

# Feedback Processes in Economic Growth: Relations between Hours Worked and Labour Productivity

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## Abstract

*This paper analyses different feedback processes arising from physical and human capital accumulation as well as from technological change, which are considered general factors to promote the growth in any economy by economic literature. A dynamic system is constructed to explain the relative influence of these factors on the rate of economic growth in a generic economy. The development of the model requires to analyse different interactions among variables linked to decisions of the agents that participate in the economy, particularly certain variables associated to the labour market are examined. Using a system dynamics simulation the conditions under which a smaller number of hours devoted to the labour market could imply a greater labour productivity are characterized.*

**Key words:** Economic growth, System dynamics, Computer simulation.

## 1. - Introduction

Since the second half of the 19th century it is empirically proven that the world economy grows. In spite of the fact that not all countries grow in the same proportion and that there are wide periods of stagnation, the growth of the world economy continues increasing. The verification of those facts motivated the search of the factors that would promote economic growth. Modern economic literature considers physical and human capital accumulation together with technological progress as necessary elements to maintain a sustained growth in any country. These ideas are relatively recent because during the 1950s and 1960s, mainstream economics was dominated by the growth model of Solow, whose main focus was on physical capital accumulation; this model could not generate sustained per capita growth unless technology was assumed to improve exogenously. The current strand is the result of different researches undertaken by influential authors such as Lucas (1988) or Romer (1990), although it also includes the development of ideas already stressed by authors such as Schumpeter or Rosenberg.

The topic of economic growth is one of the most deeply studied in current economic literature and consequently there is a wide bibliography. Some recent contributions show different issues in researches: the problem of the poverty trap is analysed by Azariadis et al. (2005); the ascent to growth of developing countries is studied by Peretto (1999), Kosempel (2004) or Kejak (2004); the convergence of the growth rates across countries is tackled by Barro and Sala i Martín (1992); the influence of specific regional factors on growth paths is analysed by Breton (2004). Not only the topic is widely researched in many directions, but the methodological approach given to the subjects is different as well.

There are models constructed only by using statistical methods and econometric techniques and in other cases, the researchers use particular insights in order to capture aspects of a complex reality. The latter approach presents certain parallelism with models constructed using the methodology of system dynamics. For example, most of those models are very complex because the matter undoubtedly presents that feature; the authors use numerous variables to interpret the patterns of behaviour of the different agents acting in economies; the interconnections among the variables have essentially a non-linear nature and, in addition, since the models aim to explain aspects of a reality in evolution, the variables vary over time. But a study in depth of the models reveals that numerous variables are interconnected by causal influences that generate feedback processes, which would explain the accumulation processes immersed in any growth process. The influence of positive loops on economic growth was pointed out by Sterman (2000, pp. 386) in reference to Romer's model who showed how growth for an economy as a whole could arise from some of the positive loops, particularly those related to research and development, learning by doing, and other investments in human capital.

As consequence of these considerations, it is possible to corroborate that many features of the growth process could be studied from a perspective of system dynamics. This point of view will allow promoting the study and the understanding of these processes due to the contributions of the methodology.

The general purpose of this paper is to analyse the dynamics arising from the interrelations among those variables, which are considered to be growth and development supports in countries by economic literature. To attain that purpose a system dynamics model is constructed. Its structure is consistent with the causal influences among the variables, which encourage physical and human capital accumulation as well as technological change. The relationships among variables are mainly supported on Peretto and Kosempel performances, who analysed an economy of which general characteristics had already been presented in the investigation of Romer. The selection of these authors is not groundless because, in some way, they built complementary models. Both of them pursue a similar aim since they try to explain the paths of growth that an economy could follow from a situation of developing to achieve a sustained economic growth. Moreover, their models have a similar structure because they consider economies in which the same agents participate and take similar decisions. However, there are some differences between them. Peretto analysed in depth the actions of the intermediate sector in the economy; but neither the human capital accumulation nor the participation of the labour productivity in the productive sector were taken into account. Furthermore, technology is accumulated by a simple linear rule. On the contrary, Kosempel considered a more realistic rule regarding the

accumulation of technology. However this author assumed that the number of intermediate remains constant over time provoking a simpler dynamics.

Although the structure of the system dynamics model is based on some developments done by the authors previously mentioned, our approach uses distinctive elements of the methodology of system dynamics. The modelling specifies particularly that certain causal influences take time. In this way, the accumulation of technology, the process of learning and the decisions that households and firms take in economy are modelled by using delays. It also seems important to emphasise that the model contains unlikely quantifiable variables, soft variables, such as human capital or even technology. Its use is necessary in order to achieve the aim pursued. Taking up Saeed' ideas (2005, pp.1) *in the complex world of today, it would be impossible to ignore these variables without losing sight of the importance dynamics that we experience in reality.* Nevertheless, the model also deals with quantifiable variables such as the interest rate, the number of firms in the economy or the gross domestic product (GDP).

From a systemic perspective, the treatment of ideas developed in models constructed from a strictly economic outlook is not pioneering in the system dynamics literature. Authors such as Saeed (1998, 2005), Stermann (2000, pp. 718), Sice et. al. (2000) used this approach to carry out the study of certain current problems. Under this position Saeed analysed relevance questions: income distribution, technological development or innovation in organizations. This author (1995, pp.1), regarding the classical economics models, affirms that *system dynamics modelling and computer simulation can be used to demonstrate the systemic perspective and the richness of these models.*

Once the model is able to produce results, it is proposed a simulation analysis for studying an emerging debate in the European Union (EU) on the effectiveness of long working hours, particularly with respect to increases of productivity. The matter arises because the data seem to confirm that long working hours are associated with various negative effects, such as decreased productivity, poor performance, health problems, and lower employee motivation. This issue can be analysed from the model because this one includes the dynamic interactions that determine human capital accumulation, which requires the analysis of some aspects of the labour market. In particular, the formation of the causal structure requires the analysis of two quantifiable variables: the labour productivity and the number of hours that workers devote to the labour market. The former is an endogenous variable while the latter can vary across countries and it is sometimes affected by national regulations. Using the connections of these two variables with other variables of the model it is possible to tackle this issue to characterize the conditions under which a smaller number of hours devoted to labour market could imply a greater labour productivity.

The remainder of this paper is organized as follows. The study of the causal influences and the formation of the stock and flow diagram are analyzed in section 2. Section 3 concentrates on results analysing the results of the simulations in particular. Conclusions and remarks are provided in section 4.

## **2. – The positive loops**

It is possible to explain, from a system dynamics outlook, the insights involved in the processes of economic growth in countries, considering the feedback loops that govern human and physical capital accumulation, as well as technology. As Sterman (pp. 406)

affirms, *the evidence suggests that the profitability of individual firms and the evolution of the economy as a whole is strongly influenced by positive loops.* He continues with *these feedbacks involve scale economies, learning, network effects, market power, and many other processes.* However he indicates that *success with one set of these positive loops can lead to inertia and rigidity.*

In order to analyse the way selected variables lead to the economic growth, a generic economy is considered. In the economy households, a set of firms that constitute the intermediate sector and a final sector take part. Each agent has a clearly defined function. The intermediate firms produce different goods using the following factors: elements of the labour market, technology and physical capital. The final sector produces a homogeneous final good using the whole intermediate production. The prices of the intermediate goods are fixed proportionally to the costs of labour and capital. Without considering fixed costs, the intermediate sector has at its disposal resources as consequence of the sale of their products to the final sector. These resources can be distributed as dividends or could be invested in activities of research and development (R&D). Then, the final output can be consumed or reinvested to accumulate technology and physical capital. The rate of growth of the final output is identified with the rate of growth of the economy.

Figure 1 shows the exchanges among economic agents in accordance with the relationships mentioned above. Households provide labour to the intermediate production. The intermediate sector contributes with technology to its own production. This sector receives amounts of final output from the final producer that are used to remunerate the contributions of households to intermediate production and the remainder is distributed partially or completely among their shareholders.

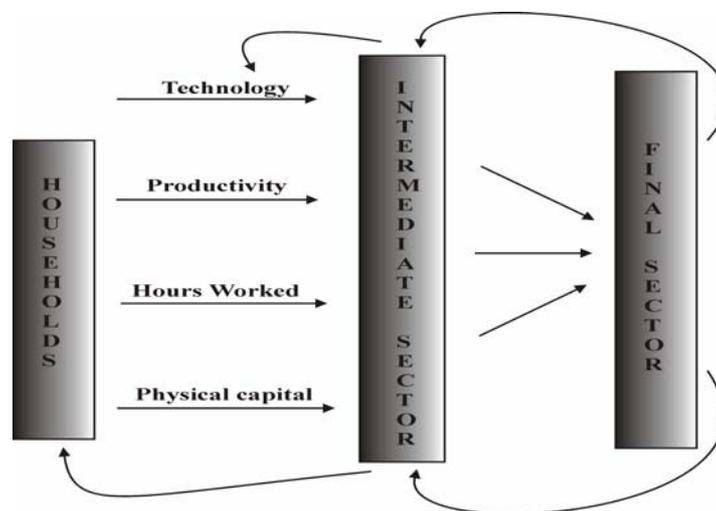


Figure 1: Limits of the system

The decisions about the contributions to the productive process that households and intermediate sector have to take over time are interconnected. Moreover, they affect the evolution of the final output, that in turn influences the solutions adopted by both economic agents. Then the variability of the final output can be described causally. Seeking the influence of positive loops on the growth of the final output, each one of the elements that promote the economic growth will be studied separately.



Households have three sources of income. Firstly, they are owners of the intermediate sector and consequently they could receive dividends. Secondly, they receive labour income, which are determined considering both the time dedicated to the labour market and their productivity and finally, their wealth not consumed is lent to the intermediate sector and as a result, they receive capital income. The price of a unit of physical capital, the interest rate in economy, is determined by its contribution to the intermediate production. Then if final output grows, the interest rate increases, but if the current stock of physical capital increases, the variable decreases. The negative influence between the stock and the interest rate does not have a decisive impact on the accumulation of the stock because a decline of the interest rate only influences the capital income that constitutes just a part of the whole households income.

As in many macroeconomic models, it is assumed that households consume a proportion of their available income depending on their marginal propensity to consume, which is considered an exogenous variable since the model does not consider expectations about future income of households. Therefore, the net change in the stock of physical capital can be positive, negative or null, hinging on the value of the propensity to consume inferior, superior or equal to one, respectively. This feature reflects the strong influence of the consumption on the accumulation of this stock.

## 2.2. –Technology and human capital evolution

While the accumulation of technology is promoted by the intermediate sector, human capital accumulation depends on decisions adopted by households. In addition to other reasons that will be considered later, intermediate firms have reasons to expect that economy has suitable technology and human capital levels. Therefore, on one hand technology is an intermediate productive factor and, on the other hand, both human capital and technology determine the labour productivity, which influences directly on the intermediate production.

Figure 3 shows the variables connected to the accumulations of human capital and technology. Next we shall explain the meaning of the different elements that participate in this figure as well as the causal influences among them.

Starting with the productivity, this variable indicates the way workers are operating in the labour market. In agreement with Kosempel, it is assumed that the labour productivity depends positively on the stock of human capital and negatively on the current level of technology:

$$Productivity = \eta \left( \frac{Human\ Capital}{Technology} \right)^\theta$$

where  $\eta$  is a positive parameter and  $\theta$  belongs to  $(0, 1)$ . The formulation of this variable will lead us to obtain some consequences. For example, if the stock of technology remains constant and the stock of human capital grows, then the productivity grows less proportionally, though. This is one of the results pursued by the specification since it attempts to reflect the following idea: experienced workers could boost slightly their operational abilities in the productive process but their efforts will run into with the current technological level. Also the specification proposed implies that if the stock of human capital remains constant and technology grows, then the productivity decreases because new technologies could not be operated efficiently in the

productive process until an investment in learning is undertaken. Finally, if both variables change, the productivity does not diminish while the stock of human capital grows faster than technology.

The estimates of the Organization for Economic Cooperation and Development (OECD) in 2004<sup>1</sup> show that Belgium, Ireland, France, Luxembourg and Norway had the highest productivity levels in the OECD area, with levels at or above those in the United States (US). In most of these countries, high labour productivity was accompanied by a low level of hours worked per capita. Japan's level of productivity was 30% below the US. Productivity levels in Germany were about 9% below the US level; Mexico and Turkey had the lowest productivity levels, just below 30% the US level.

In order to analyse human capital accumulation it should be taken into account that each worker can dedicate a fraction of his non-leisure time to intermediate production and the remainder can be invested in learning. The latter variable must be understood in a wider sense. The accumulation of human capital depends on commitment: the time invested by households in the learning process, the current level of human capital and the current level of technology in economy. The participation of two elements in this specification, human capital and time invested in the learning process, is often used in the literature of economic growth; for example, Lucas (pp. 18) has already linked these elements postulating that the growth of the stock of human capital hinges on the level already attained and the effort devoted to acquiring more. However, as in Kosempel' formulation, the specification proposed introduces the technology to assume that a technological change creates new possibilities of learning. In addition, the causal effect from the technical progress to human capital accumulation is consistent with certain evidences; for example, Heckman and Klenow (1997) admit that countries having high levels of schooling do so because of high levels of technology.

The influence of the time invested in learning on the stock of human capital is formulated as follows:

$$\text{Effect of Time to HC} = B (\text{Learning}) (\text{Human Capital})^\beta (\text{Technology})^{1-\beta}$$

being  $B$  a positive constant and  $0 < \beta < 1$ , which indicates the variation suffered by the effect when the stock of human capital varies close to 1%. In the previous formulation

$$\text{Learning} = (\text{Percentage Learning})(\text{Total Time})$$

where *Total Time* collects non-leisure time available for all workers in the economy and *Percentage Learning* indicates the fraction of *Total Time* invested in learning. Note that, if no effort is devoted to human capital accumulation, then nothing accumulates, which collects Lucas' idea.

However, it seems reasonable to think that the effort in learning does not generate results at once. Due to this conjecture, a process of adaptive expectations for modelling the accumulation of this stock is proposed. The stock will be gradually adjusted to the value that defines the effect. The speed of the adjustment depends on the value estimated for the parameter *HC Adjustment Time*.

In this instance, the flux associated to the human capital stock is formulated as follows:

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<sup>1</sup> [www.oecd.org/statistics](http://www.oecd.org/statistics)

$$\text{Change in HC} = (\text{Effect of Time to HC} - \text{Human Capital}) / (\text{HC Adjustment Time}).$$

In the model, human capital is a dimensionless variable. To be more precise, it is an index associated to the economy. The indicator Tertiary graduates in science and technology per 1,000 of population aged 20-29, published annually by Statistical Office of the European Communities (Eurostat)<sup>2</sup> may be used to show the differences across countries. The data indicate that in 2003, US or Japan had indexes next to the half of the indexes achieved by countries such as Ireland, France or United Kingdom, but close to the average in the European Union (15 countries). The lowest indexes correspond to Malta and Cyprus.

Unlike human capital, technology is a productive factor in the intermediate sector and as consequence its growth will increase the production of that sector. However, the formation of technology requires to take into consideration at least two aspects. On one hand, the production of technology is costly and its growth will only be achieved by dedicating resources to activities in R&D. On the other hand, it is assumed that R&D activities are aimed to improve the quality of existing products as opposed to create entirely new products. Then, the results of R&D will depend on the amount of resources invested and the current value of technology.

Therefore, if the intermediate sector devotes the same resources to activities in R&D at different dates, the technological change could be different due to the influence of the value achieved by the technology at each date. The result of the investments is formulated as follows:

$$\text{Effect of Resources to Technology} = (1 + a \text{ Resources to R\&D})^b \text{ Technology}.$$

The parameter  $b$  belongs to  $(0, 1)$  and as consequence, the effect does not increase at the same proportion than the resources invested; the parameter  $a$  also belongs to  $(0, 1)$  assuming that only a fraction of the resources can be transformed into technology.

It is evident that numerous activities are required to carry out any technological change: creation of ideas, organization of tasks, developments, designs, tests, evaluations, etc. Undoubtedly, the process is slow, even slower than human capital accumulation. Then in order to model the technological change process two variables of level are considered: *Technology* and *Technology in Development*; the former is the technological factor used by the intermediate producers, whereas the latter collects those technological projects that have not been used in the productive process, yet.

Both levels constitute a second order structure. The level *Technology* is fed by *Technology in Development* of which inflow is governed by the discrepancy between the effect of the resources invested and the current value of *Technology*. An adjustment time *Tech Adjustment Time* indicates the average time required to transform resources into technological factor. This parameter is divided into two to affect the inflows of both levels.

Hence the inflow associated to level *Technology in Development* is defined as follows:

$$\begin{aligned} \text{Change}_1 = \\ (\text{Effect of Resources to Technology} - \text{Technology}) / (2 / \text{Tech Adjustment Time}). \end{aligned}$$

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<sup>2</sup> <http://epp.eurostat.ec.eu>

This specification is proposed in order to capture specific characteristics about the process of accumulation of this stock: technology grows if resources are invested but, the process of maturation of the investments also takes time.

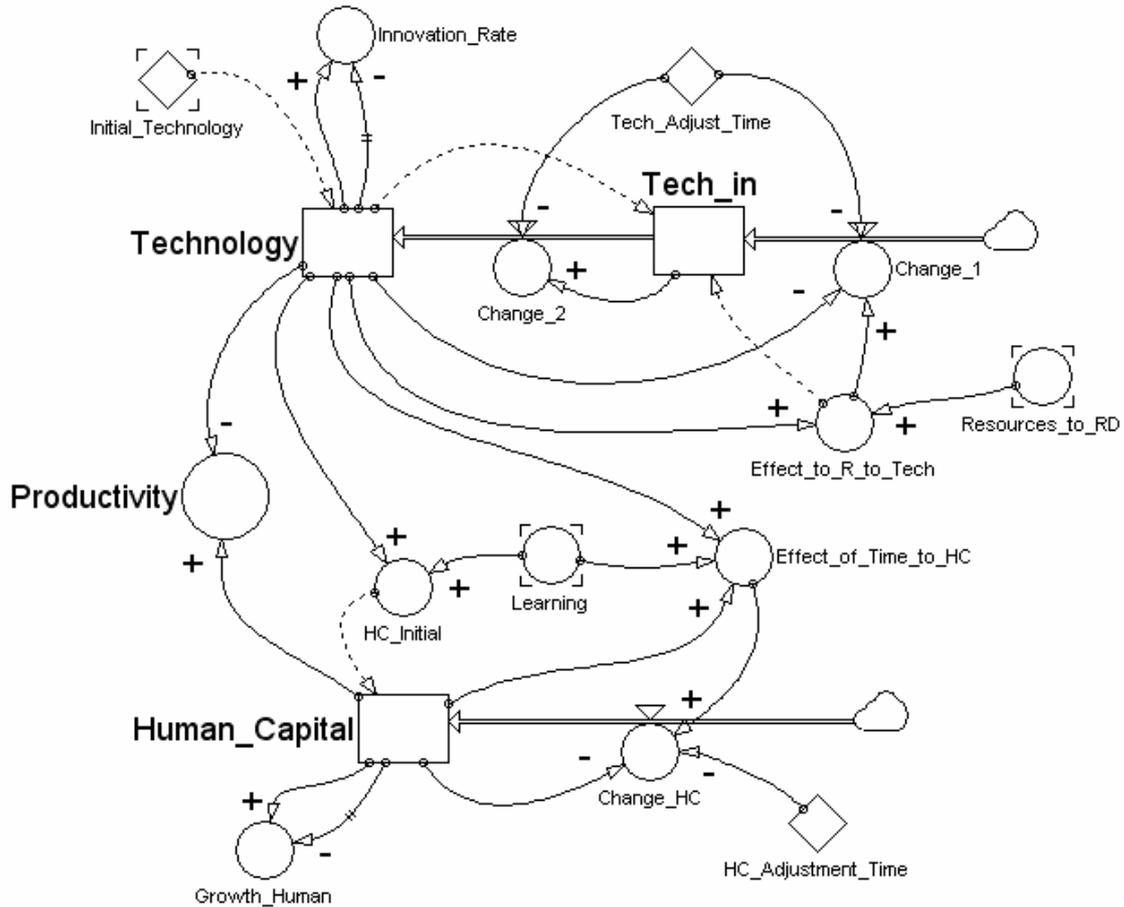


Figure 3: Variables associated to the labour productivity

The importance of activities in R&D is patent these days checked the strong correlation between R&D intensities in countries and GDP. The indicator Gross Domestic Expenditure on R&D (GERD) as a percentage of GDP, annually published by Eurostat, shows that in 2004 some countries such as Sweden, Finland and Japan devoted more than 3% to tasks of R&D; Denmark, Germany, France, Austria, Iceland and US devoted more than 2%. The average in the Euro-zone does not achieve these percentages. The East Europe countries show the lowest percentages. However, between 1994 and 2004, the historical series show that the effort in most countries is growing.

Note that, when connecting the figures 2 and 3, the introduction of the new variables generates new feedback loops. For example, a growth of technology provokes the growth of intermediate production and as a result the final production grows. Then the intermediate firms have more resources, they are either invested in R&D closing a positive loop, or they are distributed among shareholders, enlarging the positive loop linked to physical capital accumulation.

It is still possible to obtain more consequences. If technology grows and human capital remains constant, then the labour productivity decreases and, as consequence, intermediate production decreases. On the contrary, if human capital grows and technology remains constant, then productivity increases and intermediate production also grows. Therefore, depending on the behaviour of human capital, the growth of technology might affect positively or negatively on the growth of the final production. But we need to study more variables to determine the behaviour of human capital.

The three variables analysed in this subsection are considered to be dimensionless, actually they are variable indexes. At the same date, it is assumed that, technology can be used by any intermediate firms and that any workers have the same productivity.

The following subsection completes the examination of those variables associated to human capital accumulation.

### 2.3. – The Process of Learning

As was pointed out before, the model assumes that workers can dedicate a maximum number of hours per year to labour market, which was collected in *Total Time*. A share of that amount could be invested in learning, which enables us to define *Percentage Learning* and the remainder will be devoted to the labour market, which is denoted by *Percentage Working*. The introduction of these variables together with the participation of the labour productivity allows the definition of the productive factor:

$$Labour = (Total\ Time)(Percentage\ Working)(Productivity)$$

that was considered in the figure 2.

In order to analyse the proportion of non-leisure time that workers devote to the labour market, it is necessary to determine the price of an efficient work unit, the salary, which is obtained by using its marginal contribution to the intermediate production. Note that, if the salary and the number of workers of the economy are known, the labour income is determined. The specification of the salary implies that if a worker increases the time devoted to labour market or his productivity grows the salary decreases, as consequence of diminishing that time. Moreover, the salary grows while the final production increases.

It is assumed that the time dedicated to labour market is paid and the time invested in learning is not. Therefore when people devote time to learning, they have an opportunity cost. Following this idea, a worker will decide to invest time in learning on the current productive cycle if the future labour income compensates the lack of current salary. Assuming that the decision considers a temporal horizon of one year, we have:

$$Total\ Time(t)\ Percentage\ Working(t)\ Salary(t)\ Productivity(t) \geq \\ Total\ Time(t-1)\ Percentage\ Learning(t-1)\ Salary(t-1)\ Productivity(t-1)\ (1 + Interest\ Rate(t)).$$

The latter expression assuming the equality, could provoke strong instabilities in the system. To be more precise about this if the variables included in the previous specification remain more or less constant except for variables collected the percentages, then the evolution of those percentages will depend totally on initial conditions. Let us look at an example, if *Percentage Learning* at date  $t-1$  equals 0.2, then *Percentage Working* at date  $t$  could reach that value; but in the next productive cycle, the value of this variable would be 0.8 and in the following one, afresh, 0.2. The amplitudes of the oscillations could be even higher depending on the behaviour of the variables involved

in the previous expression. In this situation any variable would be affected by strong oscillations, in particular due to the influence of *Percentage Working* on the intermediate production, the final production would replicate the oscillations and, though the rates of growth of almost all the economies oscillate, the amplitudes are not so strong. Figure 4 shows the rates of growth the OCDE groups 30 member countries.

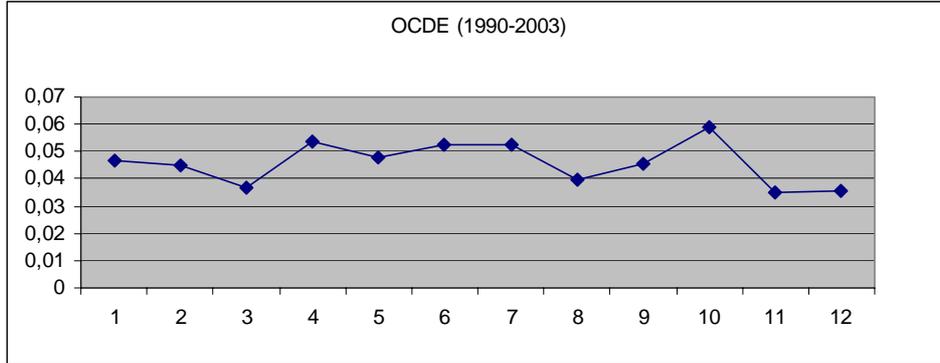


Figure 4: Rate of growth

In order to avoid this issue, it is possible to consider more criteria without eliminating the previous one, which is often used in macroeconomic literature. It seems obvious that workers would rather earn salaries such that their labour income do not diminish over time; otherwise, their consumption could decline in each productive cycle. Hence this new reason requires:

$$\frac{Total\ Time(t)\ Percentage\ Working(t)\ Salary(t)\ Productivity(t)}{Total\ Time(t-1)\ Percentage\ Working(t-1)\ Salary(t-1)\ Productivity(t-1)} \geq 1$$

Defining a new variable:

$$Desired\ Working = \min\{1, \max\{Percentage\ Learning(t-1)\ Ratio\ (1 + Interest\ Rate(t)), Percentage\ Working(t-1)\ Ratio\}\}$$

where

$$Ratio = \frac{(Salary(t-1)\ Productivity(t-1)\ Total\ Time(t-1))}{(Salary(t)\ Productivity(t)\ Total\ Time(t))}$$

is defined for simplification, the variable *Percentage Working* at date  $t$  must be higher than *Desired Working*, as consequence of the expressions verified by both reasons.

On the other hand, workers have at least a reason to invest time in learning. In fact, if human capital decreases, workers may have problems to handle technology. Then a worker would prefer to have a suitable level of human capital and his /her opportunity is to dedicate time to learning in order to:

$$Effect\ of\ Time\ to\ HC(t) \geq Effect\ of\ Time\ to\ HC(t-1)$$

Once again, it is possible to define a new variable:

$$Desired\ Learning = \min\left\{1, \left(\frac{1}{B\ Total\ Time(t)}\right) Effect\ of\ Time\ to\ HC(t-1)\ Human\ Capital(t)^{-\beta}\ Technology(t)^{\beta-1}\right\}$$

When replacing this variable into specification for the criterion, we find that *Percentage Learning* at date  $t$  must be higher than *Desired Learning*.

These three criteria determine an interval to which *Percentage Working* must belong:

$$\text{Percentage Working} \in [1 - \text{Desired Learning}, \text{Desired Working}]$$

Any point in this interval indicates the preferences of workers to activities of learning. Then, considering the convex combination of the extreme points, a new variable can be considered:

$$\text{Reference} = \alpha (1 - \text{Desired Learning}) + (1 - \alpha) \text{Desired Working} \quad (1)$$

where each  $\alpha \in [0,1]$  determines the weight that workers proportionate to each one of the variables desired. Each choice of  $\alpha$  determines the percentage of time devoted to the labour market and consequently the percentage invested in learning by workers in the economy.

Once again, assuming that the decision about the percentage of participation in the labour market takes time, the true value of *Percentage Working* is determined using the discrepancy between the variable *Reference* and the current value of the level *Percentage Working*. The parameter *Percentage Adjustment Time* is the adjustment time for the elimination of the gap between them.

Figure 5 shows the variables allowing the specification of this level.

Finally, it is assumed that, both the stock of physical capital and the time that workers devote to labour market are distributed among the intermediate firms in equal shares.

## 2.4. –The size of the intermediate sector

The intermediate sector presents some characteristics that would explain its influence in the growth of the economy. On one hand, the sector is assumed to have a free entry and exit and firms do not have legal or commercial barriers to entry or exit of it. Moreover, it is also assumed that firms sell similar, but not identical, products. Under these conditions, if a firm is profitable, it is possible to expect other firms to enter in the sector. If a new firm enters and it starts producing a product that is close substitute for the old one, then the demand for the old product would decline and this fact could imply that the firm would have to leave the sector. As consequence, the profit of any intermediate firm must be zero.

In spite of this fact, intermediate firms sell all their production to the final producer, fixing a mark-up on their variable costs. Then intermediate sector will get resources in each productive cycle. A fraction of these resources can be allocated to pay dividends and the remainder could be devoted to activities in R&D. Independently of the allocation of these resources, the decision will increase at least one of the factors used in the intermediate production and this production would consequently be improved.

But now, let us move on to others aspects related to the intermediate sector which let us solve different issues.

As we have already pointed out, final production is function of the intermediate production but that variable is also influenced by the number of intermediate firms. This is due to the fact that monopolistic competition characterizes the intermediate sector and the final producer obtains advantages of the specialization of this sector. Then a

new issue emerges, since it is also necessary to characterize the size of the intermediate sector.

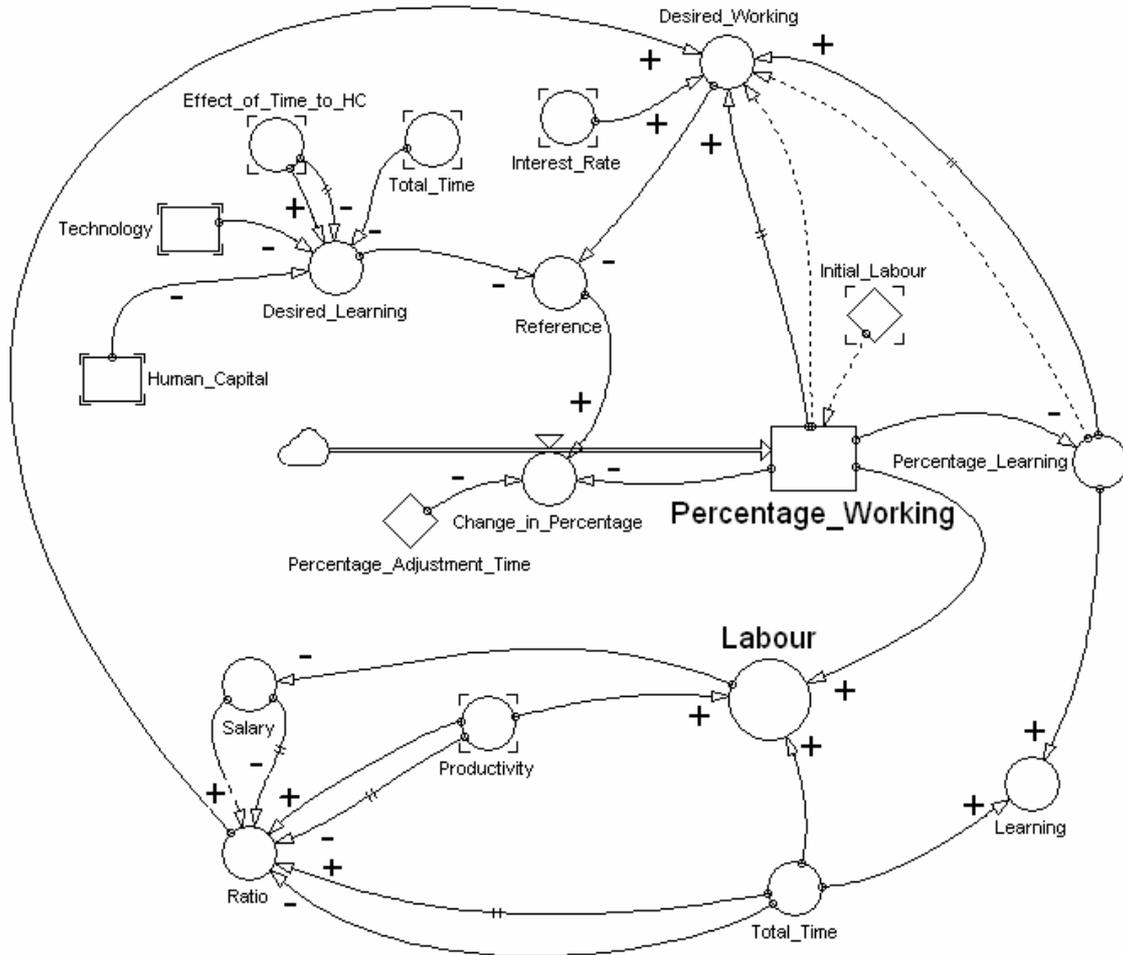


Figure 5: The auxiliary variable Labour

It is possible to check that the number of intermediate firms is related to the level of technology in economy. To find the relationship between both variables two facts are considered. On one hand, as mentioned before, each intermediate firm can allocate its available resources to two alternatives. One of them could boost the financial resources of the economy and the other one would improve the technology. On the other hand, technology is a productive factor and as consequence it may have a price. As the remainder of the productive factors, it could be paid by its marginal contribution to the intermediate production. If it was the case and final output grew, this hypothetical price would be increased and if technology grows this price should decrease. Then, this price can be used to determine whether a firm carries out activities in R&D. The idea is simple, if the price of technology for a firm coincides with the price of a unit of physical capital, this firm undertakes activities in R&D; otherwise the firm only distributes dividends. The relationship between the prices of these two factors implies that the

stock of the number of intermediate firms is directly related to the stock of physical capital but it is inversely related to the level of technology.

The previous condition can be expressed in other words: the firms' capital stock is proportional to the level of technology in economy. Then, while technology is low, the number of intermediate firms grows according to the growth of capital. If firms invest in R&D, the technology grows and the size of the intermediate sector is modified in order to adapt it to the relative growth of both stocks. These features are useful to characterize the transitional dynamics of the economies consistent with certain empirical evidences. In this regard Peretto (pp. 407) affirms: first the market grows by accumulating capital and after, technology will drive the size of the market.

Assuming that the firms' decision about to enter or exit of the sector does not have an immediate reply, once again it is assumed that the accumulation of this stock is governed by the discrepancy between a desired value and its current value. The desired value is defined assuming that the rate of return to investment equals the rate of return to R&D. The speed of accumulation of this stock is governed by a new adjustment time parameter. Then, the net change of this level will be:

$$\text{Change Firms} = (\text{Desired Firms} - \text{Number of Firms}) / (\text{Firms Adjustment Time}).$$

Note that, since final production depends on both intermediate production and the number of intermediate firms, the feedback loops in which final production participates are enlarged to consider these new relationships.

The arguments defended in this subsection completely specify the level *Firms*, but they do not determine the allocation of the available resources to each possible purpose. The assignment is assumed to be exogenous; in this way the variables associated to *Technology* are totally specified.

### 3. – Model analysis and results

Some considerations about the evolution of the variables involved in the system dynamics model can be obtained just by taking into account the causal influences among them.

The evolution of the final production is clearly influenced by the behaviour of two exogenous variables: consumption and investment in R&D. A policy of high consumption and a low flux of investments in R&D will drive to economic stagnation. Whereas if physical capital is accumulated as consequence of the level of consumption and there is no investment in R&D, the size of the intermediate sector is increased. Although the interest rate diminishes, both capital income and time percentage dedicated to labour market decrease. Then, the stock of human capital grows and as consequence the productivity grows too. These facts together with the saving growth would impel the growth of the economy in the short and medium term; however, the growth of those variables does not imply the same growth of final output and the rate of economic growth tends to a stationary situation<sup>3</sup> in the long term.

On the contrary, if the consumption is high and firms invest in R&D, technology grows in the short term but it does not grow in the long term. As the number of intermediate firms gradually diminishes and the growth of technology would not compensate the

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<sup>3</sup> Solow's model did not consider an intermediate sector.

modifications suffered by the remainder of the productive factors, this alternative will cause a gradual decline of the final production.

The saving together with a resolute policy of investments in R&D would provoke the growth of the final production; however, since the growth of the productive factors does not imply the same growth of the final output, once more, the rate of growth will tend to a stationary situation in the long term. It is clear that if similar decisions and policies are adopted by different economies, the stationary situation must not change. This characteristic allows the model to replicate certain facts stressed by Barro and Sala i Martin who affirm: *The empirical evidence support conditional convergence, which suggests that countries with similar preferences and technologies should converge to the same level and growth rate of per capital income. However, despite a tendency to converge, countries do not always follow a common path during the transition period. Some countries grow quickly during the early years of development, and then their growth slows as they approach the steady state. Other countries start off growing slowly, and then experienced a relatively short period of very rapid growth.*

The model is able to replicate these facts because a different degree of development in economies could be interpreted by differences on the initial value of the levels; a different way to do things could imply different adjustments for parameters of production or parameters associated to delays. Whereas the economies present some mix of those possibilities, the model would generate different responses about the different growth paths that economies undergo when they tend to a stationary situation.

### **3.1. – Simulating the labour productivity**

The variables related to the labour market, included in the dynamic system, allow the carrying out of a simulation analysis to study the responses of the model for two economies in which the number of hours dedicated by their workers to the labour market are significantly different.

In order to obtain simulation results, the time step is set equal to 0.25 years and the unit of time is the year. The simulations are run over a period of 45 years, the duration of a wide labour life. This period is enough to be appreciated as the rate of growth of human capital impacts on the productivity. The parameters associated to the production functions final output, intermediate output and human capital as well as the parameters associated to productivity and activities in R&D are selected using literature of economic growth. With regard to the adjustment parameters associated to delays, the corresponding to human capital and number of intermediate firms are set equal to two years; regarding technology, six years and finally, regarding percentage of time dedicated to the labour market, one year.

The initial values of the levels are selected assuming that there is a balanced equilibrium. The physical capital is initialised so that to the interest rate is close to 0.05%, value used in numerous studies in which a calibration is required. The number of intermediate firms is fixed by taking into account the initial values of both physical capital and technology. The human capital is initialised in order to make its change equal to zero and as consequence, this value depends on the initial values of both technology and percentage of time dedicated to labour market. However, the initial value of productivity is independent of the initial value of technology. Since the

analysis pursues to study the productivity rather than the rate of economic growth, it is assumed that the population is constant over time.

The initial value of the time percentage dedicated to the labour market, is directly related to the selected value for non-leisure time. An approach to determine the latter variable is done by taking into account the average worked hours in different countries. There are some problems associated to these averages because there are countries that do not publish their results and also, countries can use different methodologies to get the data. In spite of the fact that the averages vary across countries mainly due to the duration of the periods of vacations and the influence of unions, except for methodological changes, the averages remain more or less constant over time in each country. Regarding the data published by OECD, this organization states that the data are not suitable for comparing the levels of average annual hours of work for a given year among countries because of differences in their sources. According to the statistics of this organization, in 2004, the workers in South Korea put in an average of 2,380 hours, about 48 hours a week with two weeks vacation per year. The average in US is 1,812 hours similar than the Hungarians; in France and Germany the average is 1,360 hours; Netherlands has the lowest average 1,312 hours and the annual average across OECD countries is estimated in 1,925 hours that year.

It seems clear that the average worked hours are influenced by each economy's own factors, out of the scope established for this study. For this reason, the non-leisure time per year will be an exogenous variable. During the horizon of simulation it will be taken the value of South Korea. However, without enough data and without taking into account the peculiarities of a country, there are some risks involved at the conclusion of the exercise if two real countries are compared. Then from now on the references to real countries will be eliminated. The economies will be named the first and second economy.

The study assumes that the first economy has an average worked hours of 1,360 hours per year and the second 2,380 hours during the same time period. Then the initial values of time percentages dedicated to labour market in both economies will be set different to reflect that difference. That initial value, in the second economy, must be equal to 90%, which indicates that a worker devotes nearly five years of his labour life to learning. In the first economy, the initial value could be 50% doing a simple rule of three bearing in mind that both economies have a non-leisure time of 2,380 hours per year.

Most of the initial values of the levels are interconnected. The initial value of human capital depends on the initial values of the percentages selected above and the economies started with different values of human capital. But this initial gap in human capital affects to final production and as a result, the interest rate changes. Then the first economy has to modify the initial value of physical capital for achieving an interest rate next to 5%. This modification affects the initial number of firms value, which is also influenced by the initial value of technology.

The initial values of the levels associated to the formation of technology will complete the initial conditions of the levels. The technology that is not used in the productive process, is initialised so that its net change is zero. However, the productive factor has to be initialised exogenously. In addition, this initial value must be different for both economies not only the initial values of human and physical capital seem to indicate a

different level of development between both economies, but also workers devote initially a different number of hours to the labour market in each of them.

Finally, it is assumed that, during the simulation, the marginal propensity to consume takes the value 0.8 and the resources allocate to activities in R&D represent 2% of GDP. It seems to be important to emphasise that in this study, the policy of maintaining the values of the exogenous variables constants over time together with the introduction of the delays provokes that the variables follow behaviours without oscillations. The changes in consumption rather than the changes in the policy of investments in R&D would provoke instabilities on the evolution of the variables.

The system is not especially sensitive to changes of very little amplitude in parameters, initial conditions of the levels and exogenous variables. The model also behaves appropriately under extreme conditions.

The figure 6 shows the evolution of the rates of growth associated to four variables: final production, productivity, human capital and technology. The trajectories marked by one refer to the first economy. The remainder of the trajectories corresponds to the second economy when different choices with regard to preferences to learning are undertaken. Remember that different preferences imply different selections of the parameter  $\alpha$ , which was defined in (1), so the trajectory number two assumes  $\alpha = 0.9$ , number three  $\alpha = 0.8$  and so on until  $\alpha = 0.5$  that is the value adjusted for the other economy.

Looking at the trajectories collected in figure 6, it is feasible to obtain some conclusions. With regard to the paths one and two, it is worth mentioning that both economies keep the percentages of hours worked very close to their initial values during the simulation, which is a feature that is shown in any real economy. All the stocks grow in the simulations one and two, but whereas the first economy shows rates of growth stables, this characteristic is not so evident for the second economy since its rates of growth change of tendency slightly during the simulation. The different evolutions of the rates of growth together with the differences that both economies present in the initial conditions of the stocks would indicate that the economies have a different degree of development. The first economy could have already achieved a stationary situation. The second economy has more possibilities of promoting human capital accumulation and as consequence, its development.

Despite the differences between both economies, the results clearly show that similar economic decisions provoke different responses in the short and medium term, but in the long term the rates of growth converge. This was the fact stressed by Barro and Sala i Martin.

The remainder of trajectories shown in the figure 6 illustrates the growth of rates associated to the second economy. Observe that if the workers of the second economy modify their preferences in order to dedicate more time to learning, in the short and medium term, this economy shows a spectacular growth. Greater preference to learning implies greater growth of the rates. The structure of the model would justify the growth of the rates because of the growth of human capital. More time dedicated to learning implies greater growth of human capital and how the initial value of technology is low, the productivity begins to grow very rapidly.

However, the growth of the economy implies the growth of the resources allocated to R&D and the technology also starts growing. Then productivity continues growing until

technology achieves a specific level because human capital is accumulated more slowly than technology as a result of the strong economic growth. After this phase, the rates of growth, except for the innovation rate, begin to diminish as consequence of the technological progress. Once again, the convergence of the rates between both economies appears, regardless of the choice done by the workers.

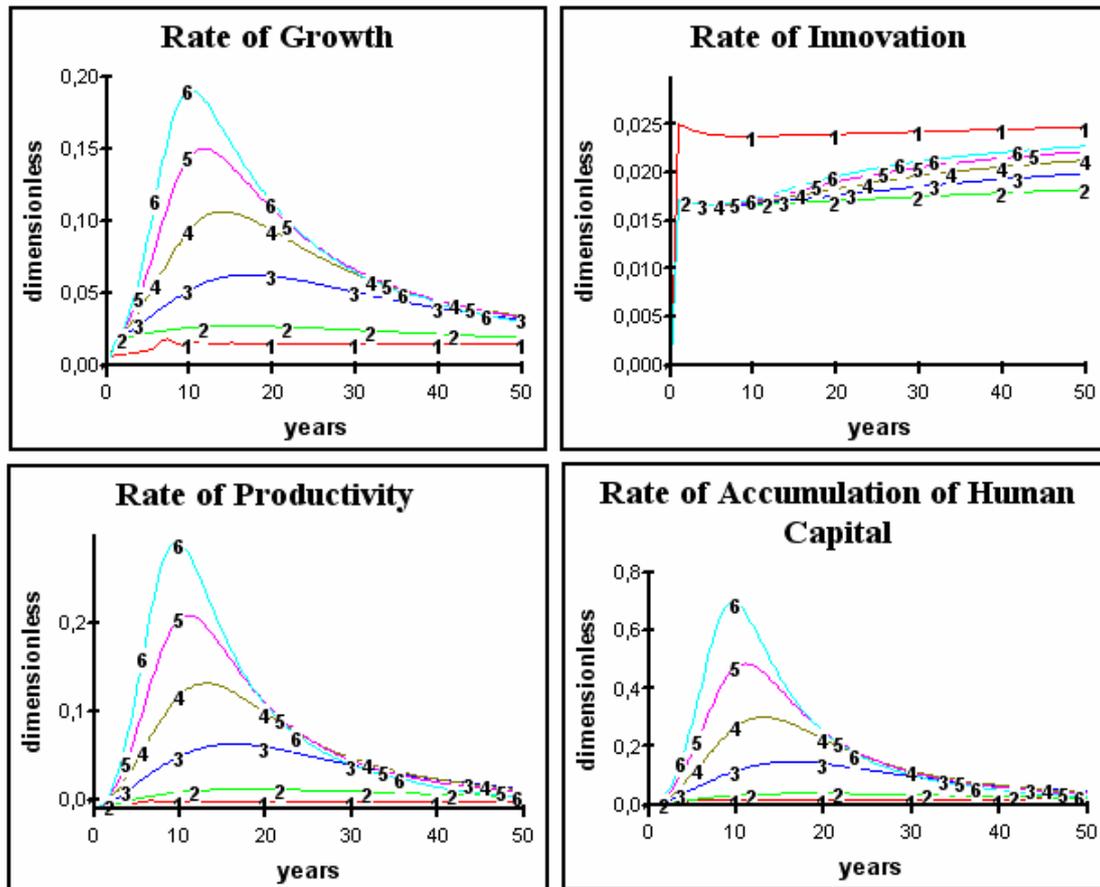


Figure 6: Rates of growth

Observe that, unlike the remainder of the rates show in figure 6, the innovation rate associated to the first economy always shows a slightly growing tendency, whereas the rates for the second economy always grow, though with different slopes depending on the preferences of the workers to learning. This is so because innovation rate evolution is independent of the relation between technology and human capital.

Finally, it seems important to emphasize that figure 6 does not show the values that the corresponding variables achieve during the simulations, otherwise its rates of growth. In this instance, a greater growth of the rates associated to the second economy implies a diminishing of the initial gaps between both economies. In spite of this fact, the results show that the differences between them are kept at the end of the simulation. However, the labour productivity of the second economy converges to the value achieved by the other economy; the gap between these variables is larger when the workers select to work more hours.

## 4. - Conclusions

This study, considering different data and references of economic growth literature, build up a dynamic system whose structure is supported by interrelations among those variables affecting to human and physical capital accumulation as well as technological change.

The model confirms why the saving together with a resolute policy of investments in R&D generate a sustained economic growth. The influence of positive feedback loops explains why the economic growth rate in the long term tends to a stationary situation, regardless of economies while they present similar preferences and structures. In this way, certain empiric facts are verified by the model.

The dynamic system model includes variables associated to the labour market, which let studying an emergent issue in the EU about the gain of decreasing the number of hours dedicated to the labour market. In order to analyse this question a simulation exercise is proposed, in which two economies are considered and compared. One economy is characterized by its workers devoting as much time to the labour market as they devote to learning, whereas the workers in the other economy, devote a high percentage of their non-leisure time to work. These differences provoke that both economies present a different level of development, which affects the evolution of their processes of growth. The results obtained show that the labour productivity of the developing economy could improve substantially if their workers modify the preferences in order to devote more time to learning. Moreover, the convergence between productivities could be a fact if developing economy workers replicate the preferences of the workers in advanced country. Despite this result, the evolution of the other variables involved in the system does not show a significant decrease of the initial gaps between both economies.

There are numerous opportunities for future research motivated by this study. Some could arise from the elimination of certain assumptions done in the study. The population growth, the creation of new products in advanced countries and their subsequent imitation by less developed countries, the diffusion of knowledge or the introduction of the unemployment are some factors that offer possibilities for enriching the analysis. These factors not only would let a contrast with real countries but also could show limitations to growth. The intervention in the economy of a public sector and its influence on taxes, corruption and regulations, just as cultural and social factors could explain different paths of growth. Finally, if the economy is open, the model would have to introduce exchanges across countries and the differentiation of products in connection with the trade would have to occupy a central role in the analysis.

## REFERENCES

- Azariadis, C. and J. Stachurski. 2005. *Poverty Traps*. Handbook of Economic Growth. P. Aghion and S. Durlauf, eds.
- Barro, R.J. and X. Sala-i-Martin. 1992. *Convergence*. Journal of Political Economy. 100, 223-251.

- Breton, T. R. 2004. *Can institutions or education explain world poverty? An augmented Solow model Provides some insights*. Journal of Socio-Economics. 33, 45-69.
- Heckman, J. J. and P. J. Klenow. 1997. *Human capital policy*. Working Paper. University of Chicago.
- Kejak, M., Seiter, S. and D. Vavra. 2004. *Accession trajectories and convergence: endogenous growth perspective*. Structural Change and Economic Dynamics, 15, 13-46.
- Kosempel, S. 2004. *A theory of development and long run growth*. Journal of Development Economics 75, 201-220.
- Lloyd-Ellis, H. and J. Roberts., 2000. Twin engines of growth: skills and technology as equal partners in balanced growth. Journal of Economic Growth, 7, 22-45.
- Lucas, R.E. 1988. *On the mechanics of economic development*. Journal of Monetary Economics, 22, 3-42.
- Meadows, D. H., D.L. Meadows, J. Randers and W. Behrens. 1972. *The Limits to Growth*. Universe Books. N.Y.
- Peretto, P. F. 1999. *Industrial development, technological change, and long-run growth*. Journal of Development Economics. 59, 389-417.
- Romer, P. M. 1990. *Endogenous technological change*. Journal of Political Economy, 98, 75-102.
- Saeed, K. 1988. *Wage Determination, Income Distribution and the Design of Change*. Behavioral Science, 33.
- Saeed, K. 2005. *Classical Economics on Limit to Growth*. Proceedings of Conference System Dynamics Society. Boston.
- Sice, P., Mosekilde, E., Moscardini, E., Lawler, L. and I. French. 2000. *Using system dynamics to analyse interactions in duopoly competition*. System Dynamics Review. 16, 113-133.
- Soto, M.D. and R. Fernández. 2005. *Trayectorias de desarrollo y crecimiento, un análisis sistémico*. 3º Congreso Latinoamericano y Encuentro Colombiano de Dinámica de Sistemas.
- Sterman, J.D. 2000 . *Business Dynamics. System Thinking and Modeling for a Complex World*. McGraw-Hill. Boston, MA.