Economic Growth, Pollution, and the Accumulation of Abatement Capital in a System Dynamics Framework

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Abstract

This paper develops a model of economic growth when emissions are generated as a byproduct of the production process. The production of goods yields income that can be used either for consumption or investment. The production of goods also induces an undesired byproduct – emissions. The emissions accumulate to a stock of pollutants which in turn impairs the economy. The consumption of good, in contrast, leads to an improvement of societal well-being. The positive impact of consumption utility and the negative impact of pollution are measured by a welfare function. In this setup of the problem there are two basic intertemporal control problems to solve: First, the society has to decide how much to consume today and how much tomorrow. Second, it has to decide how much of GDP should be invested in pollution abatement over time. The SD model developed in this paper allows simulating the consequences of policy choices and searching for optimal policy strategies. Several simulation scenarios demonstrate the scope of the modeling approach.

Keywords: Optimal Growth, Pollution, Environment

1. Introduction

The output of the economic production process consists not only of the goods which are intentionally produced but also of byproducts in the form of various kinds of emissions. Most of these emissions are (potentially) harmful to people, animals, and plants. Insofar as emissions have a negative impact on human health they decrease welfare immediately. Welfare can also be touched more indirectly: If emissions do harm to animals or plants the environmental systems changes, which may lead, possibly with a delay, to a negative impact on human health, and therefore to a decrease in human welfare. In most of the economic literature that deals with this kind of problems on a macroeconomic level a comparative static modeling approach is employed.

This paper develops a dynamic macroeconomic growth model which takes into account that as a byproduct of goods production emissions are generated. The emissions are modeled here to be directly decreasing the welfare of the economy. (Another approach – or a supplementation – would be to assume that the emissions deteriorate the production process.)

The theoretical roots of the employed approach date back to the 1970th. Early examples of papers that integrate the effects of emissions and environmental damages into neoclassical growth models in an analytical framework are Keeler, Spence and Zeckhauser (1971), Strom (1973), Bender (1976), and Buchholz and Cansier (1980). These papers assume that a part of GDP can be devoted to abatement expenditures and analyze the conditions that are required to get the economic system into a steady state not only with respect to economic growth but also with respect to pollution. Due to the analytical methods employed, this research did not analyze explicitly the dynamic processes.

A second strand of the literature asked for the conditions that have to be fulfilled for inter-temporal optimality by employing optimal control theory. Important early contributions in this category are Fisher, Krutilla and Cicchetti (1972), Smith (1972), Forster (1973a), Forster (1973b), Forster (1975), (Asako (1980), Cropper (1980). All these papers treat the optimality problem analytically, i.e. they derive the first order conditions for an optimal growth path taking into account the negative impacts of pollution and the possibility for policy to steer growth and pollution by appropriate policy measures. Only a very limited number of papers tried to model the optimal control of a macroeconomic model explicitly by employing numerical methods. Two examples are Steindl (1984) and John (1989). The main reason for this scarcity is that often even models of only moderate complexity cannot be solved by the known numerical methods because the optimal control problems in question do not constitute “simple” initial condition problems but boundary value problems, which cannot simply be numerically integrated over time.

The third strand of literature that has to be mentioned in this context starts with the famous work of Meadows et al. (1972). Certainly, this book had the biggest political influence of all the contributions to the literature mentioned above. One of the reasons that it had this enormous impact was the employment of the system dynamics methodology which allowed to model and to analyze explicitly the consequences of different policies. The World Model differs from the other mentioned models not only in using numerical integration but also in its complexity. Whereas the macroeconomic growth models are highly aggregated, the World Model is disaggregated into several sectors.

The interest in this kind of “growth and pollution” models – and in macroeconomic growth models in general – faded away during the 1970th and the 1980th. Growth theory regained interest again in the early 1990th with the contributions of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Despite of that, most of the work still uses purely analytical methods.

In the system dynamics literature there is a recent contribution of Bartoszczuk (2005). The model developed here starts with basically the same idea of integrating emissions into a neoclassical growth model which, then, is transformed into a simulation model in order to evaluate the resulting time paths. But it departs from Bartoszczuk’s model in several aspects. The three most important are the following.

First, it is based on the concept of continuous time whereas Bartoszcuk develops his model in discrete time. Modeling the macroeconomic growth in continuous time has two advantages: On the one hand, the analytical presentation is simpler, and on the other hand, continuous time fits better into the system dynamics framework.

Second, Bartoszcuk assumes that the decision makers want to maximize the sum of discounted consumption. This paper generalizes his approach by assuming that the decision makers judge the economic development and their policies by looking at the integral of a discounted welfare measure. This welfare measure comprises consumption and environmental quality.

Third, as in Bartoszcuk's paper it is assumed that emissions are a byproduct of goods production and that they can be reduced by emission abatement expenditures. But in contrast to the work of Bartoszcuk it is here assumed that emissions accumulate to a stock of pollutants and that those environmental damages do not stem from emissions directly but from the accumulated stock of pollutants. This approach seems to be more appropriate for most pollutants as only a limited number of pollutants cause damages by emissions themselves. (The most prominent example is noise. But almost all other important pollutants do accumulate.)

The following section describes the components of the model in detail.

2. The Building Blocks of the Model

The model is basically of the neoclassical growth theory type as it was introduced into the economic literature by Solow (1956). It also embodies some elements of optimal growth theory, another strand of the literature on economic growth which started with the seminal paper of Ramsey (1928): it contains a welfare function which allows discriminating between different growths paths. (But it is not a fully-fledged optimal growth model because it refrains from searching explicitly for the optimal control paths.)

I assume that the welfare of the economy depends on the amount of consumption goods which are produced and on the environmental quality. The utility generated by environmental quality is captured by its counterpart – the disutility of environmental damages generated by the accumulated stock of pollutants. Denoting consumption by C and the stock of pollutants by P the welfare U at a given point in time can be described by

$$U = U\left(C, P\right), \quad U_C > 0, U_{CC} < 0, U_P < 0, U_{PP} < 0$$

That is, we assume a decreasing marginal utility of consumption but an increasing marginal disutility of pollution. From a system dynamics perspective we are confronted with the methodological problem that actual consumption is a flow and pollution is a stock. But if C and P are interpreted as states of mind then both variables can (and should) be treated as stocks.[1] These stocks generate a flow of utility and disutility, respectively.[2] For simulation purposes the utility function has to be parameterized. The following function preserves the basic characteristics of the above stated utility function:

$$U = \frac{1}{1-\eta}C^{1-\eta} - \frac{1}{1+\pi}P^{1+\pi}, \quad 0 < \eta < 1, \pi > 0 \tag{1}$$

The parameters η and π are the partial elasticity of marginal utility with respect to consumption and the partial elasticity of marginal (dis-) utility of pollution, respectively. The consumption elasticity of utility is given by $1 - \eta$ and the pollution elasticity of utility is given by $1 - \pi$. Hence, the utility function (1) is isoelastic. A macroeconomic utility function is an artificial concept. Neither its form nor its parameter values are observable. To my best knowledge there

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[1] For a discussion of the modeling of mental states see Sterman (2000: 199).

[2] Currently, there is an interesting discussion in the literature what "utility", "happiness", "welfare", and "well-being" is (see e.g Frey and Stutzer (2002). This includes the question whether these concepts should be treated as stocks or flows. Here, I assume for sake of simplicity that utility is a flow which can be integrated over time.

are no empirical studies from which the parameter values of function (1) could be readily derived. But the happiness literature suggests that the consumption elasticity of utility should be significantly smaller than one (see Frey and Stutzer (2002: 7-10)). Due to the lack of more precise information this paper assumes for simulation purposes a consumption elasticity of 0.5. The effect of different values of the pollution elasticity is tested by simulation.

The objective of economic policy is to maximize the total stream of utility over the planning period. Formally, this means that the following dynamic optimization problem has to be solved:

$$J = \int_0^T e^{-\rho t} \frac{1}{1-\eta} C^{1-\eta} - \frac{1}{1+\pi} P^{1+\pi} dt \rightarrow \max \tag{2}$$

The maximization of *J* is subject to certain constraints that will be discussed below. In an optimal control framework the task would be to find the time path of the control variables that maximize the functional *J*. As stated in the introduction the resulting boundary value problem can, in general, only be solved for quite simple reduced form models. [3] In addition, it is questionable whether economic and environmental policy in the real world can in fact follow the calculated control path. For these reasons, this paper follows another approach: It uses the functional given by equation (2) to measure the consequences of variations of model parameters and the impact of different economic policies over time.

Two other problems have to be stated with regard to the welfare functional: First, because we integrate utility the utility function (1) cannot be interpreted as a utility *index* function. In general, it is preferable to think of (1) as an index function because in this interpretation it is only important to be able to rank different situations with respect to the generated utility. But one cannot sensible integrate the ranks given by a utility index function. Integration of utility means that we have the imagination to be able to add the “amount” of utility (measured in “utils” per time period, e.g.).

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[3] For an overview with respect to problems of this class that are tackled by using the Maximum Principle and multiple shooting algorithms see John (1989).

Second, we argue that from the perspective of the society (or a planning authority) future "utils" are less valuable than current ones as long as we assume that $\rho > 0$. Only in the case when the discount rate equals zero all generations is given equal weight. Whether future utility should be discounted and if so by what discount rate is a topic widely discussed in the literature.[4] Because the literature does not lead to a conclusion I simply assume that the discount rate is non-negative and has a value of about 3 percent. (The influence of the discount rate on the dynamics of the model can be tested by sensitivity analysis.)

The production function is modeled by a standard Cobb-Douglas-Function with constant returns to scale:

$$Y = e^{\gamma t} K^{\beta} L^{1-\beta}$$

The symbols of this equation have the following meaning: Y – output, K – capital, L – labor, β – production elasticity and γ – rate of technological progress. Because the purpose of this paper is just to analyze the consequences of policy measures when environmental damages have a negative impact we can treat labor as constant. Normalizing labor input to one, the production function can be rewritten in the following way:

$$Y = e^{\gamma t} K^{\beta} \tag{3}$$

A parameter constellation of $\beta = 0.3$ and $\gamma = 0.02$ seems to be reasonable for many countries.

Following the standard approach of neoclassical growth theory the model does not include a separate investment function but assumes that circular flow equilibrium holds and that, consequently, investment always equals savings. Saving in turn is defined as that part of income that is not used for consumption or for abatement spending. Consumption (C) is proportional to income and abatement spending (A) is taken as policy control variable. The circular flow equilibrium condition can be stated as:

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[4] Pittle (2005) goes one step further. She argues that the discount rate depends on the state of the environment and therefore should be endogenized in models of economic growth and environment.

$$Y = C + I + A \tag{4}$$

The stock of capital is increasing due to investment of firms and decreasing due to depreciation:

$$\dot{K} = I - \delta K \tag{5}$$

For simulation purposes we assume a depreciation rate (δ) of 3 percent. This is the same as to say that the average life time of a capital good is about 30 years.

We now turn to pollution and environmental damages. Pollution is modeled as a byproduct of goods production. It is assumed that with every unit of output a certain amount of a pollutants is emitted. The emissions accumulate over time to a stock of pollutants. With respect to most pollutants bio-systems have a certain self cleaning capacity. Therefore, we assume that a given fraction of the stock of pollutants is biologically degraded in every time period. With these assumptions the dynamics of the stock of pollutants can be described by the following differential equation:

$$\dot{P} = \varepsilon Y - \alpha P \tag{6}$$

In this equation ε denotes the amount of emitted pollutants relative to output and α denotes the fractional "natural" abatement rate.

Natural biological abatement can be supported by appropriate environmental policy measures. We assume that the economy can build up a stock of abatement capital just as it can build up a stock of production capital. To build up this stock the economy has to give up on some of its consumption or on some of its investment that accumulates to the stock of capital necessary for the production of goods. In order to make clear the distinction between the capital used for the production of good and the capital used for emission reduction we shall further on call the former *production capital* (or, for short, just *capital*) and the latter *abatement capital*. The dynamics of the abatement capital (M) are given by

$$\dot{M} = A - \mu M \tag{7}$$

The stock of abatement capital increases by abatement investment expenditures *A* and it decreases by wear and tear. In equation (7) a constant rate of depreciation of abatement capital is assumed. Because there is no good reason for assuming the depreciation rate of abatement capital different from the depreciation rate of production capital the base scenario of the simulation model presumes $\mu = 0.03$. The next step will be to put these building blocks together to form a system dynamics model of growth and pollution.

3. A System Dynamics Model of Economic Growth and Pollution

Figure 1 shows how the elements discussed above can be brought together in a stock and flow diagram. We start by an interpretation of the accumulation of capital used for goods production and then explain the remaining elements of the diagram clockwise.

The capital used for goods production is just called "capital" in the diagram. (The capital used for abatement purposes is called "abatement capital".) The stock of capital is increased by (gross) investment and decreased by depreciation. The fractional depreciation rate is the percentage of the capital stock that leaves the capital stock each time period due to wear and tear.

Production is modeled by a Cobb-Douglas production function. As it is the case in standard neoclassical growth theory, the only factors of production are labor and capital. We further assume that labor input is given and normalized to unity. Due to this assumption the capital stock is, in effect, the only factor of production. Production grows when more capital is employed, but at a decreasing rate. The second source for output growth is technological progress. We assume that there is a constant exogenously given rate of technological progress.

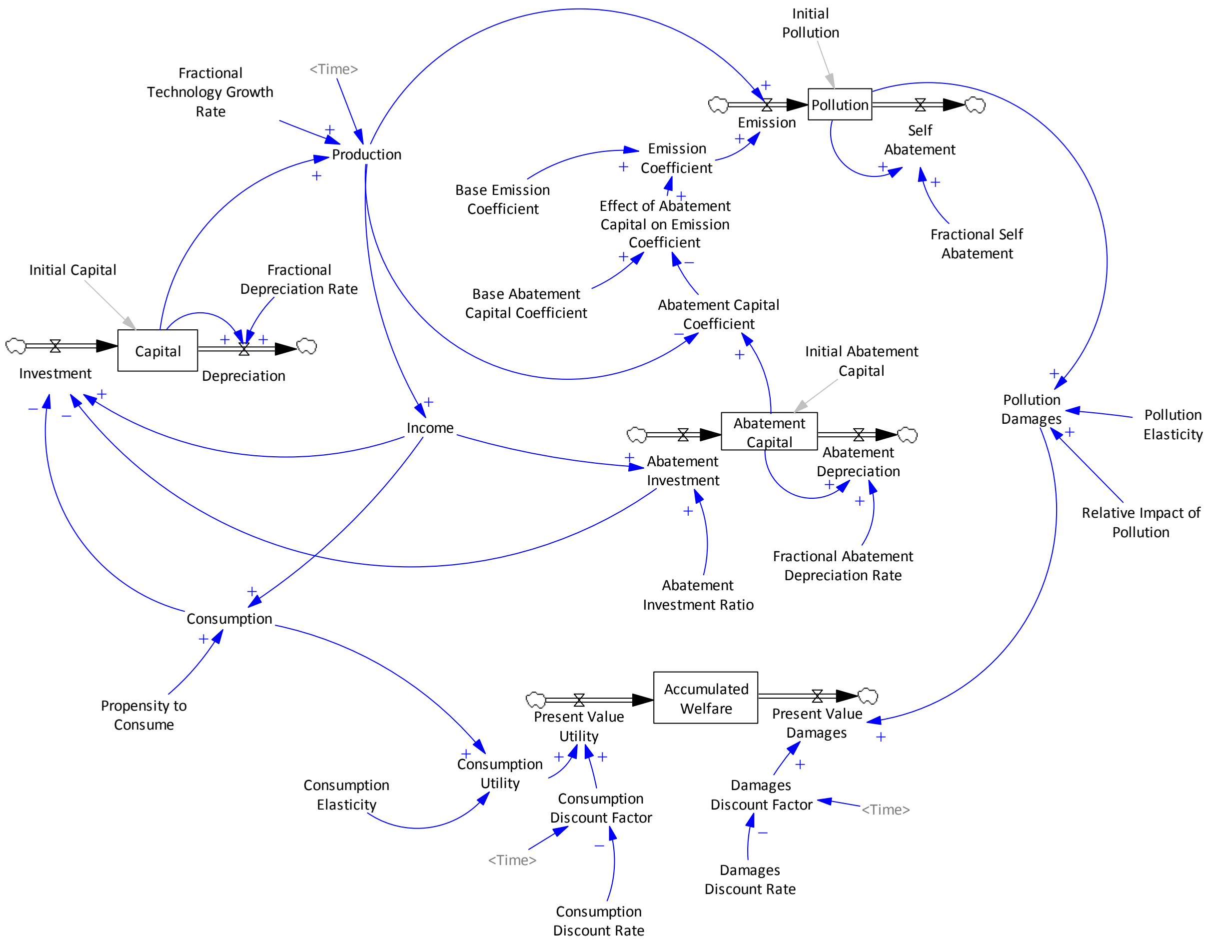


Figure 1: Stock and flow diagram

Goods production leads, as the flip side of the coin, to an equal amount of income. Departing from the standard economic growth model it is taken into account that in the production process emissions are generated as byproducts. These emissions are not considered in the systems of national accounts and, therefore, do not decrease GNP. Still, the emissions have a negative impact on societal welfare. The emissions from the production process accumulate to a stock of pollutants. The stock of pollutants is decreased by self cleaning processes of the bio-system. For a concrete example we may think of carbon dioxide emissions. Carbon dioxide accumulates in the atmosphere but a certain amount is transformed to organic matter by photosynthesis each time period. Similar processes exist for other pollutants.

As we have just stated the stock of pollutants is decreased by self abatement. In addition, the stock of pollutants can be decreased by employing economic resources. In our model it is assumed that a part of the GDP can be used for abatement investment. Abatement investment, on the one hand, reduces the amount of income that can be used for consumption or for investment in (production) capital. On the other hand abatement investment accumulates to a stock of abatement capital which has the effect to decrease emission. As examples we can envision filters or other equipment that reduces emissions generated in the production process. In this paper the impact of abatement capital on emission is modeled by a look up function which ensures that it becomes the harder to reduce emission by one further unit the more abatement capital is already employed. The stock of abatement capital suffers like the stock of production capital from wear and tear. This depreciation is modeled in the same way as in the case of production capital.

The stock of pollutants has a negative impact on the society. This effect may work via a deterioration of the production process, via a deterioration of the consumption process or via health impairment. In our model we simply assume that pollution leads to pollution damages which in turn decrease welfare. If we talk of pollution damages we have to value pollution in some way. In the model this is achieved by a damage function. The damage function uses two parameters: a scaling factor (which is called “relative impact of pollution” in the stock and flow diagram) and the pollution elasticity. The scaling factor is calculated in a way that the disutility of pollution and the utility derived from consumption are on a comparable scale. The numerical value of the pollution elasticity determines how sensible damages react on an increase of pollution. If the pollution elasticity equals zero then an increase in pollution by one unit leads to a proportional increase in damages no matter what the level of pollution is. If the pollution elasticity is a positive number then with one additional unit of pollution the damages increase the more the higher the level of pollution already is.

The societal evaluation of the utility resulting from consumption and the disutility resulting from environmental damages is done by a welfare function. In figure 1, the stock which is called "accumulated welfare" shows one inflow and one outflow. The inflow is the discounted value of consumption utility and the outflow is the present value of the environmental damages. Note that if the model is used to evaluate different policy strategies it is not the time path of accumulated welfare that counts but the value of this stock at the end of the planning horizon.

The model summarizes a basic dynamic conflict of every real economy: how to best use its scarce resources over time. More consumption leads to higher utility. But to be able to produce goods the society has to build up a capital stock first. The higher the capital stock the higher is production and the higher are the consumption possibilities. But to build up the capital stock a part of production has to be put aside as investment which in turn decreases consumption possibilities. This is the classic golden rule problem of normative growth theory dating back all the way until Ramsey's seminal work (Ramsey 1928). But if we consider the emissions that are generated in the production activities we have another dynamic problem: how much of its resources should the society devote to pollution abatement. In other words: how much of its income should the society put into abatement expenditures knowing that this part of income cannot be used for consumption or accumulation of production capital.

The model allows answering this and a lot of related questions on a macroeconomic level. In the following simulation exercise we demonstrate the scope of the model by analyzing three scenarios. The first scenario may be labeled the “ignorance” scenario because pollution exists but policy ignores it. The second scenario is labeled “base run” because the parameters are adjusted to values that seem to be quite realistic for an industrialized countries. (By evaluating the term “realistic” one has to take into account that the model we discuss here is a very simple and a highly abstract one.) The third scenario is labeled “pollution matters” because we model a system in which the pollution elasticity is assumed to be equal to 0.3 – whereas in the base run scenario it is assumed to be equal to zero. Policy is implemented in a very simple way: The authorities can decide which fraction of GDP is invested in abatement capital. As the propensity to consume is treated as a given constant the policy decision about abatement investment is by the same time a decision about the fraction of income that is invested in production capital.

4. Simulation Results

The initial values as well as the parameter values used for simulation are basically adjusted to reflect the conditions of the German economy. This is true for the size of the stock of production capital, GDP, consumption, investment. Emissions and the emission coefficients reflect the carbon dioxide intensity of the German economy. The purpose of this model is to analyze basic relationships and to analyze policy strategies. Its purpose is not to forecast the development of a real economy. Therefore, most of the more “problematic” parameters (e. g., the elasticities in the utility function) are chosen by a rule of thumb. Consequently, the simulation results should not be interpreted as representing exact time paths of the variables in question.

Figure 2 to 7 show the simulation results. In scenario 1 (“ignorance”) the capital stock accumulates fastest. As a consequence the time path of the stock of pollutants is the highest, by far. The initial stock of abatement capital is decreasing by the depreciation rate year after year because in the ignorance scenario there is no new abatement investment. Accumulated welfare is quite high compared to scenario 2 at final time. This is due to the fact that in both scenarios a pollution elasticity of zero is assumed. This means that the perceived pollution damages only grow proportionally with pollution. Even though the stock of pollutants grows considerably this is not judged as an especially serious problem. Looking at both components of welfare separately it becomes clear that the present value of consumption utility decreases over time due to discounting (but remains positive) whereas the present value of environmental damages increases due to the strong increase in pollution despite of discounting. Because the present value of consumption is higher than the present value of environmental damages in each time period the accumulated welfare increases monotonically.

Scenario 2 ("base run") keeps the parameter values of scenario 1. In particular, the value of the pollution elasticity remains equal to zero. But the model now assumes an abatement investment ratio of 0.1 (instead of zero). Because the propensity to consume remains unchanged investment in abatement capital reduces investment in production capital by an equal amount. Consequently, in figure 1 the time path of the capital stock (marker 2) runs completely below the "ignorance" time path (marker 1). Because of abatement investment pollution is all the time considerably smaller compared to scenario 1 (see figure 3), and the accumulated investment net of depreciation leads to an increase in the stock of abatement capital (see figure 4). Accumulated welfare is somewhat higher than in scenario 1 because policy makers no longer ignore environmental damages but react to them by investing into abatement capital. Compared to scenario 1 the time path of the present value of consumption utility is now below, but the path of the present value of environmental damages is also below the respective paths. In the end, the lower discounted damages outweigh the lower consumption utility and we have an increase in welfare.

The situation described by scenario 2 can be improved slightly by policy optimization. This is done be asking the software to find the value of the abatement investment ratio that maximizes the accumulated welfare. The optimal value is 7.5 percent compared to 10 percent of the non optimized base run. (The time path graphs of the optimized base scenario are marked "3".) The reduction of abatement investment means by the same token an increase in production investment. Consequently, the time path of the capital stock is now above, the time path of the abatement capital stock below, and the time path of pollution above the non-optimized path.

Accumulated welfare is slightly higher at final time. This is hard to see in figure 5 but it must be so – otherwise we had not optimized.

At this point, it is worthwhile to emphasize again that the word "optimization" in the procedure just described has a restricted meaning. Our approach includes a restriction: the policy makers of our model can only make a onetime choice at the beginning of the planning period – they choose from a given interval parameter value that maximizes accumulated welfare *if they keep that value until final time*. They cannot choose an optimal set of parameters. Nor – perhaps more important – can they choose an *optimal time path* of the control parameter. But it might very well be the case that a "real" optimal policy would imply to change the fraction of abatement investment over time. As stated above, it turns out that this "optimal control" problem is very demanding because in many cases the known optimization algorithms do not converge.

Scenario 3 differs from scenarios 1 and 2 in that society takes increasing pollution more seriously. This is modeled by increasing the value of the pollution elasticity to 0.3. In figures 2 to 7 the lines marked 4 represent the time paths for the base run value of the abatement investment ration of 10 percent and the lines marked 5 represent the time paths for the optimized ratio of 21 percent. A look at figures 2, 3, 4 and 6 reveals that the time paths for capital stock, pollution, and abatement capital stock marked "2" and "4" are identical. This should be no surprise because the abatement investment ratio is in both cases the same (10 percent). The difference is only in the judgment of environmental damages. The higher value of the pollution elasticity implies that people take environmental damages more seriously. This in turn leads to higher time path of the present value of environmental damages. As the time path of the present value of consumption utility remains the same, in total, the accumulated welfare must be below the value of scenario 2 at final time.

Again, the situation can be improved by choosing that value of the abatement investment ratio that maximizes the accumulated welfare which turns out to be 21 percent. With this comparatively higher ratio less GDP is left for accumulating production capital. Figure 2 shows that in this case the capital stock first even decreases slightly because gross investment does not outweigh depreciation. On the other hand the abatement capital stock increases quite fast and pollution is nearly kept constant over time. The optimization of the abatement investment ratio leads to a lower time path of the present value of utility from consumption but also to a lower time path of the present value of environmental damages. The latter outweighs the former resulting in a higher value of accumulated welfare at the end of the planning period.

Figure 2: Capital Stock

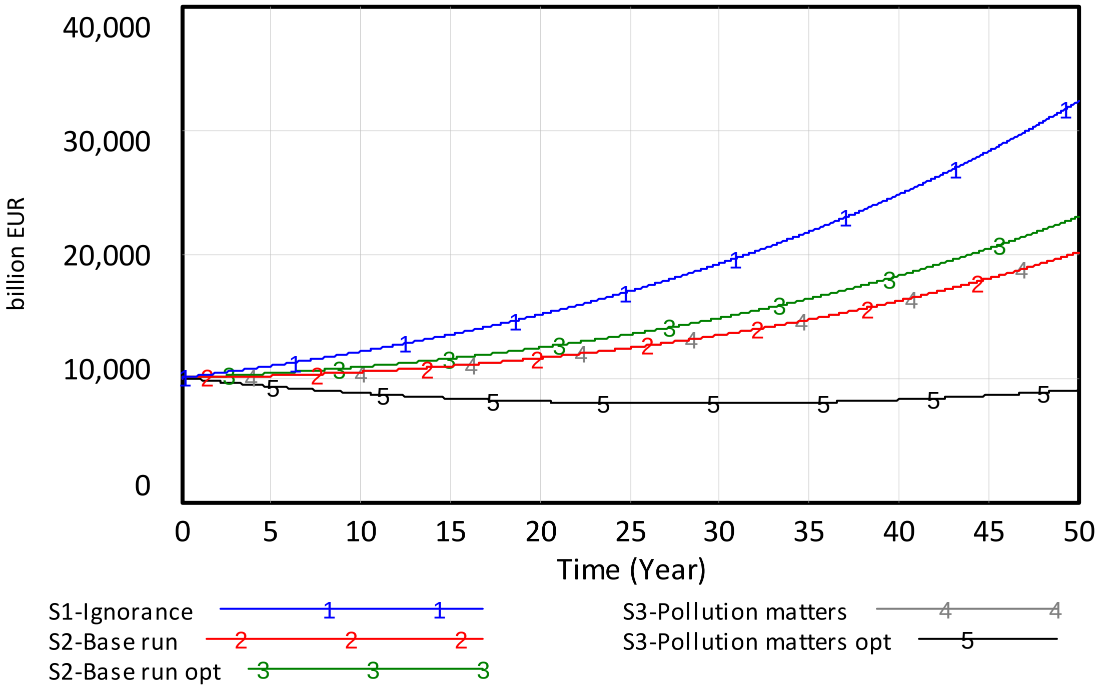


Figure 3: Pollution

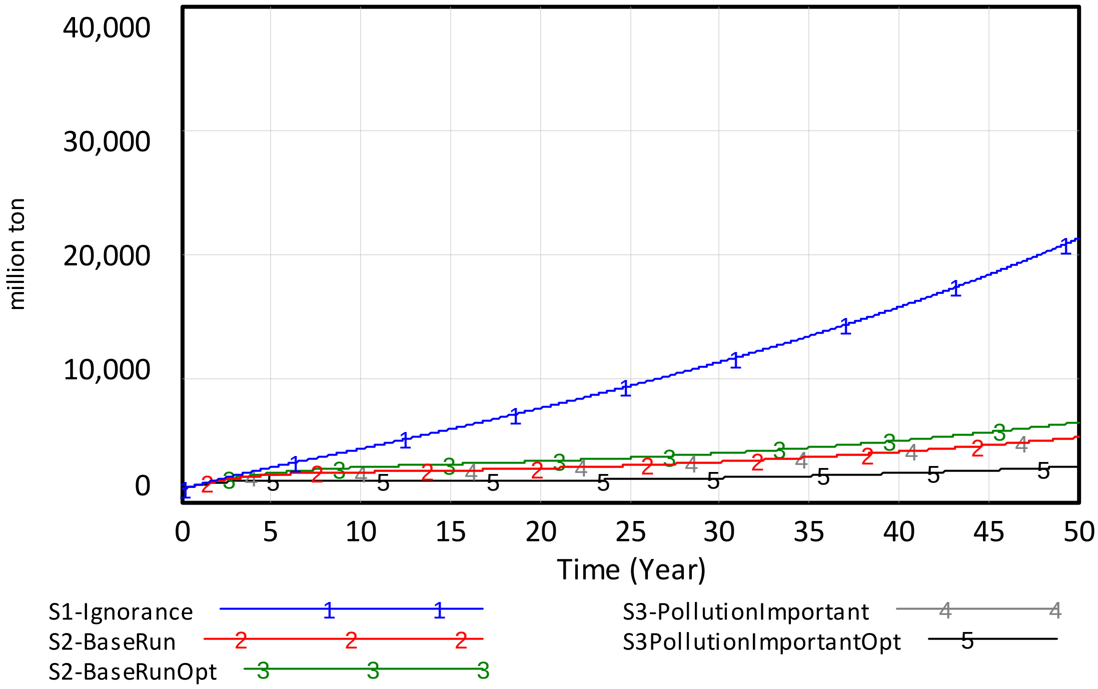


Figure 4: Abatement Capital Stock

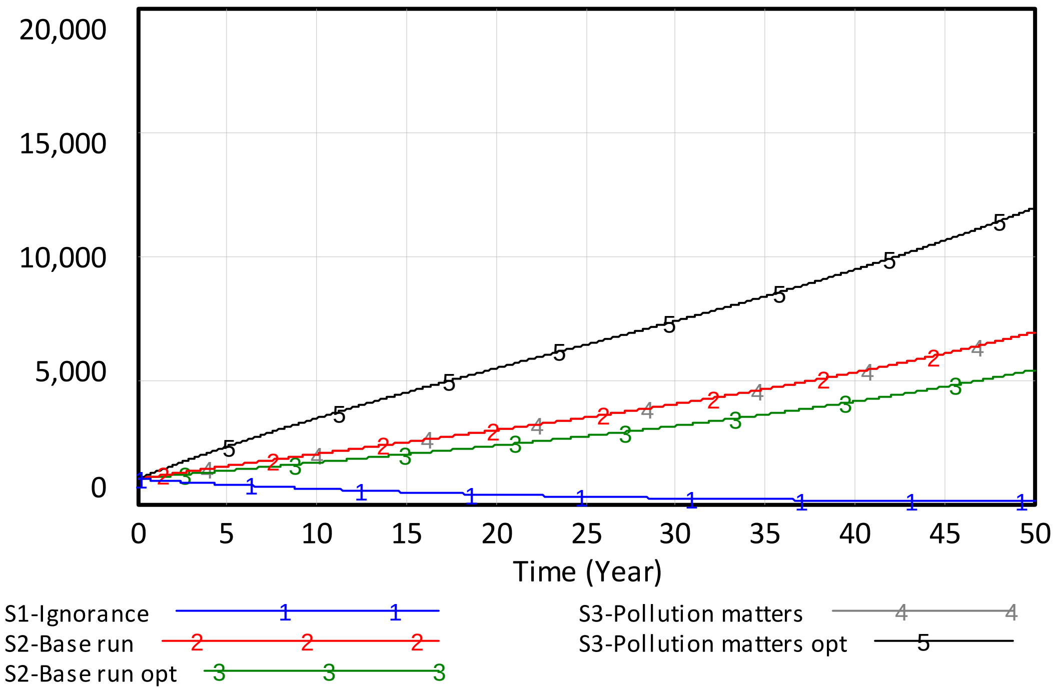


Figure 5: Accumulated Welfare

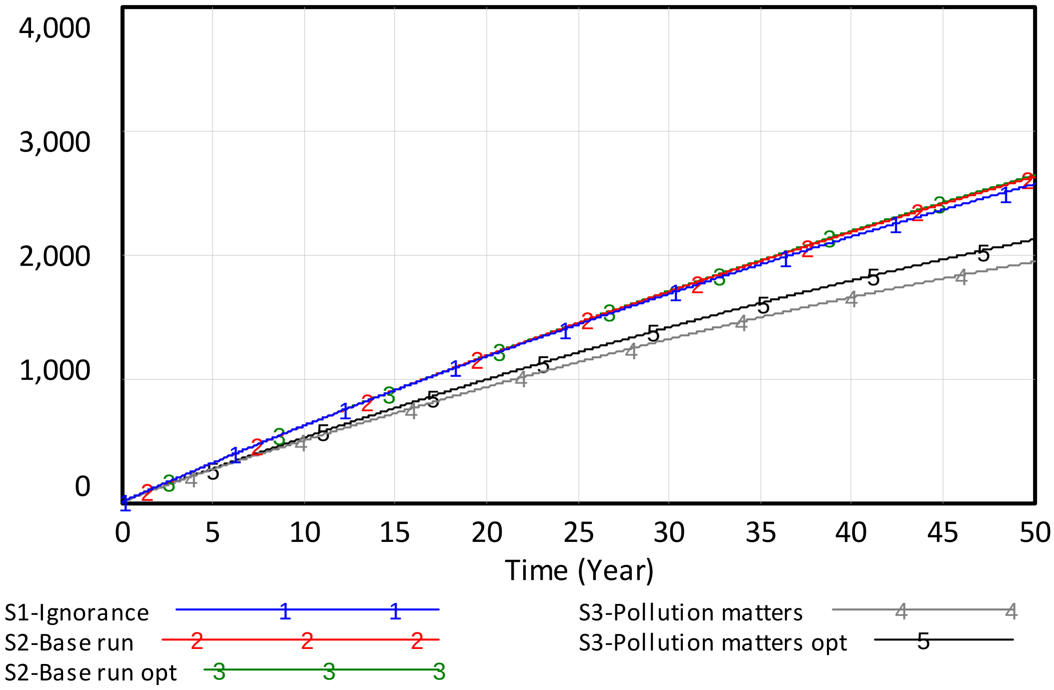


Figure 6: Present Value of Utility from Consumption

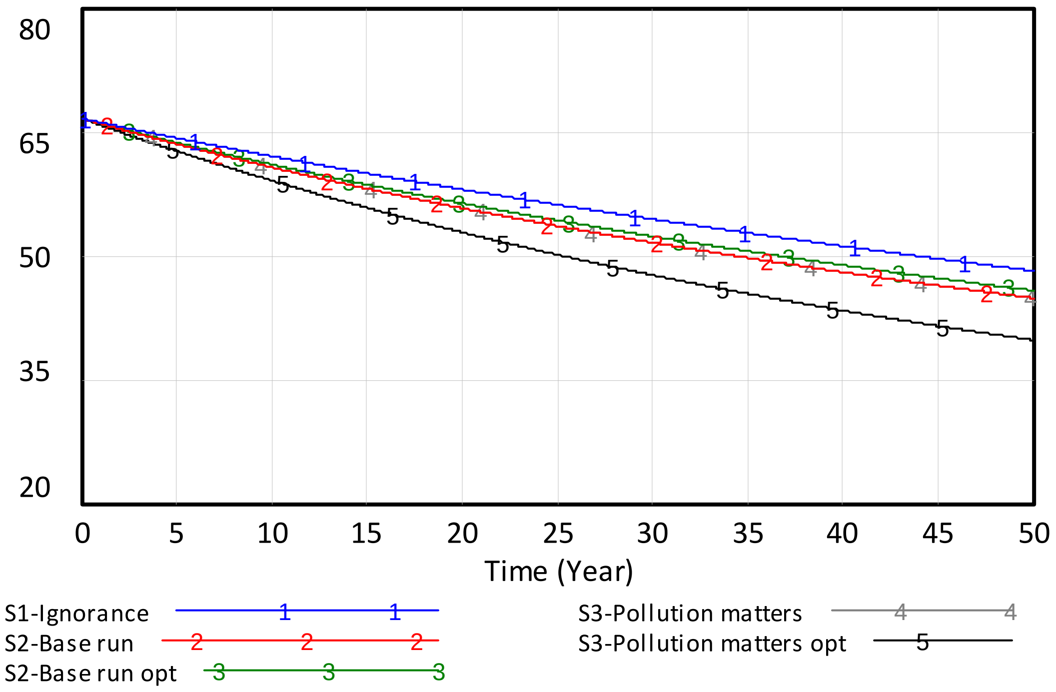
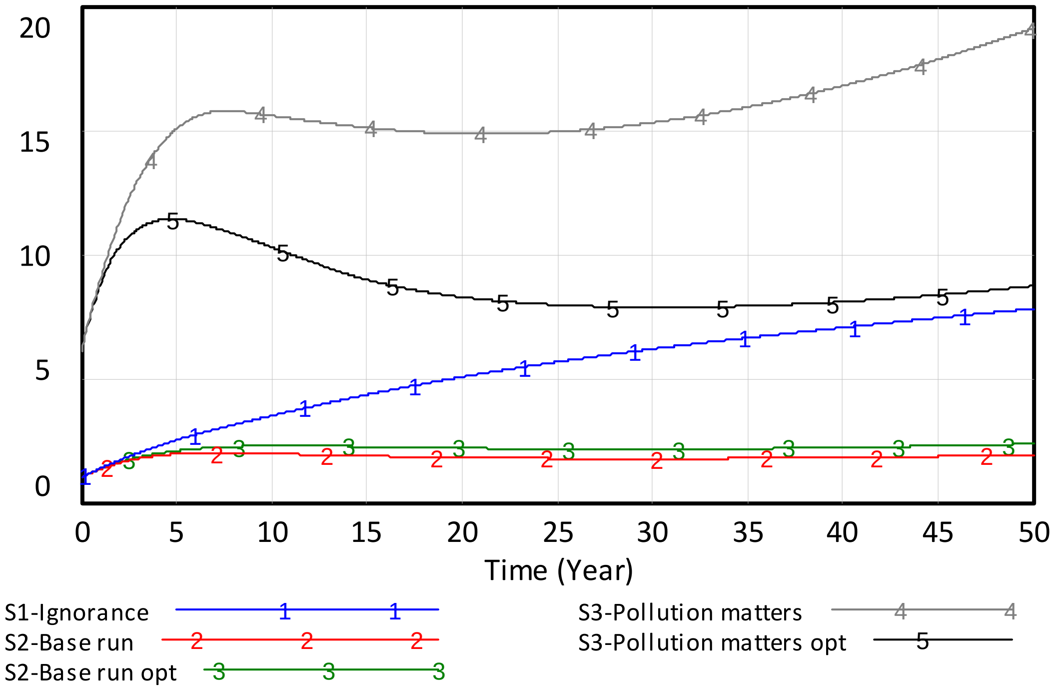


Figure 7: Present Value of Environmental Damages



5. Conclusion

The model developed in this paper integrates on a macroeconomic level growth and pollution in a dynamic framework. It improves with respect to the usually employed growth models in that it explicitly takes the environmental damages into account that are generated by the production of goods and in that it analyzes the dynamic interactions.

The model can be used to analyze different policy measures in different settings. This was demonstrated for situations in which policy makers ignore environmental damages and in which they take environmental damages into account. Further, the model was used to analyze the impact of different societal attitudes towards environmental damages.

As usual, a lot of questions remain open for the time being. For example, it is unclear whether a fully-fledged optimal control solution makes a big difference with respect to the accumulated welfare compared to the parameter optimization employed in this paper.

The model can be extended in several directions. For example, in many cases pollution makes production more expensive. If one wants to study this phenomenon on a macroeconomic scale the employed production function should be altered in a way that the environment is treated as a factor of production. Another promising extension would be the integration of more realistic instruments of environmental policy (emission taxation, command and control rules, etc.). Finally, following the ideas of new growth theory the production of knowledge and increasing returns to scale could be integrated.

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