Integrating Judgment and Outcome Decomposition: Exploring Outcome-based Learning Dynamics

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Abstract The authors present a preliminary theory of outcome-based learning under uncertainty. We created this theory by integrating social judgment theory and signal detection theory using the system dynamics approach. This integration offers a unique framework to explore the fundamental mechanisms associated with learning dynamics using outcome feedback because it allows the decomposition of the judgment process and its resulting outcomes.

1   Introduction

Learning in complex systems is an important area of scientific inquiry (Sterman 1994). It is especially difficult to learn in feedback-rich, uncertain environments. Learning from outcomes has received attention in the human judgment literature (Klayman 1988b, 1984). Studies on probability learning in laboratory settings have documented individuals' difficulties learning probabilistic relationships from outcome feedback (Klayman 1988a). However, there is also research that has shown that individuals seem to learn adequately about probabilistic relationships from outcome feedback in real-life situations (Hammond 1996). This divergence of findings suggests that the existing models of learning have limited applicability. Scholars have tried to address these limitations by building simple dynamic models which appear to explain

intermediate-term behavior better than long-term behavior (see Roth and Erev 1995; Funke 2001). Our model is further attempt to improve the explanatory power of learning models of this sort.

Learning in complex systems is often embedded in a selection-detection-action process. This process involves the identification of same-class elements in a mixed-class group and the decision to take action based on this identification. There are two important sources of error in this process, outcome decomposition and judgment. Although signal detection theory offers a process to understand uncertainty and to decompose outcome, it fails to address the mechanisms of judgment that lead to a particular selection. Social judgment theory, alternatively, offers a mechanism to decompose the judgment process that leads to detection, but fails to explore outcomes derived from decisions following the judgment. Integrating these theories may improve understanding of the selection-detection problem and associated learning dynamics in a holistic manner.

We integrate Brunswik's (1943; 1956) lens model of judgment analysis (Cooksey 1996; Hammond 1996) and signal detection theory (Egan 1975; Green and Swets 1966) using the system dynamics approach (Forrester 1961; Richardson and Pugh 1981; Sterman 2000) to understand the behavioral aspects of outcome-based learning in the selection-detection process.

We are not interested in the identification of optimal feedback control for the decision threshold, because "the optimal rule can be rejected as a descriptive model of human performance based on past research" (Busemeyer and Myung 1992, p. 177). Similarly, Green and Swets (1966) have shown that "observer[s] tend[s] to avoid extreme criteria" in the case of moving thresholds even when an extreme movement is required to achieve optimal results.

The impetus for this work has come from the 2004 Workshop of the Security Dynamics Network at Carnegie Mellon University in Pittsburgh, PA. This paper is a formal statement of the abstract decision model conceptualized at that meeting and related work.

2   Theoretical Framework

This section explains the basic tenets of the theories used in this integration effort: social judgment theory and signal detection theory.

2.1   Social Judgment Theory

Social judgment theory (SJT) comes from Egon Brunswik's (1943; 1956) probabilistic functionalist psychology coupled with multiple correlation and regression-based statistical analyses (Hammond 1996; Hammond and Stewart 2001; Hammond et al. 1975). In his work, Brunswik's (1956) main assumption is, "that the environment is, from the point of view of the organism, fundamentally probabilistic," and, therefore, uncertain (Balzer, Doherty, and O'Connor 1989, p. 431).

Decomposition of judgment is one important aspect of SJT. It is the identification of the main elements of the judgment process and the analysis of their interactions. According to SJT, judgment tasks comprise information cues, organizing principles of information integration, relative weights of information cues, and distinct functional forms that relate information cues

with judgment. Social judgment theory uses the lens model proposed by Brunswik (1956) to represent the relationships between information cues, the phenomenon being judged (distal variable), and the judgment (see Figure 1). At the center of the lens model, information cues are represented by circles and identifiers (in Figure 1, see $X_1 \ldots X_5$) that form a vector of information available to the judge. Curved lines connecting the cue circles represent the correlations, r_n, that potentially exist between pairs of cues. The left side of the model represents the focus of judgment, that is, the environment (Y_e).
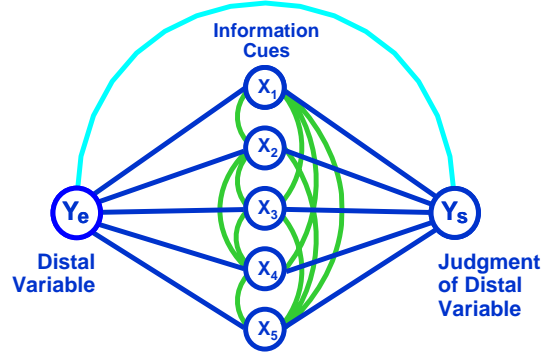

Figure 1. The Lens Model.

Information cues, when combined in a specific way, are predictors of the distal variable. In SJT, the most common way to model the environment is to use a weighted additive linear combination of the information cues complemented by bias and error parameters. The lens model, "embodies a truism that in order for a person to be able to predict some external criterion, that external criterion itself must be predictable," (Balzer, Doherty, and O'Connor 1989, p. 410). Equations 1 and 2 represent the general model for both the environment and the judge:

$$Y = \hat{Y} + e \qquad \qquad \textbf{(1)}$$

where

Y is the distal variable or the judgment of the distal variable (Y_s or Y_e in Figure 1),

$\hat{Y}$ is an estimate of Y, and

e is an indicator of the inherent unpredictability of the environment or is the degree of the judge's reliability.

$$\hat{Y} = b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + k \qquad \qquad \textbf{(2)}$$

where

$\hat{Y}$ is an estimate of Y,

b_n is the weight of information cue n on the distal variable or the judgment,

X_n is the information cue n, and

k is a bias term.

2.2   Signal Detection Theory

In detection systems, the decision-making process includes the comparison of the current status of indicators with thresholds that determine the preferred course of actions. The use of thresholds represents a simple decision rule that determines how the stimulus space is partitioned. This approach is also known as the cutoff rule. For an exploration of how decision rules are selected see Busemeyer and Myung (1992). The threshold model of decision making is well known in basic and applied research on decision making (Jones and Ryan 1997; Hammond 1996, 2000; Mumpower, Nath, and Stewart 2002). Signal detection theory is useful when decision outcomes depend on discrete events (Swets 1973; Swets, Dawes, and Monahan 2000; Hammond et al. 1992).

One area for the application of SDT is the detection of potential attacks on computer systems in an environment of uncertain information. Figure 2 represents a detection problem related to the identification of attack events. The X axis represents the probability that an unusual or suspicious event is the result of an attack. This probability may be an indicator produced by an automated system or an individual's subjective judgment based on observations of system activity. The bell-shaped curve on the left represents the distribution of that indicator for non-attack events. The bell-shaped curve on the right represents the distribution for attack events. The areas are not necessarily proportional to the probabilities of these events. The decision threshold line represents the point at which the decision maker decides to treat the event as an attack. Four possible outcomes can occur depending on the decision maker's action. Two of these outcomes—unrecognized attacks and normal events identified as attacks—are errors (see shaded areas in Figure 2). The decomposition of outcomes into four categories—true positives, true negatives, false positives, and false negatives—allows for the creation of an outcome-based feedback mechanism that can inform detection processes in the future. See Erev (1998; 1995) for studies that have examined SDT empirically as a behavioral theory.
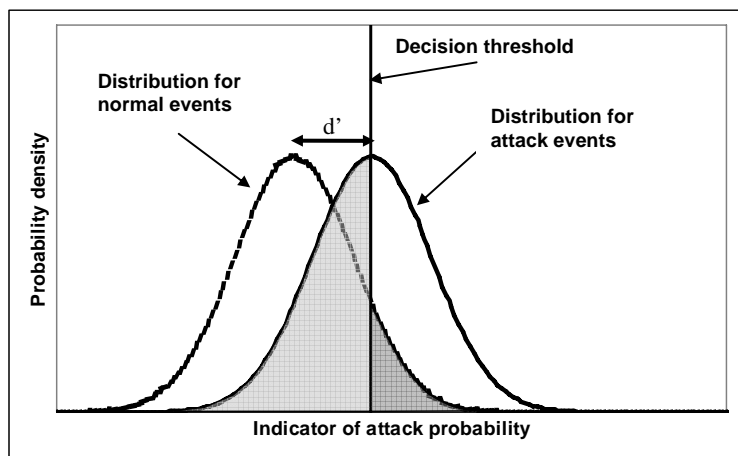

Figure 2. Signal Detection Theory.

2.3 System Dynamics

System dynamics is a type of computer simulation. System dynamics uses ordinary differential equations to create computer simulations that provide endogenous explanations for feedback behavior exhibited by systems (Forrester 1961; Richardson and Pugh 1981; Sterman 2000). Scholars have used system dynamics for theory integration efforts. Studies of this sort that use system dynamics are varied and diverse, including financial theory (Imai 1972), human resources theory on job security (Repenning 2000), service operations (Oliva 1996), and firm survival (Scholl 2002).

Using system dynamics as architecture to link SDT and SJT provides two benefits. First, system dynamics supports explicit feedback from prior system states to current ones. In our model, we use this type of feedback to capture the learning process. In addition, system dynamics allows us to model how cue weights shift based on outcomes because it allows nonlinear and accumulating effects to be represented.

3   Judgment and Decision-Making Model

The model presented in this paper has three basic mechanisms—judgment, decision making, and threshold modification (based on outcome identification and use). Figure 3 is a schematic of the model structure. There are four categories of outcomes: true positives, true negatives, false positives, and false negatives.

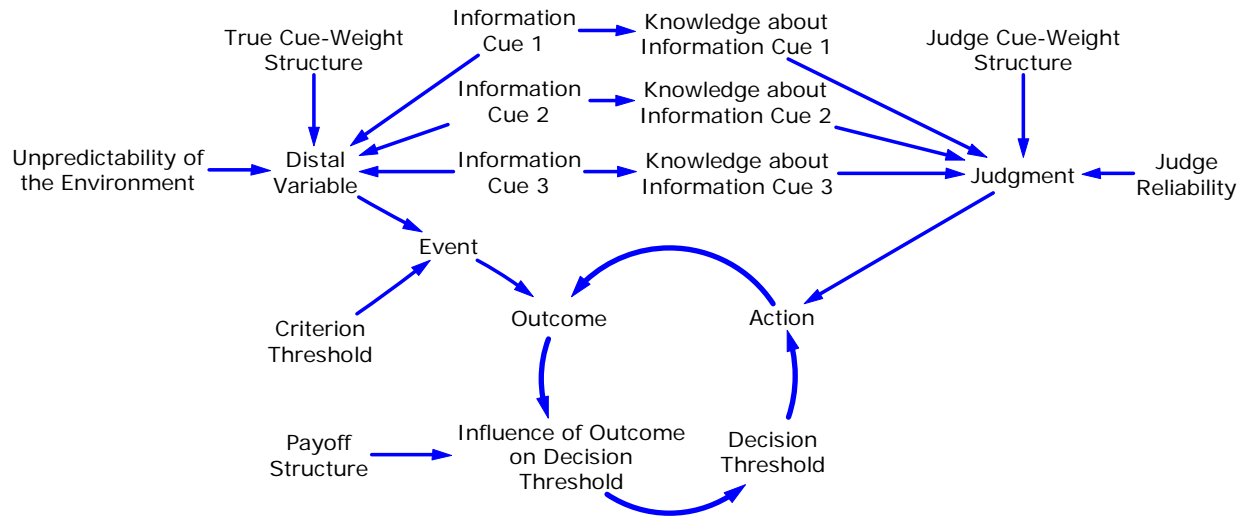**Figure 3.** Simplified Model Structure

3.1 Structure of the Model

We model judgment and decision-making processes as a modified version of the lens model framework and a threshold-based decision process. In the model, we use a three-cue structure

(*Information Cue 1*, *Information Cue 2*, and *Information Cue 3*) to identify the behavior of the distal variable (the phenomenon to be detected). Information cues are variables that identify relevant aspects of certain phenomena. The three cues are formulated as stochastic variables. The statistical characterization of the descriptors of the distal variable is identical for the three cues. The cues follow a truncated normal distribution that varies between –3 and +3 centered on 0 with a standard deviation of 1. The distal variable is also determined by an error term that captures its inherent unpredictability. In the case of a perfectly predictable environment, the associated error term is 0.

Attacks are determined by comparing the value of the distal variable with the criterion threshold: the definition of what constitutes an attack. When the distal variable is greater than or equal to the criterion threshold (CTh), an attack is created. The criterion threshold may be altered to adjust the number of attacks generated from the random cues.

$$E = \begin{cases} 1 & \text{if } Y_e > CTh \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Where

E is a binary variable indicating the occurrence, or not, of an attack,

Y_e is the *Distal Variable* in the form of Equation 2, and

CTh is the *Criterion Threshold* $[-3 \leq CTh \leq +3]$.

In the model, judges do not have access to the true descriptors of the distal variable. Information acquisition is represented by the introduction of a modified set of information cues—*Knowledge about Information Cue*—that captures the reliability of the information acquisition process by incorporating measurement error. Judges generate their judgments based on whatever knowledge they are able to acquire related to the true descriptors of the distal variable. If measurement error is zero, the reliability of information acquisition is 100%, representing the case of perfect information in the judgment process. The judge model also depends on a parameter that captures judge bias, and on a parameter that captures judge reliability (modeled as a stochastic variable).

When the judgment of the distal variable Y_s is equal to or greater than the corresponding decision threshold DTh, an action A is launched as a response to the identification of an attack. In this sense, the judgment process begins with an evaluation of cues. After the judged result is compared to a predefined decision threshold, a decision to act, or not, is made. Judgments are precedents to decisions. Decisions are made based on both the outcome of the judgment process and the judges' definition of what constitutes an attack. Through the decomposition of the judgment process and of the threshold-based decision mechanism, different sources of error can be identified: poor judgment capability and poor decision-threshold setting. Equation 4 shows the decision-threshold model for determination of action.

$$A = \begin{cases} 1 & \text{if } Y_s > DTh \\ 0 & \text{otherwise.} \end{cases} \qquad \textbf{(4)}$$

Where

A is a binary variable indicating the occurrence, or not, of an action,

Y_s is the *Judgment of the Distal Variable*, and

DTh is the *Decision Threshold* [$-3 \le DTh \le +3$].

Outcomes, the results of actions, are modeled using a binary formulation in which "1" is assigned if the conditions for that type of outcome are met and "0" is assigned otherwise. Four types of outcomes are possible: true positives ($E = 1 \wedge A = 1$), true negatives ($E = 0 \wedge A = 0$), false positives ($E = 0 \wedge A = 1$), and false negatives ($E = 1 \wedge A = 0$).

Social judgment theory (Hammond 1996, 2000; Hammond et al. 1992) distinguishes between feedback from knowledge of the results of actions (outcome feedback) and from knowledge of cognitive mechanisms that created those results (cognitive feedback). In the model, outcome feedback is used to modify the decision threshold as a result of the relative influence of the four types of outcomes generated in the decision process.

Each type of outcome has a different relative influence on the position of the decision threshold. The distribution of outcomes, however, is a function of the level of the decision threshold. Thus, modifications to the decision threshold change the distribution of the outcomes that are being used to modify the threshold. This situation can lead to the identification and adoption of an optimal decision threshold or to the adoption of a suboptimal decision threshold. The final decision threshold is a function of the updating mechanism and of the payoff matrix associated with the different types of outcomes. We use a simplified version of Erev's (1998) cutoff reinforcement learning model and a modified version of the Erev et al. (1995) outcome payoff matrix in our model. We use a simplified version of Erev's (1998) cutoff reinforcement model to represent a reactive detection process that is influenced by outcomes as soon as they are perceived. Equations 5 and 6 show the model for the decision threshold.

$$DTh = \int (DThCh)dt + DTh_{t0} \qquad \textbf{(5)}$$

Where

DTh is the *Decision Threshold*,

$DThCh$ is the *Change to the Decision Threshold,* and

DTh_{t0} is the value of *Decision Threshold* at time zero (in this case, 0).

$$DThCh = \begin{cases} \dfrac{\sum_{i=1}^{4}\left(PIO_i \cdot O_i\right)}{\tau} & \text{if } MinDTh < DTh < MaxDth \quad \textbf{(5)} \\ 0 & \text{otherwise.} \end{cases}$$

Where

$DThCh$ is the *Change to the Decision Threshold,*
PIO_i is the *Payoff Influence of Outcome "i,"*

O_i is the *Outcome "i,"*

τ is the *average time to change the decision threshold*,
$MinDTh$ is the *minimum level of the decision threshold* (in this case -3),
$MaxDTh$ is the *maximum level of the decision threshold* (in this case +3), and
DTh is the *Decision Threshold* [$MinDTH \leq DTh \leq MaxDTH$].

3.2   Numerical Simulations

The model is parameterized to create two base runs (*fixed base*, *dynamic base*) in which cautious judges select a conservative decision threshold and apply it consistently as a decision policy over time. In these runs, the judges have neither perfect information (measurement error exists) nor the optimal judgment weights. The judges have a fixed decision threshold set at 0 standard deviations from the mean, and produce a selection rate of 50% as well as a degree of judgmental consistency R_s (see Stewart 2001) of 50%. The environment has a criterion threshold set at 1.2815 standard deviations from the mean of the cumulative probability distribution, a base rate of 10%, and a degree of predictability R_e (see Stewart 2001) of 80%. With this parameterization, the judges are 36.8% accurate in getting the optimal cue weights.

In the *fixed-base* run, there is no outcome feedback present and all judgments are compared to the same decision threshold of 0. 100 weeks of transactions are simulated, and of the 40 events generated, the judges are able to correctly identify 26, yielding an accuracy of judgment (also called sensitivity) of 65% (measured as the percentage of events correctly detected of the total number of events). The judges generate 216 actions (190 false positives, and 26 true positives) and 204 total errors (190 false positives and 14 false negatives). Most of these errors are not problematic (the false positives), but the 14 false negatives can have tremendous consequences if they are attacks. These numerical results suggest that there is a more suitable decision-threshold level that reduces the total number of errors. To find this decision threshold we also simulate a dynamic decision threshold.

In the *dynamic-base* run, the judges start the judgment and decision-making processes with a decision threshold of 0 and adjust it over time. Knowledge from past outcomes drives this adjustment process. The introduction of outcome feedback and the consequent dynamic decision threshold yields a higher accuracy of judgment (77.50%) than in the *fixed-base* run. The judges are able to correctly identify 31 of the 40 simulated attacks by generating 274 actions (58 actions

more than in the *fixed-base* run). Of the 274 actions, the judges make 243 errors (false positives), allowing 252 total errors to be produced (including 9 false negatives).

To continue exploring the parameter space, four additional scenarios are simulated—with both fixed and dynamic decision thresholds. See Table 1 for the results of all the simulation runs.

- ***Perfect Information***. By eliminating measurement error of information cues, this set of runs simulates the availability of perfect information for the judges.
- ***Perfect Weights.*** The degree of fit of the judges' model to the optimal cue weights is 100%.
- ***Consistent Judge***. The degree of judgmental consistency is set to 100%.
- ***Combined***. This set of runs includes all the changes of the previous runs, simulating a perfect judge who has access to perfect information and who has perfect knowledge of the true weights for the predictors of the distal variable. The environment, however, remains 80% predictable, not 100%.

Under the conditions of perfect information, using a fixed threshold yields a sensitivity of 50% (which matches the selection rate), while, with a dynamic threshold, the sensitivity grows to 77.5%. The introduction of perfect information produces a marginally negative impact on outcomes, at least in this region of the parameter space, compared to the base run (see Table 1). The consistent, perfectly reliable judges, trigger fewer detection actions and fewer total errors, yielding a similar accuracy of judgment (55% and 77.5%). In these runs, improvement comes from the increased reliability of judgments, which creates more consistent judgment patterns, and thus, more true positives. The assumption of very consistent judges is more realistic than that of judges with perfect information.

Simulating the introduction of optimal weights is akin to saying that the relationship of the information cues and the distal variable is knowable. These simulations show that accuracy of judgment improves from 65% to 75% with fixed thresholds and from 77.5% to 85% with dynamic thresholds. Additionally, the number of false negatives decreases.

Table 1. Numerical Simulation Output

	Fixed	Dynamic	Fixed Perfect Info	Dynamic Perfect Info	Fixed Perfect Weights	Dynamic Perfect Weights	Fixed Consistent Judge	Dynamic Consistent Judge	Fixed Combined	Dynamic Combined
True Positives	26	31	20	31	30	34	22	31	39	37
False Negatives	14	9	20	9	10	6	18	9	1	3
False Positives	190	243	193	269	179	226	168	229	162	170
True Negatives	169	116	166	90	180	133	191	130	197	189
Events (TP + FN)	40	40	40	40	40	40	40	40	40	40
Actions (TP + FP)	216	274	213	300	209	260	190	260	201	207
Number of Errors (FP + FN)	204	252	213	278	189	232	186	238	163	173
Non-Events (TN+FP)	359	359	359	359	359	359	359	359	359	359
Total Judgments	399	399	399	399	399	399	399	399	399	399
Sensitivity (TP/Total Events)	65.00%	77.50%	50.00%	77.50%	75.00%	85.00%	55.00%	77.50%	97.50%	92.50%
Specificity (TN/Total Non-Events)	47.08%	32.31%	46.24%	25.07%	50.14%	37.05%	53.20%	36.21%	54.87%	52.65%

Asymmetrical Payoff Structure

True Positives	0
False Negatives	(10)
False Positives	(1)
True Negatives	0

In the combined simulation runs there is perfect information and there are perfectly reliable judges with perfect knowledge of the optimal cue weights. These runs generate an accuracy of judgment of 97.5% in the fixed run and 92.5% in the dynamic run, with the correct identification of 39 and 37 of the 40 generated attacks, respectively. The difference between the two cases is worth exploring to see if it is due to chance or structural components. This result is interesting because the inherent predictability of the environment is 80%. However, even under these benign conditions, the total number of actions and errors generated is very large (201 and 207, and 163 and 173, respectively). Although almost all errors are false positives, these are errors nonetheless.

The simulated detection process does not efficiently define a threshold that could eliminate errors, even when it detects events efficiently (sensitivity of 97.5% and 92.5% in the combined case).

4   Conclusions and Future Research

Incorporating judgment decomposition and outcome decomposition in a dynamic framework holds promise to enhance understanding of the judgment-decision-action process. The approach taken in this research has a number of opportunities and problems associated with it. Theory integration is a delicate enterprise, with potential yield for understanding the mechanisms present in detection processes. The research presented here still requires refinement of the model's structure and exploration of current decision threshold–adjustment behavioral theories (Erev 1998; see Erev et al. 1995). Additionally, data must be collected to identify descriptive time-series data to continue with the process of confidence building. Although "all models are wrong," as Sterman (2002) claims, the creation of an integrated model of judgment and decision making for detection problems like the one presented here can aid in the understanding of the implications and complications of these processes in complex situations.

Acknowledgments

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