoveries reflects the behavior of mature markets rather than of early exploration stages. A majority of economic models of resource extraction (Pindyck, 1987; Pesaran, 1990; Halvorsen and Smith, 1991; Black and La France 1998) take discoveries as depending on current and past exploratory effort and although they assume that discoveries decline with resources depletion, the randomness of the sequence and size of discoveries is not considered.

Engineering models approach reserves discoveries differently. In engineering models, reservoirs' production history and geological and engineering principles are the basis for estimating reserves. Some of the engineering and geological models also predict the sequence and size of discoveries based on the probability distributions first studied by Barouch and Kauffman in 1976 (Eckbo et al., 1978).

Although this approach directly addresses the randomness of resource discoveries, its application for forecasting reserves in regions with little exploratory history is limited because such engineering models of discoveries require detailed information on geology and resource production. In addition, engineering models of discoveries do not usually include the economic rationality behind the exploration decision and therefore cannot be used alone to predict reserves behavior. A better approach is to combine engineering estimates of reserves and supply with economic models of supply. As proposed by Nesbitt (1993), engineering and probabilistic assessments of reserves can be used to estimate long-run natural gas supply curves. These curves are inputs for forecasting natural gas prices in comprehensive market models; if the modeled prices differ from expected long-term levels, they are used to revise reserves assessments in an iterative process.

Another approach to linking economic relationships to discoveries is found in Naill (1973), Naill et al. (1992), Davidsen et al. (1990). Naill et al. (1992) simulated the dynamic interactions of natural gas demand and supply in the USA, including reserves evolution. This model and other system dynamic models, such as the oil model of Davidsen et al. (1990), replicates well the dynamics of gas and oil discoveries in the mature U.S. market, but does not explain well how reserves evolved in the early stages of these markets, in particular in the natural gas market.

Our paper studies the dynamic relationships between markets, exploration, and stochastic natural gas discoveries. It differs from the literature discussed above in that 1. it directly addresses the issue of randomness in the size and sequence of discoveries and 2. it addresses the dynamic relationships between formation of natural gas markets and natural gas discoveries.

We believe that natural gas discoveries respond differently to market signals petroleum discoveries do because natural gas discoveries are more closely related to the development of local and regional markets than oil discoveries are. Unlike petroleum, natural gas markets are not true commodity markets. Large transportation costs, along with high storage costs prevent arbitrage and natural gas markets are local and regional rather than global.

Our hypothesis is that market maturity and geological knowledge acquired during exploration impact discoveries and reserves growth and similarly, random discovery of new reserves may impact market development and public policies. To study the effect of early random discoveries in the formation and development of natural gas markets, we incorporate the uncertainty of discoveries into a dynamic market model for natural gas.

We explain uncertainty of discoveries by two different stochastic hypotheses. According to the first hypothesis, size and sequence of discoveries are two concurrent stochastic processes whose parameters are invariant during simulation. As expected, the two stochastic processes of discovery size and timing result in large discoveries that cannot be explained by exploration activity alone. With this hypothesis, however, large discoveries are as likely to occur during early exploration as during late exploration stages. Because this behavior contradicts observed discoveries sequences, we propose a second stochastic hypothesis in which reserves are found in a sampling without replacement process. This second hypothesis correctly reproduces the sequence of size and timing of discoveries, thus supporting our original hypothesis.

Unlike the deterministic dynamic models of discoveries, adequate for marginal reserves additions in mature markets, these stochastic models are able to explain high reserves growth in emerging markets, in which natural gas consumption is often incipient and cannot absorb large quantities. By introducing a stochastic component to the market model, we can reproduce past behaviour while being consistent with the investor rationality of market models. Then, the approach we present increases our understanding of recently formed markets and contributes to a better modelling of future market development while improving policy analysis.

The double stochastic model for natural gas discoveries we present is a model of emerging natural gas markets, in which supply zones have been little explored. In our model, the uncertainty on the size of discoveries is high at the beginning of exploration but it decreases as the market develops, demand grows and investment in exploration increases in order to offset the perceived depletion of reserves. Before presenting the model, in the next section, we discuss how the dynamic interaction of supply and demand can explain natural gas discoveries. This is the dynamic hypothesis we modify later to include uncertainty in the sequence and size of discoveries resulting from investment in exploration. Finally, we apply this model to the Colombian case and compare its results to the historical behavior of reserves.

1. Dynamic hypothesis

The dynamic hypothesis explains exploratory activity and reserves additions as the result of investment, which is activated by price expectations and other market signals. Although exploration results are random, continued exploration identifies new reserves that are added to the proven reserves base[1].

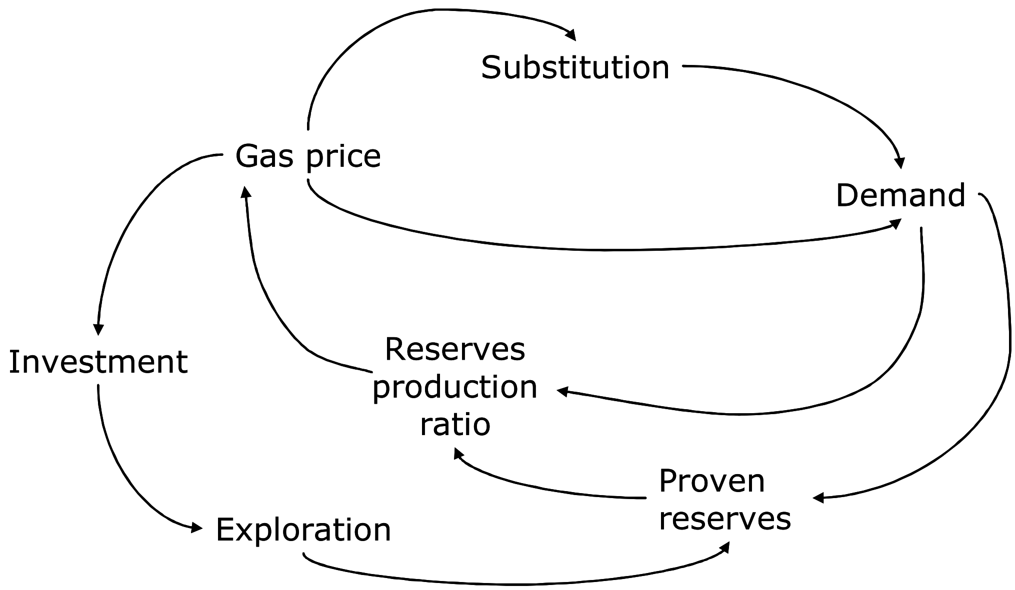


Figure 1. Dynamics of discoveries

As shown in Figure 1, proven reserves are extracted to satisfy demand. In the simplified figure above natural gas demand depends only on natural gas price and on the price of its substitutes only and other factors, such as economic growth, are exogenous.

Natural gas production declines as cumulative production increases and extraction costs above market price indicate exhaustion of resources (Hubbert, 1969). Exhaustion of reserves is economical rather than physical for extraction always leaves a fraction of remaining gas reserves underground; therefore, resource scarcity is associated with increasing long run prices and with decreasing demand.

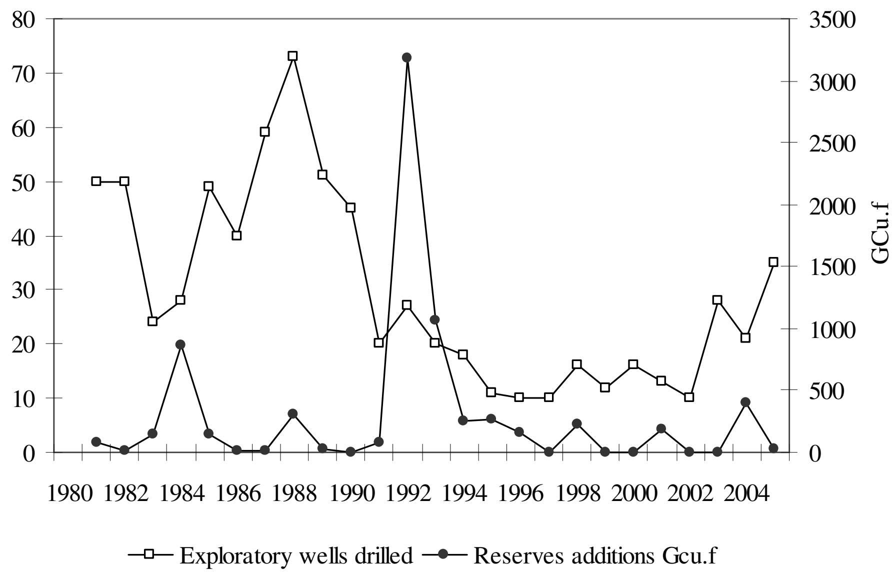
---

[1] There are several systems to classify natural gas and petroleum reserves according to their degree of uncertainty (Nesbitt, 1973; Dolton et al., 1993; Tobin, 1996; Morehouse, 1997). In this work we use two categories: proven reserves and probable reserves. Proven reserves are defined as the estimated volumes that can be produced under the current technological and economic conditions. Probable reserves are the identified remaining resources that can be discovered by exploration.

To simplify the dynamic hypothesis, we use the reserves-production ratio as signal for increasing exploration. Low reserves-production ratios suggest that demand may surpass production in the short run, unless new discoveries are made, and attract investment on exploration. On the contrary, high reserves production ratios suggest that the resource is relatively abundant which signals low investment levels.

Price expectations are the only determinant of investment shown in Figure 1. In real decisions, other factors such as geological potential, economic and politic risk are evaluated by investors. As discussed before, when reserves are depleted, exploration and production (including recovery) costs increase, reducing returns to investment. This tendency of increasing costs is offset but technological advances[2].

The case of Colombia illustrates the relationship between transportation availability, development of gas markets and evolution of reserves we discussed before. After discovering a 7 Tcf free gas reservoir in the late 1970s, the Colombian government supported the construction of a national gas transportation network and implemented price policies for increasing domestic consumption of natural gas. Natural gas consumption grew slowly between 1980 and 1987, but the natural gas market consolidated and new discoveries were made in 1992, adding about 100 million of cubic meters of gas to proven reserves (Figure 2). These discoveries of the early 1990s seem too large to be explained as the result of exploratory activity only.



[2]See Naill (1992), Alazard (1996), Berg (1999) and Davidsen et al. (1990), Livernois and Uhler (1987), among other authors for further analysis of the dynamics of technology, reserves development, extraction costs and prices of non-renewable resources

Figure 2.Exploratory wells drilled and natural gas reserves additions in Colombia between 1981 and 2005. Source: ACPET, Boletín Estadístico Mensual.

Olaya and Dyner (2005) built a deterministic system dynamics model based on the dynamic hypothesis depicted in Figure 1. Exploration and production costs in their model increase with reserves depletion, but can also decrease with technological progress, following Berg's approach (Berg, 1999). The results of Olaya and Dyner (2005) reflect the impact of market growth on exploration and discoveries in Colombia, but they are not entirely satisfactory because they do not capture large increases in reserves from unexpected discoveries.

The appeal of explaining reserves behaviour as the result of market dynamics relies on its insight and on the fact that deviations have negligible second-order errors. This approach, however, does not seem to fully capture high uncertainty in recently formed markets. In other words, our question is if a deterministic approach can adequately describe the behaviour of new markets.

In the next sections, we analyze the problem of reserves growth differentiating mature from immature markets. The basic assumption is that immature markets are more volatile and that uncertainty plays a very important role in them, whereas mature markets tend to be relatively more stable –less volatile. In the particular case of exhaustible resources, like gas, it is reasonable to assume that when markets mature, cumulative discoveries grow more slowly than in immature markets as in the former case the largest reservoirs have been already detected in the early stages of market formation.

2. Dynamic hypotheses of exploration with stochastic discoveries

Discovering hydrocarbon deposits is similar to a sampling without replacement process, in which all reservoirs have the same probability of being found, but in which the largest reservoirs have higher probability of being discovered first than the smaller reservoirs (Nesbitt, 1993). Although simple, Nesbitt's model is also based on estimations of area and gas volumes for all reservoirs in a play, which difficult its application in Colombia, where geological information about the sedimentary basins is scarce, and where the only production data available are from dry gas fields discovered during the 1970s.[3]

In the next sections we model natural gas discoveries sequence and size as two concurrent stochastic processes. In the first version of the model, we assume that the parameters governing the probability distributions of discoveries sequence and size are constant in time. This assumption is equivalent to have equal probability of discovering a field of any size regardless of the level of cumulative discoveries. Because it has been

[3] Most of associated gas production before 1980s was flared or vented, and thus, not accounted for as natural gas.

observed that the probability of finding a large field is larger at the beginning of exploration, in the second version of the model, we propose a process of sampling without replacement, similar to the one described by Nesbitt (1993), but requiring less information.

2.1. Hypothesis 1: discoveries from two concurrent stochastic processes

The analysis of historical discoveries suggests that the sequence of discoveries can be approximated by two concurrent stochastic processes: one Poisson processes governing the number of discoveries –or an exponential process for the time between discoveries- and a second process for the discovered volumes, which follow a log-normal distribution. Table 1 shows the parameters estimated for Colombia.

Table 1 Parameters of the concurrent stochastic processes

Process	**Probability distribution**	**Parameters**
Time between discoveries	Exponential	$\lambda = 3.84$
Size, Million m^3	Log-normal	$\mu = 12770$ $\sigma = 18750$

Figure 3 shows some simulation results using the distributions described in Table 1 for modeling the sequence of discoveries in a dynamic model for the Colombian natural gas market previously built by the authors (Olaya and Dyner, 2005) and which is based on the relationships illustrated in Figure 1 and on equations (1) to (2).

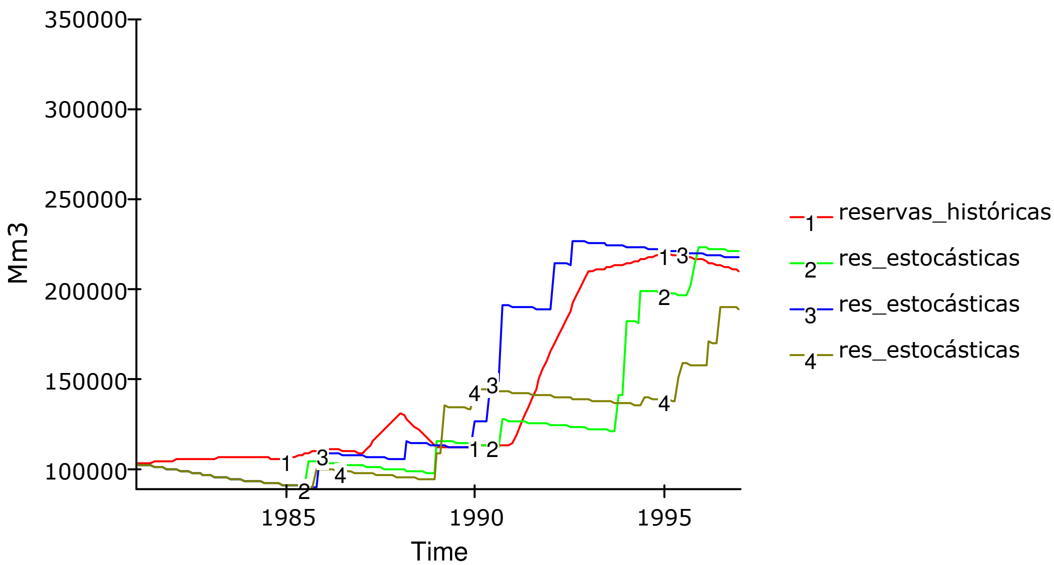


Figure 3 Comparison of natural gas reserves for Colombia, simulated with a stochastic model for discoveries within a dynamic market model (lines 2 to 4) and the historical reserves growth (line 1)

Lines 2 to 4 in Figure 3 show some of the simulated reserves found incorporating the double stochastic model of discoveries to a dynamic market model for Colombia (Olaya and Dyner, 2005). The large random reserves additions of the stochastic model are consistent with the actual historical behaviour of reserves depicted by line 1 in Figure 3. The dynamic hypothesis above then replicates historical reserves. When we increase the simulation time, however, we find that the double stochastic process does not accurately describe the actual discovery sequence. As can be seen in Figure 4, as long as there are market signals triggering exploration, it is possible to discover a large field at any stage of exploration, which contradicts observations of discoveries' sizes decreasing as exploration advances.

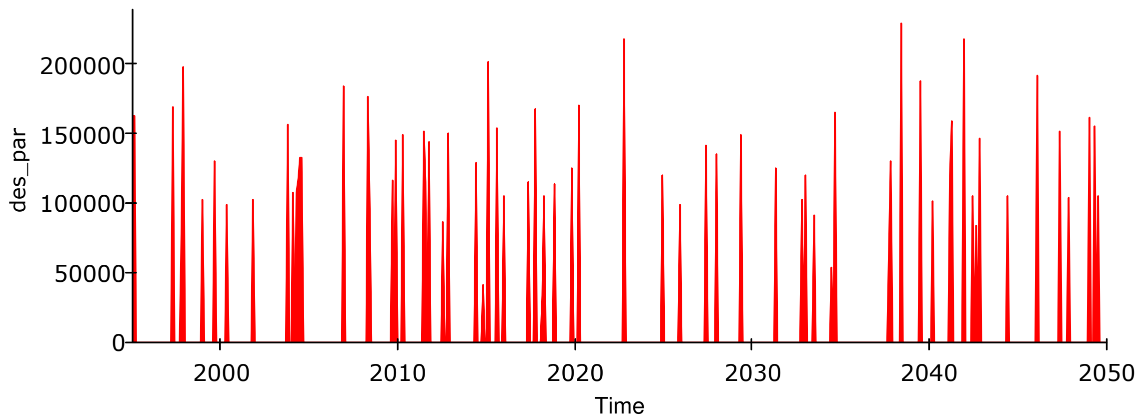


Figure 4. Discovery sequence simulated with the double stochastic process of hypothesis 1.

Since the double stochastic process does not fully represent the sequence of discoveries, we propose a second dynamic hypothesis closer to the sampling without replacement process discussed before. This second hypothesis aims to explain the differences in the volume of discoveries made in the early and late exploration stages.

2.2 Hypothesis 2: discoveries as a sampling without replacement process

As discussed before, the random sequence of discoveries might be appropriately modelled using an approach of sampling without replacement of the type proposed by Nesbitt (1993). Because data on sizes and areas of discovered reservoirs in Colombia are insufficient to fit the probability functions *f* and *SA* of Nesbitt's model, we replicate the discovery process making the following assumptions:

- Exploratory wells are drilled depending on perceived depletion, expressed as the reserves production ratio or the number of years that current production can be sustained. There is an inverse relationship between exploration and depletion: the lower the level of reserves, the higher the incentives for exploration are.

- Drilling an exploratory well is similar as a sampling without replacement process and has three possible findings: a dry well, a small reservoir or a large reservoir
- Reservoir size (in gas volume) distribution is log-normal, such as in Nesbitt's model. Similarly, the surface function, SA, is asymmetrically high for large reservoirs
- The number of dry, small and large reservoirs is adjusted in trial simulations, until cumulative production reaches a level that reflects the estimated potential reserves

Prospect fields (types: dry, small, big) are discovered after drilling exploratory wells, as illustrated in Figure 5.

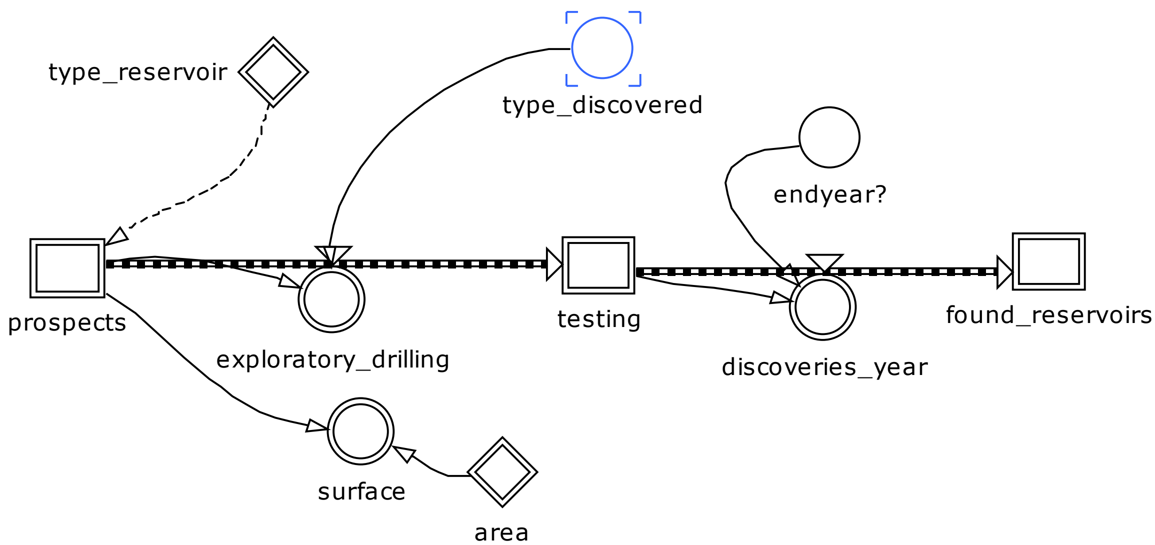


Figure 5. Simple model of discoveries as a sampling without replacement proces. .

Once the type of reservoir found is determined, we find the volume of the reservoir using the probability distributions of Table 3 and the total volumes are added to the proven reserves base as shown in Figure 6. Proven reserves and the production rate determine the reserves production ratio (rpr) which determines investment and the number of exploratory (A3) wells drilled.

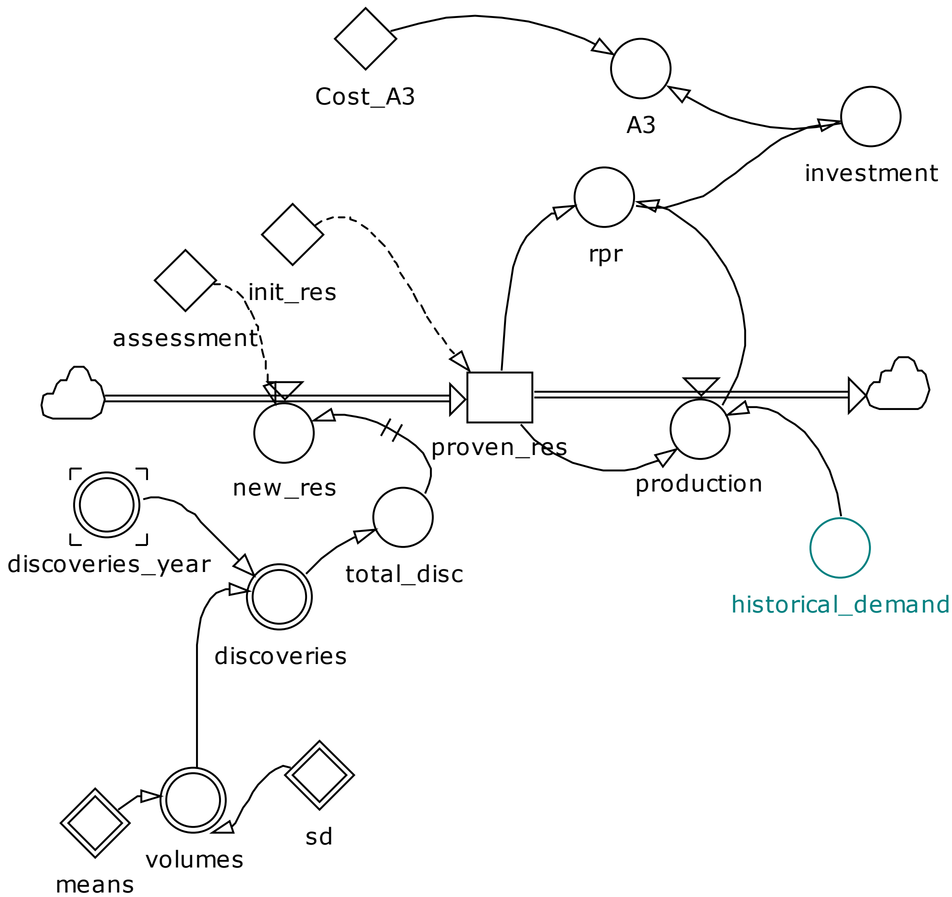


Figure 6. Simple model of reserves addition

Based on the assumptions above and on fitting of historical data, we define the model parameters as indicated in Table 2.

Table 2 Parameters for reservoir size distribution and surface area, and adjusted number of prospective reservoirs for the stochastic model with parameter adjustment

Reservoir	Reservoir size Million m^3	Surface Area	Number of reservoirs
Dry	--	1	150
Small	Log-normal μ =100 σ= 50	1	46
Large	Log-normal μ =1500 σ= 800	3	2

Figure 7 compares several simulated series of proven reserves to the actual proven reserves. Because the historical series we used for validation is short and it includes the discovery of two giant fields, it does not clearly show other discoveries in the sequence.

Simulated reserves however, have random, unexpected increases, also observed in the historical proven reserves series.

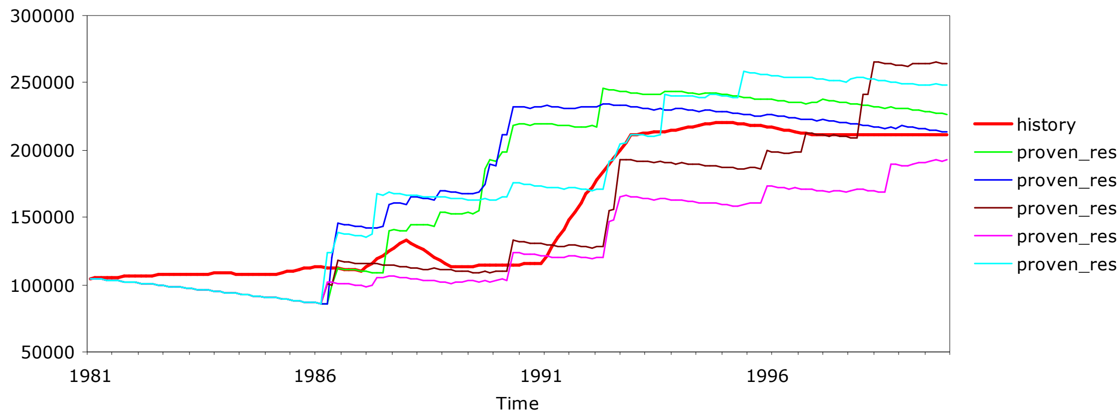


Figure 7 Comparison of historical reserves and reserves simulated with the stochastic model with parameter adjustment

As can be seen in Figure 8, there are large discoveries early in the simulation, when the exploratory effort is small, and the size of discoveries decreases with time as the cumulative exploration effort increases. This sequence of discoveries is consistent with empirical observations and theoretical predictions regarding the sequence of hydrocarbon discoveries, while it also captures the effects of market signals on exploratory activity.

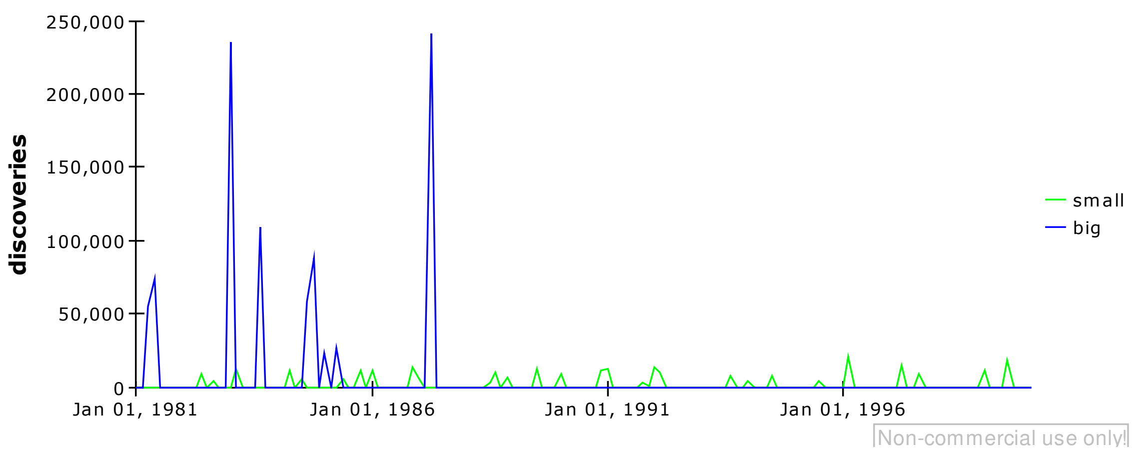


Figure 8. Discovery sequences for big and small reservoirs, simulated with the stochastic model with parameter adjustment

Figure 9 shows simulated proven reserves for Colombia using a deterministic system dynamics model of discoveries (line 3) and compares these reserves with simulations using the dynamic hypotheses 2 (line 1) and 1 (line 2). All simulations assume reserves potential of 800 Gm^3 of natural gas.

Proven reserves simulated with the deterministic dynamic market model of Olaya and Dyner (2005) are depicted in line 3. Proven reserves in the deterministic model decrease when the market begins to develop and large discoveries occur only after the market signals are strong enough to stimulate exploration levels above reserves replacement.
Line 2 of Figure 9 represents the proven reserves modelled with discoveries resulting from the two concurrent stochastic processes of hypothesis 1 and a dynamic market model for exploratory activity.

Because large and medium size discoveries are possible at any exploration stage, proven reserves with the dynamic hypothesis 1 decrease at a lower rate than proven reserves modelled with the dynamic hypothesis 2 (line 2). Discoveries made at the beginning of the exploration are larger with the dynamic hypothesis 2 (line 2) than with the other two hypotheses; this increases the availability of natural gas and stimulates consumption, thus accelerating market development and reserves depletion.

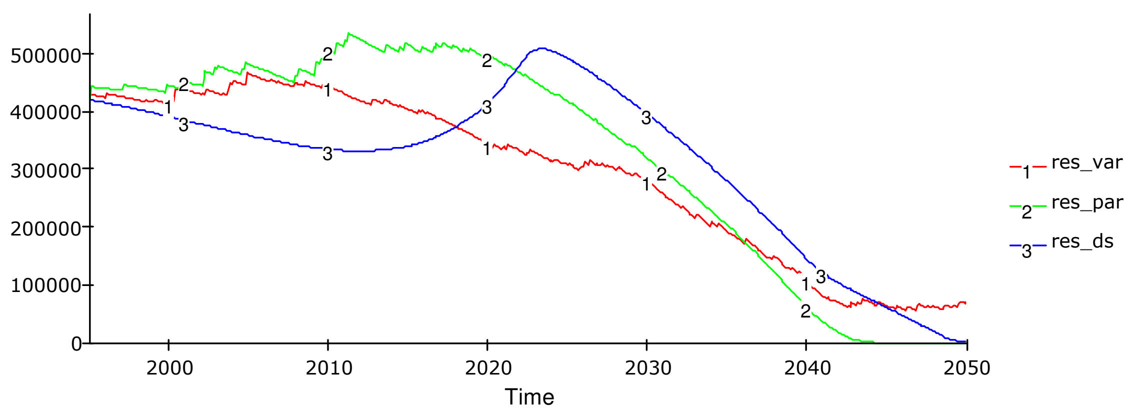


Figure 9. Comparison of proven reserves simulated with a deterministic system dynamics model (3) and proven reserves simulated with hypotheses 1 and 2

Production rates are more important determinants of the long-run evolution of reserves than it is the particular discoveries sequence, as can be appreciated in Figure 9. In the short run, however, the particular sequence of discoveries has important effects on the industry development; we develop these ideas in the next section.

3. Deterministic and stochastic discoveries in dynamic natural gas markets

As we discussed before, in isolated or closed natural gas markets, resource availability can be approximated by the reserves-production ratio and it is reflected in gas prices. When reserves are low with respect to production, prices increase non-linearly. On the other hand, when reserves are abundant relative to production, prices fall and are close to long-run average costs. High prices and expectations of high prices drive investment and with a delay, reserves growth. Such systems create expansion and contraction cycles, primarily caused by delays between the perception of price signals and the construction of additional capacity. These effects have been reported in the literature of electricity systems by Bunn and Larsen (1997).

As natural gas markets and industry mature, individual discoveries contribute to reserves growth only marginally, and the probability of finding giant fields is low. This is the case of gas reserves evolution in the USA, where the market has reached maturity and domestic production is declining as imports increase.

When we take a look at gas markets behaviour rather than at historical reserves only, we find that a system dynamics model that does not consider uncertainty is not consistent with market rationality. A deterministic market model allows investors to continue searching and developing new natural gas prospects, even if there are not available transportation networks and access to markets.

Figure 10 shows that a stochastic model for discoveries can reproduce market behavior under uncertainty more closely than a deterministic model. In a competitive closed market, without export or import capacity, market rationality does not allow continued investment in exploration when the reserves-production ratio is above 30 or 40 years as in Figure 10 below.

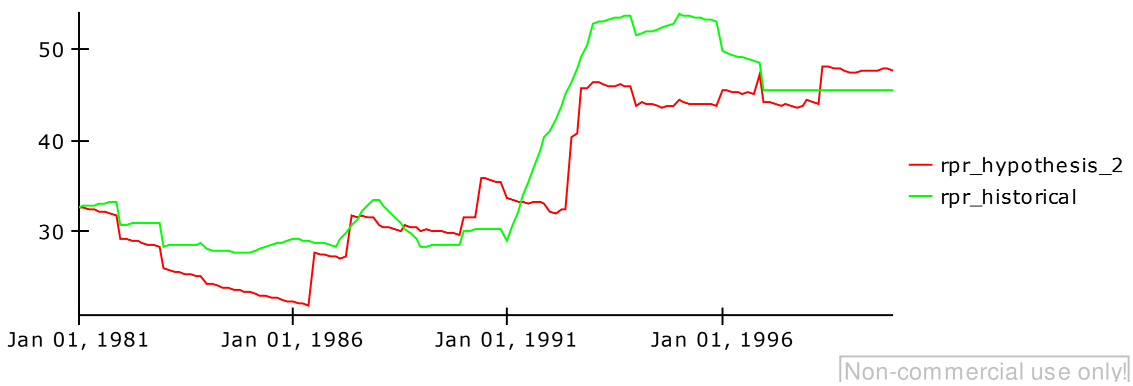


Figure 10 Historical reserves production ratio (green) compared to reseves production ratio simulated with the stochastic discovery hypothesis 2 (red).

Since rational agents search for new reserves when market signals high expected prices and demand, only random discoveries can explain the high reserves-production ratios observed in non-mature markets illustrated in Figure 10.

Furthermore, deterministic market models seem to be insufficient to explain the behaviour of isolated immature markets in which the randomness of new discoveries generates high uncertainty. When reserves are abundant, investments in exploration research and development are harder to justify because they produce marginal benefits only. A stochastic model by contrast, explains the possible dynamics better, as suggested by series 3 to 5 of Figure 10. Figure 10 shows that reserves production ratios vary sharply between reasonable and very high as a result of random discoveries and not as a consequence of ordinary market business.

Figure 11 summarizes these ideas. In recently formed natural gas markets, reserves levels respond to supply and demand interaction in the market, but there are also highly uncertain. Random new discoveries draw from probable reserves to increase proven reserves and success rates of exploration change with time.

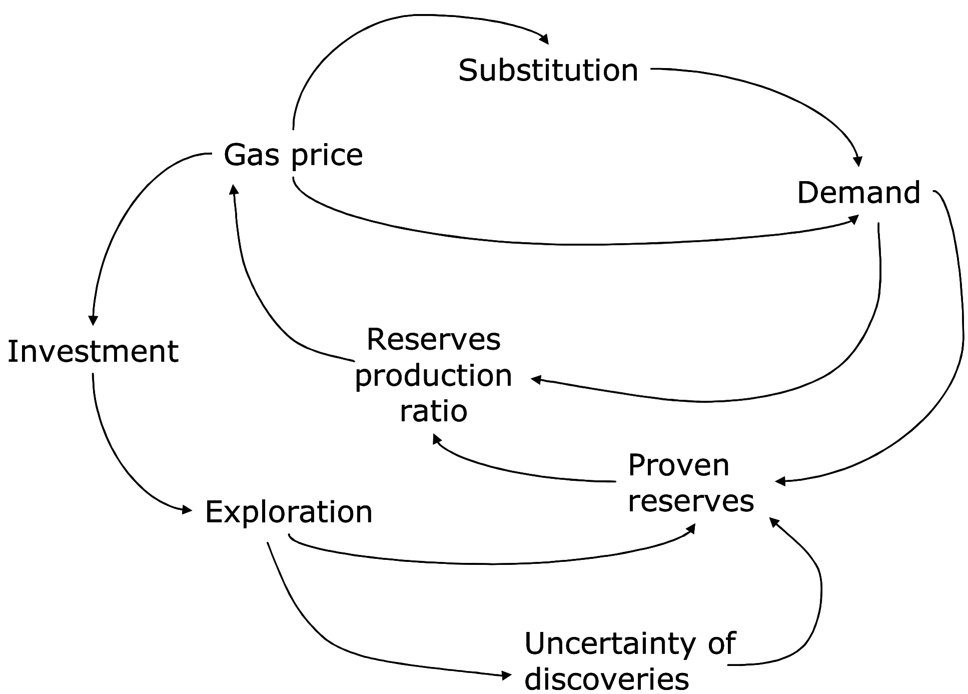


Figure 11. Uncertainty and dynamics of discoveries

When exploration progress is low, there is little knowledge about potential reserves and thus uncertainty of discoveries is very high. As exploration progresses, potential reserves are better identified and new discoveries are made, improving quality of information and decreasing the uncertainty of discoveries. Figure 11 suggests that a stochastic model for discoveries captures this uncertainty and, along with the dynamic market model, explains the behaviour of the system.

4. Conclusions

Frequently, mature markets exhibit patterns of expansion and contraction that have been explained as a result of business cycles. Non mature markets, however, are also exposed to higher volatility and randomness of market events and conditions.

To analyze growth of natural gas reserves in inmature markets, we propose double stochastic and sampling models for natural gas discoveries. In our stochastic hypotheses, uncertainty on the size of discoveries is high, but it decreases as the market develops, demand grows and investment in exploration increases in order to offset the perceived depletion of reserves. Whereas the double stochastic hypothesis (hypothesis 1) reproduces well the historical behavior, it does not fully reproduce the size and time sequence of discoveries. By contrast, when describing exploration as a sample without replacement process (hypothesis 2) the sequence of size and timing of discoveries is correctly reproduced.

Unlike the deterministic models of discoveries, adequate for marginal reserves additions in mature markets, these stochastic models are able to explain high reserves growth in emerging markets, in which natural gas consumption is often incipient and cannot absorb large quantities.

Our research illustrates some of the arguments above for the Colombian case and suggests a modelling approach to analyze policies and craft strategies. We show that a deterministic approach can capture the historical trend of growing reserves, but it cannot explain investor rationality and market behaviour appropriately. By introducing a stochastic component to the market model, we can reproduce past behaviour while being consistent with the investor rationality of market models. Then, the approach we present increases our understanding of recently formed markets and contributes to a better modelling of future market development while improving policy analysis.

Modelling to support policy analysis and strategic decision making in the natural gas industry could benefit from directly incorporating scenario analysis and stochastic behaviours, depending on market maturity and on the probability of finding large reservoirs. . In a more advanced stage of this research, we could investigate the interaction of other market elements, such as transportation capacity or the role of petroleum price expectations in natural gas discoveries.

References

Alazard, Nathalie. (1996) Le progres scientifique et technique en exploration-production: impact sur les réserves et les coûts. *Revue de l'Energie*, No 476 pp. 157-166

Berg, Elin. Kverndokk, Snorre. Rosendahl, Knut. (1999) *Optimal Oil exploration under climate treaties*. Statistics Norway, Discussion paper No 245 Jan 1999.

Black and La France (1998), Is Hotelling's rule relevant to domestic oil production? Journal of Environmental Economics and Management. Vol. 36 No.2, pp. 149-169

BP. (2006) Statistical review of world energy 2006. BP. Available at http://www.bp.com

Bunn D W and Larsen E R. (1997) Systems modelling for energy policy. Wiley and son Ltd. United Kingdom

Davidsen Pål I, Sterman John, Richardson George. (1990) Petroleum life cycle model for the U.S. with endogenous technology, exploration, recovery and demand. *System Dynamics Review* Vol. 6 No.1 (Winter) pp. 66-93

Dolton, Gordon L., Donald L. Gautier, Richard F. Mast and David H. Root, (1993). U.S. Geological survey estimates of natural gas energy resources. In Howell David G: The Future of Energy Gases, U.S Geological Survey professional paper N. 1570 pp:495-506

Eckbo, P.L., H.D Jacoby, J.L. Smith. (1978) Oil Supply Forecasting: A Disaggregated Process Approach. *The Bell Journal of Economics*, Vol. 9, No. 1 (Spring), pp. 218-235

Halvorsen, R. and Smith, T.R. (1991) A test of the theory of exhaustible resources. *The Quarterly Journal of Economics*, Vol. 106, No. 1. (Feb.), pp. 123-140.

Livernois, John and Uhler, Russell. (1987) Extraction costs and the economics of nonrenewable resources. *Journal of Political Economy*. Vol. 95 No.1 (Feb.) pp. 195-203

Morehouse, David F. (1997) The intricate puzzle of oil and gas reserves growth. Energy Information Administration Natural Gas Monthly Jul.1997. Available at www.eia.doe.gov

Naill, Roger G. (1973) The discovery cycle of a finite resource: a case study of U.S. Natural gas. In Meadows: Toward global equilibrium. Cambridge Massachusetts: Wright Allen Press

Naill, Robert G. Belanger, S. Petersen, E. (1992) An analysis of the cost effectiveness of U.S. energy policies to mitigate global warming. *System Dynamics Review* Vol. 8 No.2 pp: 111-128

Nesbitt, Dale M. (1993) An economic approach that links volumetric estimates of resources with cost and price information. In Howell David G: The Future of Energy Gases, U.S. Geological Survey professional paper N. 1570 pp:749-786

Olaya Y. and I. Dyner (2005). Modelling for policy analysis in the natural gas industry. *Journal of the Operational research Society*. Vol 56 No.10, pp 1122-1131

Pesaran, M.H. (1990) An econometric analysis of exploration and extraction of oil in the U.K. Continental Shelf. *The Economic Journal*, Vol. 100, No. 401. (Jun.), pp. 367-390.
Pindyck Robert (1978) The optimal exploration and production of nonrenewable resources. *The Journal of Political Economy*. Vol. 76 No.5 (Oct), pp. 841-861
Sterman, J. and G. Richardson. (1985). An experiment to evaluate methods for estimating fossil fuel resources. *Journal of Forecasting*, Vol 4, 197-229.
Tobin, J.C. (1996) Virtual reserves -And other measures designed to confuse the investing public. *SPE Reservoir Engineering*, Vol. 11 No.1, pp. 34-36