# Stochasticity in electricity markets : Combining system dynamics with financial economics

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## 1 Abstract

In this paper, we combine a fairly complex system dynamics model of the Nord Pool power market with stochastic price models from financial economics. The idea is to combine the fundamental relationships (given by the system dynamics model) with a good representation of stochasticity from stochastic price models. The purpose is to provide long-term price prognoses for investment decisions as an alternative to the current approach of using scenarios for long-term prices generated by fundamental, partial equilibrium models.

Our paper describes a case study at Agder Energi, a Norwegian utility operating in the Nord Pool market.

## 2 Introduction

Recent experience that electricity prices have become more sensitive to external factors, such as the development of fuel prices, CO2-markets and related political processes. With deregulation, electricity markets interact with other markets, (in particular energy- and environmental markets). Existing decision support models for utilities need to incorporate these new uncertainties, perhaps at the expense of the detailed, deterministic optimisation approach.

In a monopolistic regime, prices were determined on a cost basis. In a liberalised market, a large number of factors influence prices. Special characteristics of the power supply make these prices especially volatile.

Scenarios can to some extent capture (long-term) uncertainties, but simpler, stochastic financial models better describe the statistical properties of both short-term and long-term uncertainties.

In the following sections, we give an overview of the use of decision support models at Agder Energi, a Norwegian utility operating in the Nord Pool market. In our case study, we test a system dynamics models capability of providing long-term price prognoses with uncertainty. Although system dynamics models main purpose (including Kraftsim) are policy design, we have reasons to believe that the SD, as a by-product, can provide long-term price prognoses with a good description of uncertainties when combined with financial models.

Section 3 describes Agder Energi's use of models to support decisions on trading, generation scheduling and investments.

Section 4 provides a description of Kraftsim, a system dynamics model of the Nord Pool power market.

Section 5 introduces financial models for stochastic prices that will be used to address stochasticity in fuel prices, and describes hydro inflow and wind as physical sources of uncertainties important for the Nordic power market.

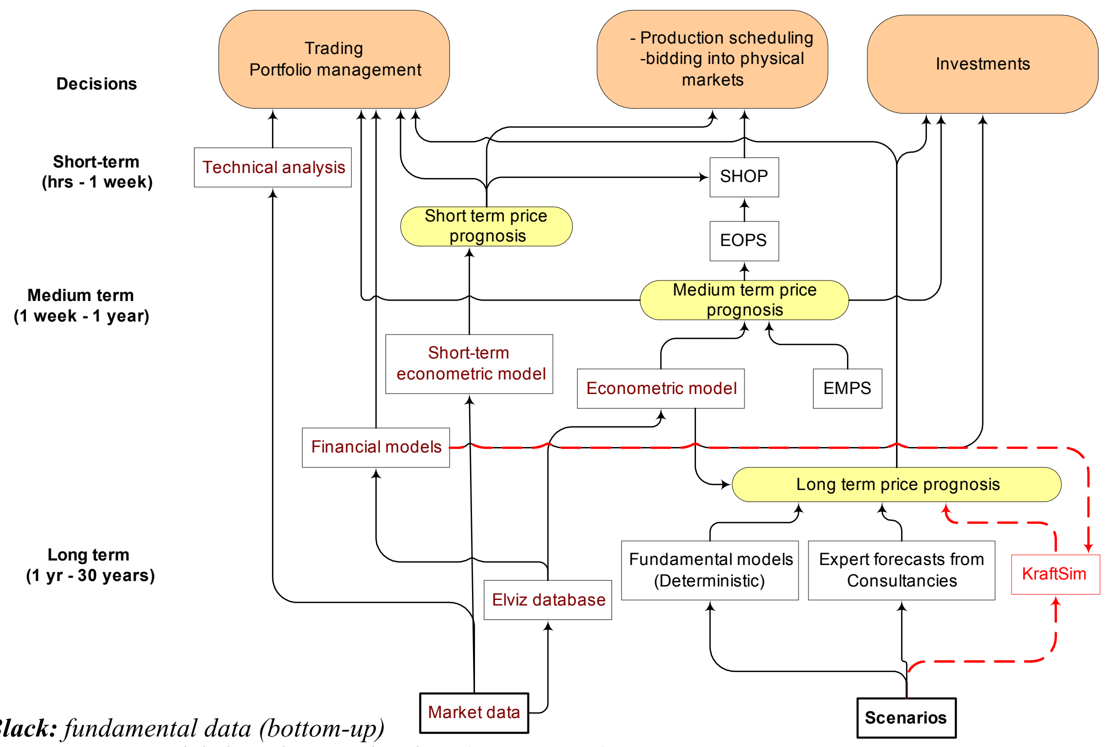
Section 6 provides simulation runs with the stochasticities described in section 5 included in the Kraftsim model, and section 7 concludes the use of SD models for long-term price prognosis by comparing with our current practice and methods.

## 3 Decision support models in restructured electricity markets

Utilities are heavy users of complex decision support models, probably due to the complex nature of production scheduling and transmission that was previously governed by engineers. After liberalisation, financial economists have entered the scene, as market mechanisms have replaced some of the detailed coordination and planning of various activities.

Agder Energi[1] is a typical user of decision support models for electricity generation. *Figure 1* illustrates the variety of our models for decision support on *trading*, *generation scheduling* and *investments*.

*Figure 1 Decision support models at Agder Energi for trading, generation scheduling and investments.*




***Black:*** *fundamental data (bottom-up)*
***Purple color:*** *Models based on market data (econometric)*
***Red:*** *Implementation of Kraftsim*
***Yellow:*** *price prognosis as important model outputs*

The decision support models are interdependent, differ by time span (short, medium and long-term models), and by the level of details. The arrows illustrate the information flow between the models. Some of these models are fundamental techno-economic bot-

1. Norway's third largest utility in terms of hydropower

tom-up models building on extensive information of the physical system and the market mechanisms, while other, financial models heavily rely on available market information in terms of financial market price data and historical price development.

Trading and portfolio management involves short-term, hourly trading in the financial market, where the market psychology plays an important role. Some traders use technical analysis to support their decisions. Portfolio management concerns hedging for risk management using financial instruments, mainly forwards. *Figure 2* shows Agder Energi's trading desk at work.

*Figure 2 Agder Energi's trading desk*

Physical production scheduling in Agder Energi concerns hydropower scheduling and bidding into physical markets, (i.e. the spot market and the power balance market). Detailed optimisation models of interconnected hydropower stations are in use, where hourly production and seasonal reservoir management are the decisions of interest.

Investments deal with the profitability of new plants, transmission links or long-term contracts that necessitates long-term prognoses for profitability assessment. Today, expert forecast / analyses from consultancies are used, combined with deterministic, long-term fundamental models. Another alternative is to use (simpler) financial models of stochastic price processes to generate price distributions. Such financial models are favoured among financial economists for valuating derivatives of the electricity market.

## 4 Financial price models versus bottom up-models

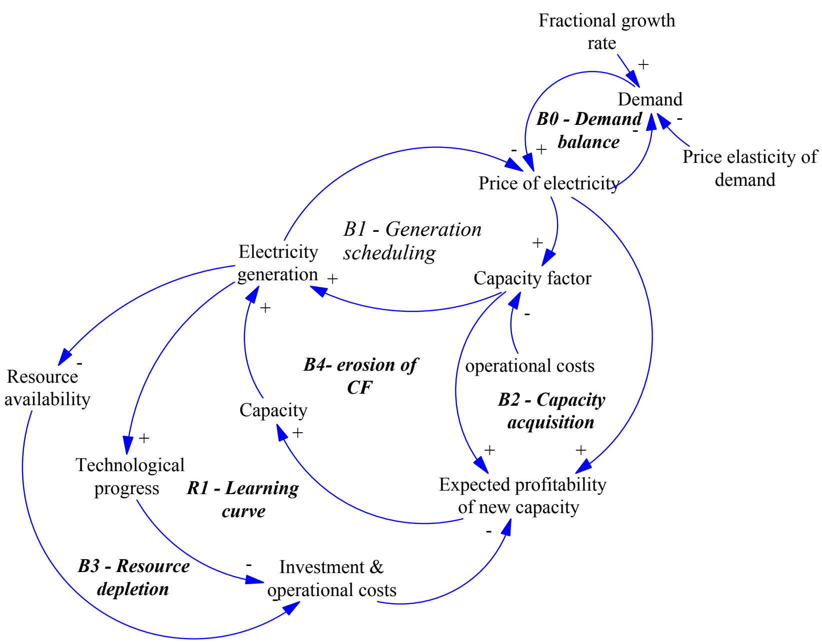
Fundamental bottom-up models have traditionally been used for investment analyses. Uncertainty is usually addressed by creating scenarios assigned with a probability (usually a base scenario, and a high and low scenario). The shortcomings of this method, is that it does not give a good description of the stochastic price process, as prices will move between the high- and low scenarios over time.

Financial economists are concerned with valuing options and risks. Stochastic differential equations have been the preferred tool. Simple stochastic price models can be solved analytically, and can conveniently be used to generate price scenarios. Financial price models do not, however provide fundamental prices but rely on historical data to fit statistical parameters.

Our idea is to enhance the deterministic fundamental KraftSim system dynamics model with stochastic inputs from fuel prices, in addition to stochastic hydro inflow and wind. In doing so, we are able to capture both market fundamentals and the stochastic price distributions propagating through the power market feedback structure.

## 5 The Kraftsim model

*Figure 3 Main loops of the Kraftsim electricity market model*




Kraftsim's development started by Botterud et al (2000, 2002). Its development continued in Vogstad et al. (2002), Vogstad et al. (2003), and Vogstad (2004) and the model is documented in Vogstad (2005). KraftSim is a model of the Nord Pool electricity market including Norway, Sweden, Finland and Denmark, with exchange to Europe. The market price coordinates generation scheduling in the short term and capacity acquisition in the long-term for the nine technologies considered : *nuclear*, *coal*, *gas*, *gas peak load*, *gas with CO2-sequestration*, *hydro* (with reservoirs), *bio*, *wind* and *wind offshore*.

KraftSim differs from partial equilibrium models by being descriptive, rather than prescriptive. Long-run equilibrium is potentially a result of the policies and model structure, not an assumption underlying the model. Behavioural assumptions of investments are boundedly rational, and the large delays involved in acquisition of new capacity as well as the expectation formation in markets are explicitly represented. Technological progress and resource availability are partly endogenous, which is of importance for eval-

uating policies for stimulating new technologies.
The model was built to test various implications of current energy- and environmental policies, pointing out counterproductive consequences and flaws in market designs (Vogstad 2004, 2005). *Figure 3* shows the main causal loop diagram of the KraftSim model and each feedback loop is described below:

***B0 - Demand balance***. Electricity demand responds to price changes. Higher electricity prices tend to reduce demand and vice versa.
***B1 - Generation scheduling*** coordinates the capacity utilisation of each technology type according to price.
***B2 - Capacity acquisition*** describes the process of applying for permits, investing and building new production capacity.
***R1 - Technology progress*** is the cost reduction that takes place as experience cumulates. Technology progress is partly exogenous for investments, and entirely exogenous for improvements in efficiency.
***B3 - Resource depletion*** keeps track of the remaining available resources for development. Resource availability is also partly exogenous, i.e. no constraints on fossil fuels.
***B4 - Erosion of CF*** When new capacity comes on line, the new capacity replaces some of the more expensive units in operation, reducing their capacity utilisation. Ideally, the power market is in long-run equilibrium when the market price (on average) equals the long-run marginal costs of new capacity.

The above page displays the corresponding stock & flow diagram of the KraftSim model. A complete documentation of the Kraftsim model can be found in Vogstad (2005).

We will now turn to the description of stochastic price processes to be implemented in the KraftSim model.

# 6 Stochastic price models for energy commodities

Early stochastic models of commodity prices are based on the geometric Brownian motion adapted from the models originally used for modelling financial markets (Black (1976) and Brennan and Schwartz (1985)). The geometric Brownian motion suggests that the log of the commodity price behaves as a (possibly drifting) random walk (possibly with drift). For commodities, mean-reversion models have more economic logic than the geometric Brownian model. If a commodity price gets very high or low, supply and demand side pressure would tend to push the price back to some long run equilibrium level. Cortazar and Schwartz (1994) introduced mean reversion in the lognormal modelling set up. Multifactor extensions of the initial mean reverting model include, among others, Gibson and Schwartz (1990) and Schwartz and Smith (2000). Schwartz (1997) provides a review of the different models, and he calibrates the models to forward prices of oil.

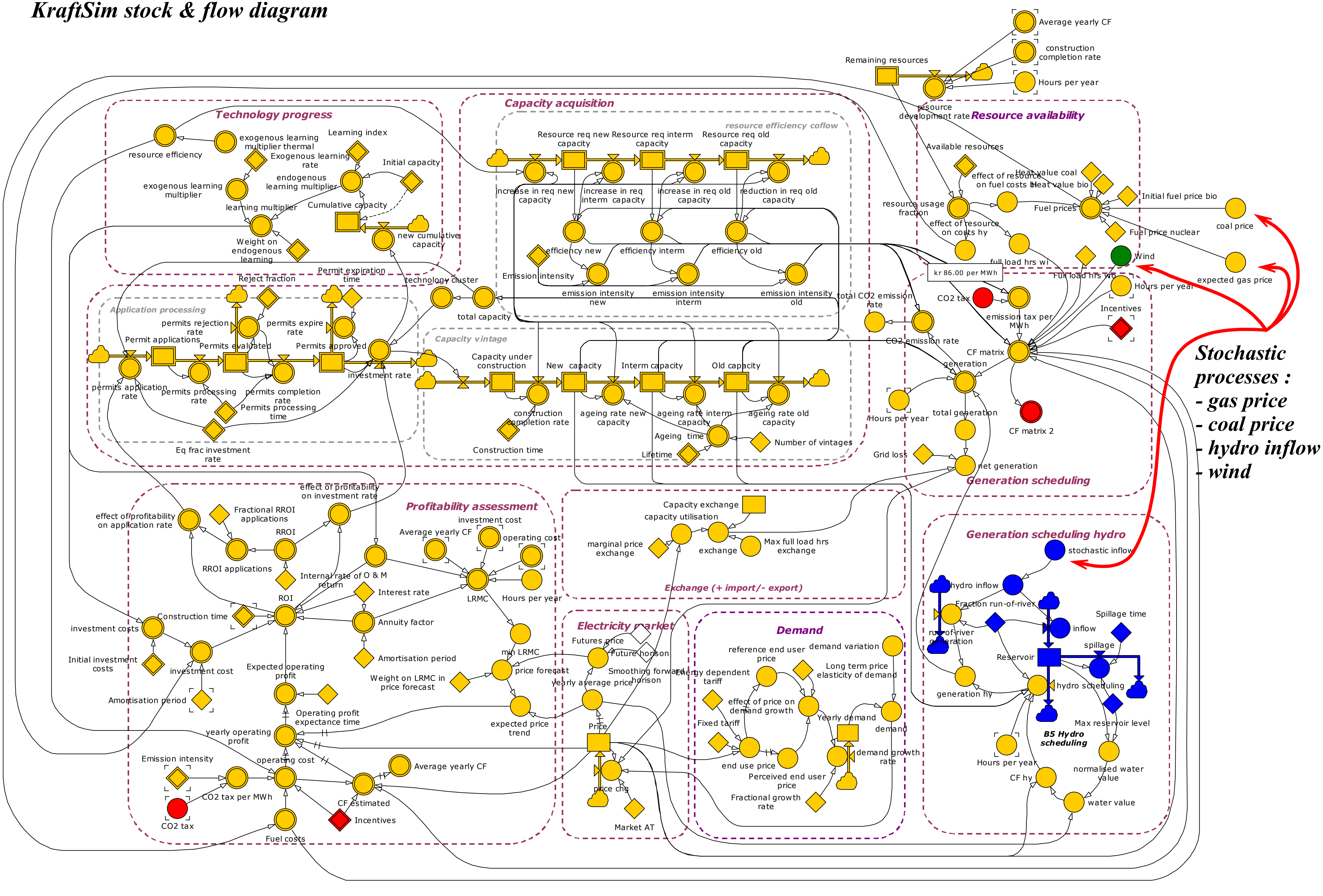
Cooke (2004) demonstrated how the above-mentioned stochastic price processes can be formulated using system dynamics.

## 6.1 Characterising stochastic price movements in fuel prices

Energy markets have become increasingly more integrated and liquid. The gas price development is one of the most important factors influencing electricity prices, because gas power plants have been the backstop technology in recent years.

Gas is sold at different hubs in Europe for day-ahead and quarterly deliveries. Utili-

# KraftSim stock & flow diagram



sation and flexibility of gas supply depends on the infrastructure and conditions of contracts. Daily price movements of around 15% on quarterly contracts occur. However, it is not clear to which extent rapid price movements affects the generation scheduling decisions of gas power producers. It is reasonable to believe that gas power producers act on quarterly or longer term contract prices. Short - term price movements in the gas markets may, however still influence traders and analysts psychologically in the electricity market.

Coal prices are more stable, with price movements of about 3 %. However, China's growth and occasional cold winters in Germany do affect coal prices in the short term, along with freight rates. In the long-term, however, coal reserves are abundant, the market is global and there are less uncertainty concerning the long-term prices.

Stochastic fuel price processes should :

- Reflect the short - term price movements as in the day-ahead or weekly market $\surd$
- Exhibit some long-term mean reversion characteristic, reflecting demand- and supply responses to price changes $\surd$
- Utilise available market data from forward markets $\surd$
- Maintain possible cross-correlations between other sources of stochastic variables
- Reflect long-term uncertainties

The symbol $\surd$ indicates the fuel price properties that we have taken into account. The remaining properties are possible to implement at a later stage, and are commented in the next section.

## 6.2 Characteristics not implemented in the stochastic fuel price model

Gas and coal are partly substitutes. If prices of one fuel increase, one can expect the price of other to increase as well. Demand side pressure suggests that the fuel price spread cannot get too great. Then switching to the cheaper fuel would occur, effectively lowering the price of the pricier fuel and increasing the price of the cheaper fuel. Such market integration suggests a positive correlation

Forward price quotations for next years will drift, and these long-term uncertainties are not represented in our model. Uncertainties of long-term forward prices can be incorporated by extending the price model to a two-factor model where the additional stochastic variable represents the long-term uncertainty. Some formulations have been proposed to represent long-term stochasticities in forward markets, (Schwartz, 2004). This means the long-term fuel price uncertainty here is underestimated.

## 6.3 Stochastic fuel price model

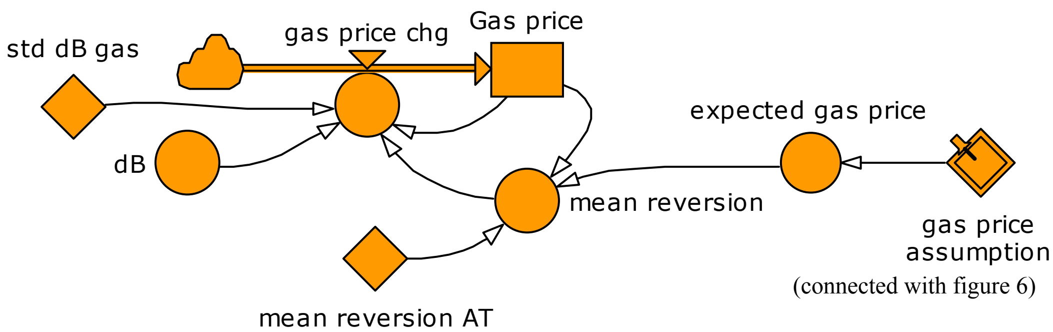
As a starting point, we define a stochastic price process with mean reversion and Brownian motion of the spot price P :

$$\frac{dP}{P} = \kappa(\ln(\mu) - \ln(P(t)))dt + \sigma dB(t) \qquad \textit{(i)}$$

Here, $\kappa$ represent the speed towards the long run equilibrium level of $\mu$ to which the process reverts. The spot price volatility is $\sigma$. The Brownian motion is a standard normal variate with mean 0 and a variance proportional to the time interval $dt$ : $dB \sim N(0, \sqrt{dt})$.

Taking the above equation as a starting point, we develop a system dynamics formulation of the price process :

*Figure 4 Stochastic browninan motion price process with mean reversion*




***(1) Stochastic Brownian motion price process with mean reversion***

*1.1* $\quad$ *Gas price$_t$ = Gas price$_0$ + $\int$ gas price chg$_t$·dt* $\quad$ *[€/MWh]*

*1.2* $\quad$ *gas price chg$_t$ = mean reversion + dB$_t$* $\quad$ *[€/MWh/da]*

*1.3* $\quad$ *mean reversion = (ln(expected gas price)-ln(Gas price))/Mean reversion AT* $\quad$ *[€/MWh/da]*

*1.4* $\quad$ *mean reversion AT = 3* $\quad$ *[yr]*

*1.5* $\quad$ *expected gas price = GRAPH CONTROL({13.24,17.1,11,13.24,19.1,24,25,27.73 ,25.80,24.65,23,21.5,19.6,17.00,15.00,13,12,11,11,11,11,11,11,11,11,11,11,11,11, 11,11}) // see Figure 5* $\quad$ *[€/MWh]*

*1.6* $\quad$ *dB = N(0, $\sqrt{dt}$) // Normal distribution* $\quad$ *[1]*

*1.7* $\quad$ *std db gas = 0.15* $\quad$ *[1/wk]*

Gas price expectations in eq. 1.5 are taken from available market data on forward contracts up to 2012. After 2012, however, we must rely on scenarios from fundamental model or expert forecasts, in this case Markedskraft's long-term scenario (2005). The main idea here, is that the user specifies *price expectations* and the mean reversion process will generate price scenarios around this expected price development path.

The same price process is made with respect to coal, but with different parameters for standard deviation in coal day ahead price movements (*std dB coal = 0.03 [1/da]*) and mean reversion adjustment time (*Coal mean reversion AT = 2 [yr]).*

*Figure 7* and *Figure 8* shows the Monte Carlo simulations of the stochastic price process, for which their stochastic properties are characterised by the market forward curves (*Figure 5* and *Figure 6*) and their respective short-term stochasticity given by the Brown-

*Figure 5 Gas prices. Historical data, forward prices quotations and long-term forecasts. Source : Market data quoted from BP and Montel*

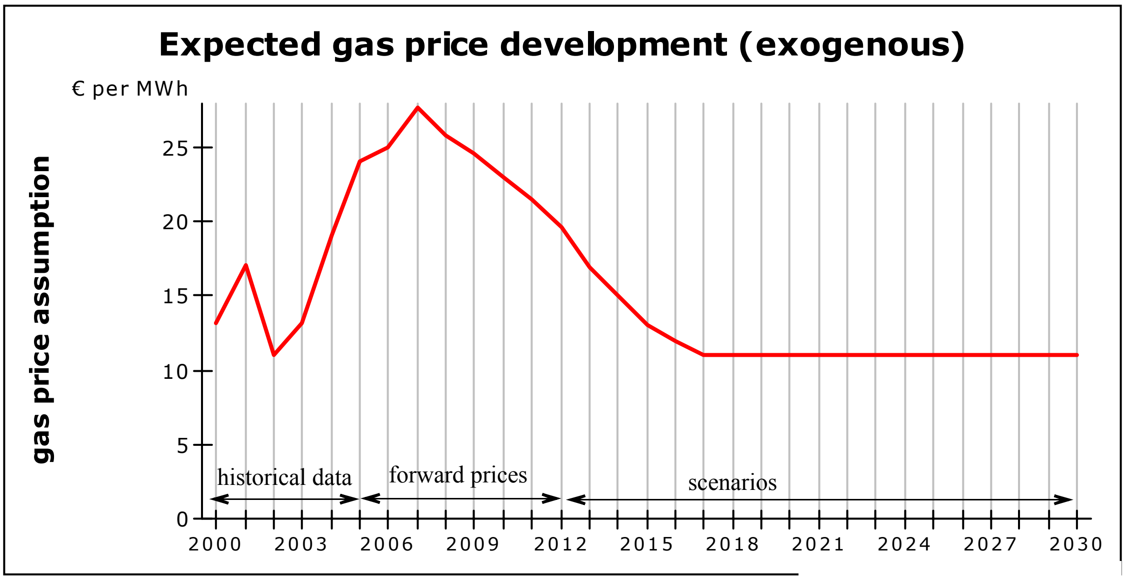
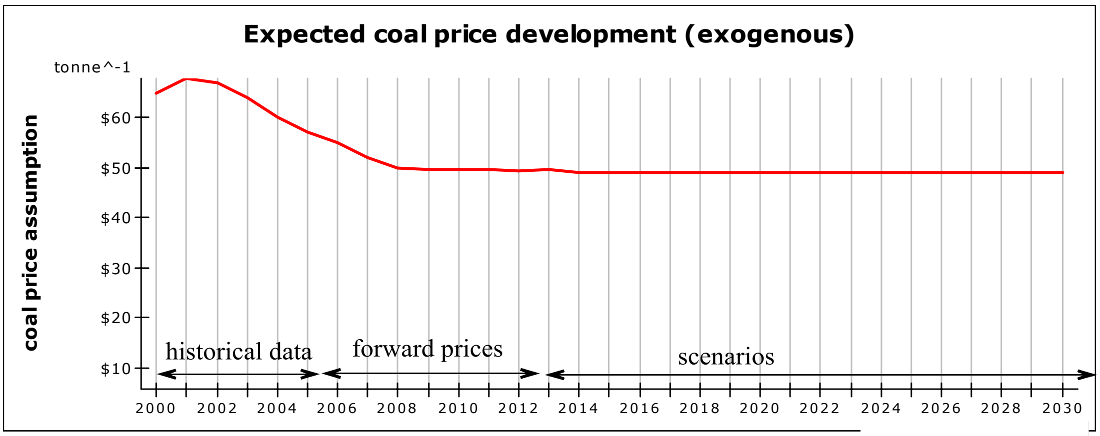



*Figure 6 Coal price historical data, forward market data and long-term scenario.*




ian motion and the standard deviation of each.

## 7 Stochasticity of inflow and wind

50% of the Nordic supply is hydropower, which in turn depends on hydro inflow conditions from year to year. Yearly precipitation can vary as much as +/- 30% from year to year, and there is a significant seasonal variation in hydro inflow. The stochasticity of hydropower makes hydro scheduling a complex task, which is addressed using stochastic dynamic programming (SDP). The inflow stochasticity has been (and still is) the main

*Figure 7     Price development gas (percentiles)*

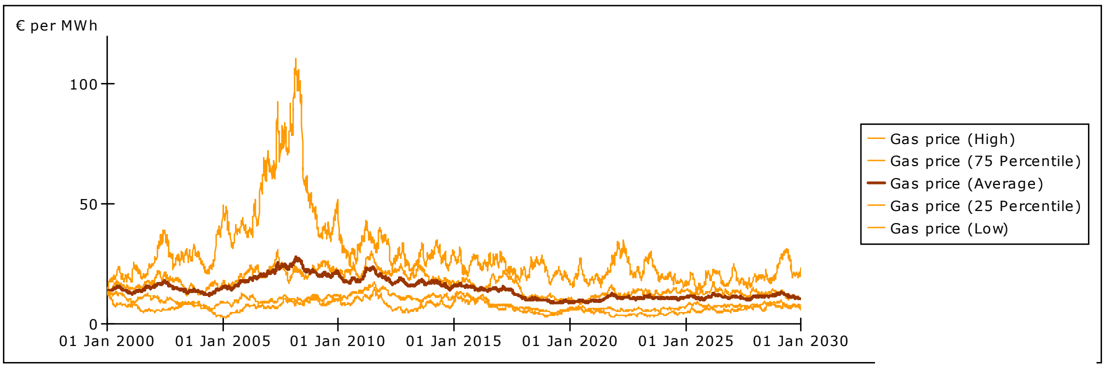
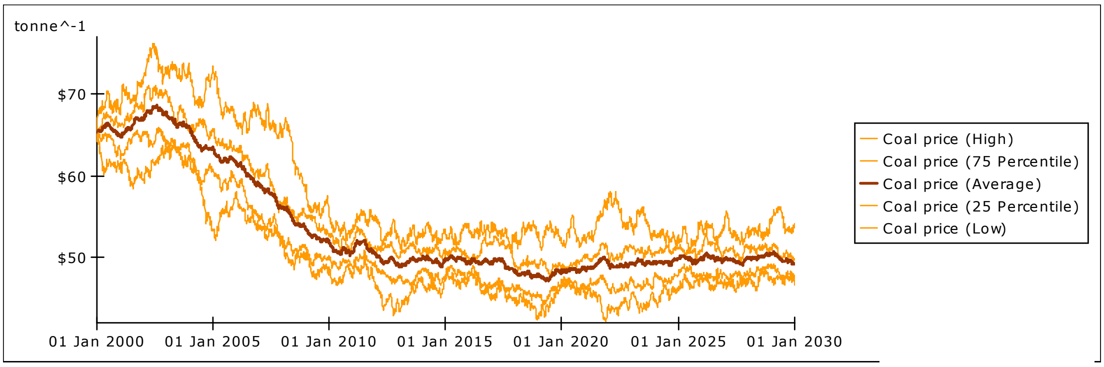



*Figure 8     Price development coal (percentiles)*



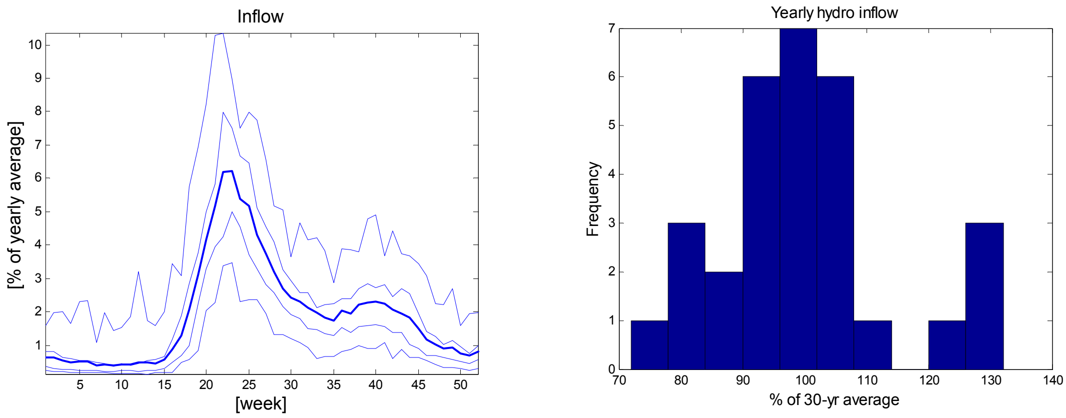

concern, and is well incorporated into the decision support tools.

With increasing shares of wind power, the intermittency of wind will affect market prices, as has been observed in areas with high concentration of wind power, such as Jutland (Western Denmark) and Northern Germany. Hydro inflow is represented with weekly resolution. Planning tools (such as the EMPS and the EOPS[1] model) represent hydro inflow using historical time series, i.e. sequences of data with weekly resolution), and we adopt this approach by randomly picking 52-week series from the historical database in order to maintain the seasonal profile of hydro inflow. The yearly inflow level is independently drawn from a normal distribution with $\mu$ and $\sigma$ fitted to historical data[2].

Wind is characterised by shorter-term variations, important for generation and therefore market prices. As a first approach however, we use the same approach as with hydro inflow: Picking yearly series of 52 weeks from the historical database in order to maintain

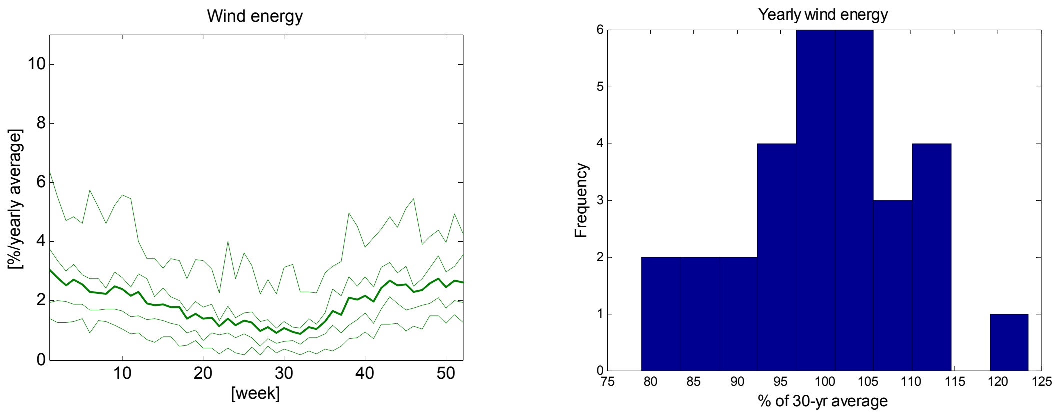
1. EMPS - Sintef's Multiarea power market simluator. EOPS - Sintef's one-area production scheduling. These models are the most commonly used tools for price prognosis and seasonal planning of hydropower. The tools are based on perfect competition, and uses stochastic dynamic programming to optimise weekly hydro scheduling with a yearly planning horizon.
2. Yearly inflow and wind distribution are lognormal distributed, but for the purpose of this paper, but we are at this stage using approximating with a normal distribution.

*Figure 9 Left: Hydro inflow statistics from 1961 - 90 in percentiles (0,25,mean, 75,100%). Right: Yearly inflow distribution ($\mu = 100\%, \sigma = 13.8\%$) Source: Vogstad et al 2001*




the seasonal profile.

*Figure 10 Left: Wind energy statistics from 1961-90 in percentiles (0, 25, mean, 75 and 100%). Right: Yearly wind energy distribution. ($\mu = 100\%, \sigma = 10.5\%$) Source: Vogstad et al. 2001.*




# 8 Investment decisions under uncertainty

Key factors for investments in liberalised markets are future price expectations. Price expectations are in system dynamic models often represented as some simple forecast process based on adaptive expectations. Such behaviour however, is a poor description of the investment process in the Nord Pool market. Direct observations from investors in the Nordic market suggest the following process for investment analyses :

1. Scenarios are made for fuel price development, demand and energy policies to be used as exogenous inputs to detailed models of the electricity market. Several scenarios are made to capture long-term uncertainties of, for instance fuel price develop-

ment and new energy policies[1]. Usually, the reference scenario is made consistent with information from forward markets on gas, coal and oil.

2. Partial equilibrium models are then run with the above mentioned scenario assumptions to establish electricity price scenarios. These models may involve short-term uncertainties such as inflow. In some models, investments are endogenous, while most of the models presently in use need exogenous inputs on capacity investments. Price scenarios in partial equilibrium models typically converge towards the long run marginal costs of the cheapest available technology. The reference price scenarios are usually made consistent with available information from forward markets.
3. Uncertainty is addressed through various scenarios of the exogenous assumptions, plus the short-term uncertainties that are endogenous in the partial equilibrium model.
4. Electricity price scenarios plus exogenous fuel price scenarios is used in a net present value analysis, which incorporate details of taxes, required returns, financing of the project, and other relevant information of importance for the profitability assessment.
5. Sensitivity analyses or additional stochastic simulations are made to assess the risk of the project.
6. Project is presented to the board for investment decision.

What can be observed from this process, is that price scenarios are not simple extrapolations, but will behave as a goal seeking processes where partial equilibrium models are used to establish long-term price prognosis.

A way of examining the expectation formation, is to look at the forward market data, as we will do in the next section.

## 9 Financial market data

*Figure 11* shows 10 years of market data from Nord Pool, organised as daily price quotations along the time axis (x-axis), and as the different financial products along the y-axis. The forward contracts are offered as follows :
Weekly forwards from 1 to 8 weeks ahead; monthly forwards, starting from the next month; quarterly forwards and yearly forwards up to 5 years ahead. A contract with 365 days to maturity means that the delivery date of the contract is 1 year from now. *Figure 11*, shows that prices are more volatile in the short end (short time to maturity), and less volatile in the long end (long time to maturity), as illustrated in *Figure 11* and depicted in *Figure 12*, (right graph). The market believes that the prices eventually will converge towards the long run equilibrium price, which in fact represents a *goal seeking* process. Prices in the short term, however, may fluctuate wildly, without affecting forward prices significantly in the long-term.

These observations from the forward market suggest that price expectations are not simple extrapolations of current trends, but a result of fundamental, rational decision support models. This poses a challenge for modeling investment behaviour.

## 10 Representing investment decisions in Kraftsim

Representing the price expectations according to the steps 1-6 as outlined in *section 8* turns out to be a difficult task. One approach is to represent future price expectations

1. Energy policies, turns out to be one of the most stochastic and unpredictable processes seen from an investor's point of view, for instance new subsidy schemes or permits.

*Figure 11 Historical forward price data 1995-2005, nominal values 8.2 NOK = 1 €*
*Source: Nord Pool*

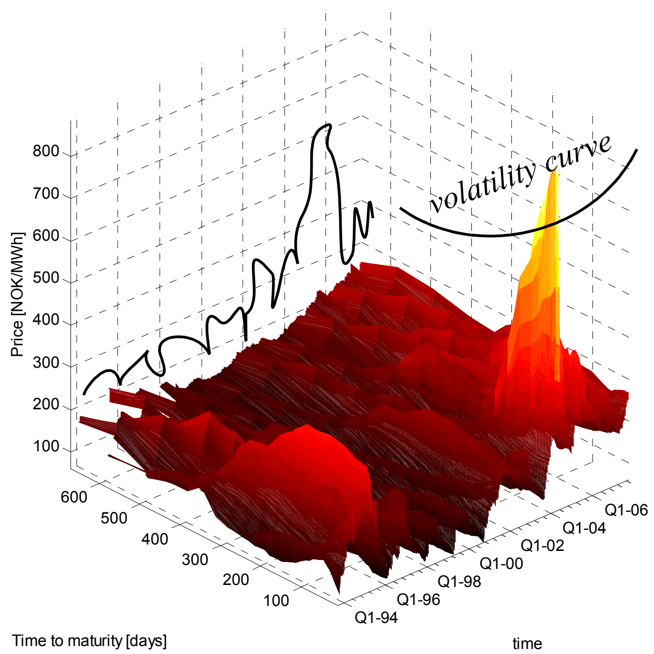
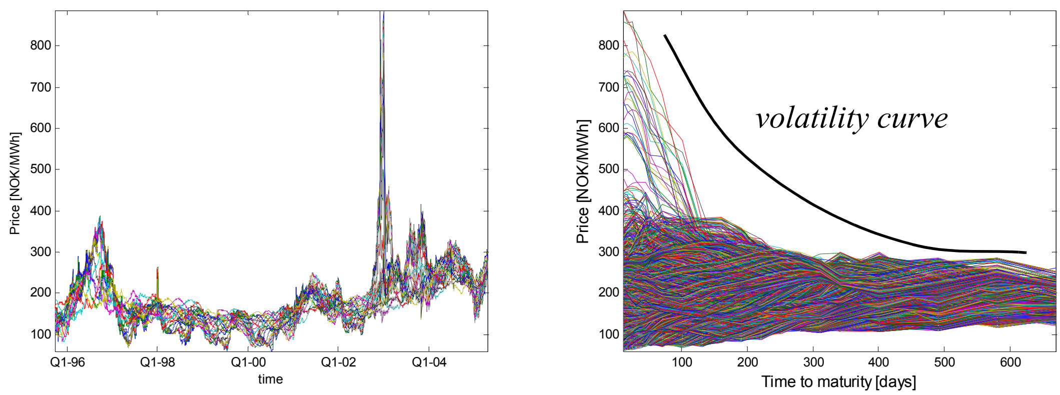



*Figure 12 Left: price history for 21 forwards. Right: Price distribution for the same forwards on time to maturity. Price data from Nord Pool, 1995-2005.*




for fuel and spot prices as state variables, preferably with associated uncertainty for each future expected price path (which would require additions sets of state variables for each scenario and each future period). These future price paths can then be used to calculate expected future revenue stream (being the difference between spot price and operational costs).

The price structure defined as variation of prices over the day, season and years is important for the profitability of some of the generating technologies. Peak load generators with low investment costs and high operational costs can be profitable in markets with many price spikes, even though average prices may be low. The price structure is therefore important to include into the investment analysis for some of the technologies.

Representing both future price expectations, its associated uncertainty in terms of future price scenarios, and the price structure does not seem to be very practical within the standard system dynamics software, the main difficulty being representing price expectations as future states. We have therefore retreated to make a simplified representation, while a more thorough representation of investment decisions will be implemented in another modeling tool. It should be pointed out, however, that our approach of representing future price expectations are not in conflict with the system dynamics method or theory, but rather a practical implementation problem of the SD software.

## 10.1 Future price expectations

The Nord Pool forward market provides an indicator for prices up to four years, and more importantly enable the possibility to hedge risk.

Investment must, however consider time horizons longer than four years. Three to four years ahead, fundamentals such as new capacity or demand are not likely to change (except for reservoir levels). The current state of the reservoirs can influence prices several years into the future.

In the longer term, however - new capacity and development on the demand side, as well as new environmental regulations or market regulations can change prices significantly. These factors must be brought into consideration when moving beyond the time horizon of the forward market.

The long-run marginal costs of each technology, which is subject to changes in fuel prices, technology progress and resource availability change slowly. A perfect market will tend to converge towards long-term equilibrium, and so the long-run price in the market should converge towards the most competitive technology, which (at present) appears to be combined cycle gas power. Naturkraft (2003) held this view at a seminar organised by Montel[1].

Because of this expectation formation, long-term prices should converge towards the long run marginal cost of the cheaper technology - in this case gas - depending on the required return on investments and the gas price.

Assuming investors pay attention to both the forward market, and long-run marginal costs of technologies (possibly from model simulations[2]) as a basis for price expectations we model a price expectation as outlined in equation set (2) below.

In 2.1, *Price forecast* is a weighted average of *forward prices* and the most competitive technology, taken as the minimum of $LRMC_i$ (2.2) of the nine technologies $i$ considered. The weight factor (2.3) states how much investor emphasise the forward market versus the long-run marginal costs in their expectation formation, considering the discounting factor. If we discount using a 10 required return on investment, the first 5 years would account for 1/3 of the NPV, while the remaining 15 years would comprise approx-

1. Montel, a magazine for the electricity business, www.montel.no
2. Nordmod-T, (NOU, 1998) provide such scenarios for long-term price development

imately 2/3.

***(2) Price forecast***

*2.1 Price forecast = Forward price·(1-Weight on LRMC in price forecast)+MIN(LRMC$_i$)·Weight on LRMC in price forecast* *[NOK/MWh]*

*2.2 LRMC$_i$ = energy investment cost$_i$·Annuity factor$_i$+operating costs$_i$+O&M$_i$* *[NOK/MWh]*

*2.3 Weight on LRMC in price forecast = 0.75* *[1]*

## 10.2 Return on investments, ROI

The first version of the profitability assessment submodel was developed and implemented by Botterud et al. (2001) in his Kraftsim model. Later versions of the model incorporated changes, from Vogstad et al. (2002) to Vogstad (2004). Utilities invest when the *expected* present value of a project is positive, that is:

$$\Pi_i = \pi_i(t) - IC_i = \int_0^{T_{a,i}} (\pi_i(t) - \text{O\&M}) \cdot e^{-r(t + T_{c,i})} dt - IC_i > 0 \qquad \textbf{\textit{(ii)}}$$

where $\pi_i(t)$ is the expected yearly operating profits in [NOK/MW/yr], $IC_i$ the investment costs at time *t*, O&M is the operation and maintenance costs independent of the capacity utilisation, *r* is interest rate, $T_{c,i}$ and $T_{a,i}$ is the *construction time* and *amortisation time*, respectively.

At break even, operating profits equal investment costs:

$$\Pi_i(t) = \int_0^{T_{a,i}} ((\pi_i(t) - \text{O\&M}) \cdot e^{-r(t + T_{c,i})} dt) = IC_i \qquad \textbf{\textit{(iii)}}$$

Furthermore, we simplify into:

$$\Pi_i(t) = (\pi_i(t) - \text{O\&M}) \cdot \int_0^{T_{a,i}} e^{-r(t + T_{c,i})} dt = IC_i \qquad \textbf{\textit{(iv)}}$$

Solution of the integral gives:

$$-\frac{1}{r}(e^{-r \cdot (T_a + T_c)} - e^{-r \cdot T_c}) \qquad \textbf{\textit{(v)}}$$

Inserting (v) into (iv) and then divide by the annuity factor $a = \frac{1 - e^{-r \cdot T_a}}{r}$, we can rearrange (iv) into the *return on investments ROI$_i$*:

$$ROI_i = \frac{(\pi_i(t) - \text{O\&M}) \cdot e^{-rT_c} \cdot a}{IC_i} = 1 \qquad \textbf{\textit{(vi)}}$$

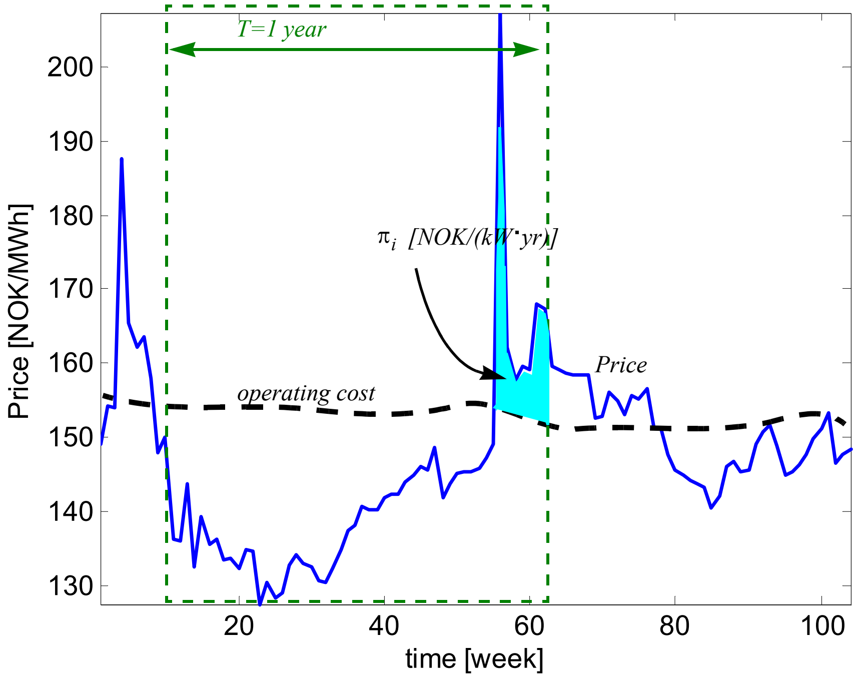
Expected *operating profits*, $\pi_i(t)$ has not been defined yet.

Operating profits$_i$ $\pi_i(t)$ depend on the difference between price and operating costs and capacity utilisation, CF$_{i,new}$. Since we do not know prices or price distributions, we make some expectations about future profits based on experience. We calculate the recent years operating profits over the period $T = 1$ yr for technology i as:

$$\pi_i(t) = \int_{t-T}^{t} \left((\text{Price} - \text{ operating cost}_i) \cdot CF_{i,new}\left(\frac{\text{Price}}{\text{operating cost}_i}\right)\right) \cdot dt \quad \forall i \qquad \textbf{\textit{(vii)}}$$

$CF_{i,new}$ range between 0 and 1 depending on the whether the price is above or below operating costs. *Figure 13* demonstrates the operating profits over the one-year interval *T*.

*Figure 13 Operating profits is the moving window of recent year's profit per energy unit expressed in [NOK/(kW·yr)].*




In the model, operating profits are updated by the simulation running in continuous time. Operating profits capture the price distribution within a year, depending on the resolution of the simulation. Variations from year to year are not included. If hourly load patterns are included, the profit calculation will also contain the resulting price distribution. *Figure 13* shows weekly spot prices at Nord Pool.

Furthermore, we can adjust expected operating profits by using the *price forecast*:

$$\pi_{i(t)} = \int_{t-T}^{t} \left(\left(\text{Price}_t \cdot \frac{\text{Price forecast}_t}{\text{Average price}_t} - \text{operating cost}_i\right) \cdot CF_{i,\,new}\left(\frac{\text{Price}}{\text{operating cost}_i}\right)\right) \cdot dt \quad \forall i \qquad (viii)$$

Rather than only relying on the recent year, we can take into account previous year's by exponentially averaging $\bar{\pi}_i$ of yearly operating profits $\pi_i$:

$$\bar{\pi}_i(t) = \bar{\pi}_i(0) + \int_0^t (\pi_i(t) - \bar{\pi}_i(t)) \cdot \frac{1}{T_s} dt \qquad (ix)$$

where $T_s$ is the smoothing time.

Using $\bar{\pi}_i(t)$ as an estimate of future *operating profit$_i$, ROI$_i$ can be rewritten to:*

$$ROI_i = \frac{(\bar{\pi}_i(t) - \text{O\&M}) \cdot e^{-rT_c}}{a \cdot IC_i} \qquad (x)$$

Hence, (x) yields the expected return on investments taking into account the price distribution and price expectations.

The return on investment formulation here incorporates price distribution and price expectations. The price distribution is endogenously calculated from previous year's data. Normally, price distributions are computed from outside the model.

. The below *submodel Return on investments* restates the *ROI* model formulation outlined in (ii) - (x). 2.4 corresponds to (x), 2.5- 2.7 to (viii) and (x).

***Return on Investment (ROI)*** $\forall\ i \in$ ***T:***

2.4 $ROI_i$ = *discount factor from construction delay·(expected operating profit*$_i$ *- O&M*$_i$*)/(Investment cost*$_i$*·annuity factor)* [1]

2.5 *Expected operating profit*$_i$ *= DELAYINF(yearly operating profit*$_i$*,* $T_s$*)* [NOK/MW/yr]

2.6 $T_s = 3$ *//smoothing time* [yr]

2.7 *yearly operating profit*$_i$ *= SLIDINGINTEGRATE(price·(price forecast/Yearly average price)-operating cost*$_i$*)·estimated CF*$_i$*, 1)* [NOK/MW/yr]

2.8 *operating costs*$_i$ *= fuel cost*$_i$*/resource efficiency*$_i$*-Incentives*$_i$*+CO2 tax per MWh*$_i$ [NOK/MWh]

2.9 *fuel cost*$_i$ *= expected fuel price*$_i$ [NOK/MWh]

2.10 *Incentives*$_i$ *= [0 0 0 100 0 0 100 100 100 ]* [NOK/MWh]

2.11 *O&M = [162 40 37 40 25 0 40 162 226.8]* [NOK/MW/yr]

2.12 *discount factor from construction delay*$_i$ *= exp(-interest rate·Construction time*$_i$*)* [1]

2.13 *annuity factor*$_i$ *= (1-exp(-interest rate·*$T_a$*))/interest rate* [1]

2.14 *Investment cost*$_i$ *= Initial investment cost*$_i$*·learning multiplier*$_i$ $\forall\ i \neq hy$ [NOK/kW]

2.15 *Initial investment cost*$_i$ *= [22.5 11 5.5 10.45 4 0 10 6.5 8.45]* $\forall\ i \neq hy$ [NOK/kW]

2.16 *Investment cost*$_{hy}$ *= effect of resource on costs hy* [NOK/MWh]

The CO2 tax depends on the type of fuel and conversion efficiency for each plant:

2.17 *CO2 tax per MWh = Emission intensity /efficiency*$_i$*·CO2 tax* $\forall\ i \in$ *{co,ga,gc,gp}* [NOK/MWh]

2.18 *Emission intensity*$_i$ *= [0,0.3,0.2,0.02,0.25,0,0,0,0]* [kg CO2/kWh]

## 10.3 Long run marginal costs LRMC

Long run marginal costs *LRMC* can be calculated as follows:

***Long run marginal costs (LRMC)*** $\forall\ i \in$ ***T:***

2.19 $LRMC_i$ *= (Investment cost*$_i$*·annuity factor+O&M*$_i$*)/(Hours per year·Average yearly CF*$_i$*)+ operating cost*$_i$ $\forall\ i \neq hy$ [NOK/MWh]

2.20 $LRMC_{hy}$ *= Investment cost*$_{hy}$ *+ O&M*$_{hy}$ [NOK/MWh]

The main shortcomings of the above-simplified formulation, is the inability to take into account long-term uncertainty, and an inadequate representation of the price structure. However, investment analyses as they are carried out in practice, have difficulties in prognosing the price structure in the future, as price structures need a detailed model of the capacity mix, which imply endogenous investment decisions.

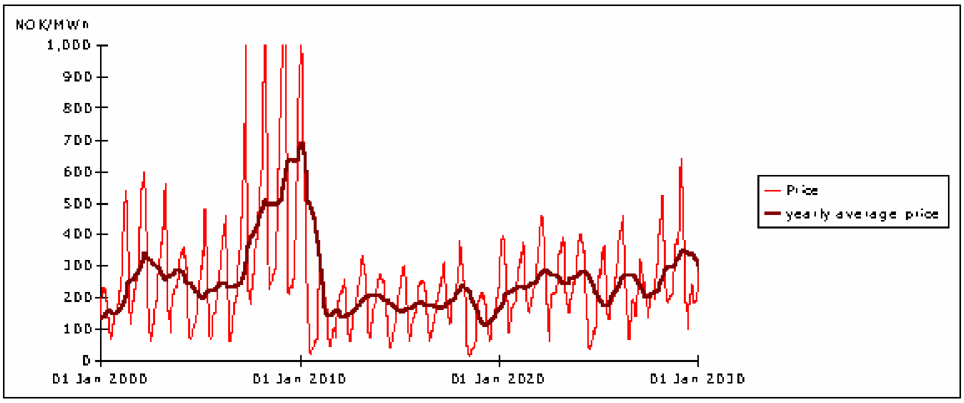
The stochastic variables in the investment decisions are the average expected values. Ideally, each expected price scenario should be calculated in the investment decision, but this operation has been left out, as we reached the conclusion that a detailed representation of investment behaviour in the system dynamics software was not practically feasible in our case.

# 11 Simulations results

## 11.1 Deterministic reference runs of the Kraftsim model

*Figure 14* shows the price development for a single run with stochastic inflow, where the bold line indicates yearly average price and the thin red line include seasonal variations in the price. The seasonal variations stems primarily from the combination of hydro

*Figure 14 Price formation in Nord Pool market*



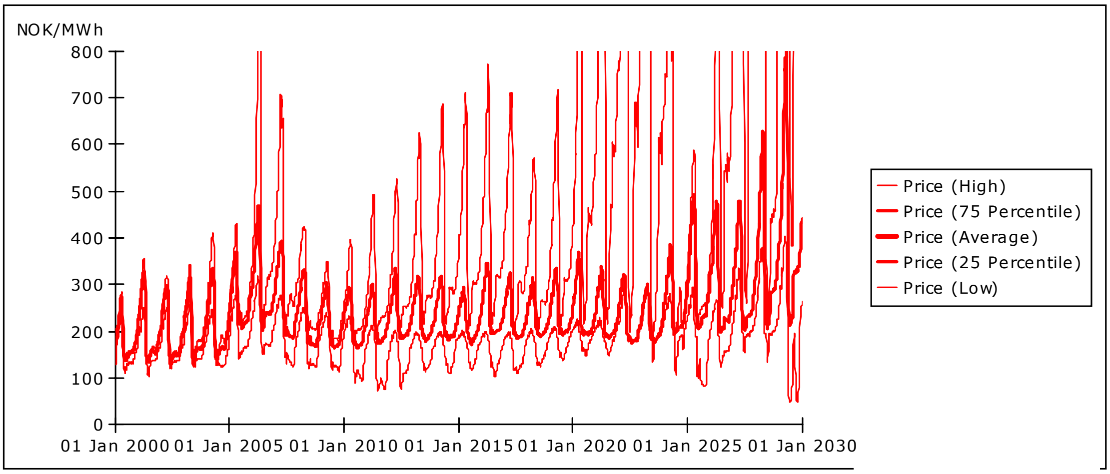

inflow characteristics, reservoir storage and seasonal demand pattern, where prices peak in winter with low inflow and high consumption and are low in summer during reservoir filling and low demand.

## 11.2 Price prognoses with stochastic data

*Figure 15* shows the resulting price distribution for stochastic runs of the Kraftsim model. The seasonal price variations that can be observed are too strong in comparison to what we historically have observed in the Nordic market. This is partly due to a poor calibration of the water value calculation (i.e. the production strategy for the hydropower units), partly the insufficient representation of investment decisions that result in inadequate capacity mix.

With respect to price prognosis, the model needs to be improved by (1) iteratively updating water value calculations, (2) adequately represent long-term uncertainty in fuel prices through a simulation of the forward market, and (3) include these uncertainties into

*Figure 15 long-term price distribution from Kraftsim with uncertainties of gas, coal wind and hydro inflow. (there are some flaws in the model and the price processes that needs to be corrected at this stage)*




the investment analysis.

## 12 Conclusion

Combination of SD models with financial models is theoretically feasible, but not practically feasible with standard SD software. The main difficulty is to represent the future states of price expectations, as the number of state variables will grow large when you introduce uncertainty in terms of scenarios / price paths, and additionally need one state variable for each future period of contracts. A high number of future time periods might be required to represent the price structure.

It is of course possible to give a more simplified representation of the investment decision process, but the simplified representation should be compared with the more detailed representation to check how good the simplification is.

We have concluded that a model with a detailed, realistic description of investment behaviour that takes into account both long- and short-term uncertainty was not practically feasible with standard SD software, although there our approach itself does not conflict with the SD paradigm. Our future approach will be to implement the Kraftsim model in the Matlab environment, which is a more flexible, general-purpose modeling language.

Our approach will be to simulate future price expectations by generating price paths using financial models and their related uncertainty. The simulated future price expectations generated are then used in a NPV calculation.

## 13 Bibliography

Black, F (1976): The pricing of commodity contracts Journal of Financial Economics. pp. 3-79, 1976.

Brennan, M and ES Schwartz (1985): Evaluating Natural Resource Investments. Journal of Business 58, 2 (April), pp. 135 157.

Cortazar, G and ES Schwartz (1994), The Valuation of Commodity-Contingent Claims. The Journal of Derivatives, pp. 27-39.

Gibson, R and ES Schwartz (1990): Stochastic Convenience Yield and the Pricing of Oil Contingent Claims. The Journal of Finance, XLV(3) pp. 959-976.

Schwartz, ES (1997): The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. The Journal of Finance, LII(3), pp. 922-973.

Schwartz, ES and JE Smith (2000): Short-Term variations and long-term Dynamics in Commodity Prices, Management Science 46(7), pp. 893-911

Cooke DL (2004) : *Using system dynamics models to enhance the visualiztion of stochastic price processes*. Proceedings, 22st International conference of the system dynamics society.

Botterud, A., M. Korpås, K. Vogstad and I. Wangensteen. 2002. *A Dynamic Simulation model for Long-term Analysis of the Power Market.* Proceedings, Power Systems Computation Conference, 25th-28th June, Sevilla, Spain.

Forrester, J.W. 1961: Industrial Dynamics. MIT Press. Cambridge MA

Sterman, J.D., 2000: “Business Dynamics: Systems Thinking and Modeling for a Complex World”, McGraw-Hill. Book website: www.mhhe.com/sterman

Vogstad, K., A. Botterud, K.M Maribu and S.G. Jensen. 2002. *The transition from a fossil fuelled towards a renewable power supply in a deregulated electricity market*. Proceedings, System Dynamics Conference, 28th-1st Aug, Palermo, Italy.

Vogstad, K., H. Holttinen, A. Botterud and J.O. Tande 2000. System benefits of coordinating wind power and hydro power in a deregulated market. Proceedings "Wind Power for the 21st Century, 23-25 Sept. 2000, Kassel Germany.

Vogstad K (2005): *A system dynamics analysis of the Nordic electricity market : The transition from fossil fuelled toward a renewable electricity supply within a liberalised electricity market.* PhD thesis 2005:15, Norwegian University of Science and Technology, Trondheim.

Vogstad K, A Santiago, HI Skjelbred and LN Grimsmo (2005): Experimental economics for market design. In Proceedings, 23rd International conference of the sytem dynamics society.

Vogstad K, I Slungård and O Wolfgang (2003): Tradable green certificates : the dynamics of coupled electrictiy markets. Proceedings, 21st International conference of the sytstem dynamics society.