

Formal Theory Building for the Avalanche Game: Explaining counter-intuitive behaviour of a complex system using geometrical and human behavioural/physiological effects

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In ‘Avalanche’ a rod is lowered to the ground, team members staying in contact throughout. Normally the task is easy. However, with larger group sizes counter-intuitive behaviours appear, including the object’s ascending.

A formal theory for the geometric element and other human behaviour effects gives insight into the behaviours. Each player has two balancing loops, one involved in lowering the object, the other ensuring contact. For more players these loops interact and these can allow intermittent dominance by reinforcing loops, causing the system to chase upwards towards an ever increasing goal.

Analysis indicates that there is only a narrow region in which the system is able to move downwards. An analogy is drawn between the co-operative behaviour required in this system and Prisoners’ Dilemma situations. Sensitivity analysis gives further insight into the system’s modes and their causes. Reflections on the benefits of formal theory building close the paper.

Key words: System dynamics, simulation modelling, systems thinking, management games, co-operation, prisoners’ dilemma

N.B.

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INTRODUCTION

This paper concerns the ‘Avalanche’ game (Booth-Sweeney & Meadows, 1995). What happens when groups play Avalanche is the phenomenon at the heart of this paper. In this section the game is first outlined. Then explanations for the phenomenon are described; those currently used and those that formal theory building might create.

The Avalanche game

‘Avalanche’ is a management exercise in which a group is asked to manipulate a light physical object to achieve an aim, subject to certain conditions, or rules. The physical object may be a large hoop, perhaps one meter in diameter – children’s hulla-hoops are usually employed. Alternatively, a long rod, some 2 metres in length may be used; e.g. tent poles or broom handles. It is the hoop version of the game that is outlined below.¹

A group of four to twelve individuals is asked to space itself around the hoop and, with each person using one finger, to support the hoop somewhere between waist and chest height, keeping it as level as possible. The group is then given the task of lowering the hoop to a height closer to the ground, perhaps 10 cms. Group members are given a clear condition: the lowering of the hoop must be achieved whilst all members stay in contact with the object throughout. Silence is encouraged during the game.

With only three people this task is easily accomplished (see Fig. 1); the hoop descends steadily, taking between half and one minute to do so. This is the system’s desired reference mode (Randers, 1980; Richardson & Pugh, 1981).

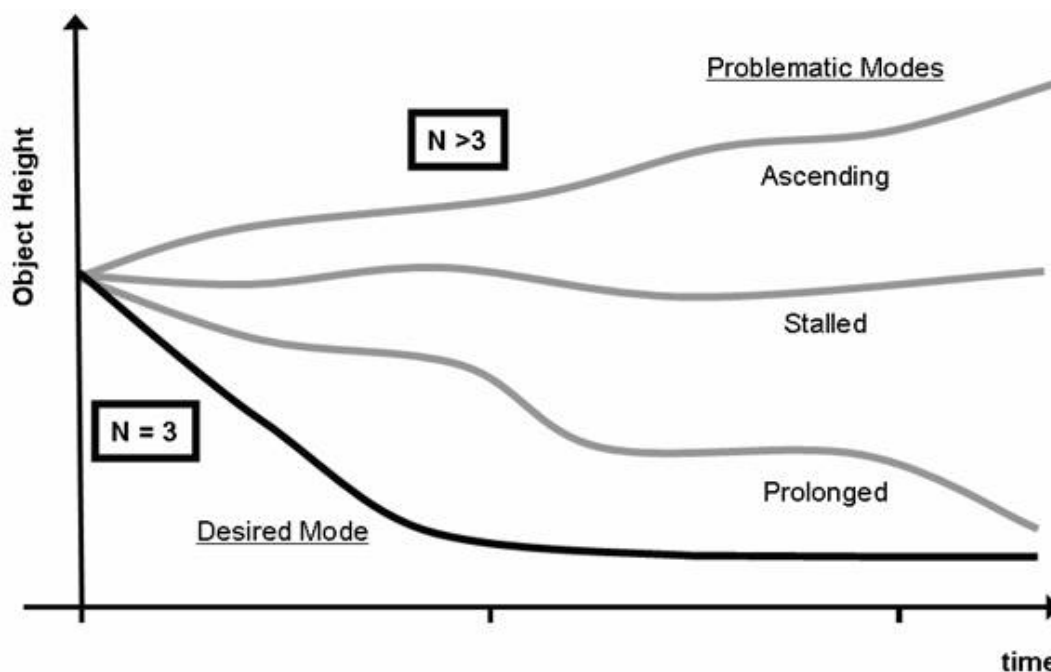


Fig. 1. The four observed reference modes for the Avalanche game: one desired mode and three problematic ones (grey).

1. The author first saw the game played in a demonstration by Dennis Meadows at the 2005 International Conference of the System Dynamics Society, in Boston. This experience kindled his interest in the game. He has since facilitated the game himself.

However, when the size of the group is increased interesting phenomena are observed as problems with fulfilling the task arise and ‘problematic behaviours’ are generated (ibid.). The counter-intuitive behaviours may be divided into three: prolonged, stalled and ascending. (see Fig. 1). In the first, the task is accomplished but it takes much longer. Alternatively, the hoop may simply remain close to its original height, merely displaying small and seemingly random, inconclusive upward and downward movements. In the third problematic mode the hoop rises upwards, to the consternation of the players. In these three cases players may react verbally, trying to direct or encourage other group members. These reactions are discouraged. If players break the rule and lose contact with the hoop then they are asked to declare this. The facilitator also monitors for loss of contact during the game. In either case the task is started again.²

These modes are examples of the counter-intuitive behaviour of social systems, particularly the last case since the outcome produced by the group is in direct opposition to the task set (Forrester, 1970).

On possible explanations

Avalanche is an amusing and involving experience. However, it aspires to be more than this. Caution is required when claiming that a gaming/simulator delivers learning, the record of achievement being chequered (Neuhauser, 1976). However, Avalanche emerges from the system dynamics community, wherein there has always been an aspiration to create learning by exposing people to models (Forrester, 1961). The field has been consistent in its concerns about what is gained (Senge, 1990; Sterman, 1994) and in avoiding previous errors (Lane, 1995; Größler, 2004). In terms of paraphernalia, protocol and purpose, Avalanche has been carefully thought through. Meadows’ goal with such experiences is to achieve specific learning aims (Meadows, 1989; 1998).

The Avalanche documentation contains clear ideas about the purpose and outcome of the game (Booth-Sweeney & Meadows, 1995). Firstly, the exercise illustrates the difficulties of managing the emergent, self-organized behaviour of a complex social system. Consistent with this, Meadows says it illustrates that small changes in parameters can produce large changes in behaviour.³ Second, Avalanche is a metaphor for exploring the relative importance of co-operative and competitive behaviour (making an interesting link to ‘Prisoners’ Dilemma’). It supports discussion about individual behaviour and group goals, and about the role of rule breaking in achieving aims. Avalanche introduces players to such lessons in as little as 10 minutes.

Current versions of the game do not – as far as this author is aware – explore the causal mechanisms in play. The description refers to the ‘escalation archetype’ (Meadows, 1982; Senge, 1990), implicitly invoking a causal loop diagram but it goes no further. This is curious because of the game’s system dynamics roots. For example, the remark about changes in parameters is an example of the general idea that social systems have only a few, key parameters that can alter behaviour (Forrester, 1969). A central assumption of the field is that inferring behaviour is a perilous business; system dynamicists emphasise the importance of using a fully formulated simulation model rigorously to deduce the behavioural consequences of causal linkages (Forrester, 1956; 1960). This at least hints at the idea that a more formal approach might be able to generate increased and more robust insight regarding the sources of behaviour in Avalanche.

A broader issue concerns the nature of theorising about social systems. Some forms of social theorising – and Avalanche is a long way from this – offer explanations based solely on

2. Sometimes this condition is emphasised by using small slips of paper between fingers and the hoop. If contact is lost then the slip can be seen to fall away.

3. Comment made at the 2005 session.

complex verbal accounts. Criticisms of this approach are widespread, e.g. Mills (1959), but this form of ‘explanation’ continues in modern ‘social theory’, the resulting neologism-laden word spinning being termed derisively a ‘vast conceptual quilt’ (van den Berg, 1998). Such approaches are said not to create accounts of mechanisms, “do not possess conceptual tools for explaining social behavior” (ibid., p. 220), and are even accused of misleading researchers about the very nature of scientific explanation (Abell, 1994; Mills, 2000). In contrast, system dynamics offers a form of scientific inquiry which illuminates subtle social phenomena using formal, empirically-grounded theories expressed in appropriate conceptual tools (Homer, 1996; Lane, 1999; 2001). So, although the existing explanations for Avalanche are more firmly grounded in detailed thinking, this paper is part of the project of moving from the ‘conceptual quilt’ approach to ‘theory building’ to the formal, social mechanism-centred approach (Elster, 1989; Hedström & Swedberg, 1998).

This paper therefore proposes a formal theory for the causal mechanisms that are in operation in this system. The aim is to provide an explicit, testable hypothesis for the source of the observed modes of behaviour of the Avalanche game and to gain insight into the relative importance of various strengths of effect believed to be in play.

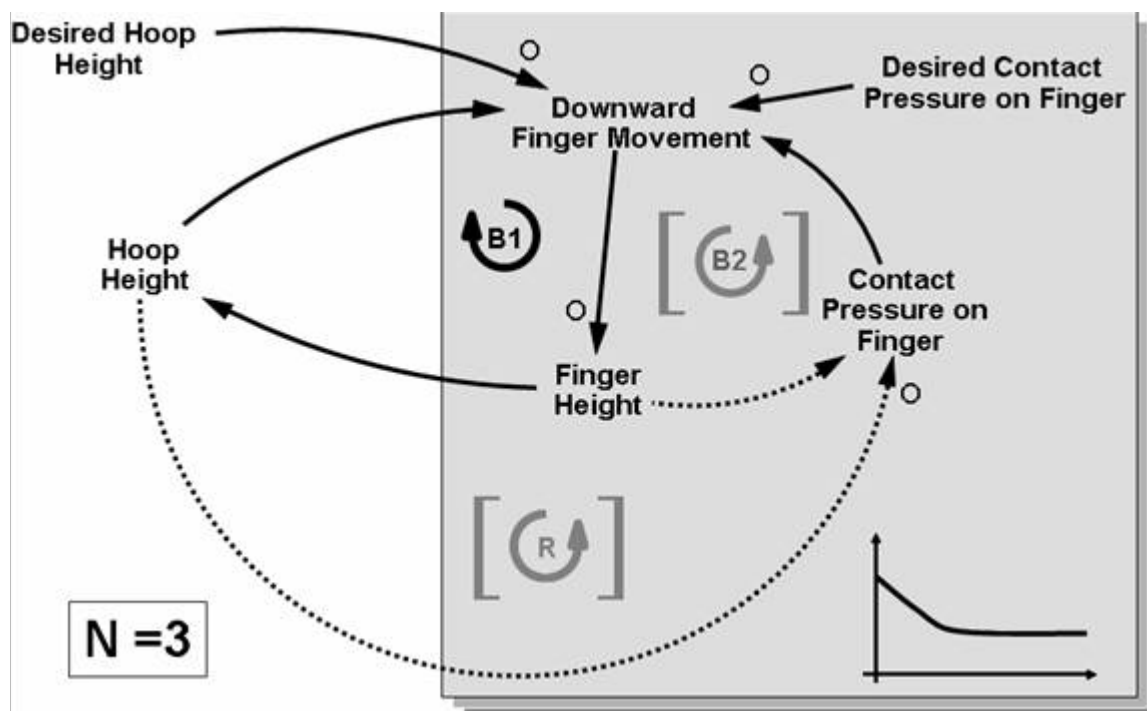


Fig. 2. Causal loop diagram of Avalanche, as played with a hoop by three people.

TOWARDS A THEORY

This section first offers a map-based analysis of the links and loops that may be in operation in Avalanche. To this is then added a range of human effects considered as possible sources of explanation.

Mapping a theory

Lowering the hoop is the central concern for Avalanche participants. This is shown in Fig. 2 as balancing loop B1, a goal-seeking process. A player adjusts downwards the height of their finger, so lowering the hoop until it descends to its desired height.

However, there is also the condition or rule that a player must stay in contact with the object. This is conceptualised as loop B2 in Fig. 2. A player has a desired level of pressure on the finger with which he/she is manipulating the hoop. As the actual pressure that he feels reduces, he becomes cautious about moving his finger further downward for fear that the pressure will fall to zero, indicating a loss of contact. Eventually he may move his finger upward to increase contact pressure.

There are two further features of this causal loop diagram. First, establishing the actual contact pressure on a player's finger requires a link from 'Hoop Height', shown at the bottom of Fig. 2. The hoop rests on a player's finger, compressing its surface slightly and generating a counteracting force. As the height of the hoop increases with respect to the height of one's finger the contact pressure falls.

The second feature is that these two loops exist for every player. This is illustrated by the layered effect in Fig. 2. So the actions of each player are governed by two balancing loops, one representing the task, the other the constraint, or rule. Hence we refer to loops B1_i and B2_i, the subscripts indicating players α , β and γ . In contrast, the 'Desired Hoop Height' is specified by the facilitator and the 'Hoop Height' emerges from the collective behaviour of the group.

Potentially this CLD has additional loops R_i. However, with three players this loop is inactive. The argument runs as follows. When one player reduces the value of his 'Finger Height', this has two consequences, indicated by the two causal paths from that variable. First, the direct link to the right would, *ceteris paribus*, reduce the 'Contact Pressure on Finger'. Second, the link to the left reduces 'Hoop Height' which would then, *ceteris paribus*, increase the contact pressure. This is the critical point: with three players the 'Hoop Height' is the 'Finger Height'. In consequence, the two dotted causal links cancel each other out and 'Contact Pressure on Finger' is unchanged for all players. Hence, loops R_i are dormant and loops B2_i are not conscious parts of the players' actions; they concentrate instead on the operation of loops B1_i. The hoop descends - the CLD being consistent with this desired behaviour.

The CLD reveals the importance of geometry to this system. With three players all are involved in setting the hoop height because three support points are required for this two dimensional object.⁴ With more than three players this no longer applies.

The system for more than three players is shown in Fig. 3. The crucial difference is that one of the key causal mechanisms may cease to operate at times. Consider the following. Imagine that player α moves his finger downwards. Perhaps he moves with a speed greater than the others. Alternatively, a wobble in a finger's position might cause the downward movement. Whatever the reason, Fig. 3 indicates that this movement should then produce a reduction in 'Hoop Height'. However, as indicated, this mechanism may not operate: if the other players have not moved and there are enough to support the hoop then player α 's changed finger

4. This is merely a form of the geometric points define a plane.

height does not influence the height of the hoop.

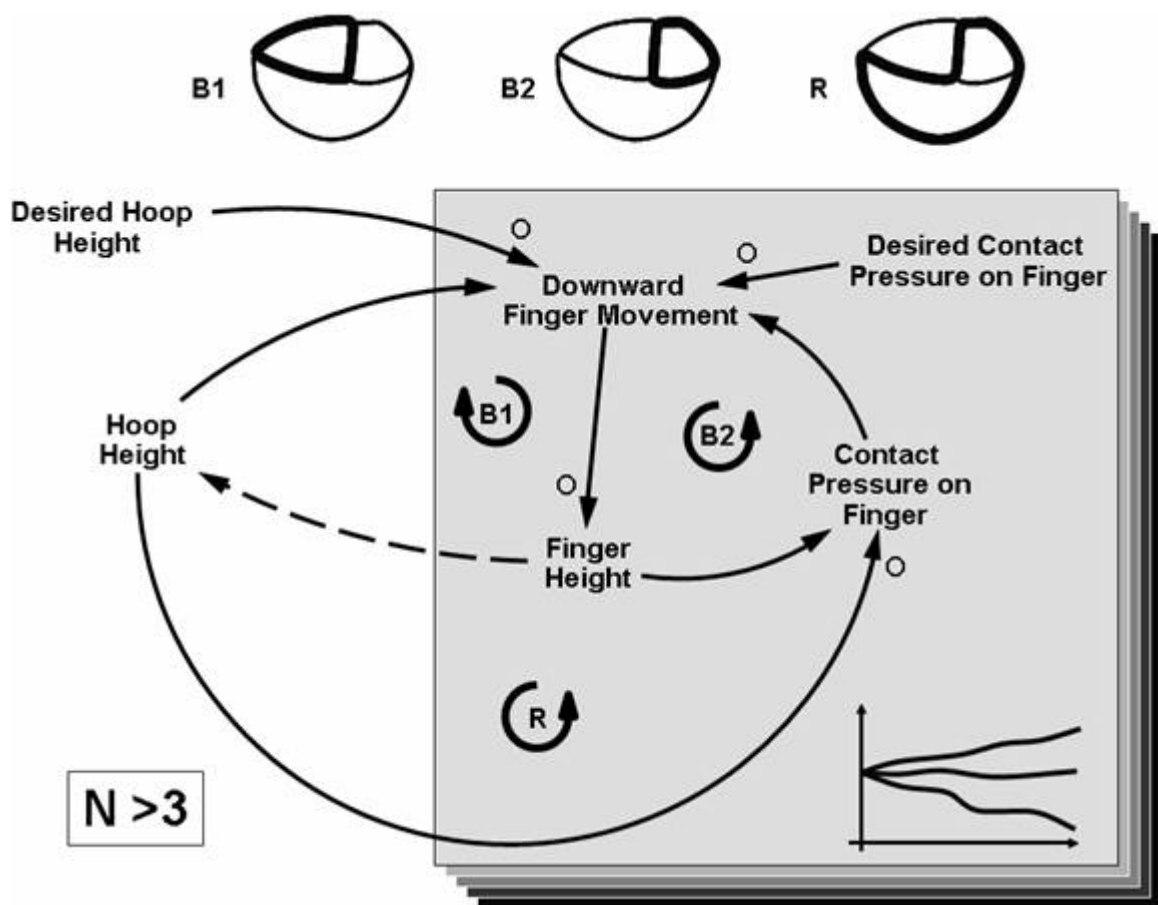


Fig. 3. Causal loop diagram of Avalanche, as played with a hoop but now by more than three people.

Various consequences might follow. First, loop $B2_\alpha$ dominates player α 's actions. This could simply bring his finger back into contact with the hoop. However, over-reaction and/or finger positioning errors might increase player α 's finger height such that the link to 'Hoop Height' is re-established. Player α lifts the hoop. In this case one of the other players who was in contact experiences a reduction in contact pressure. That player's B2 loop now operates as for player α . What can happen now is that the hoop rises further, all players terminate the operation of their loop B1, concentrating instead on their loop B2. The result of this can be the activation of one or more of the loops R_i . Eventually all players may be raising their fingers simply to avoid losing contact with the hoop. This is a rational response to the upward motion, a rational operationalisation of the condition set by the facilitator. Unfortunately, the players are using their B2s to chase a goal which is itself increasing as a result of their joint efforts. Of course, loops $B1_i$ may regain dominance and bring the system under control but this may be very difficult to co-ordinate and effect. The loops $B2_i$ and R_i are dominating the behaviour of the system.

The underlying logic of this set of reinforcing loops seems to offer an explanation for how the counter-intuitive 'ascending' behaviour is generated. Indeed, any effect which activates

one or more of the loops B_{2i} may cause other loops B_{2i} to awaken and may cause the R_i loops to be activated. If this happens - even for a short period before loops B_{1i} control the system again – then this has the potential to delay, or nullify, or discard and reverse the achievement of the game’s objective: bringing down the hoop. In this way the CLD gives insight into the structural source of all three problematic behaviours described earlier.

Other sources of behaviour

Clearly the number of players and the interaction with geometry is a critical feature of any theory. However, observing the conduct of the game there are a number of other effects that could be involved in the behaviours. Grouped together here using the collective term ‘human physiological and behavioural effect’, or HPB effects, these are also candidates for inclusion in any theory. They are features or parameters of the situation that might be changed to investigate their importance. These may be divided into two: homogenous and heterogeneous.

Homogenous Effects

The consequences of these effects are the same for all players. Five are listed below.

A) Weight of object. The hoops and rods used in Avalanche are light. The object rests lightly on the players’ fingers and only small movements are needed for contact pressure to be lost. This may be a source of the effects that activate the loops B_{2i} and R_i .

B) Finger positioning errors. Players introduce ‘wobbles’ to the hoop via their fingers because a single supporting hand held out palm downwards requires constant muscle action to keep the fingers horizontal and this unnatural position seems to introduce random variations into the action of each player. The amplitude of these random movements may merit investigation.

C) Speed of height adjustment. No time scale for the task is given. However, it is clear that some groups try to achieve the task faster than others.

D) Speed of pressure correction. A second speed parameter in the system is that with which players seek to increase contact pressure. Again, one observes teams who are very aware of the condition and move quickly to ensure it is complied with, whilst others are much less concerned.

E) Finger sensitivity to pressure. Individuals are monitoring the pressure on their fingers and they may require a certain scale of change for it to register. One can imagine a group with tough skin which is less sensitive, responding only to large changes, and a group of players who are very sensitive in their ability to detect even small pressure changes.

Heterogeneous Effects

These are effects which are different for different players. From observing the game one can posit that items (B) to (E) can manifest themselves in this way. Hence: different players may find it more or less difficult to position their finger accurately (B); may seek to fulfil the task at different speeds (C) or respond differently to loss of contact pressure (D); and players may have varying abilities to sense that pressure (E). These differences may be critical features of a theory.

MODELLING AND TESTING A THEORY

The paper now moves to the formulation of a simulation model as a platform to express and explore the geometric and HPB effects described above.

Purpose and Dimensions of the Theory

This section describes a formal theory for the Avalanche phenomena. By this is meant a precise statement of the causal mechanisms believed to be in operation. Together the model and the four behaviour modes represent a dynamic hypothesis: that the specified mechanisms can generate and therefore explain the modes. The purpose of developing the formal theory is that it allows a rigorous test of that hypothesis via simulation and that the construction and simulation of the model gives insight into the source of the modes.

The description earlier in the paper concerns the hoop variant of Avalanche. In this case the geometry of the situation indicates a difference between groups of three players and those larger than this. In the case of the rod, two is the critical group size. Conceptualising the hoop as a two-dimensional object (an annulus) and the rod as one-dimensional (a line) allows two important points to be made.

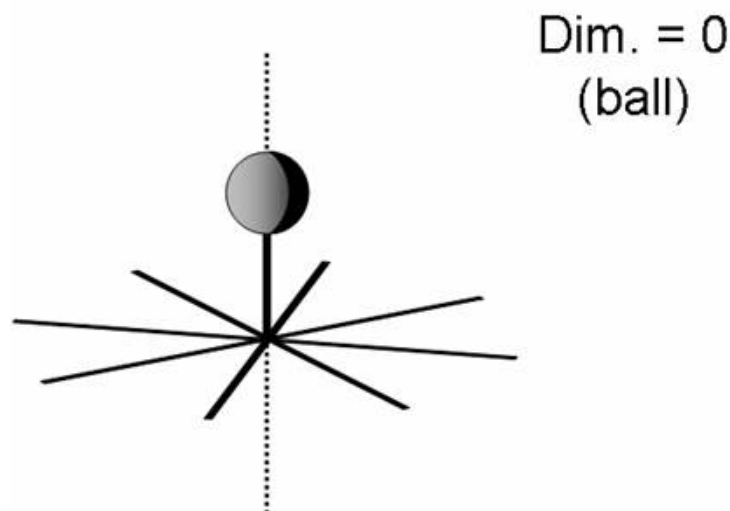


Fig. 4. A zero-dimensional version of Avalanche.

First, one can conceive a zero-dimensional variant of Avalanche. Illustrated in Fig. 4, this is a light ball mounted on a rod which emerges from the centre of a rimless (similarly light) spoked wheel. A fixed vertical rod runs through these components, causing the movement of the ball and spoke to be confined to upward and downward movements and therefore avoiding the ball's tipping over. At the end of the spokes players support the ball's weight using a single finger. Hence, in principle, the number of spokes must equal or exceed the number of players. This curious contraption allows for considerable simplification of the theory – a point returned to at the close of the paper. However, it allows the geometric property of the system and HPB factors to be included and renders the subsequent stability analysis more transparent.

The second point follows directly. The ball version requires only one person to set the height of the object. There are now three examples – hoop, rod and ball – for which the following statement applies:

$$\text{Threshold Player Number} = \text{Dimension of Object} + 1$$

This suggests that when the number of players equals this threshold number then only the desired behaviour mode should be generated, even in the presence of any and all HPB effects. When the threshold number is exceeded, some combination of HPB effects might then generate the three problematic modes. The above relationship therefore represents a theory for how the geometric aspects of Avalanche influence its behaviour and is the first fruit of this analysis.

As will be deduced, the theory that now follows applies to the ball version of Avalanche. A ‘zero-dimensional’ theory is therefore being put forward.

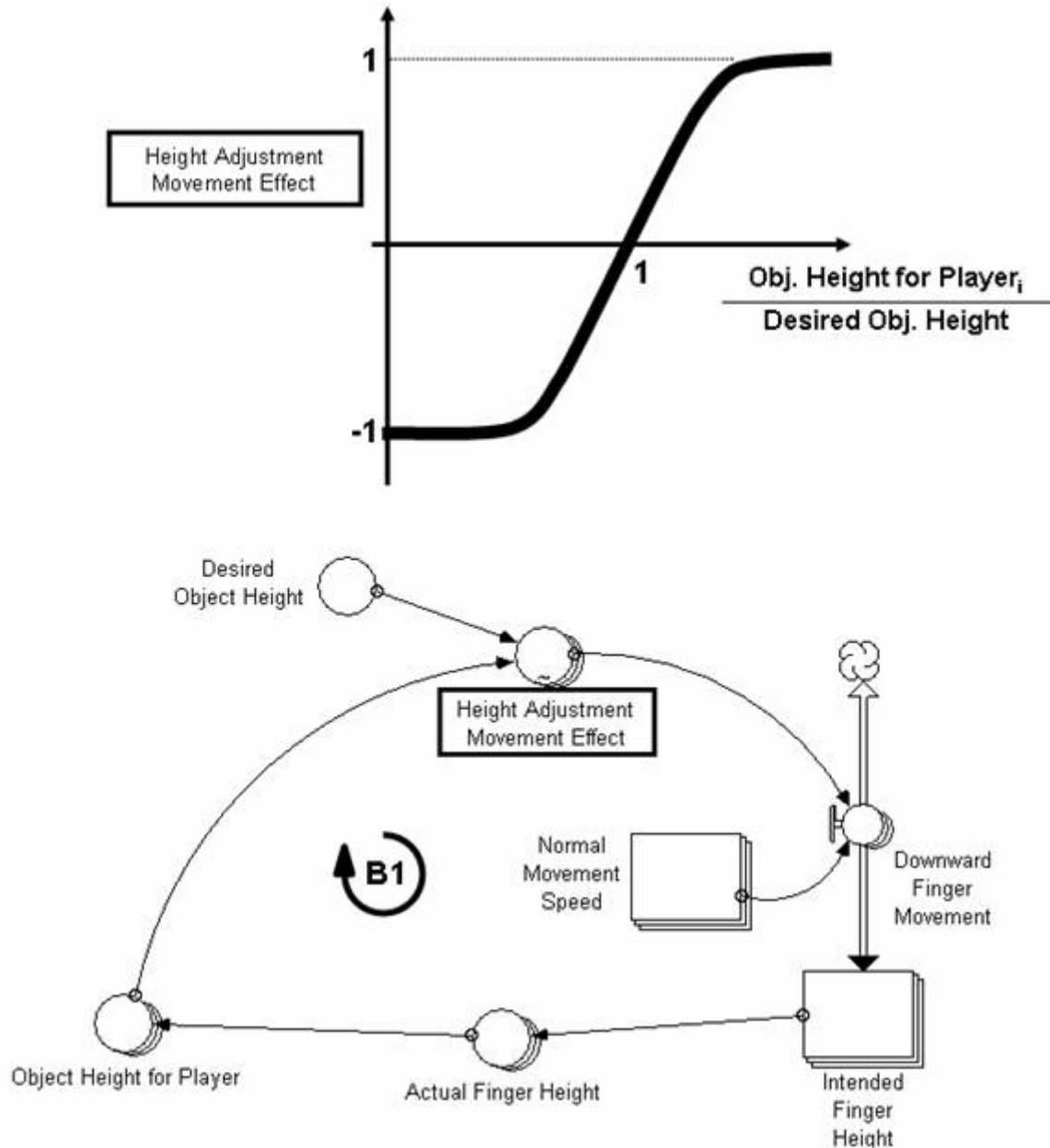


Fig. 5. Model structure for the mechanisms concerned with the task of lowering the object (left), with key table function at right.

Specifying a Theory

Here the model structure is developed using plausible mechanisms and parameter values. However, it is noted that alternative formulations are possible. A full equation listing of the theory specified here can be seen in Appendix 1.

Height adjustment to achieve the task

The mechanisms concerned with the finger heights for the players and the height of the ball are described first. In combination, these structures relate to the basic task of lowering the object. These are shown in Fig. 5. Note the layering of some of the icons, indicating that these structures and their underlying equations are reproduced for each player. However, for simplicity subscripts/array indicators are generally not shown in the equations below.

1. INITIAL Intended_Finger_Height = 1000 [mm]
2. Normal_Movement_Speed = 30 [mm/second]
3. Height_Adjustment_Movement_Effect = GRAPH(Object_Height_for_Player/Desired_Object_Height)
4. Downward_Finger_Movement = Normal_Movement_Speed*Height_Adjustment_Movement_Effect
5. Actual_Finger_Height = Intended_Finger_Height + Noise
6. Object_Height_for_Player = MAX_i(Actual_Finger_Height_i)

There are three main effects. Players all start one metre high (1) and can move their fingers at the same speed (2). This speed of movement is adjusted when the ball is close to the desired height (3 & 4). This adjustment is achieved by comparing the actual with the desired height and using the table function in Fig. 5 to reduce the speed. This mechanism therefore represent to desire to achieve the main task of the game. (In fact (4) is only an intermediate stage; below a further effect will be added to this formulation.)

The second effect concerns the distinction between intended and actual finger position (5). Here a noise term is introduced, representing the finger positioning errors described earlier. In the full model the amplitude of the noise term can be varied and the series re-created via a noise seed (see Appendix 1).

The final effect here shows how the players' finger heights establish the height of the object (6). For this sole equation subscripts are shown. However moving to the ball case removes much tedious geometric concerns, leaving the simple and intuitive relation that the highest finger sets the height of the ball.

Finger pressure constraint

Mechanisms representing the constraint are now added. The additional equations theorising how players act to try to keep in contact with the ball are listed below, again without subscripts/array indicators. The full model is shown in Fig. 6.

7. Pcvd_Pressure_Indent_on_Finger = SMTH1(Object_Height_for_Player-Actual_Finger_Height,1)
8. Pressure_Length_for_Object = 2 [mm]
9. Pcvd_Finger_Pressure_Ratio = GRAPH(Pcvd_Pressure_Indent_on_Finger
/Pressure_Length_for_Object)
10. Base_Effect_on_Normal_Movement_from_Finger_Pressure = GRAPH(Pcvd_Finger_Pressure_Ratio)
11. Actual_Effect_on_Normal_Movement_from_Finger_Pressure
=1+2*Base_Effect_on_Normal_Movement_from_Finger_Pressure
12. Downward_Finger_Movement = Normal_Movement_Speed*Height_Adjustment_Movement_Effect
*Actual_Effect_on_Normal_Movement_from_Finger_Pressure

Here (7) calculates the perceived pressure indent on each player's finger.⁵ This indent is the distance – or amount of 'bite' – that the object presses into each supporting finger, so generating the upward thrust needed to support its weight. First order information smoothing with response time of one second introduces a simple perception effect (Forrester, 1961). Now the maximum possible value of this 'bite' is related to the weight of the object; heavy objects 'bite' more into the supporting fingers. For a light plastic object a figure of 2mm is used (8).⁶ The question is then what pressure each player feels and how this compares with what is desired or expected. This is achieved by comparing the perceived indent with the desired (9). The graph has the simple form $Y=1-X$ and so produces a ratio of the pressure perceived to that which the player desires.

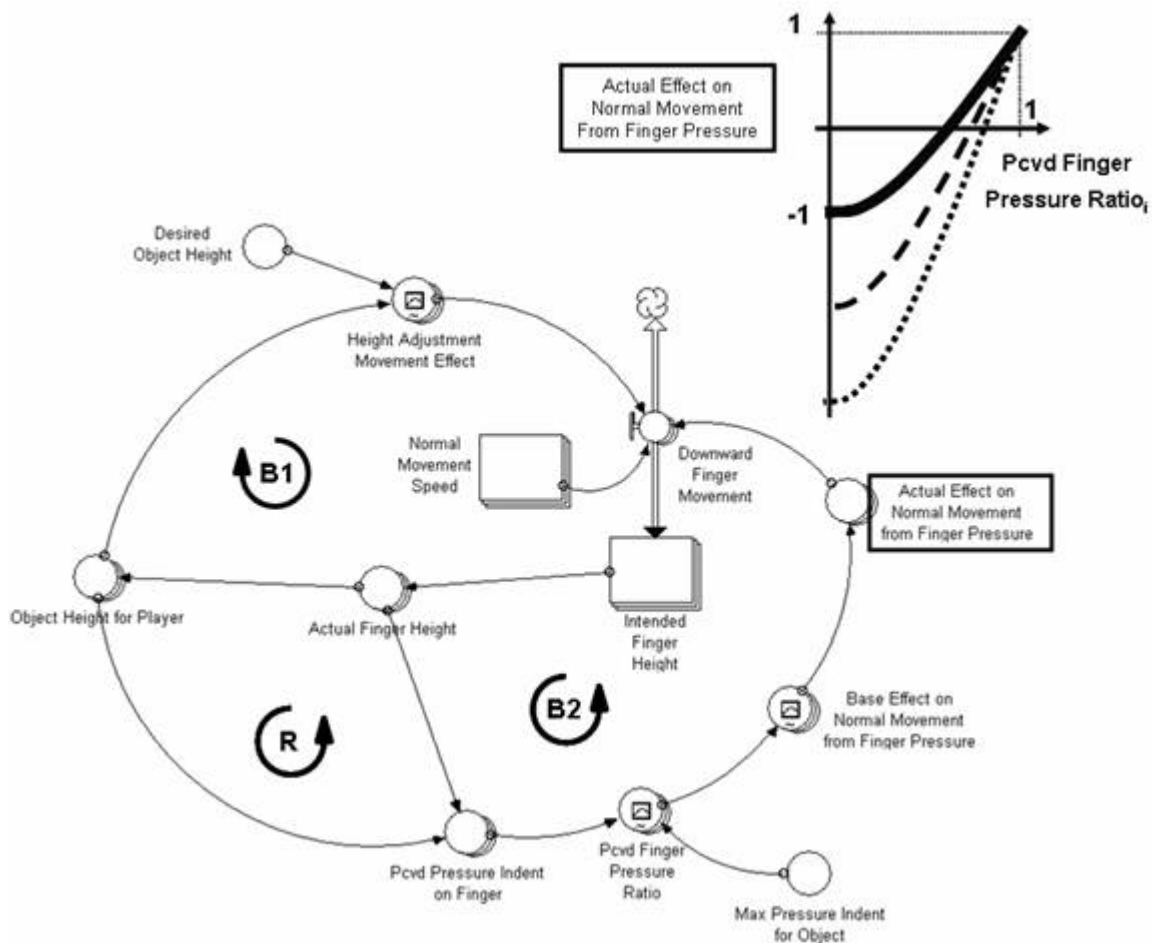


Fig. 6. Model structure now including the mechanisms concerned with the finger pressure constraint. The graph shows the net influence of pressure on downward movement, with amplified responses to falling pressure indicated using dotted lines.

5. At first sight this equation might seem to admit negative values, thereby failing the ‘extreme condition test’ (Forrester & Senge, 1980). However, equation (6) removes this possibility.

6. A minor formulation point concerns the definition of object height and its relation to this pressure length. One approach is to measure the object height from its lowest point. However, this would be inconsistent with (6) since the object cannot ‘bite’ into the supporting fingers without settling at a slightly lower level. A solution would be to alter (6) to read:

$$\text{Object_Height_for_Player} = \text{MAX}_i(\text{Actual_Finger_Height}_i) - \text{Pressure_Length_for_Object}$$

However, for simplicity this was rejected. The current formulation works if the object height is understood to be the height of its lowest point plus the appropriate pressure length.

Now these equations require some reflection. Their full form is contained in the Appendix but only their net effect is discussed here – and illustrated in Fig. 6. The previous formulation (4) is completed by the addition of a multiplicative function (12). The idea is simple. When the finger pressure ratio equals one, a player is content that the constraint is fulfilled and concentrates on the task of lowering the ball. Hence the graph has the point (1,1), indicating that pressure has no change on downward movement and B1 operates. When the pressure ratio falls to zero a player is no longer connected and concentrates solely on obeying the constraint. Downward movement is reversed into upward movement, hence the graph has the point (0,-1), indicating that pressure (or lack of it) has caused B2 to dominate.

As can be deduced from the graph, at some intermediate ratio the task and the constraint cancel each other out: the curve passes through zero, freezing all movement and indicating an equal contribution from loops B1 and B2.

(In this formulation players will only ever move upwards as fast as they can move downwards (2). However, in the version of the model listed in Appendix 1, this limit is weakened and a more complex formulation employed. The purpose is to allow the model to include effect (D) - speed of pressure correction. This is achieved as follows. Returning to (11), the full model replaces the ‘2’ with the term ‘1+FPA’, FPA standing for ‘Finger Pressure Amplification’. This formulation allows the value of, say FPA=3 to be given and this in turn sets a maximum upward movement that is three times the maximum downward movement speed of (2). Examples of this amplification are shown in Fig. 6.)

The inclusion of a finger pressure mechanism completes the model, its three sets of loops – B1, B2 and R – being shown in Fig. 6.

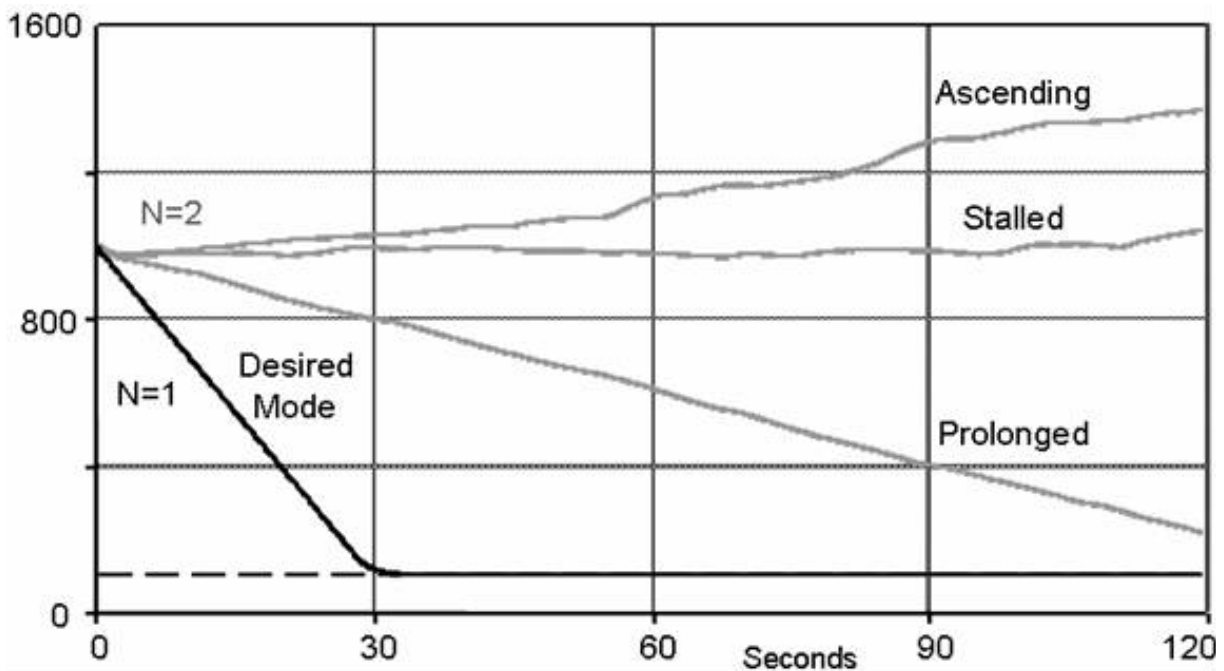


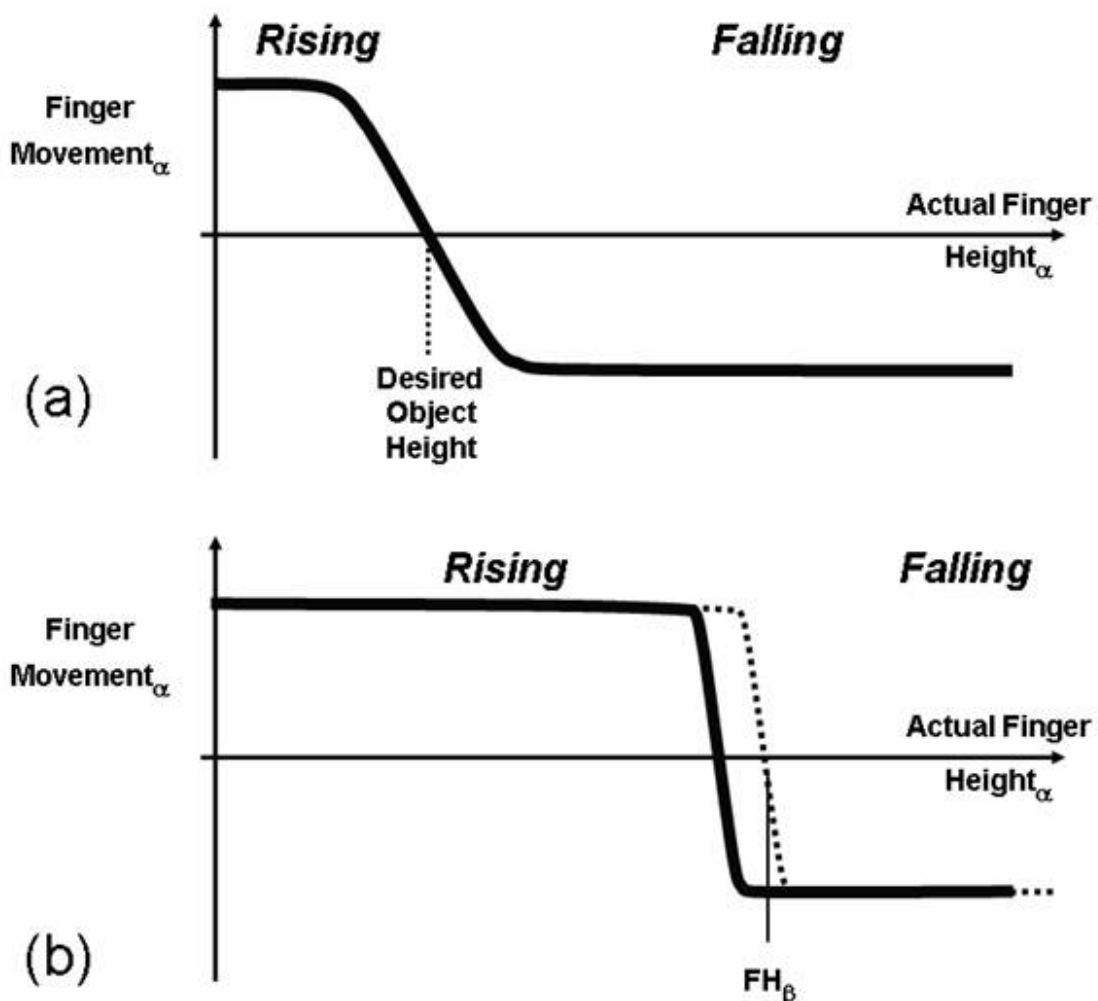
Fig. 7. Model simulations, showing - with one player – the desired mode and with two players the three problematic modes (grey).

Testing the theory

The theory is rigorously tested by simulating the model to see if it does generate the observed behaviour. However, a choice needs to be made about which of the HPB effects to include. The runs discussed here include homogenous effects (A) and (B); a light object with the single HPB effect of stochastic errors in the positioning of players' fingers.⁷ Geometrical effects are included by varying the number of players.

The first run has only one player (Fig. 7). The model reproduces the observed, desired mode of behaviour. Moving to two players, the mode is dependent on the random number stream used. The model output's becoming a distribution is considered latter.⁸ However, amongst the distribution it is straightforward to find runs corresponding to the three observed, problematic modes – prolonged, stalled and ascending (Fig. 7, grey).

The model therefore passes these tests. It is indeed a plausible theory for what is going on in the Avalanche game.



Figs. 8. Rate/level diagram for player α . (a) Single player only, or $FH_\alpha \gg FH_\beta$ case. (b) Diagram for $FH_\alpha \sim FH_\beta$ case.

7. See Rudolph & Reppenig (2002) for a similar example of a model being stimulated with stochastic effects to assist exploration of its behaviour modes.

8. See Fig. 10a for the CDF.

EXPLANATION AND MORE TESTS

Merely pointing at the model and mutely offering it as an explanation is insufficient. The question now is whether the existence of the model can be used to aid our understanding of why the system behaves in this way. That question is considered in this section.

The Narrow Descent Region

Simple rate/level diagrams are useful in understanding model behaviour (Forrester, 1968; Sterman, 2000). Extending this to qualitative analysis using multi-dimensional rate/level diagrams gives insight into the Avalanche modes.

Simple rate/level diagrams

Staying with the zero-dimensional system, we start by considering the situation from the perspective of player α . Consider the rate/level diagram for the game with only that one person, shown in Fig. 8a.⁹ This diagram shows the dominance of loop $B1_\alpha$ in this system: the player's finger moves downwards when above the desired object height, upwards when below.

However, the diagram also holds in the two player case - if player α 's finger is much higher than player β 's ($FH_\alpha \gg FH_\beta$). In this case player α is controlling the ball in the manner given by this diagram. Of course, player β will be experiencing the operation of loop $B2_\beta$. Nevertheless, player α 's movements are governed by $B1_\alpha$.

In contrast, when the two are roughly equal, $FH_\alpha \sim FH_\beta$, there is an interaction between the two players. If player α is too far below player β then loop $B1_\alpha$ is dominated by loop $B2_\alpha$; the player seeks to reconnect with the object. It might be thought that this means that player α has a new goal height that is equal to that of player β (see dotted line in Fig. 8b). This is not quite true. Player α merely wishes to re-establish some degree of pressure and this is done by moving to a point somewhere within the 'pressure length' for the object – see (8) above. Hence the actual relationship is shifted to the left by an amount less than or equal to the pressure length (solid line).¹⁰ Nevertheless, the interval in which player α moves upwards is greatly extended.

Three dimensional rate/level diagrams

When considering the two player game the discussion above merely represents slices of the three-dimensional rate/level diagram that actually governs the system. The form of that diagram is developed here and in Figs. 9.

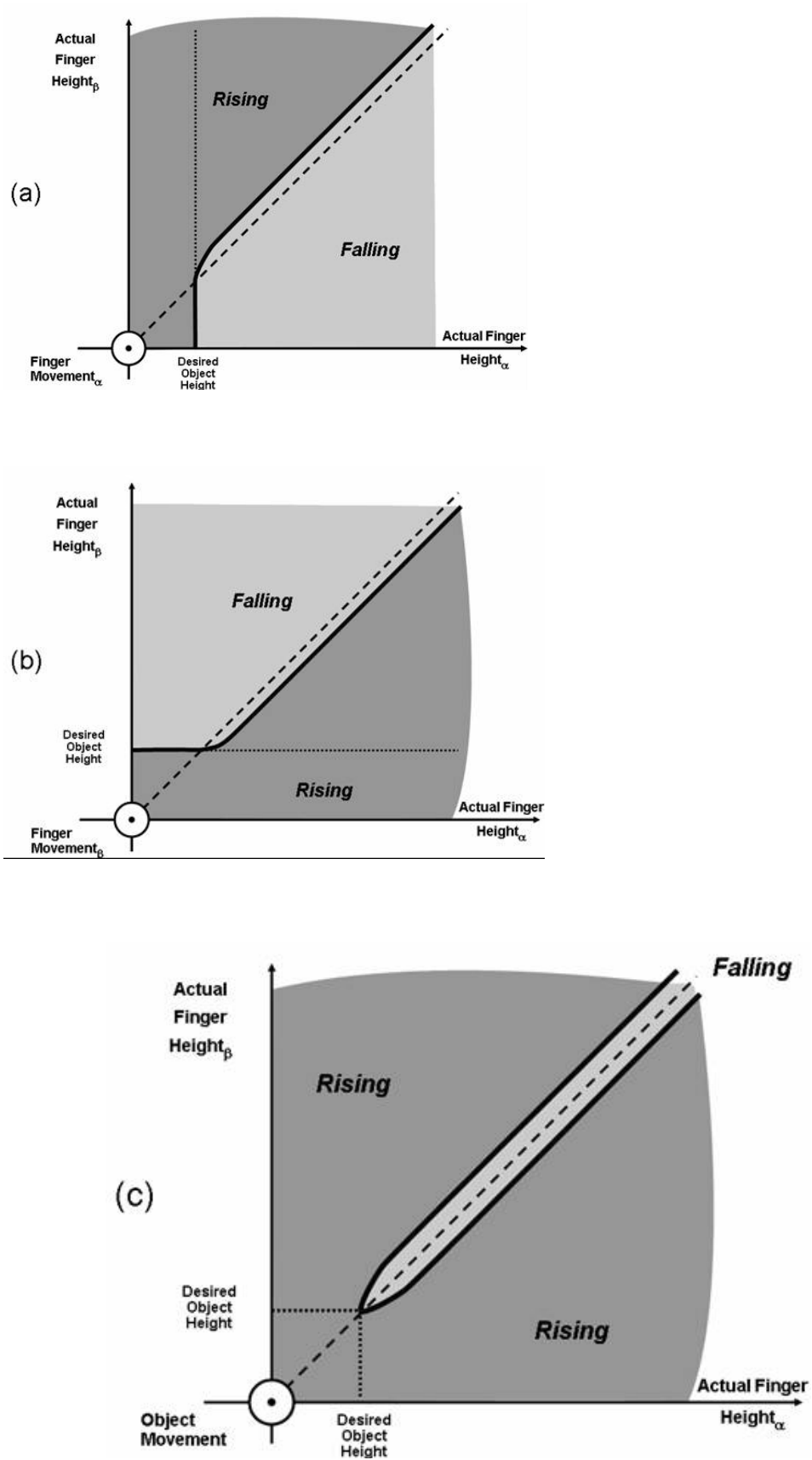
Combining the results developed above, the response for player α is shown in Fig. 9a. The finger movement axis points out of the page and the area maps the space of finger positions for both players. The player's finger rises in the regions marked 'Rising', falls in those marked 'Falling'. Note that the changeover does not occur on the $Y=X$ line but is offset by roughly the pressure length.

A similar diagram can be drawn for player β (Fig. 9b). Now the axis emerging from the page is the finger movement for player β .

Recall now the simple height setting mechanism in the ball case – see (6) above. This allows the two separate relationships to be combined to create a diagram which gives some insight into the direction of movement of the object itself as a function of the two finger heights (Fig. 9c).

9. This is simply an up/down reversed version of the function in Fig. 5. That function shows downward movement, whilst in Fig. 8 net change in height is being considered.

10. Note also that the curve has a different gradient. This is because the function defining the curve is no longer that of (3) but the combined effects of (9)-(12).



Figs. 9. (a) Three dimensional rate/level diagram for player α . (b) Three dimensional rate/level diagram for player β . (c) Three dimensional rate/level diagram for the object itself.

Insights from the rate/level diagram

The geometry of the game – the coupling of the players via the ball – means that there is only a very narrow region in which downward movement is possible. The geometric element of the game causes the balancing loops representing the goal of the task to dominate only in a narrow hyper-volume, or region of space. Only in this ‘narrow descent region’ do loops $B1_\alpha$ and $B1_\beta$ dominate. Outside this region one or more of the $B2_\alpha$ or $B2_\beta$ operates which, in turn can lead to the dominance of the reinforcing loops as the system chases an never-increasing goal.

This means that with two players any and all HPB effects that push the system outside this ‘narrow descent region’ act against downward movement. The system must find its way back into the region before the game’s prime task (lowering the object) is able to dominate behaviour again. This gives an important insight into why, in the context of the two player geometry, the HPB’s have the potential to generate the counter-intuitive ‘ascending’ behaviour, or simply to stall or prolong descent. In the first case the system is spending a considerable proportion of time outside the descent region; ascent results. In the other two cases the proportion of time is lower but enough to produce zero net movement, or smaller net downward movement.

Testing Sensitivity

In the light of the above explanation sensitivity analysis of some of the system’s features are performed: first on an HPB, then on geometry. The results - presented using cumulative density functions - are consistent with the intuition created by the analysis from the rate/level diagrams.

Finger positioning errors

Stochastic errors in finger positions for the players are a critical feature of the theory. As stated above, the model output becomes a distribution and is best represented as such. The full range of results for 1000 runs of the model as originally calibrated is shown in Figs. 10: (a) as a CDF and (b) as three points on a spider plot, used here to show the minimum, average and maximum heights achieved after 120 simulated seconds.

The theory had two players and errors of maximum amplitude 1 mm. It is clear from the figure that the dominant mode here is the ascending one. The CDF is almost all to the right of the start height of 1000 mm (Fig. 10a) and the average is greater than that value (Fig. 10b). However, some ‘prolonged’ and ‘stalled’ modes are present, as indicated by the minimum value’s being almost down to the goal height of 100 mm.

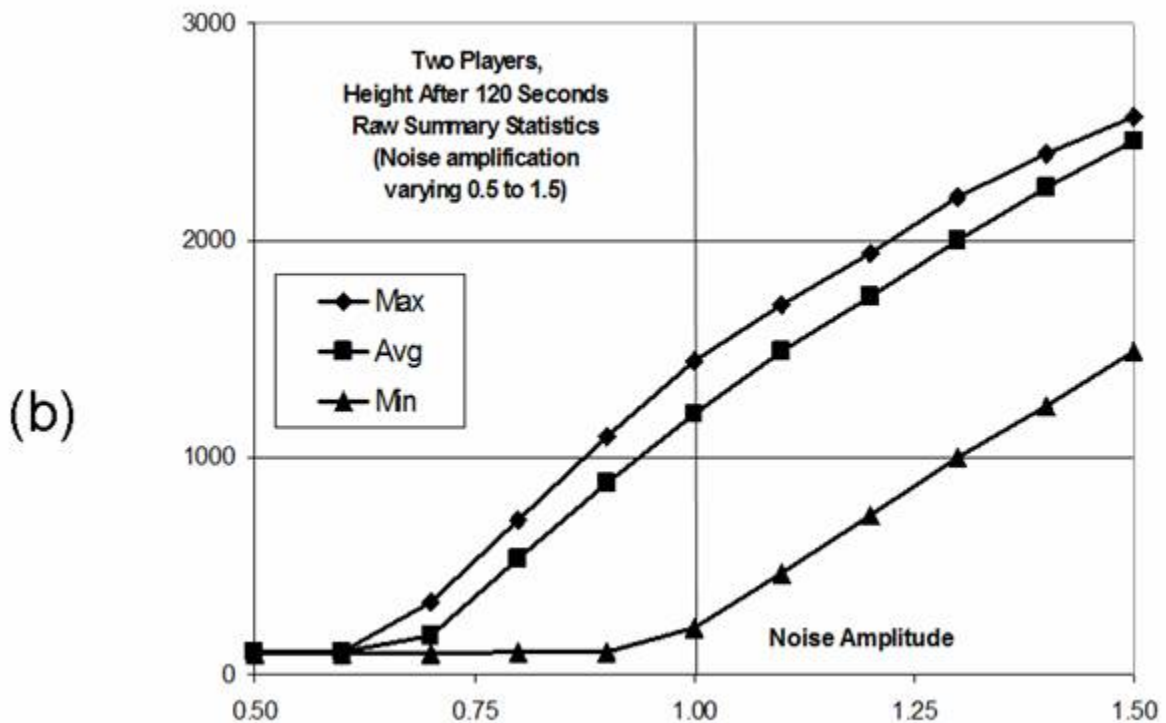
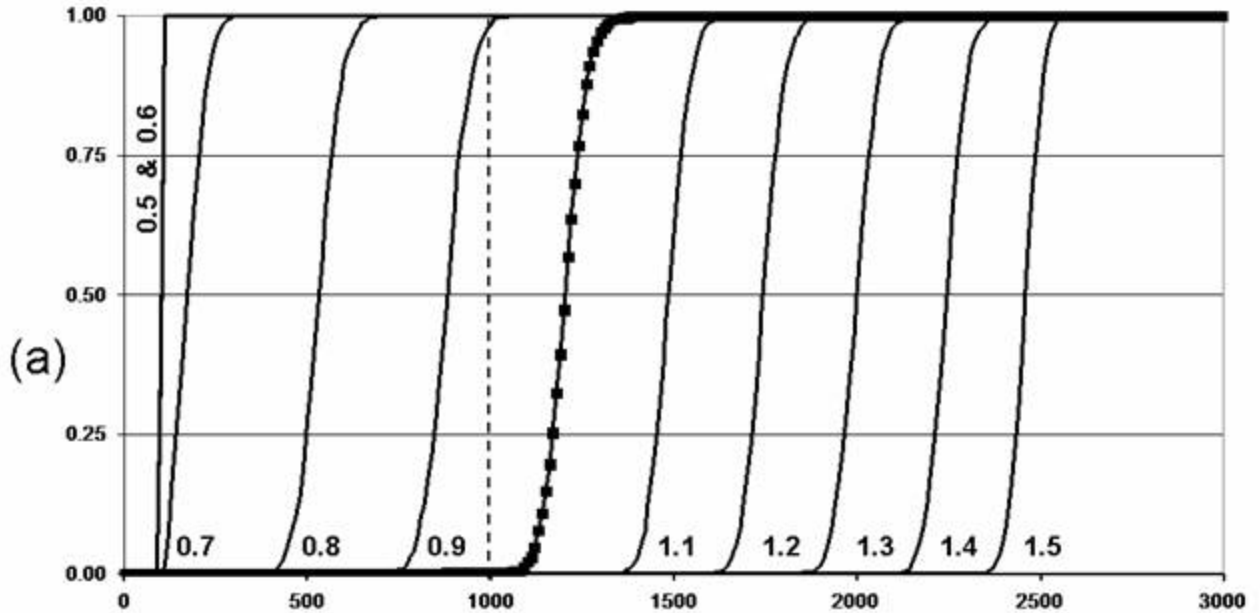
Sensitivity analysis is conducted by applying multiplying factors to the normal amplitude – hence the use of spider plots in Fig. 10b (Eschenbach, 1992). The results are in line with the intuition derived from the diagramming of the narrow descent region. As the amplitude increases the likelihood of ascending behaviour increases. Up to an amplitude 1.3 there is a mix of prolonged, stalled and ascending modes, with the last consistently becoming more prevalent. Around this value the prolonged mode seems to vanish (the minimum observed is the start height) and after this the stalled mode does the same, leaving only the ascending mode.¹¹

Reducing the amplitude reverses these effects: the ascending mode becomes more rare,

11. Some comment is needed on those results with values above the start height. First, anatomically human beings cannot support an object at a height in excess of $3 - 3\frac{1}{2}$ m. In this sense the model excludes a balancing loop effect which shuts down further upward movement when players have their arms stretched fully upwards. However, note that the runs in Figs. 10 are chosen so as not to exceed this height. Second, these values are arbitrary in that they represent the heights attained after the chosen 120 seconds: changing the run time would change these values. Such output is therefore best considered as an indication that after a reasonably long period the ascending mode is present.

seemingly vanishing around 0.8 as the maximum falls below the start height. For values of 0.6 and below the problematic modes vanish and the model output appears to collapse into the desired mode.

These results are entirely consistent with the narrow descent region argument above.



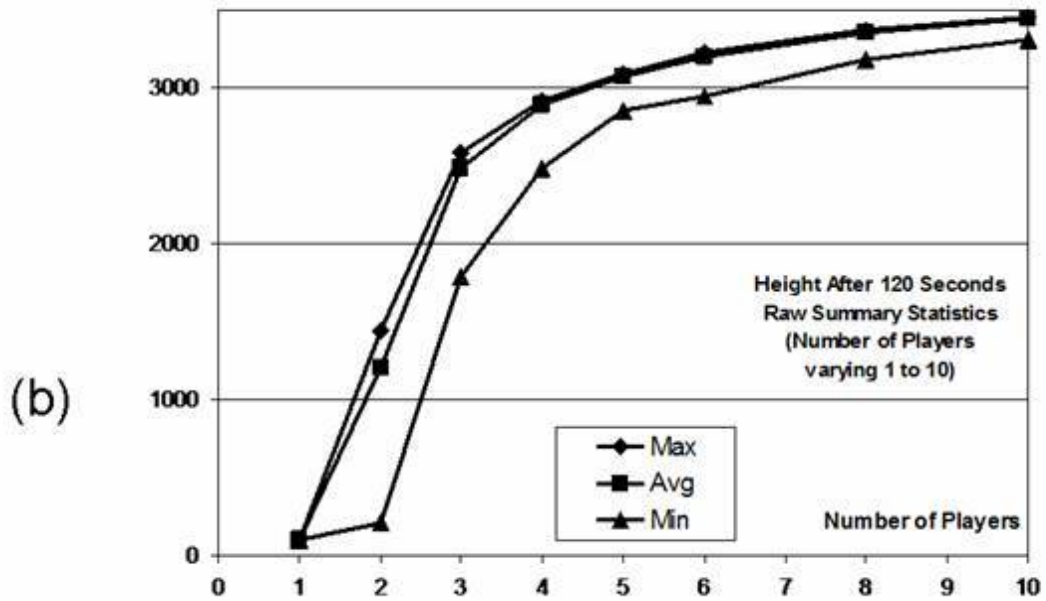
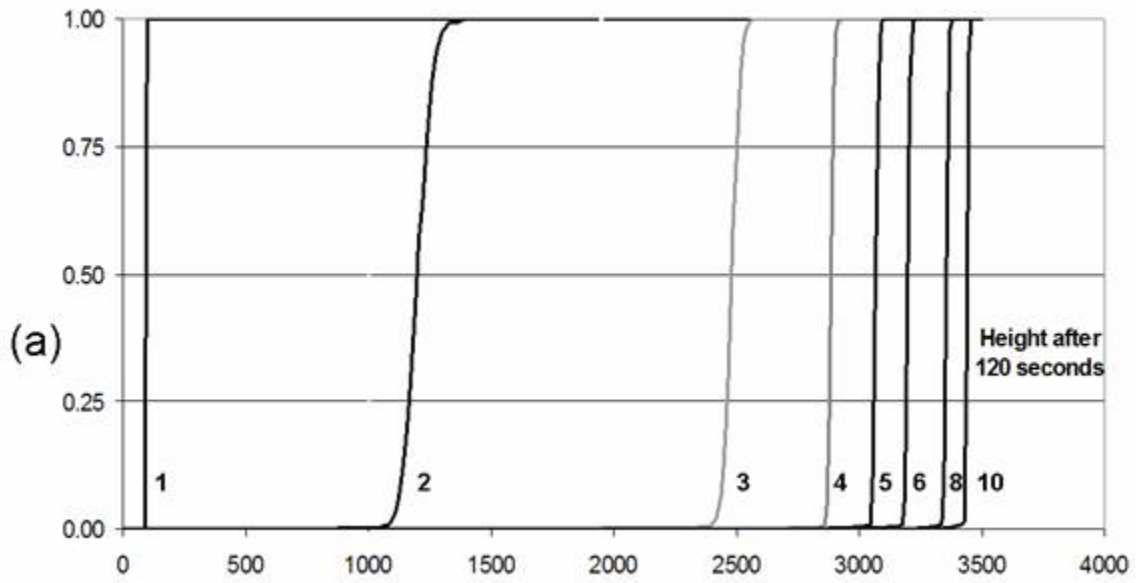
Figs. 10. Effects on the two player system when the amplitude of finger positioning errors is changed. Results from 1000 simulations run to 120 secs. represented as: (a) CDF of all realisations; (b) spiderplot of minimum, average and maximum height.

Number of players

A considerable benefit of the zero dimensional conceptualisation of Avalanche is that increasing the number of players is a straightforward formulational matter. The results are shown in Figs. 11.

Reducing to one player removes all three problematic modes. The behaviour is simply governed by the rate/level diagram of Fig. 8a. Increasing the number of players produces dramatic results. For the present parameterisation increasing to only three removes all but the ascending mode. The results of further increases are less dramatic in marginal terms, appearing to display a saturation effect.

The narrow descent region argument is useful in explaining qualitatively the effects of increasing the number of players. The situation is similar to that shown as a square in Fig. 9c but with three players one must visualise a cube, with axes given by the finger heights for the three. The descent region is then a thin cylinder lying long the leading diagonal of the cube. The region is small – in proportional terms it is smaller than in the two player case. Excursions outside it serve to delay, stall or even reverse downward movement, in line with the three problematic behaviour modes.



Figs. 11. Effects of changing the number of players. Results from 1000 simulations run to 120 secs. represented as: (a) CDF of all realisations; (b) spiderplot of minimum, average and maximum height.

TAKING STOCK

We close with thoughts on what further experiments might be possible and what benefit they might bring and by drawing some conclusions on the work so far.

On Further Investigations

More HPB Effects

Many more explorations of the theory are possible based on experiments concerning the contributions of different HPB factors. With reference to the list of factors labelled (A) – (E), it would appear that only factor (B), finger positioning errors, has been explored – and only the homogeneous case to boot. This appearance is deceptive. Having formulated the model and considered its dynamics via the rate/level diagrams it is now possible to see how two of the other effects are related.

First, it is possible to suggest that changes in factor (A), the weight of the object, would produce results not unlike those of (B) shown in Figs. 10. The reasoning is that the amplitude of the finger positioning errors must be judged against the pressure length for the object. However, increasing the weight of the object increases the pressure length. Therefore, we would expect an increase in weight to have a qualitative effect that is the same as a decrease in error amplitude.

Second, experiments with factor (E), finger sensitivity to pressure, are subject to similar deductions. To be more sensitive to pressure changes is to detect even small changes and so have a greater length scale of movement across which a response is possible. This is similar to increasing the pressure length, similar to increasing the weight, and hence similar to decreasing the error amplitude. Therefore, we would expect an increase in pressure sensitivity to have a qualitative effect that is the same as a decrease in error amplitude.

Formally, these deductions should be transformed into observations via simulation of the model. Simulations would also be needed to explore the effects of the remaining two factors. Even this is only to deal with the homogeneous cases. Considering these factors in an heterogeneous way potentially involves exploring a large number of different combinations of factors (B)-(E). One example might be that player α displays larger finger positioning errors, player β tries to lower the height more slowly, and player γ corrects for pressure quicker etc. This is just one amongst many combinations which could, in principle, be formulated and tested and so represented as a theory regarding the source of the observed behaviours. Careful structuring of this large number of possible combinations would be needed (Clemson, Tang, Pyne & Unal, 1995). The critical question is whether further explorations would generate significant insight about the system. This point is considered below.

Higher Dimension Geometric Effects

The ball version of Avalanche would work physically. So this analysis of the zero dimensional game – which does include the appropriate geometric effects that bind the players together – is valid. Extending the analysis to the one and two-dimensional versions, the rod and hoop, seems an obvious next step. However, two points must be made.

First, these cases are more complex in terms of the formulation needed to express the ‘physics’ of the game. The essentially point is that now one must establish the position of the object for each player. Consider a game with the rod and five players spaced equally along its length. Only two players are needed to support the rod. To work out the position of the rod for any of the players one needs to establish whether that player is supporting the rod and, if so, with which other player. In that case the player’s finger height equals the object height for them. However, the alternative is that a given player is not supporting the rod but that its

height is being set by two others. In this case the object height for the player is above that player's finger height and will depend on which of various possible pairs of players are setting the object's height. Hence, the simplicity of equation (6) is lost. Instead the height of the object for each player is a complex function of the finger heights of all of the players, bearing in mind their number and position along the rod. The situation would be even more complex with the hoop: three players would be involved in setting the height and vector cross-products needed to solve for the range of possible positions of the hoop for any player, under any choice of the three supporting players. From the brief outline given here, the reason for studying a zero dimensional system may now be clearer to the reader.

The second point is that although this physics problem is considerably more complex, one can see now that the dynamic characteristics of these higher order systems may not be greatly different from the zero dimensional case studied here. The comments about the narrow descent region and the consequences of the HPB effects still apply. Again, the critical question arises of whether a rod or hoop-based theory would generate significant further insight.

Tentative Conclusions

Summary comment can now be offered on the source of the observed problematic behaviours in 'Avalanche'. This comes as six conclusions.

First, in understanding the Avalanche modes one can now see that two conditions arise: the number of players must exceed the threshold number for the object, that is, the dimension of object + 1; and at least one of HPB effects (B)-(E) must be present. The three problematic modes arise if and only if both conditions hold.

The reason for this is illustrated by the narrow descent region argument. A smooth and rapid descent for the object is achieved by moving down the $Y=X$ line in two dimensions, $Z=Y=X$ in three dimensions, etc., that movement being governed by loops B1. However, the geometry of the system and the presence of the pressure constraint introduces loops B2 which begin to operate if the system moves only a small distance from these lines. Any effect that causes the system to move outside this region momentarily has the potential to lift the object. This may subsequently be reversed. However, it may be that the action of the B2 loop for one player serves effectively to lift upwards the goal for another player's B2 loop. Hence, although these are goal-seeking feedback effects, their net result may be that the various goals are consistently moved upward, causing the system to chase upwards further in accordance with the loops R_i (c.f. Milsum, 1968).

The second conclusion is that the formal theory presented here provides the explanation for these dynamics. Formulating equations, selecting parameter values, observing simulated behaviour provides not only an unambiguous expression of a theory, not only a rigorous test of its ability to reproduce behaviour modes. These actions also serve to clarify thinking about possible underlying mechanisms and lead to the rate/level diagram argument and dynamic insights presented here. Useful as it is in developing the model, the CLD of Fig. 6 can achieve very few of these things (Homer & Oliva, 2001).

This is but one model of the Avalanche game. A third conclusion is therefore that experiments with different formulations are needed. This is consistent with normal system dynamics practice (Forrester & Senge, 1980) and, in exposing the existing theory to further tests, is the appropriately scientific approach (Bell & Senge, 1980).

A fourth conclusion concerns the interactions of the players in Avalanche. Clearly the geometry of the system is critical in that it makes the outcome for each player dependent on the actions of all of the players because the behaviours are mediated by the geometry and physics of the object. A metaphorical link therefore might be made with Prisoners' Dilemma situations and also with debates in which co-operative/competitive behaviour is mediated by the structure of the underlying game. Such comparisons may be useful in creating insights,

even if – and this should be emphasised - only at a metaphorical level (c.f. Morecroft, Larsen, Lomi & Ginsberg, 1995).

However, metaphors are elusive beasts and one might wish for a stronger link between Avalanche and some real world phenomena (c.f. Lane, 1998). The fifth conclusion is that there may be other phenomena that are isomorphic to Avalanche. These are divided here into a specific example and a general proposal.

Since system dynamics aspires to create transferable insights, it is interesting that the lessons derived from the formal theory for Avalanche can illuminate the specific behaviour of a Ouija board. The board is decorated with letters and numbers and participants each place a finger on a planchette, or pointing device. During a séance the planchette is observed to move - propelled by ‘metaphysical forces’ - and so spell out messages from a purported ‘spirit world’. As with Avalanche, a possible explanation for this behaviour may also be random finger movements combined with a wish by participants to keep in contact.¹² Indeed, it is standard to explain the phenomenon as an ideomotor effect (Randi, 1995). However, a further insight derived from the Avalanche work is that the behaviour would cease if a heavier planchette was used. Unfortunately the hypothesised metaphysical forces are further described as being very weak, this ad hoc hypothesis thereby conveniently excluding this potential refutory experiment.

A more general proposal suggests a use of the work here that is rather more important than trying to move various objects. With Avalanche one has two goals; height and pressure, possessed by a number of players. The actions of the players are mediated by the geometry of the system. The results show that the goals are in tension and circumstances arise in which one may overwhelm the other in ways which harm all players. This situation may be related to models of competing companies. Here the players become companies and the pair of goals might be sales volume and price. Rather than the resulting behaviours being mediated by the geometry and physics of a physical object, they are then mediated by the regulations of the market in which the companies operate. One can imagine a desired mode in which sales volume (c.f. pressure) is maintained whilst price increases (c.f. height decreases). However, one can also imagine an undesirable behaviour in which the desire to sustain sales volume frustrates attempts to increase price and price may actually fall (c.f. height increases). A similar argument might apply to companies concerns with both market share and sales volume. A desire to increase sales volume whilst maintaining market share might be desirable and achievable by all companies if they are able to grow a market.¹³ However, if one or more company clings too hard to the market share goals (perhaps wishing to sustain market dominance) then one might expect a reduction in sales volume, akin to the increase in object height in Avalanche.

There are two points here. First, this linkage further explains the author’s resistance to the idea of exploring the higher dimension theories because such explanation must involve itself with the minutiae of the physics concerned with the mediating geometry. However, if a more powerful application of this work involves a market as the mediating mechanism then it seems rather less than pressing to go down that particular tangent. Second, understanding the interactions of companies in markets is a real problems upon which considerable research exists. The potential is that the formal theory here may be useful in illuminating such situations.

Having constructed the theory, a final conclusion is that it can serve as a platform for further explorations, consideration of the full range of HPB factors, as described earlier. However,

¹² I am grateful to LSE Masters student Natalie Haynes for this suggestion.

¹³ One example might be the co-operation of small rival companies who together can create a new industry standard and increase the sales volume of both (Größler & Thun, 2002).

whether these would generate significant new insight about the system is a moot point. The author would argue that the insights achieved so far might well be all that this mere game deserves, that many of the new results that might be generated would add little to what can be surmised from this analysis. What can be said is this, however. Having attempted to provide an example of the benefits that formal theory can yield, this paper is perhaps best seen as a more general encouragement for researchers to take a similar approach to the important problems that have always been the concern of system dynamics, and thus to improve our understanding of the policy options that are available to us.

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APPENDIX 1: EQUATION LISTING

Run specifications: t = 0 to 120. DT=0.01

- Intended_Finger_Height[Player](t) = Intended_Finger_Height[Player](t - dt) + (- Downward_Finger_Movement[Player]) * dt
INIT Intended_Finger_Height[Player] = 1000
OUTFLOWS:
 ✦ Downward_Finger_Movement[Player] = Normal_Movement_Speed[Player]*Height_Adjustment_Movement_Effect[Player]*
 Actual_Effect_on_Normal_Movement_from_Finger_Pressure[Player]
- Normal_Movement_Speed[Player](t) = Normal_Movement_Speed[Player](t - dt)
INIT Normal_Movement_Speed[Player] = Base_Normal_Movement_Speed
- Actual_Effect_on_Normal_Movement_from_Finger_Pressure[Player] =
1+(1+Finger_Pressure_Amplification)*Base_Effect_on_Normal_Movement_from_Finger_Pressure[Player]
- Actual_Finger_Height[1] = Intended_Finger_Height[1]+Noise_Amplitude*RANDOM(-1,1,Noise_Seed+1)
- Actual_Finger_Height[2] = Intended_Finger_Height[2]+Noise_Amplitude*RANDOM(-1,1,Noise_Seed+2)
- Base_Normal_Movement_Speed = 30
- Desired_Object_Height = 100
- Finger_Pressure_Amplification = 1
- Net_Change_in_Finger_Height[Player] = -Downward_Finger_Movement[Player]
- Noise_Amplitude = 1
- Noise_Seed = 1
- Object_Height_for_Player[Player] = MAX(Actual_Finger_Height[1],Actual_Finger_Height[2])
- Pcvd_Pressure_Indent_on_Finger[Player] = SMTH1(Object_Height_for_Player[Player]-Actual_Finger_Height[Player],1)
- Pressure_Length_for_Object = 2
- Base_Effect_on_Normal_Movement_from_Finger_Pressure[Player] = GRAPH(Pcvd_Finger_Pressure_Ratio[Player])
(0.00, -1.00), (0.1, -0.995), (0.2, -0.985), (0.3, -0.955), (0.4, -0.925), (0.5, -0.88), (0.6, -0.83), (0.7, -0.755), (0.8, -0.555), (0.9, -0.295), (1, 0.00)
- Height_Adjustment_Movement_Effect[Player] = GRAPH(Object_Height_for_Player[Player]/Desired_Object_Height)
(0.00, -1.00), (0.2, -1.00), (0.4, -1.00), (0.6, -0.8), (0.8, -0.4), (1.00, 0.00), (1.20, 0.4), (1.40, 0.8), (1.60, 1.00), (1.80, 1.00), (2.00, 1.00)
- Pcvd_Finger_Pressure_Ratio[Player] = GRAPH(Pcvd_Pressure_Indent_on_Finger[Player]/Pressure_Length_for_Object)
(0.00, 1.00), (0.1, 0.9), (0.2, 0.8), (0.3, 0.7), (0.4, 0.6), (0.5, 0.5), (0.6, 0.4), (0.7, 0.3), (0.8, 0.2), (0.9, 0.1), (1, 0.00)