Stable Stock Management Heuristics when the Outflow is Proportional to the Control Stock[1]

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In this paper, the typical anchor (expected value of the outflow or expected loss) used in the most popular decision rule of the stock management modeling, the "Anchoring and Adjustment Rule" is studied for structures including a decaying stock. A new anchor (equilibrium value of loss) is proposed and compared with the expected loss formulation. We demonstrate that equilibrium value of loss formulation helps bringing the control stock to its desired level more rapidly. In addition, we show that managing a decaying stock in a stable way is difficult when the supply line is discrete. Standard stock adjustment and supply line adjustment terms anchored around expected loss can yield highly unstable oscillations. Counter-intuitively, for some cases, ignoring the supply line adjustment term may completely eliminate unwanted oscillations. If equilibrium value of loss is selected as the anchor and when the decay time (life time) is small enough, management of the stock can even be done by ignoring all the adjustment terms.

Keywords: stock management, anchoring and adjustment rule, decaying stock, expected value of flow, equilibrium value of flow, counter intuitive behavior, anchor, stock adjustment term

INTRODUCTION

Anchoring and adjustment rule is a well known rule in the literature. Consider management of a stock. People, mostly, first call a reference point that we call "anchor" and make necessary corrections that we call "adjustment" to determine the desired order rate (Sterman, 1987, 1989; Özevin, 1999; Barlas and Özevin, 2004). If there is no anchor term in the decisions, this will yield steady state error and if there is no adjustment term, seeking the desired level (goal) is not possible, so both terms are required to make the stock achieve its goal. Note that, goal seeking without any adjustment term is possible, if anchor is *Equilibrium value of loss* instead of *Expected loss*.

In stock control management, it is important to bring the stock to its desired level rapidly, yet in a stable manner. In System Dynamics literature, the effect of adjustment terms (*Stock adjustment* and *Supply line adjustment*) on the behavior of the control stock is well discussed (Forrester 1961, 1973; Sterman, 1989, 2000; Yasarcan 2003). It is known that decreasing *Stock adjustment time* makes the control stock more responsive, but this may bring instability. To prevent instability, *Supply line adjustment* term must also be included in the decision equation (*Control flow* equation). A specific and important case is when the

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Supply line adjustment time is selected to be equal to the *Stock adjustment time*, then the control stock and its supply line effectively reduces to a single stock, and it is a well known fact that a single stock can not oscillate (Sterman, 2000; Yasarcan 2003). A stable and fast response in the control stock can be obtained with the help of this fact. Note that, this statement is not valid for a decaying stock with a discrete supply line.

Firstly, the effect of the parameters will be summarized in this paper and then we will focus on the anchor term. The outflow of the stock can be independent from the stock or it can be dependent to the stock like in decaying stock case. In most of the System Dynamics models the outflow (smoothed value of the outflow; *Expected loss*) is included in the control equation as the anchor for both independent and dependent outflow (i.e. decaying stock; outflow proportional to the control stock) cases. *Expected loss* is a good anchor in most of the cases but for decaying stock case, we propose using a new anchor that we call *Equilibrium value of loss*. We demonstrated that for a decaying stock with a second order supply line, the *Equilibrium value of loss* is more effective than *Expected loss* in bringing the control stock to its desired level, when there is a minimum limit for the value of the *Stock adjustment time*. Furthermore, in obtaining a good behavior one has to play with the value of the *Weight of supply line* (ratio of *Stock adjustment time* and *Supply line adjustment time*) if he is using expected loss as the anchor. This is not necessary when equilibrium value of loss is the anchor. Note that, if *Weight of supply line* is equal to zero, this means that *Supply line adjustment* term is ignored completely in the *Control flow* equation, if it is equal to one (*Supply line adjustment time* is equal to *Stock adjustment time*), this means that the *Supply line* has equal weight with the *Stock* in the *Control flow* equation.

Another case is when the *Supply line* is discrete (infinite order). We demonstrated that, managing a decaying stock with a discrete supply line is not easy. Counter intuitive results can be observed; unstable behavior can be observed even when two adjustment times (*Stock adjustment time* and *Supply line adjustment time*) have the same value (*Weight of supply line* is equal to one) or ignoring the supply line (*Weight of supply line* is equal to zero) may eliminate instability. For the discrete supply line case, if the decision maker selects *Expected loss* as the anchor, he has to play with two parameters, which are the *Stock adjustment time* and the *Weight of supply line*, to obtain good results, but if he selects the *Equilibrium value of loss* as the anchor, he can control the *Stock* with playing a single parameter, which is the *Stock adjustment time*. Furthermore, if *Life time* (decay time) of the control stock is small enough, *Equilibrium value of loss* necessitates no adjustment term in the decision flow.

A GENERAL MODEL STRUCTURE WITH A SECOND ORDER SUPPLY LINE AND A DECAYING STOCK

Consider the model in Figure 1. The aim is to bring the *Stock* to its desired level (goal) that is the *Desired stock* and maintain *Stock* at that level. For this, orders are placed at *Control flow* and after passing two stages they reach to *Stock*.

For completeness, first of all, the level (stock) equations will be given:

$$Stock(t) = Stock(t-DT) + (Acquisition\ flow_2 - Loss\ flow) \bullet DT \quad (1)$$

$$Supply\ line_1(t) = Supply\ line_1(t - DT) + (Control\ flow - Acquisition\ flow_1) \bullet DT \tag{2}$$

$$Supply\ line_2(t) = Supply\ line_2(t - DT) + (Acquisition\ flow_1 - Acquisition\ flow_2) \bullet DT \tag{3}$$

$$Expected\ loss(t) = Expected\ loss(t - DT) + Expectation\ adjustment\ flow \bullet DT \tag{4}$$

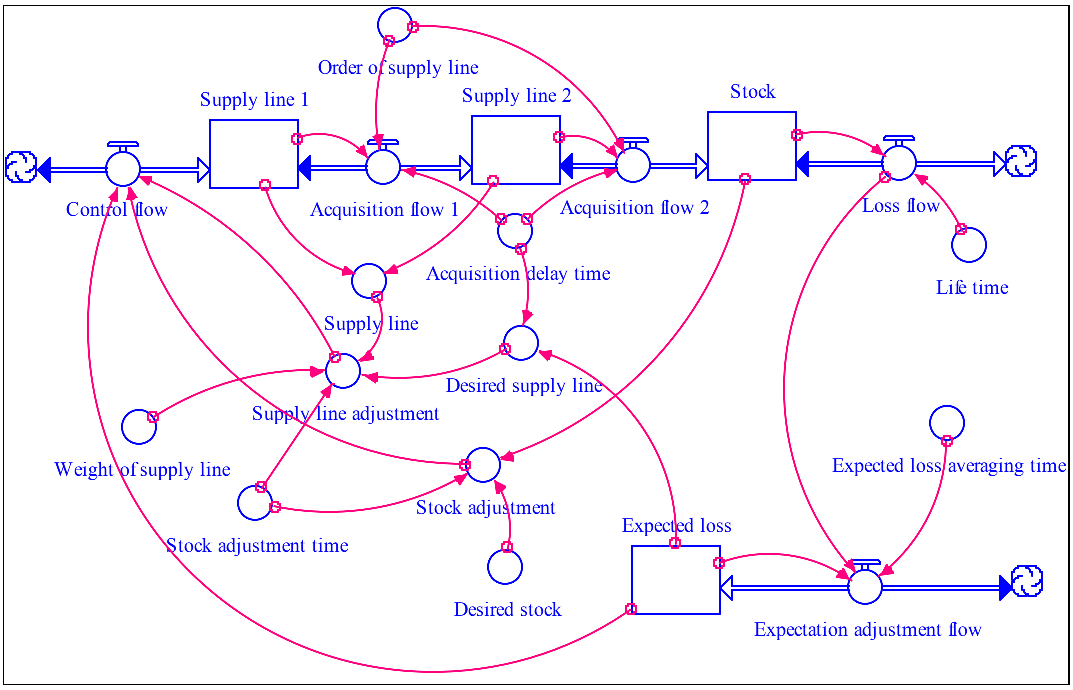

Figure 1. A stock management structure involving second order material supply line and a decaying stock

The flow equations are as follows:

$$Control\ flow = Expected\ loss + Stock\ adjustment + Supply\ line\ adjustment \tag{5}$$

$$Acquisition\ flow_1 = \frac{Supply\ line_1}{Acquisition\ delay\ time\ /\ Order\ of\ supply\ line} \tag{6}$$

$$Acquisition\ flow_2 = \frac{Supply\ line_2}{Acquisition\ delay\ time\ /\ Order\ of\ supply\ line} \tag{7}$$

In the *Control flow* equation, the anchor is *Expected loss* and the adjustment term consists of two parts; *Stock adjustment* and *Supply line adjustment*. *Expected loss* is obtained by exponential smoothing:

$$Expectation\ adjustment\ flow = \frac{Loss\ flow - Expected\ loss}{Expected\ loss\ averaging\ time} \tag{8}$$

Stock is decaying proportional to its own level:

$$Loss\ flow = \frac{Stock}{Life\ time} \tag{9}$$

The other equations are:

$$Desired\ supply\ line = Acquisition\ delay\ time \bullet Expected\ loss \tag{10}$$

$$Order\ of\ supply\ line = 2 \tag{11}$$

$$Stock\ adjustment = \frac{Desired\ stock - Stock}{Stock\ adjustment\ time} \tag{12}$$

$$Supply\ line = Supply\ line_1 + Supply\ line_2 \tag{13}$$

$$Supply\ line\ adjustment = Weight\ of\ supply\ line \bullet \frac{Desired\ supply\ line - Supply\ line}{Stock\ adjustment\ time} \tag{14}$$

Supply line adjustment can also be given as

$$Supply\ line\ adjustment = \frac{Desired\ supply\ line - Supply\ line}{Supply\ line\ adjustment\ time} \tag{15}$$

Equations 14 and 15 are identical since

$$Weight\ of\ supply\ line = \frac{Stock\ adjustment\ time}{Supply\ line\ adjustment\ time} \tag{16}$$

Weight of supply line is the weight given to *Supply line* with respect to *Stock* by the decision maker. Here we prefer to use Equation 14 instead of Equation 15 to see the relative importance of *Supply line* with respect to *Stock*, clearly.

All the level variables are initiated at their equilibrium levels:

- $Stock(0) = Desired\ stock$
- $Expected\ loss(0) = Loss\ flow$
- $Supply\ line_1(0) = Desired\ supply\ line\ /\ Order\ of\ supply\ line$
- $Supply\ line_2(0) = Desired\ supply\ line\ /\ Order\ of\ supply\ line$

Desired level of *Stock* is set to nine and it is increased by one unit at time five arbitrarily to perturb the system from its equilibrium:

$$Desired\ stock = 9 + \mathrm{STEP}(1,5) \tag{17}$$

THE EFFECT OF THE MODEL PARAMETERS ON THE BEHAVIOR

Now we will demonstrate the effects of the model parameters; *Acquisition delay time*, *Stock adjustment time*, *Life time*, *Expected loss averaging time* and *Weight of supply line* on the model in Figure 1. We will examine the parameters one by one and set the others to some selected values so that the effect of the parameter under examination will be seen clearly.

Effect of Acquisition Delay Time

If the value of *Acquisition delay time* is low, to obtain a stable and a fast response in the control stock is easier. If it is high it may even cause unstable oscillations. When it gets bigger, the strength and the period of the oscillations grow. The runs for the different values of *Acquisition delay time* can be seen in Figure 2. Note that if value of *Weight of supply line* is one, we will not have unwanted oscillations (this true in general but as will be present, decaying stock with a discrete supply line is an exception). In stable case, i.e. *Weight of supply line* is one, the only effect of *Acquisition delay time* is when it is increased, the delay in the response of the control stock will also be increased. Note that the effect of the *Acquisition delay time* that we summarized here is also valid for the non-decaying stock case.

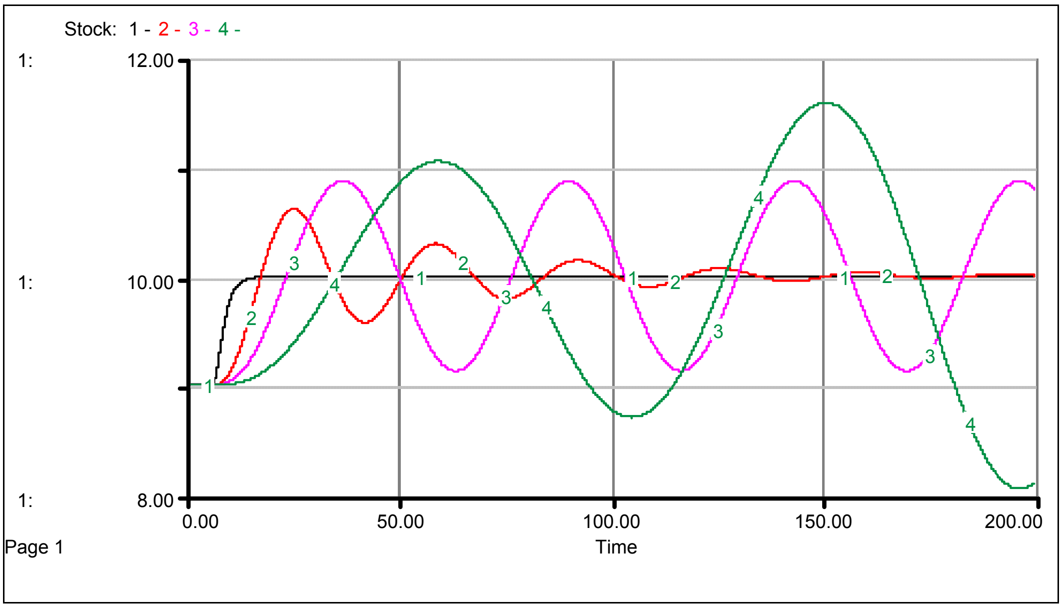

Figure 2. Runs for *Acquisition delay time* equal to 1 (goal seeking), 10 (stable oscillation), 21.4 (neutral oscillation) and 50 (unstable oscillation) respectively for the 1st, 2nd, 3rd and 4th runs

The values of the parameters (except *Acquisition delay time*, since the different values for this parameter are already given in the name of the figure) for the runs in Figure 2 are:

- *Stock adjustment time* = 3
- *Life time* = 40

- *Expected loss averaging time* = 2
- *Weight of supply line* = 0

Effect of Stock Adjustment Time

If the value of *Stock adjustment time* is low, this may result in unstable oscillations, and if it is high, this may result in slow response in the control stock. When it is decreased, the period of the oscillations also decreases but the strength of the oscillations increases. The runs for the different values of *Stock adjustment time* can be seen in Figure 3.

Note that if value of *Weight of supply line* is one, we will not have unwanted oscillations and decreasing the value of the *Stock adjustment time* will make the response of the control stock faster. However, the response in the control stock can only be fast to a point that is determined by the value of the *Acquisition delay time*. The effect of the *Stock adjustment time* that we summarized here is also valid for the non-decaying stock case.

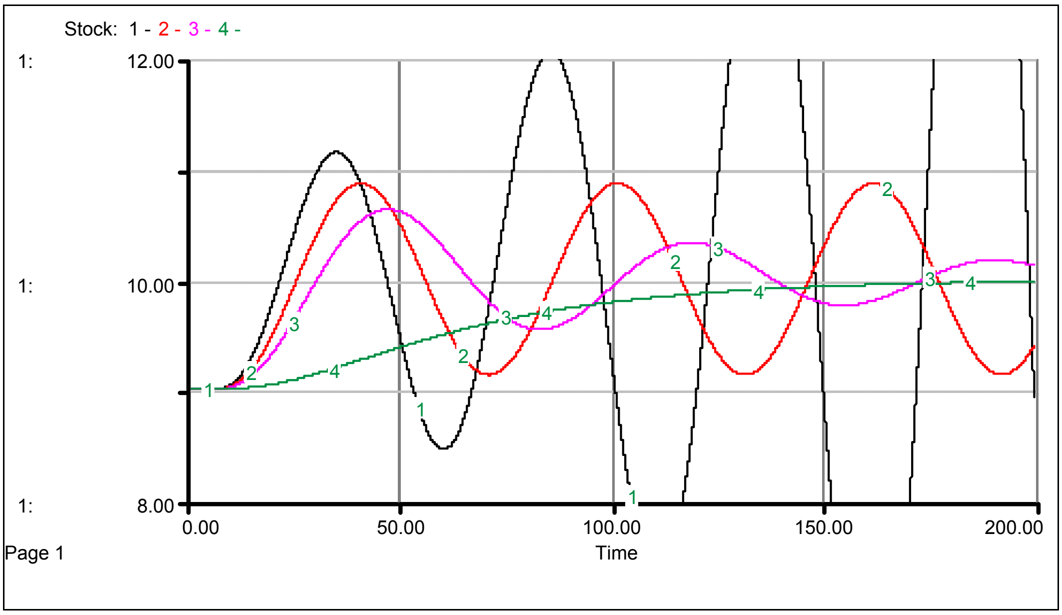

Figure 3. Runs for *Stock adjustment time* equal to 2 (unstable oscillation), 3.25 (neutral oscillation), 5 (stable oscillation) and 40 (goal seeking) respectively for the 1st, 2nd, 3rd and 4th runs

The values of the parameters (except *Stock adjustment time*) for the runs in Figure 3 are:

- *Acquisition delay time* = 25
- *Life time* = 40
- *Expected loss averaging time* = 2
- *Weight of supply line* = 0

Effect of Life Time

Decreasing the value of *Life time* results in more stable behavior in the control stock. If the behavior is already stable, i.e. *Weight of supply line* is one, then decreasing *Life time* may result in over damped behavior. The runs for the different values of *Life time* can be seen in Figure 4.

The values of the parameters (except *Life time*) for the runs in Figure 4 are:

- *Acquisition delay time* = 25
- *Stock adjustment time* = 3
- *Expected loss averaging time* = 2
- *Weight of supply line* = 0

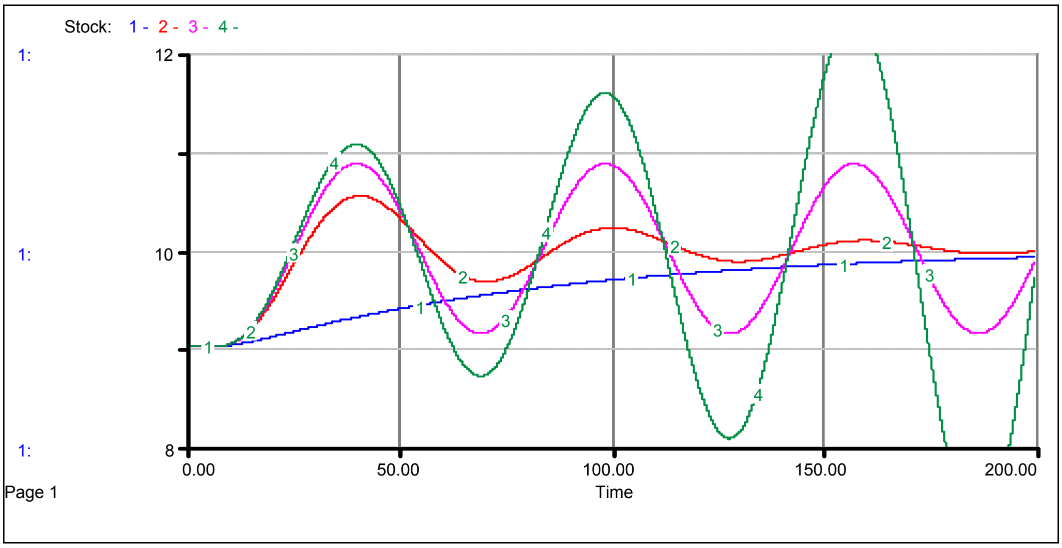

Figure 4. Runs for *Life time* equal to 1 (goal seeking), 16 (stable oscillation), 34.7 (neutral oscillation) and 70 (unstable oscillation) respectively for the 1st, 2nd, 3rd and 4th runs

Effect of Expected Loss Averaging Time

The behavior is not very sensitive to *Expected loss averaging time*. Increasing *Expected loss averaging time* just creates a bigger delay in perceiving *Loss flow* that slows down the response of the control stock. The runs for the different values of *Expected loss averaging time* for goal seeking (*Weight of supply line* is one) and oscillatory (*Weight of supply line* is 0.1) cases can be seen in Figure 5.

The values of the parameters (except *Expected loss averaging time* and *Weight of supply line*) for the runs in Figure 5 are:

- *Acquisition delay time* = 25
- *Stock adjustment time* = 2
- *Life time* = 40

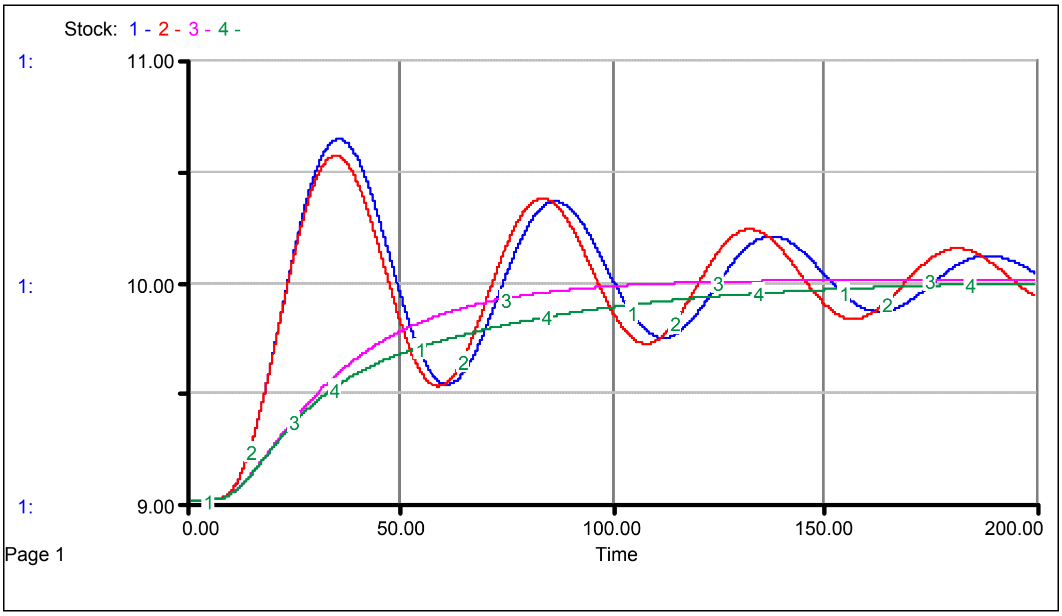

Figure 5. Runs for *Expected loss averaging time* equal to 1, 20, 1 and 20; *Weight of supply line* equal to 0.1, 0.1, 1 and 1 respectively for the 1st, 2nd, 3rd and 4th runs

Effect of Weight of Supply Line

Weight of supply line is a very effective control parameter in stock management decisions. If it is equal to one, the behavior of the control stock will be stable even when *Acquisition delay time* is high and *Stock adjustment time* is low (this true in general but as will be present, decaying stock with a discrete supply line is an exception). *Weight of supply line* equal to one means that the *Supply line adjustment time* is equal to *Stock adjustment time* (see Equation 16). When the two adjustment times are equal to each other, the stock and its supply line effectively reduces to a single stock, and it is a well known fact that a single stock can not oscillate (Sterman, 2000; Yasarcan 2003). For independent outflow (i.e. non-decaying stock) case, in general it can be said that, the behavior of the control stock can not be oscillatory for *Weight of supply line* greater than or equal to one (Yasarcan 2003). Practically, most problematic behavior is obtained when *Supply line* is completely ignored; i.e. *Weight of supply line* is zero. If *Weight of supply line* is greater than one, the result is over damped behavior (see 5th run in Figure 6). Note that, non-oscillatory behavior is also possible for *Weight of supply line* smaller than one, depending on the values of the other parameters (see 3rd run in Figure 6). The runs for the different values of *Weight of supply line* can be seen in Figure 6.

The values of the parameters (except *Weight of supply line*) for the runs in Figure 6 are:

- *Acquisition delay time* = 25
- *Stock adjustment time* = 2
- *Life time* = 40
- *Expected loss averaging time* = 2

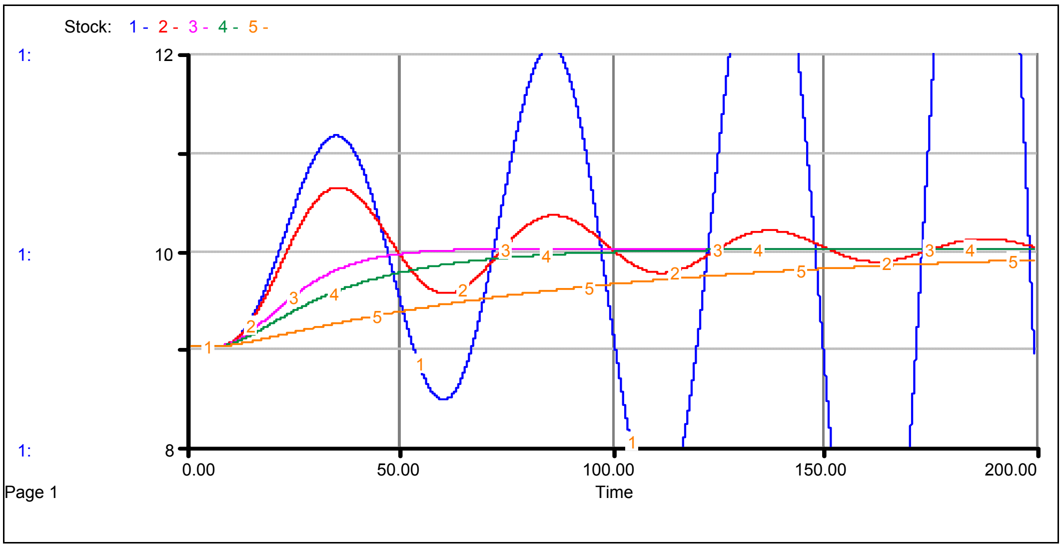

Figure 6. Runs for *Weight of supply line* equal to 0 (unstable oscillation), 0.1 (stable oscillation), 0.6 (goal seeking), 1 (goal seeking) and 3 (over damped) respectively for the 1st, 2nd, 3rd, 4th and 5th runs

DECISION AND NON-DECISION PARAMETERS

Some of the five parameters that we mentioned in this paper, can easily be changed by the decision maker, while some of them either can not be changed or can be changed between a range with great effort. Now we will see these parameters one by one:

Acquisition Delay Time

The value of this parameter can only be changed by great effort. i.e. if the supply line represents delay for the goods coming from the supplier, than *Acquisition delay time* may stand for the production and transportation times. This parameter is the basic parameter that makes the control easy or hard. It must be as small as possible, but this necessitates long term efforts. Practically it may be very hard or impossible to change the value of this parameter. Therefore, in this paper, we do not consider this parameter as a short term decision parameter.

Stock Adjustment Time

The value of this parameter can be chosen freely by the decision maker. It does not have physical limitations, it can take any value greater than zero. Note that, to get meaningful behavior in the control stock it must chosen carefully by the decision maker. In most of the cases, it is good to choose it as small as possible (after setting *Weight of supply line* to one). In practice, decisions can not be updated continuously, there is a smallest time period that can decisions can be updated. If this parameter is smaller than the revision period of control decisions, the behavior of the control stock most probably will be unstable oscillations.

We consider *Stock adjustment time* as the primary decision parameter. We assume that the smallest revision period of the decisions is one, so we will not assume a value for *Stock adjustment time* smaller than one, starting from this point of this paper.

Life Time

In this paper, we assume that this parameter stands for the life time of goods or similar objects and can only be changed by long term efforts, which may mean technological improvements. If it has high value, managing the control stock is easier. If it has low value, either it creates over damping for low order supply line (i.e. second order) or it creates counter intuitive behavior for high order supply line (i.e. infinite order; discrete) as will be presented in this paper.

Sometimes a decaying structure can be used to control a stock with adjusting the value of this parameter but in those cases, another name must be chosen instead of *Life time*. In practice, many decision makers tend to use both inflow and outflow control at the same time but simultaneous inflow and outflow control is harder than controlling a single inflow or outflow (Fey, 1974).

In this paper, we do not consider *Life time* as a short term decision parameter.

Expected Loss Averaging Time

This parameter stands for both perception delay and smoothing time. We assume that perception delay can not be changed but smoothing time can take any value. Therefore, this parameter is assumed to have a minimum level and at that minimum it only represents the perception delay. In our models, we used single order information delay (single level variable in the expectation formation structure) to form *Expected loss*. One may propose to use second order information delay structure, but the effect of such a change will be negligible with respect to our research aim. We assume that this parameter is partially under control of the decision maker; i.e. *Expected loss averaging time* can not be smaller than some given value (i.e. perception delay).

If outflow (loss) is deterministic, we prefer *Expected loss averaging time* to be small but if it is stochastic, *Expected loss averaging time* must be selected big enough to smooth the control decisions so that the behavior of the control stock becomes also smoother. In this paper, we are working with deterministic structures, so with respect to the limits of our research, this parameter is not a basic decision parameter.

Weight of Supply Line

The value of *Supply line adjustment time* can be chosen freely like *Stock adjustment time* so this means that, their ratio, which is *Weight of supply line* can also be chosen freely. Although there is no limitation in selecting a value for *Weight of supply line*, as we concluded before, we prefer to set it as one, to have a stable and fast behavior in the control stock. In addition, we also concluded that it is easier to manage an effectively first order stock system. We consider *Weight of supply line* as one of the basic decision parameters.

AN ALTERNATIVE ANCHOR FOR DECAYING STOCK CASE

We propose *Equilibrium value of loss* to be the anchor of the decision formulations for the decaying stock case. The proposed equation for *Equilibrium value of loss* is:

$$Equilibrium\,value\,of\;loss = \frac{Desired\;stock}{Life\,time} \tag{18}$$

If *Equilibrium value of loss* is the anchor, the following Equations 19 and 20 are used instead of Equations 5 and 10.

$$Control\;flow = Equilibrium\,value\,of\;loss + Stock\;adjustment + Supply\,line\,adjustment \tag{19}$$

$$Desired\;supply\,line = Acquisition\,delay\,time \bullet Equilibrium\,value\,of\;loss \tag{20}$$

COMPARING EXPECTED LOSS AND EQUILIBRIUM VALUE OF LOSS AS ANCHORS FOR A DECAYING STOCK

Expected loss is a good anchor in many cases but we argue that *Equilibrium value of loss* is a better anchor in decaying stock case. Assume that *Acquisition delay time* is high and *Life time* is small. We will try to obtain a fast and stable behavior in the control stock towards its goal by selecting good values for the three decision parameters (*Stock adjustment time*, *Weight of supply line* and *Expected loss averaging time*). The model in Figure 1 will be used. The values of the parameters for the runs in Figure 7, 8 and 9 are:

- *Acquisition delay time* = 25
- *Life time* = 4
- *Stock adjustment time* ≥ 1
- *Expected loss averaging time* ≥ 1

Life time is small enough to create extra damping and furthermore, there is no stochasticity in the model so it is good to set *Expected loss averaging time* to its minimum value that is one. Under these conditions, instability can easily be eliminated if there is any, so to obtain a fast response, it is good to set *Stock adjustment time* to its minimum value that is one. Only one parameter remains to play with, which is *Weight of supply line*.

It can be said that, best run for *Expected loss* as the anchor can be obtained if *Weight of supply line* has a value around 0.1 (see 3rd run in Figure 7) and best run for *Equilibrium value of loss* as the anchor can be obtained if *Weight of supply line* has a value around 1 (5th run in Figure 8). The heuristics of selecting parameter values in obtaining the best performance can be given as:

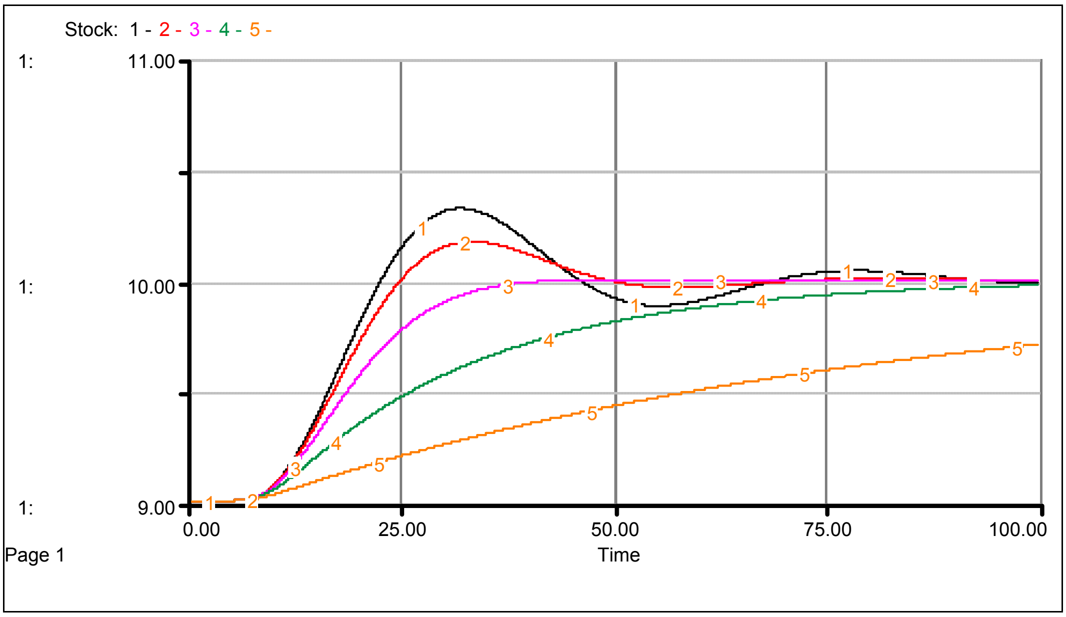

Figure 7. Runs for *Expected loss* as anchor and *Weight of supply line* equal to 0, 0.03, 0.1, 0.3 and 1 respectively for the 1st, 2nd, 3rd, 4th and 5th runs

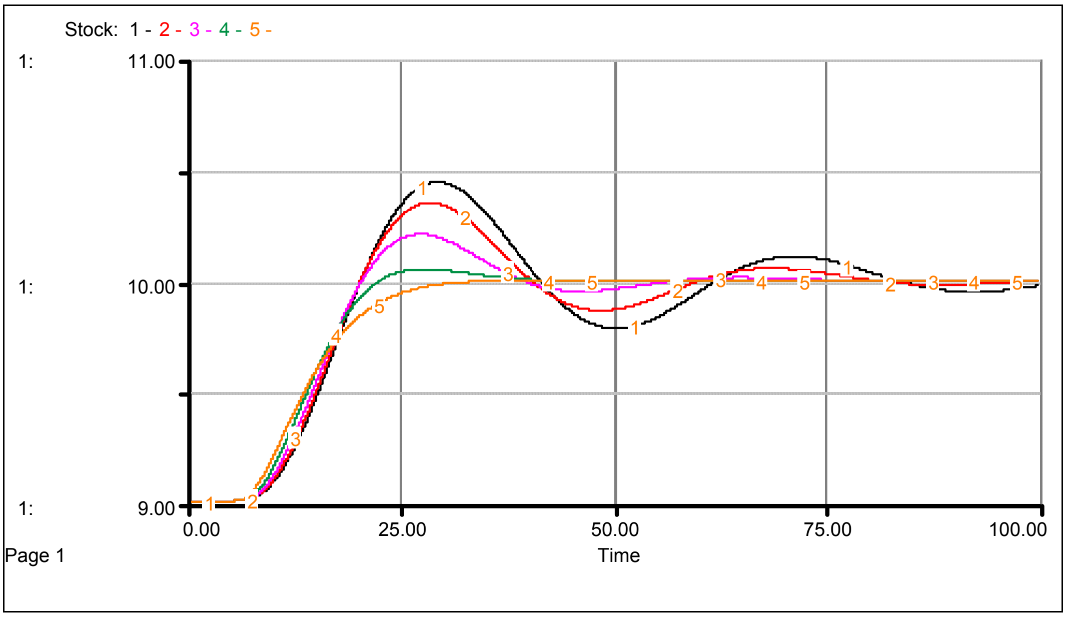

Figure 8. Runs for *Equilibrium value of loss* as anchor and *Weight of supply line* equal to 0, 0.03, 0.1, 0.3 and 1 respectively for the 1st, 2nd, 3rd, 4th and 5th runs

Parameter value selection heuristic when *Expected loss* is the anchor:

Select the minimum possible value for *Stock adjustment time* and play with *Weight of supply line* assigning a value between zero and one, and select the value giving the best performance.

Parameter value selection heuristic when *Equilibrium value of loss* is the anchor:

Set *Weight of supply line* to one and select the minimum possible value for *Stock adjustment time*.

As it can be seen the parameter value selection heuristic for *Equilibrium value of loss* is easier than the heuristic for *Expected loss*.

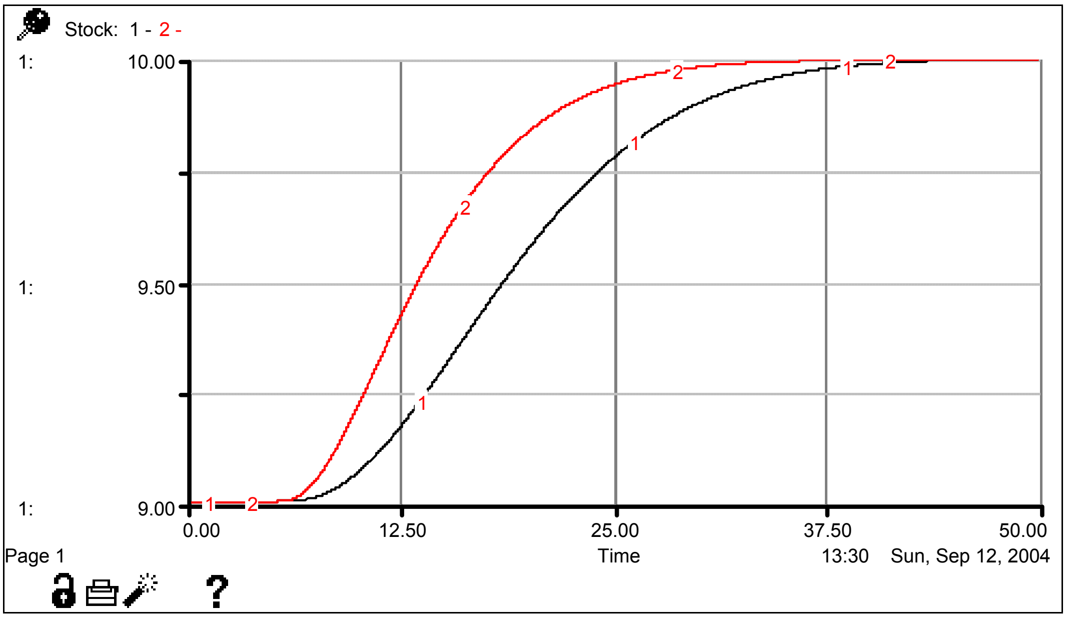

Figure 9. First run; *Expected loss* is the anchor and *Weight of supply line* is equal to 0.1, and second run; *Equilibrium value of loss* is the anchor and *Weight of supply line* is equal to 1

The best runs of the both anchors (3rd run in Figure 7 and 5th run in Figure 8) are re-sketched in Figure 9. It is obvious for this example that the performance obtained by choosing *Equilibrium value of loss* as the anchor is better than choosing *Expected loss*. This result is true in general for the decaying stock case. Either the performances of the both anchors are close to each other (if *Life time* has a high value) or *Equilibrium value of loss* produces a better result (if *Life time* has a low value).

A MODEL WITH A DISCRETE SUPPLY LINE AND A DECAYING STOCK

Consider the model in Figure 1. The aim is to bring the *Stock* to its desired level that is *Desired stock* and maintain *Stock* at that level. For this, the orders are placed at *Control flow* and after passing two stages they reach to *Stock*. Assume the number of the stages are infinite with the same delay type. This kind of supply line is said to have infinite order or simply it is called discrete supply line. The supply line of the model in Figure 10 is a discrete supply line instead of a second order supply line, which is the only difference between the two models.

Even though most of the equations of the model in Figure 10 are the same with the equations of the model in Figure 1, we will give all equations of the model in Figure 10, for sake of completeness. The level (stock) equations are as follows:

$$Stock(t) = Stock(t - DT) + (Acquisition\ flow - Loss\ flow) \bullet DT \tag{21}$$

$$Supply\ line(t) = Supply\ line(t - DT) + (Control\ flow - Acquisition\ flow) \bullet DT \tag{22}$$

$$Expected\ loss(t) = Expected\ loss(t - DT) + Expectation\ adjustment\ flow \bullet DT \tag{23}$$

The flow equations are as follows:

$$Control\ flow = Expected\ loss + Stock\ adjustment + Supply\ line\ adjustment \tag{24}$$

$$Acquisition\ flow(t) = Control\ flow(t - Acquisition\ delay\ time) \tag{25}$$

$$Expectation\ adjustment\ flow = \frac{Loss\ flow - Expected\ loss}{Expected\ loss\ averaging\ time} \tag{26}$$

$$Loss\ flow = \frac{Stock}{Life\ time} \tag{27}$$

The most important difference between the first model in Figure 1 and the second model in Figure 10 is the equations of *Acquisition flow*. The rest of the equations are similar.

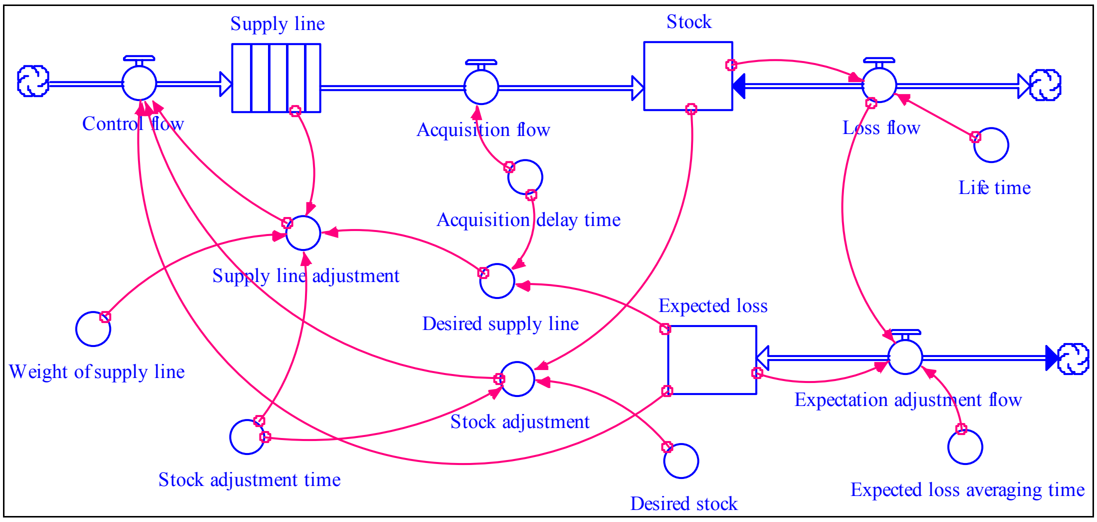

Figure 10. A stock management structure involving infinite order material supply line (discrete supply line) and a decaying stock

The other equations are:

$$Desired\ supply\ line = Acquisition\ delay\ time \bullet Expected\ loss \tag{28}$$

$$\text{Stock adjustment} = \frac{\text{Desired stock} - \text{Stock}}{\text{Stock adjustment time}} \tag{29}$$

$$\text{Supply line adjustment} = \text{Weight of supply line} \bullet \frac{\text{Desired supply line} - \text{Supply line}}{\text{Stock adjustment time}} \tag{30}$$

All the level variables are initiated at their equilibrium levels:

- $\text{Stock}(0) = \text{Desired stock}$
- $\text{Expected loss}(0) = \text{Loss flow}$
- $\text{Supply line}(0) = \text{Desired supply line}$

Desired level of *Stock* is set to nine and it is increased by one unit at time five arbitrarily to perturb the system from its equilibrium:

$$\text{Desired stock} = 9 + \mathrm{STEP}(1,5) \tag{31}$$

SURPRISE BEHAVIOR WITH EXPECTED LOSS AS THE ANCHOR FOR A DECAYING STOCK AND A DISCRETE SUPPLY LINE

The only difference between the first model in Figure 1 and the second model in Figure 10 is that, the order of the supply line is two in the first and infinite (discrete) in the latter. One may not expect big differences, but counter intuitively the difference in the behavior of the control stock is great.

Instability When Weight of Supply Line is One

For example lets assume the following parameter values:

- *Acquisition delay time* = 16
- *Life time* = 7
- *Stock adjustment time* = 5
- *Expected loss averaging time* = 1
- *Weight of supply line* = 1

Weight of supply line is one, so we expect a stable behavior. Moreover, *Life time* is small so we expect over damping in the behavior of the control stock. The first run (second order supply line) in Figure 11 produces an over damped behavior as one should expect. However, the second run (discrete supply line) in Figure 11 produces an unstable behavior that is contradictory to what we expect.

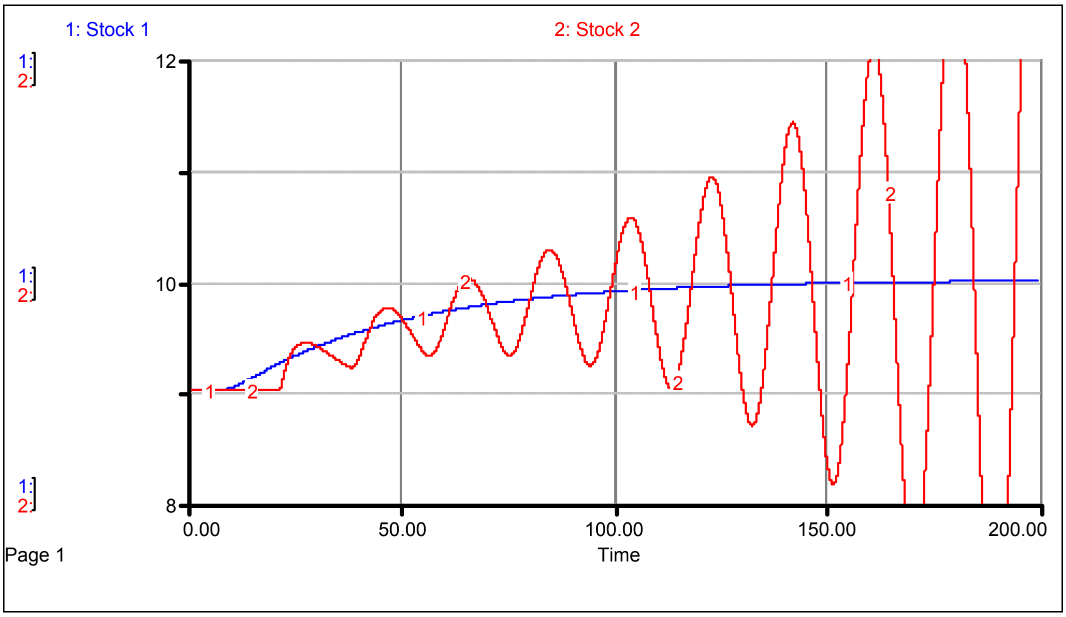

Figure 11. Runs for *Expected loss* as anchor in models with second order supply line (first run) and with discrete supply line (second run)

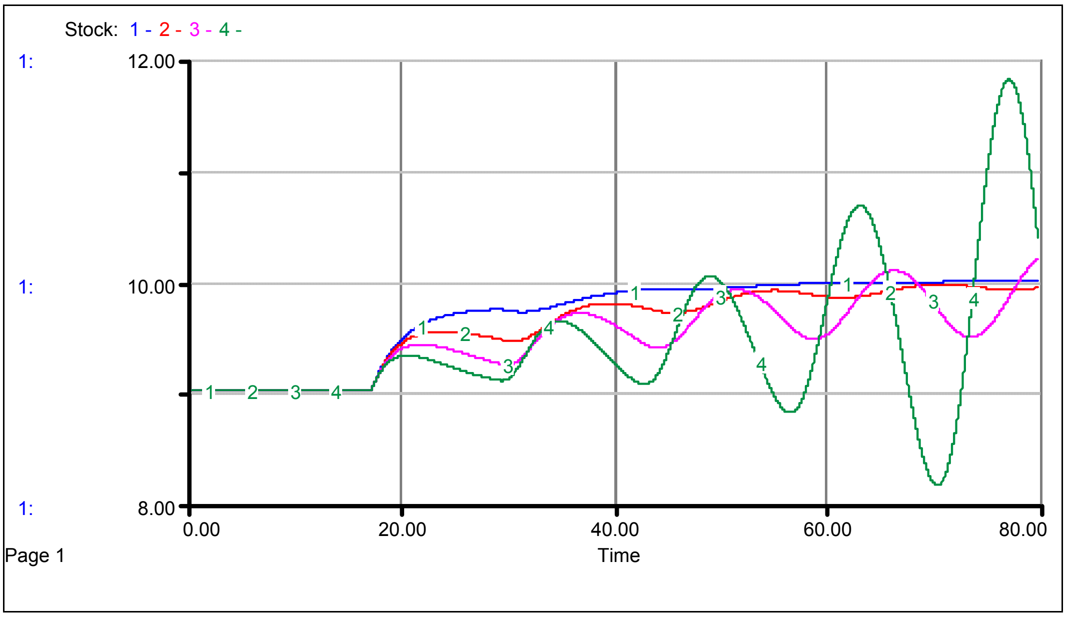

Figure 12. Runs for discrete supply line; *Expected loss* is the anchor and *Weight of supply line* is equal to 0, 0.2, 0.5 and 1 respectively for the 1st, 2nd, 3rd and 4th runs

Stability in the Behavior When the Supply Line is Ignored Completely

For example lets assume the following parameter values:

- *Acquisition delay time* = 12
- *Life time* = 3

- *Stock adjustment time* = 4
- *Expected loss averaging time* = 1

In Figure 12, there are four runs; in the first run *Weight of supply line* is zero and counter intuitively the behavior is stable; in the fourth run *Weight of supply line* is one and again counter intuitively the behavior is unstable. The second and the third runs demonstrate the transition from stability to instability as *Weight of supply line* increases. These results are contradictory with the previous general results. All the system dynamists would argue that supply line must not be ignored to have a stable behavior in the control stock but the results in Figure 12 is just the opposite. Here we must make a warning that *Weight of supply line* equal to zero does not necessarily give stable results for all parameter values (see first run in Figure 13).

Strange Behaviors for Different Values of Weight of Supply Line

For example lets assume the following parameter values:

- *Acquisition delay time* = 25
- *Life time* = 4
- *Stock adjustment time* = 1
- *Expected loss averaging time* = 3

The runs in Figure 13 demonstrate that, for some parameter values, whatever the value of *Weight of supply line* is, the resulting behavior is always oscillatory. The behavior of a decaying stock is very reach, when the supply line is discrete and the anchor is *Expected loss*.

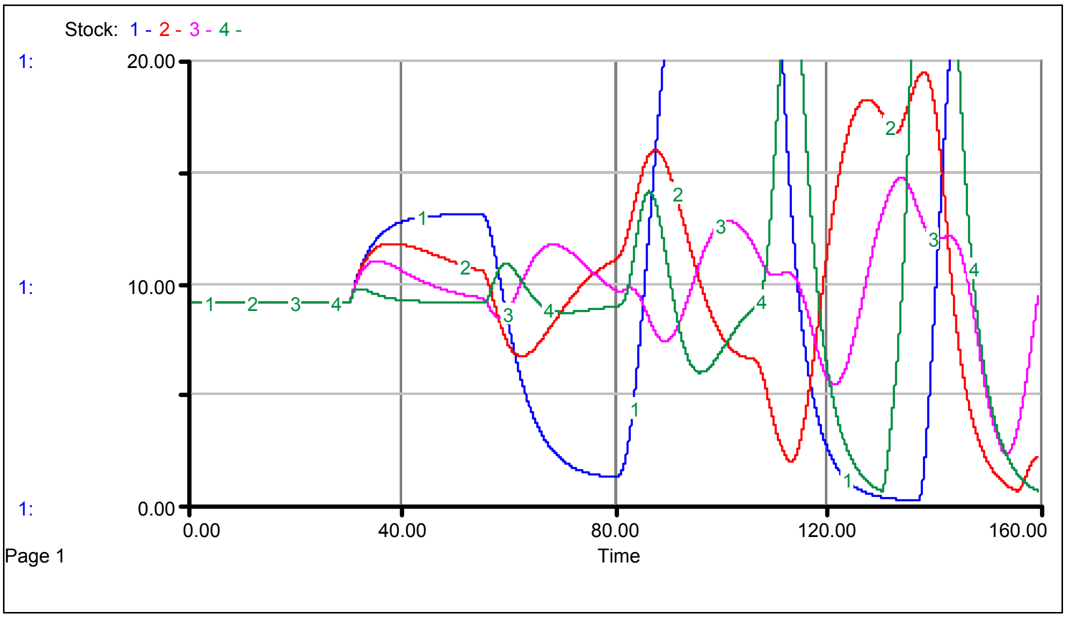

Figure 13. Runs for discrete supply line; *Expected loss* is the anchor and *Weight of supply line* is equal to 0, 0.05, 0.15 and 1 respectively for the 1st, 2nd, 3rd and 4th runs

The Possible Reason for the Surprise Behaviors Observed

The causal loop diagram of a decaying stock and *Expected loss* as the anchor can be seen in Figure 14. There are two positive loops that might be the cause of the unexpected behavior. We believe that discrete supply line is increasing the effect of these positive loops. Note that when the outflow (*Loss flow*) is independent from the stock (i.e. constant outflow), there is no positive loop. For a decaying stock we propose using *Equilibrium value of loss* as the anchor, which terminates the positive loops.

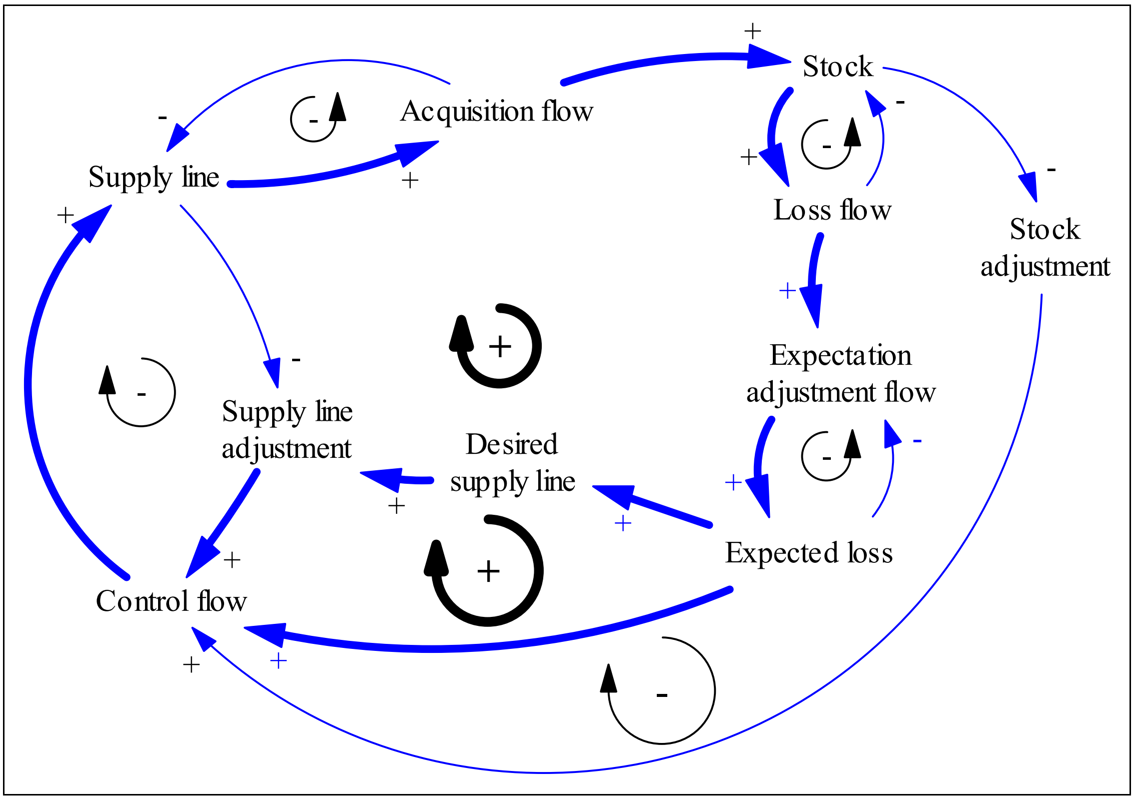

Figure 14. Causal loop diagram of a structure with a decaying stock and *Expected loss* as the anchor

EQUILIBRIUM VALUE OF LOSS AS THE ANCHOR FOR A DECAYING STOCK AND A DISCRETE SUPPLY LINE

If *Equilibrium value of loss* is the anchor, Equations 19 and 20 are used instead of Equations 24 and 28. The behavior can also be problematic, but not like the behavior for *Expected loss*. If supply line is ignored (*Weight of supply line* is zero) the behavior is mostly oscillatory and can be unstable (see first run of Figure 15). One is a good value for *Weight of supply line*, but still stable oscillations can be observed (see third run of Figure 15; see first and second runs of Figure 16). Note that, if *Weight of supply line* is one, unstable oscillation is not possible, whatever the values of the other parameters. If value of *Stock adjustment time* is low, oscillations (stable) can be observed (see first and second runs of Figure 16). For low values of *Life time*, it is not necessary to include any adjustment term in the *Control flow* equation, just using the anchor is enough to obtain a good behavior (see fourth run of Figure 16). For the fourth run of Figure 16, the following equation is used instead of Equation 19.

$$Control\ flow = Equilibrium\ value\ of\ loss \tag{32}$$

Parameter values for the runs in Figure 15:

- *Acquisition delay time* = 16
- *Life time* = 7
- *Stock adjustment time* = 3

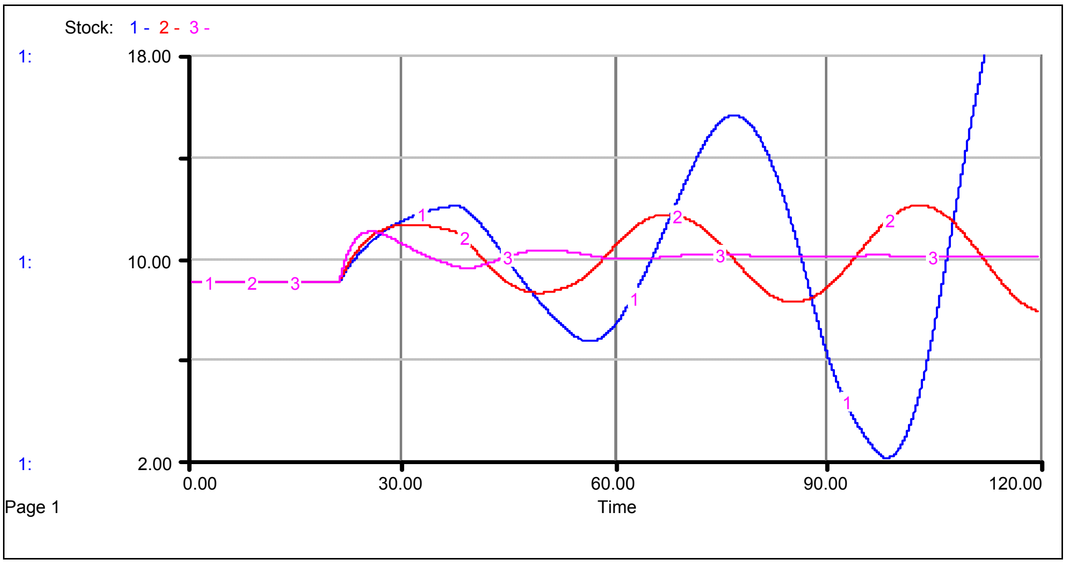

Figure 15. Runs for discrete supply line; *Equilibrium value of loss* is the anchor and *Weight of supply line* is equal to 0, 0.2 and 1 respectively for the 1st, 2nd and 3rd runs

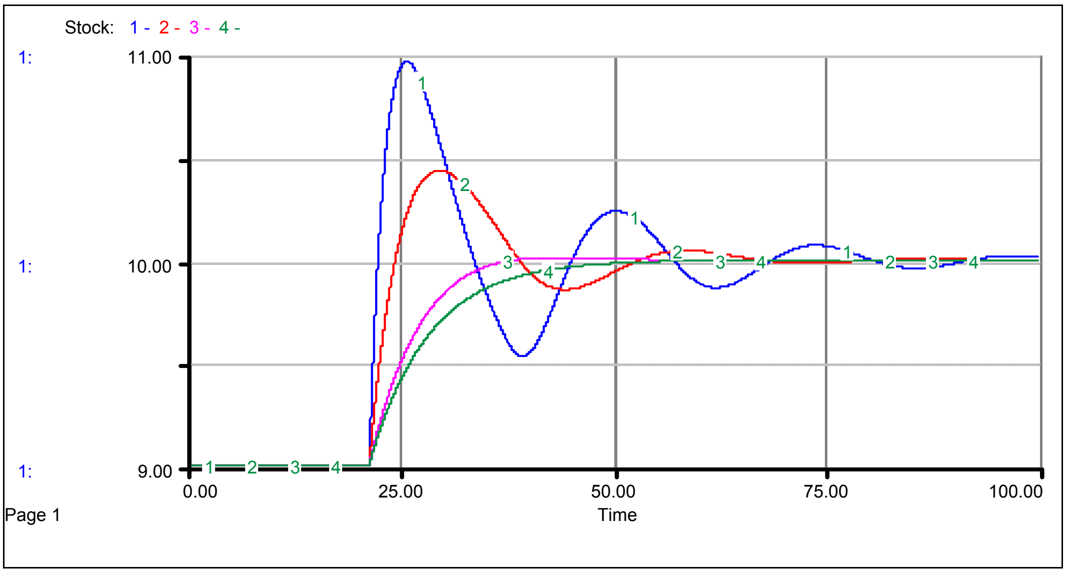

Figure 16. Runs for discrete supply line; *Equilibrium value of loss* is the anchor and *Stock adjustment time* is equal to 3, 10, 100 and ∞ respectively for the 1st, 2nd, 3rd and 4th runs

Parameter values for the runs in Figure 16:

- *Acquisition delay time* = 16
- *Life time* = 7
- *Weight of supply line* = 1

COMPARING EXPECTED LOSS AND EQUILIBRIUM VALUE OF LOSS AS ANCHORS FOR A DECAYING STOCK WITH A DISCRETE SUPPLY LINE

Our experiments showed that, if the following heuristics are used, it is possible to obtain good results with both anchors.

Parameter value selection heuristic when *Expected loss* is the anchor:

Set *Stock adjustment time* to be equal to *Life time* and *Weight of supply line* equal to zero (completely ignore the *Supply line*). In most of the cases (i.e. *Life time* value not high), this setting will give a good starting point and the optimum value will be close. Decrease the value of *Stock adjustment time* and seek a good *Weight of supply line* value by increasing it. Continue decreasing the value of *Stock adjustment time* and seeking better *Weight of supply line* value, until the behavior is satisfactory.

Parameter value selection heuristic when *Equilibrium value of loss* is the anchor:

Set *Weight of supply line* to one and set the value of *Stock adjustment time* to infinity (completely ignore the adjustment term). In most of the cases (i.e. *Life time* value not high), this setting will give a good starting point and the optimum value will be close. Decrease the value of the *Stock adjustment time*, until the behavior is satisfactory.

As it can be seen, like in the second order supply line heuristics, the parameter value selection heuristic for *Equilibrium value of loss* is again easier than the heuristic for *Expected loss*.

The starting points of the two heuristics will give very similar results (see first runs of Figures 17 and 18). The final near optimum behavior is also very similar (see second runs of Figures 17 and 18).

Parameter values for the runs in Figures 17 and 18:

- *Acquisition delay time* = 20
- *Life time* = 10
- *Expected loss averaging time* = 1

Note that, both the starting point and the final near optimum behaviors may show strange shocks, if the value of *Expected loss averaging time* is high and when *Expected loss* is the anchor (see the starting point run comparison of the two anchors in Figure 19).

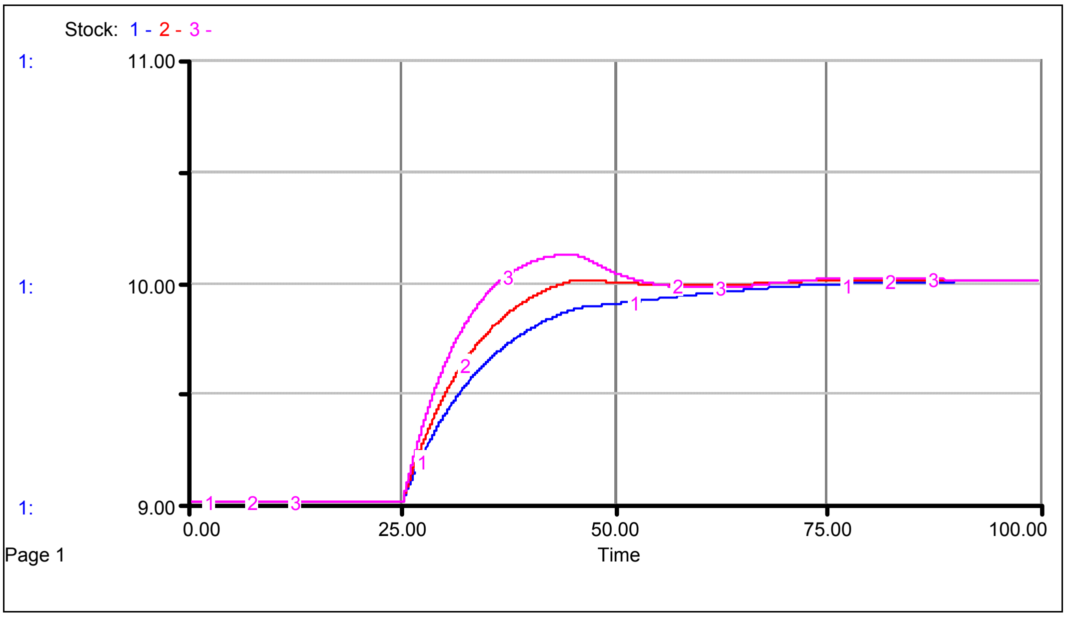

Figure 17. Runs for *Stock adjustment time* equal to 10, 8 and 6; *Weight of supply line* equal to 0, 0.05 and 0.12 respectively for the 1st, 2nd and 3rd runs; *Expected loss* is the anchor

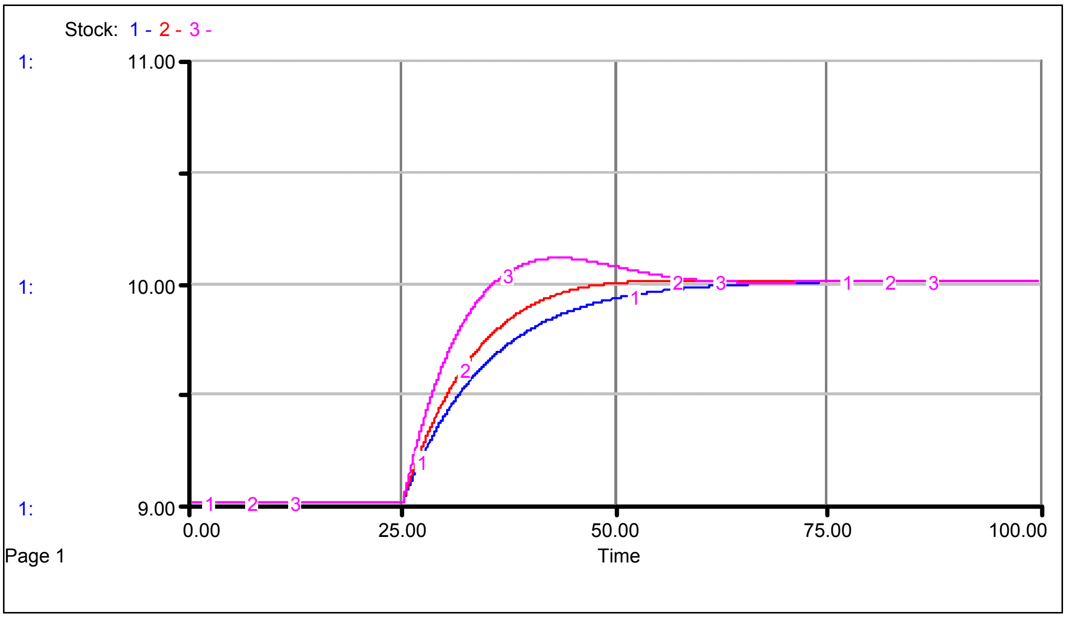

Figure 18. Runs for *Stock adjustment time* equal to ∞, 140 and 40 respectively for the 1st, 2nd and 3rd runs; *Weight of supply line* equal to 1; *Equilibrium value of loss* is the anchor

Parameter values for the runs in Figure 19:

- *Acquisition delay time* = 20
- *Life time* = 2
- *Expected loss averaging time* = 10

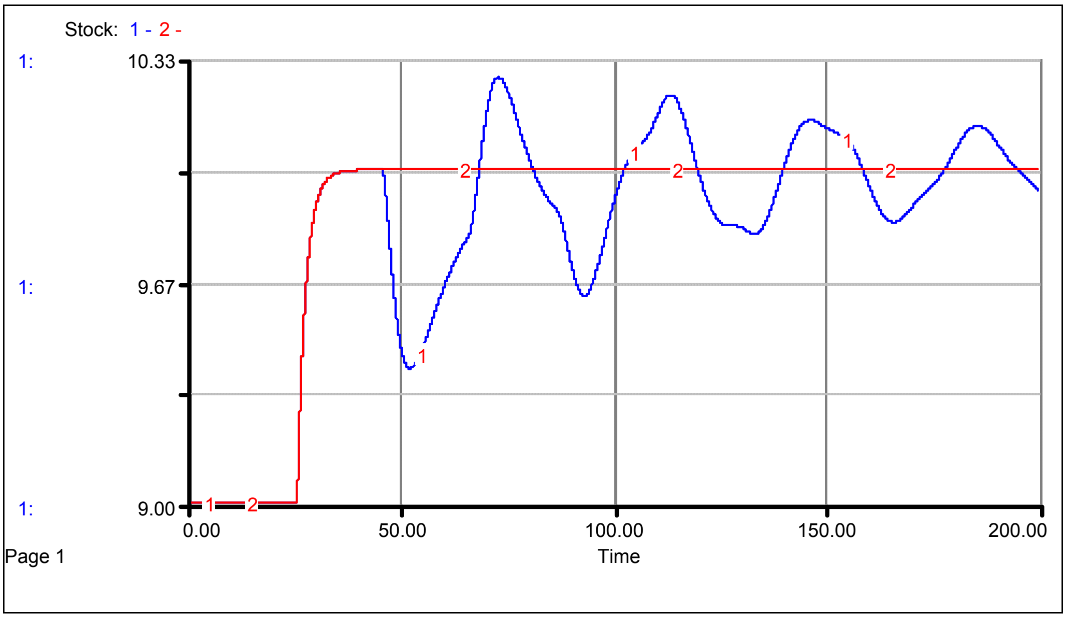

Figure 19. First run; *Expected loss* is the anchor, *Stock adjustment time* is equal to *Life time* (2) and *Weight of supply line* is equal to 0; second run; *Equilibrium value of loss* is the anchor, *Stock adjustment time* is equal to ∞ and *Weight of supply line* is equal to 1

CONCLUSIONS

In this paper, we showed that decaying stock necessitates special care. The usual anchor, the *Expected loss* can give unexpected results, especially when the *Supply line* is discrete. For discrete supply line case, there is a counter intuitive finding; completely ignoring the supply line (*Weight of supply line* equal to zero) eliminates oscillations, provided that *Stock adjustment time* is equal to *Life time*. Unexpectedly, setting *Supply line adjustment time* equal to *Stock adjustment time* (*Weight of supply line* equal to one) may produce unstable oscillations, depending on the values of the other parameters. We argue that the reason for the surprise behavior is a result of the two positive loops (formed by using *Expected loss* as the anchor) acting together with the discrete supply line.

We developed another anchor that we call *Equilibrium value of loss* that gives better results than *Expected loss*. This anchor is much more robust in terms of parameter value selections. If *Life time* is small enough, control is even possible only with the anchor and without adjustment terms.

For both discrete and non-discrete supply line cases and for both type of anchors, we developed parameter value selection heuristics that help to make the control stock to be responsive and non-oscillatory.

We did not focus on what may happen if *Life time* is an unknown parameter to the decision maker. We believe that, with a good estimation procedure, *Equilibrium value of loss* can be used without losing much in the resulting behavior of the control stock. This will be investigated as future research. Another further research topic is potential destabilizing effects of stochastic elements in the outflow.

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