Aggregate Demand Equilibria and Price Flexibility – SD Macroeconomic Modeling (2) –

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Abstract

This is a second paper of a series of macroeconomic modeling that tries to model macroeconomic dynamics such as the determination of GDP (Gross Domestic Product) and money supply from system dynamics perspective. Following the first paper on the money supply and creation of deposits, this second paper tries to model dynamic determination processes of GDP, interest rate and price level on the same basis of the principle of accounting system dynamics developed by the author. For this purpose, a simple Keynesian multiplier model is constructed as a base model to examine a dynamic determination process of GDP. It is then expanded to incorporate the interest rate, whose introduction enables the analysis of aggregate demand equilibria as well as transactions of savings and deposits, and government debt and securities. Finally, a flexible price is introduced to adjust an interplay between aggregate demand equilibrium and full capacity output level. A somewhat surprise result of business cycle is observed from the analysis.

1 Macroeconomic System Overview

This is a second paper of a series of macroeconomic modeling. In the first paper, I have written

> Macroeconomic variables such as GDP, inventory, investment, price, money supply, interest rate, etc, could be more precisely presented

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> by using a basic concept of stock and flow in system dynamics. Moreover, using SD modeling methods, determination of GDP and creation process of credits and money supply - two essential ingredients of macroeconomics- could be more precisely described as dynamic macroeconomic adjustment processes, compared with a traditional static approach. [7]

Led by this vision, creation of credits and money supply are analyzed in the first paper. In this second paper, we try to model dynamic determination processes of GDP, interest rate and a price level.

Let us start with an overview of macroeconomic system. System dynamics approach requires to capture it as a wholistic system consisting of many parts that are interacting with one another. Specifically, macroeconomic system is viewed here as consisting of six sectors such as central bank, commercial banks, consumers (households), producers (firms), government and foreign sector. Figure 1 from the first paper illustrates an overview of such macroeconomic system and shows how these macroeconomic sectors interact with one another and exchange goods and services for money.

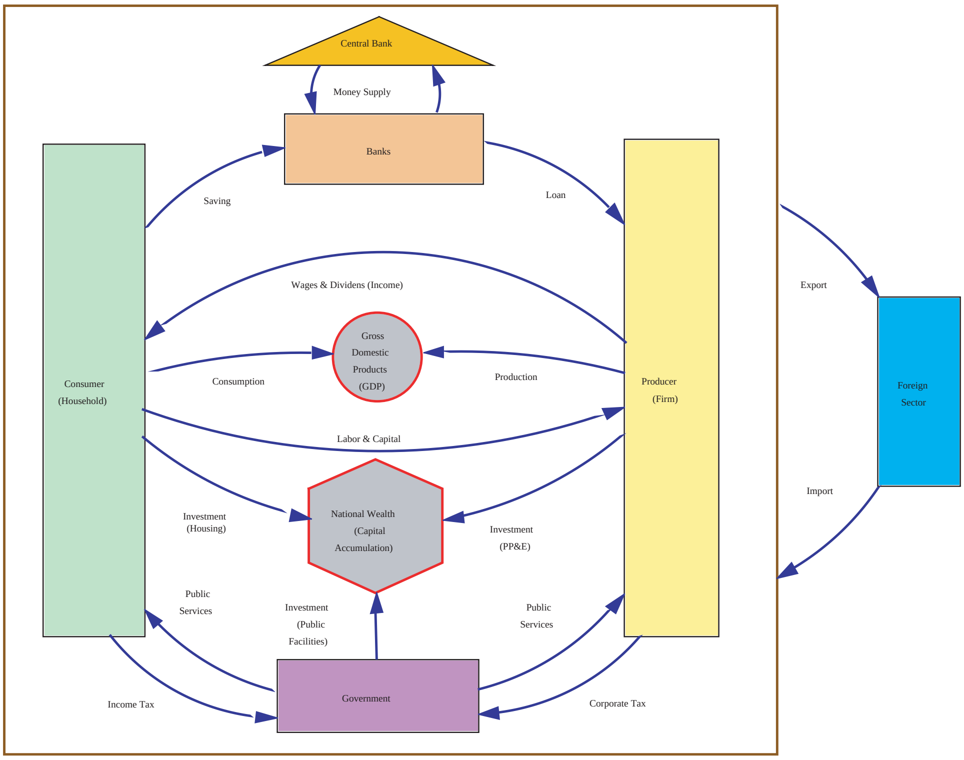

Figure 1: Macroeconomic System Overview

In the previous analysis of money supply and creation of deposits, the above six sectors are regrouped into three sectors: the central bank, commercial banks and non-financial sector consisting of consumers, produces and government. For the analysis of aggregate demand and supply in this paper, we need at least four sectors such as producers, consumers, banks and government. Since money supply is assumed to be exogenously determined in this paper, central bank is excluded. We also analyze a closed macroeconomic system and foreign sector is not brought to the analysis.

How can we describe transactions among four sectors, then? The method we employ here is the same as the one used in my first paper; that is, the use of financial balance sheet. Balance sheet is an accounting method of keeping records of all transactions in both credit and debit sides so that they are kept in balance all the time. The reader is referred to the paper [6] for the detailed explanation of a principle of accounting system dynamics.

2 A Keynesian Model

Since macroeconomics is one of the major subjects in economics, many standard textbooks are in circulation. As references, textbooks such as [1], [2], and [3] are occasionally used to examine a standard approach to macroeconomics;

A simple Keynesian macroeconomic model is described as follows.

$$Y = AD \quad \text{(Determination of GDP)} \tag{1}$$

$$AD = C + I + G \quad \text{(Aggregate Demand)} \tag{2}$$

$$C = C_0 + cY_d \quad \text{(Consumption Decisions)} \tag{3}$$

$$Y_d = Y - T - \delta K \quad \text{(Disposable Income)} \tag{4}$$

$$T = \bar{T} \quad \text{(Tax Revenues)} \tag{5}$$

$$I = \bar{I} \quad \text{(Investment Decisions)} \tag{6}$$

$$G = \bar{G} \quad \text{(Government Expenditures)} \tag{7}$$

$$\frac{dK}{dt} = I - \delta K \quad \text{(Net Capital Accumulation)} \tag{8}$$

$$Y_{full} = F(K, L) \quad \text{(Production Function)} \tag{9}$$

$$Y_{full} = Y \quad \text{(Equilibrium Condition)} \tag{10}$$

This macroeconomic model consists of 10 equations with 9 unknowns; that is, $Y_{full}, Y, K, AD, C, I, G, Y_d, T$, with 7 exogenously determined parameters $(L, C_0, c, \bar{T}, \bar{I}, \delta, \bar{G})$.[1] Obviously, one equation becomes redundant. A possible redundant equation is equations (1) or (10). Which equation should be deleted from the analysis of macroeconomic model?

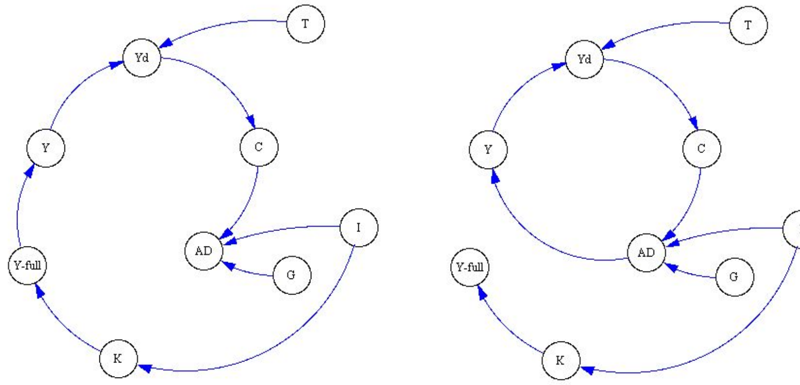

Figure 2: Causal loops of Neoclassical and Keynesian Models

According to the neoclassical view, supply creates its own demand in the long run, and in this sense the equation (1) becomes redundent. Left-hand diagram of causal loops in Figure 2 illustrates how full capacity supply and aggregate demand are separately determined without the equation(1). Therefore, in order to complete this neoclassical logic, we need to add another equation of price mechanism which adjusts discrepancies between Y_{full} and AD such as [2]

$$\frac{dP}{dt} = \Psi(AD/P - Y_{full}). \tag{11}$$

In this way, we have 10 unknown variables and 10 equations. The equilibrium attained this way is called neoclassical long-run equilibrium.

On the other hand, according to a Keynesian view GDP is determined by the aggregate demand in the short-run. In this sense, the equation (10) becomes redundant. Right-hand diagram shows that GDP is determined by the aggregate demand without the equation (10). In this case, the level of GDP is nothing but equal to the level of aggregate demand, and needs not be the same as the amount of output produced by the economy's production function (9). Contrary

[1] In this model, demand for and supply of labor, L, is not analyzed. To do so we need to add another equation of population (labor) growth such as

$$\frac{dL}{dt} = nL$$

which will be done in our following research.

[2] Whenever price is explicitly introduced, all variables have to be expressed (or interpreted) as real values.

to the neoclassical view, the economy has no autonomous mechanism to attain an equilibrium in which output produced by the equation (9) is equal to the aggregate demand; that is, a neoclassical long-run equilibrium. This is because price is regarded as sticky in the short-run, and cannot play a role to adjust a discrepancy between aggregate supply of output and aggregate demand. Hence, Keynesian economists argue that such a neoclassical long-run equilibrium could only be attained in the short run through changes in aggregate demand made possible by monetary and fiscal policies.

Can we create a synthesis model to deal with these controversies between neoclassical and Keynesian schools[3]? From a system dynamics point of view, macro-economy is nothing but a system and different views on the behaviors of the system can be uniformly explained as structural differences of the same system. This is what we like to pursue in this paper so that an effectiveness of system dynamics modeling can be demonstrated.

Keynesian Adjustment Process

Let us start with a Keynesian approach by deleting the equation (10). We have now 9 equations with 9 unknowns; that is, $Y, AD, C, I, G, Y_d, T, K, Y_{full}$ with 7 exogenously determined parameters $(C_0, c, \bar{T}, \bar{I}, \delta, \bar{G}, L)$.

A level of GDP that holds $Y = AD$ is obtained in terms of the parameters as follows:

$$Y^* = \frac{C_0 - c\bar{T} + \bar{I} + \bar{G}}{1 - c} \tag{12}$$

Let us assign some numerical values to these parameters $(C_0, c, \bar{I}, \bar{G}, \bar{T}) = (24, 0.6, 120, 80, 40)$, then we have $Y^* = 500$.

How can such a Keynesian equilibrium GDP be attained if aggregate supply and aggregate demand are not equal initially? The Keynesian model assumes that aggregate supply is determined by the size of aggregate demand. Fig 3 illustrates how an initial GDP of Y_0 continues to increase until it catches up with the aggregate demand, and eventually attains a Keynesian equilibrium Y^*. In this way the equilibrium can be always gained at a point where aggregate demand curve meets aggregate supply curve. Comparative statics is a well-known analytical method in standard textbooks to compare with two points of equilibria for two different levels of aggregate demand.

To model these static comparisons dynamically, the determination equation of GDP (1) has to be replaced with the following differential equation:

$$\frac{d\,Y}{dt} = (AD - Y)/AT \tag{13}$$

where AT is an adjustment time. In system dynamics this process is known as goal-seeking dynamics in which aggregate demand plays a role of goal and

[3] I once posed this question in the book [5]. At that time, I was unaware of system dynamics and unable to model my general equilibrium framework for simulation.

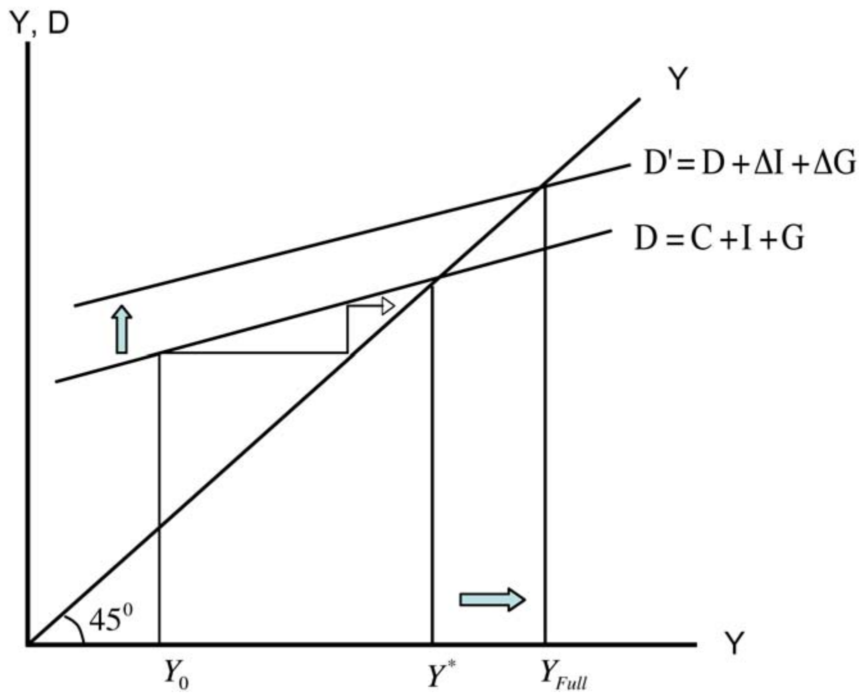

Figure 3: Keynesian Determination of GDP

aggregate supply tries to catch up with it. Figure 4 illustrates a SD model of such Keynesian process, in which an aggregate demand forecasting mechanism is additionally introduced without changing an essential mechanism of Keynesian adjustment process.

Left-hand diagram of Figure 5 illustrates how an initial GDP is smoothly increased to attain the Keynesian equilibrium GDP at $Y^* = 500$. In the right-hand diagram investment and government expenditures are respectively increased by 10 at the periods 5 and 10, while tax is reduced by 10 at the period 15. Again, GDP is shown to increase smoothly for attaining new equilibrium levels of aggregate demand.

From the production function (9) the maximum amount of output is produced by fully unitizing the existing capital stock K and labor force L.

$$Y_{full} = Y(K, L) \tag{14}$$

Obviously, there is no guarantee that the Keynesian equilibrium GDP of Y^* is equal to Y_{full}, and the equilibrium equation (10) is met. When it is less than the maximum output level, capital stock is under-utilized and some workers are unemployed; that is, the economy is in a recession. In other words, the Keynesian aggregate demand equilibrium is no longer an equilibrium in the sense that capital and labor are fully utilized.

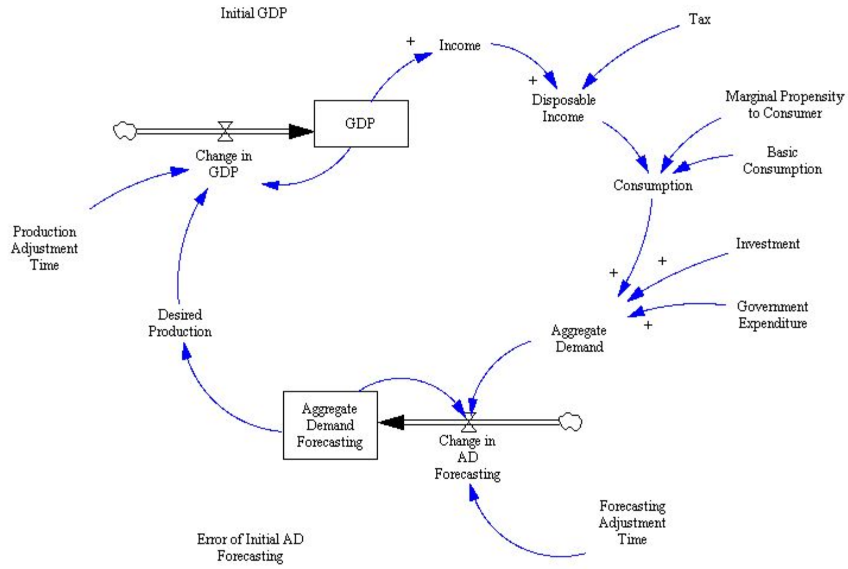

Figure 4: Keynesian SD Model of GDP

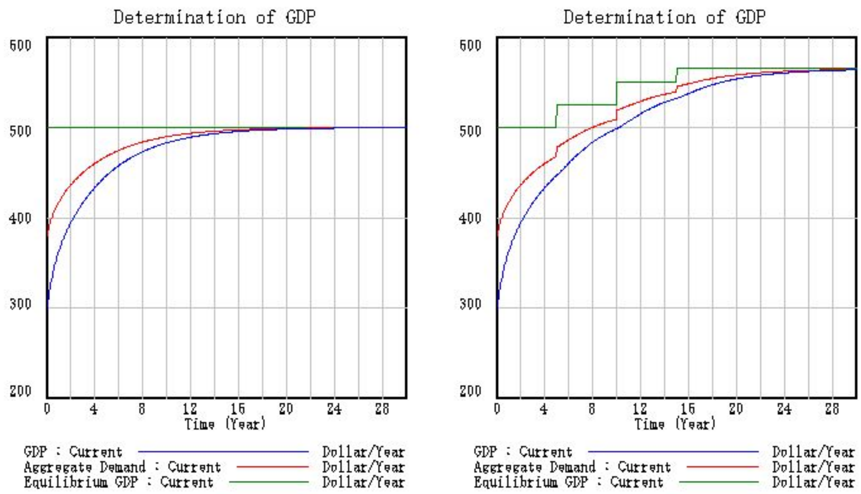

Figure 5: Keynesian Determination of GDP

According to the Keynesian theory, the underutilization is caused by deficiencies of effective demand, and to gain full capacity and full employment equilibrium, additional effective demand has to be created by increasing investment and government expenditures, or decreasing taxes.

How much increase in the effective demand is needed, then? The answer lies in the Keynesian multiplier process. From the equilibrium equation (12), we have

$$\Delta Y = \frac{-c\Delta T + \Delta I + \Delta G}{1-c} \tag{15}$$

Thus, multipliers for I, G and T are calculated as follows:

$$\frac{\Delta Y}{\Delta I} = \frac{\Delta Y}{\Delta G} = \frac{1}{1-c} = 2.5 \ ; \ \frac{\Delta Y}{\Delta T} = \frac{-c}{1-c} = -1.5 \tag{16}$$

Suppose that $Y_{full} = 560$. Then, to attain a full capacity level of GDP, we need to increase $\Delta Y = Y_{full} - Y^* = 60$. This could be done by increasing the investment or government expenditure by 24 (that is, $\Delta Y = 2.5 \cdot 24$), or decreasing tax by 40 (that is, $\Delta Y = (-1.5) \cdot (-40)$). Figure 3 illustrates how Y_{full} is attained by increasing aggregate demand such as investment and government expenditures.

System Dynamics Adjustment Process

The above Keynesian adjustment process is very mechanistic and does not reflect how actual production decisions are made by producers. More realistic decision-making process of production is to introduce an inventory adjustment management as explained in Chapter 18 of John Sterman's book [4]. In reality a discrepancy between production and shipment (or aggregate demand) is adjusted first of all as a change in inventory stock. Hence, the introduction of inventory as a stock is essential for SD modeling of macroeconomic system. The reason why inventory is not well focused in a standard macroeconomic framework may be because inventory is always treated as a part of (undesired) investment and output becomes in this sense identically equal to the aggregate demand.

Keynesian adjustment process (13) now needs to be revised as follows:

$$\frac{d\ I_{nv}}{dt} = (Y - AD) \tag{17}$$

with the introduction of inventory stock, I_{nv}. This adds another new unknown variable to the macroeconomic system. Accordingly, we need an additional equation to solve the amount of inventory. To do so, let us first define the amount of desired production as a sum of the amount of inventory replacement and aggregate demand forecasting:

$$Y^D = (\text{Desired Inventory} - \text{Inventory}) + \text{AD Forecasting} \tag{18}$$

where desired inventory is an exogenous parameter and set to be zero in our model. Then, redefine the aggregate supply as

$$Y = Y^D \quad (\text{Desired Production}) \tag{19}$$

Figure 6 illustrates our revised SD model of the Keynesian model. When this model is run, we observe that aggregate demand and production overshoot an equilibrium as illustrated by the left-hand diagram of Figure 7. This overshooting behavior vividly contrasts with a smooth adjustment process of the Keynesian model.

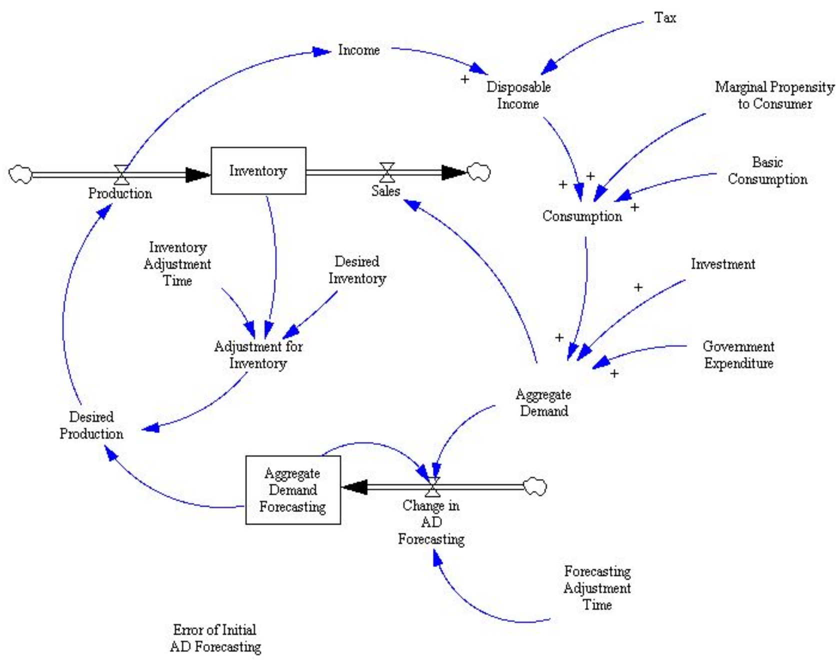

Figure 6: Revised Keynesian SD Model of GDP

In the right-hand diagram investment and government expenditures are increased by 10 at the periods 5 and 10, while tax is reduced by 10 at the period 15, exactly in the same fashion as the right-hand diagram of Figure 5. However, output and aggregate demand do not catch up with new equilibrium levels smoothly here, instead they are shown to overshoot the equilibrium levels. This suggests that Keynesian adjustment process is intrinsically cyclical or fluctuating off equilibrium, rather than smoothly adjusting as illustrated by many standard textbooks. This behavior may be the first finding in our SD macroeconomic modeling against standard Keynesian smooth adjustment process.

3 Aggregate Demand (IS-LM) Equilibria

In the above Keynesian macroeconomic model, taxes, investment and government expenditures are assumed to be exogenously determined. To make it

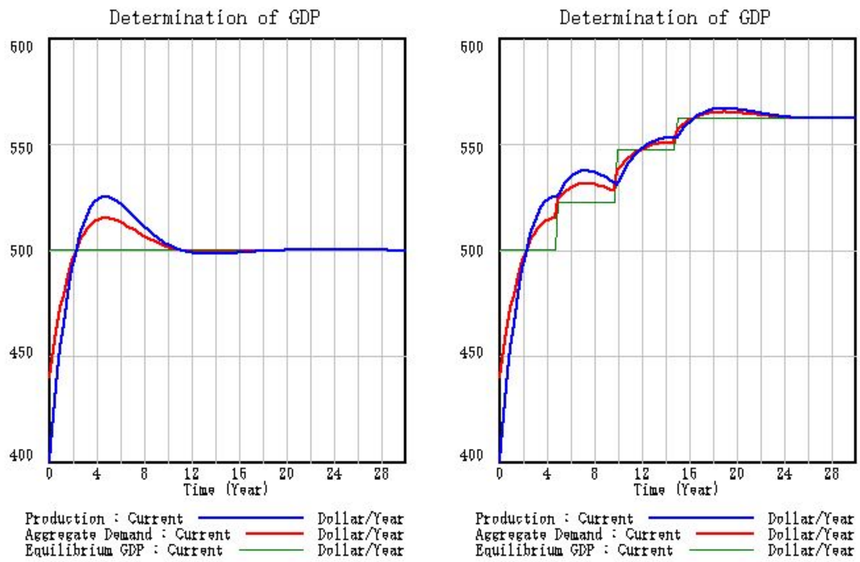

Figure 7: SD Determination of GDP

more complete, we now try to construct these variables to be endogenously determined. Let us begin with government taxes by assuming that they consist of three parts: lump-sum taxes such as property taxes (T_0), income taxes that are proportionately determined by an income level, and government transfers such as subsidies (T_r):

$$T = T_0 + tY - T_r \tag{20}$$

where t is an income tax rate.

Next, investment is assumed to be determined by the interest rate:

$$I(i) = I_0 - \alpha i \tag{21}$$

where α is an interest sensitivity of investment. We have now added a new unknown variable of the interest rate to the model, and hence an additional equation is needed to make it complete. According to the standard textbook, it should be an equilibrium condition in money market such that real money supply is equal to the demand for money:

$$\frac{M^s}{P} = aY - bi \tag{22}$$

where a is a fraction of income for transactional demand for money, and b is an interest sensitivity of demand for money. P is a price level and it is treated as a sticky exogenous parameter.

From the equilibrium condition in the goods market, a relation between GDP and interest rate, which is called IS curve, is derived as follows:

$$Y = \frac{C_0 + I_0 + G + c(T_r - T_0)}{1 - c(1-t)} - \frac{\alpha}{1 - c(1-t)} i \tag{23}$$

On the other hand from the equilibrium condition in the money market, a relation between GDP and interest rate, call LM curve, is derived as

$$Y = \frac{1}{a}\frac{M^s}{P} + \frac{b}{a} i \tag{24}$$

Equilibrium GDP and interest rate (Y^*, i^*) are now completely determined by the IS and LM curves. For instance, the aggregate demand equilibrium of GDP is obtained as

$$Y^* = \frac{C_0 + I_0 + G + c(T_r - T_0)}{1 - c(1-t) + \alpha(a/b)} + \frac{\alpha/b}{1 - c(1-t) + \alpha(a/b)}\frac{M^s}{P} \tag{25}$$

This is a standard Keynesian process of determining an aggregate demand equilibrium of GDP in the short run in which price is assumed to be sticky. Figure 8 illustrates how IS and LM curves determine the equilibrium GDP and interest rate (Y^*, i^*).

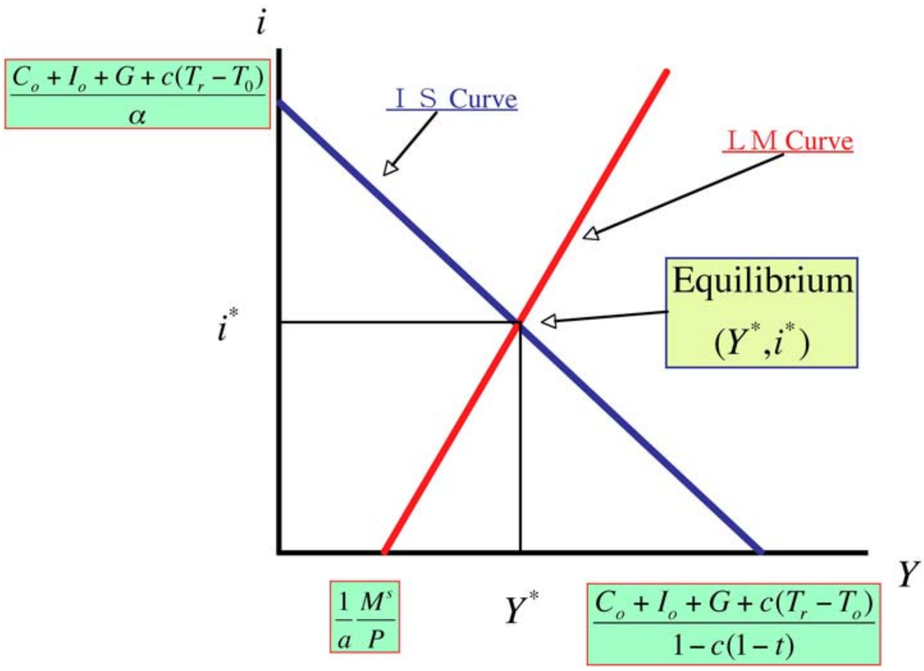

Figure 8: IS-LM Determination of GDP and Interest Rate

As discussed in the previous section, GDP thus determined needs not be equal to the full capacity output level, Y_{full}. The Keynesian model only specifies GDP as determined by the level of aggregate demand. This is why it is called aggregate demand equilibrium of GDP. To realize a full capacity equilibrium $Y^* = Y_{full}$, price needs to be flexibly changed in the long run. On the contrary the Keynesian model we presented so far lacks this price flexibility.

Endogenous Government Expenditures

We have successfully made variables such as T and I endogenous. The only remaining exogenous variable is government expenditures, G. They are usually determined by a democratic political process, and in this sense could be left outside the system as an exogenously determined parameter.

Instead, we try to make it an endogenous variable. First approach is to assume that the government expenditures are dependent on the economic growth rate, $g(t) = \Delta Y(t)/Y(t)$, such that

$$\frac{d\,G}{dt} = g(t)G. \tag{26}$$

This approach seems to be reasonable because many governments try to increase government expenditures proportionally to their economic growth rates so that a run-away accumulation of government deficit will be avoided.

Second approach is to assume that government expenditures are dependent on its tax revenues, since the main source of government expenditures are tax revenues which are endogenously determined by the size of output or income level. Then government expenditures become a function of tax revenues:

$$G = \beta T \tag{27}$$

where β is a ratio between government expenditures and tax revenues, called primary balance ratio here. When $\beta = 1$, we have a so-called balanced budget, while if $\beta > 1$, we have budget deficit.

With the introduction of the government expenditures in either one of these two ways, all exogenously determined variables such as T, I, and G are now endogenously determined within the macroeconomic system.

Let us analyze the second case furthermore. In this case IS curve becomes

$$Y = \frac{C_0 + I_0 + (\beta - c)(T_0 - T_r)}{1 - c - (\beta - c)t} - \frac{\alpha}{1 - c - (\beta - c)t} i \tag{28}$$

By rearranging, the aggregate demand equilibrium of GDP is calculated as

$$Y^* = \frac{C_0 + I_0 + (\beta - c)(T_0 - T_r)}{1 - c - (\beta - c)t + \alpha(a/b)} + \frac{\alpha/b}{1 - c - (\beta - c)t + \alpha(a/b)} \frac{M^s}{P} \tag{29}$$

How does the introduction of tax-dependent expenditures affect behaviors of the equilibrium? Let us consider, as one special case, how a tax reduction in lump-sum taxes, T_0, affect the equilibrium GDP under a balanced budget; that is, $\beta = 1$. In this case, we have from the equation (29)

$$\frac{dY}{dT_0} = \frac{1-c}{(1-c)(1-t)+\alpha(a/b)} > 0 \tag{30}$$

On the other hand, in the case of the exogenously determined expenditures, we have from the equation (25)

$$\frac{dY}{dT_0} = \frac{-c}{1-c(1-t)+\alpha(a/b)} < 0 \tag{31}$$

This implies that under a balanced budget a reduction in lump-sum taxes will discourage GDP, contrary to a general belief that it stimulates the economy. This counter-intuitive feature seems to be deemphasized in standard textbooks in which tax cut is usually treated as stimulating the economy.

4 Modeling Aggregate Demand Equilibria

Now we are in a position to construct our SD macroeconomic model of aggregate demand equilibria based on IS-LM curves. To do so, the equilibrium condition (22) in the money market needs to be replaced with a dynamic adjustment process of interest rate as a function of excess demand for money:

$$\frac{d\ i}{dt} = \Phi\left((aY - bi) - \frac{M^s}{P}\right) \tag{32}$$

With this replacement, we could directly build a SD macroeconomic model of aggregate demand model in a mechanistic way such that IS and LM curves interact one another as developed in Figure 6 in the previous section. This could be a better approach than the comparative static analysis in which IS and LM curves are manually sifted to observe how aggregate demand equilibrium of (Y^*, i^*) is changed as usually done in the standard textbooks.

However, from a system dynamics point of view, this mechanistic approach of modeling aggregate demand equilibria may incur many causal loopholes. For instance, when consumers save, they receive interests from banks. If government spends more than receives, its deficit has to be funded by consumers as a purchase of government securities, against which they also receive interests. Whenever the macro-economy is viewed as a wholistic economic system, these transactions play important feedback roles and such feedback effects need not be neglected. Therefore, as a complete system it should include those transactions among consumers, producers, banks and government from the beginning.

Due to the existence of these causal loopholes, standard macroeconomic framework has resulted in offering many open spaces which macroeconomists are invited to fill in with their own ideas and theories. We believe these open spaces have been intrinsic causes of many macroeconomic controversies such as the one between neoclassical and Keynesian schools of economics. These controversies, moreover, give us an impression that their macroeconomic models are mutually exclusive like oil and water and cannot be integrated.

On the contrary, as system dynamics researchers we believe that macroeconomy as a system could be modeled as an integrated whole so that controversies such as descried above are nothing but different behaviors caused by slightly different conditions of the same system structure. In this sense, its system dynamics model, if built completely, could synthesize these controversies as different macroeconomic system behaviors, rather than the behaviors of different economic systems. This has been the main motivation for constructing a wholistic SD macroeconomic model in this series.

For this construction of synthetic model, a double entry accounting system of corporate balance sheet turns out to be very effective for describing many transactions among macroeconomic sectors. For some reader this approach seems to make our modeling unnecessarily complicated compared with the standard macroeconomic framework. We pose, however, that this is the simplest way to describe complicated macroeconomic behaviors per se.

Producers

Let us now describe some fundamental transactions which are missing in the standard textbook framework. We begin with producers. In the macroeconomic system, they face two important decisions: production and investment. We have already assumed that production decision is made by the equation (19) following a system dynamics approach of inventory management, while investment decision is assumed to be made by a standard macroeconomic investment function (21).

Based on these decisions, major transactions of producers are, as illustrated in Figure 9, summarized as follows.

- Producers are constantly in a state of cash flow deficits. To continue new investment, therefore, they have to borrow money from banks and pay interest to the banks.
- Out of the revenues producers deduct the amount of depreciation and pay wages to workers (consumers) and interests to the banks. The remaining revenues become profits before tax.
- They pay corporate tax to the government.
- The remaining profits are paid to the owners (that is, consumers) as dividends.

Consumers

Consumers have to make two decisions: how much to consume and how to invest the remaining income between saving and government securities - a portfolio

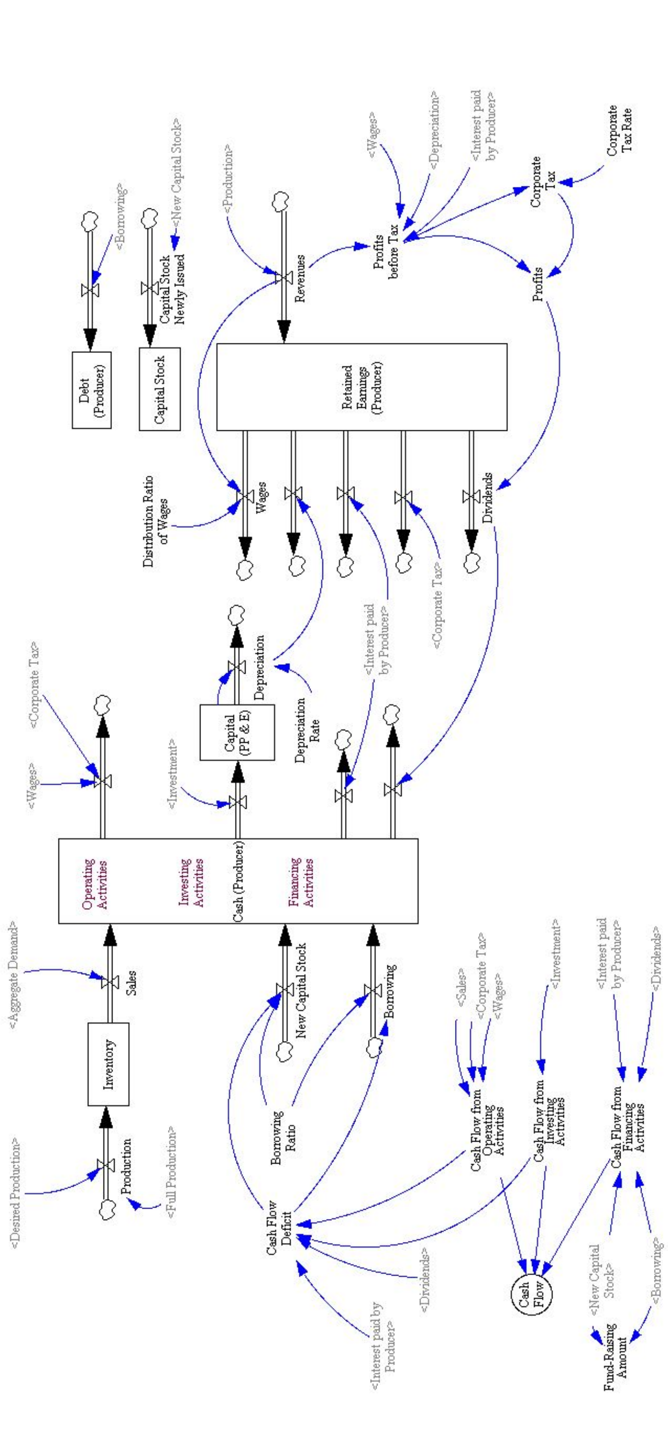

Figure 9: Transactions of Producers

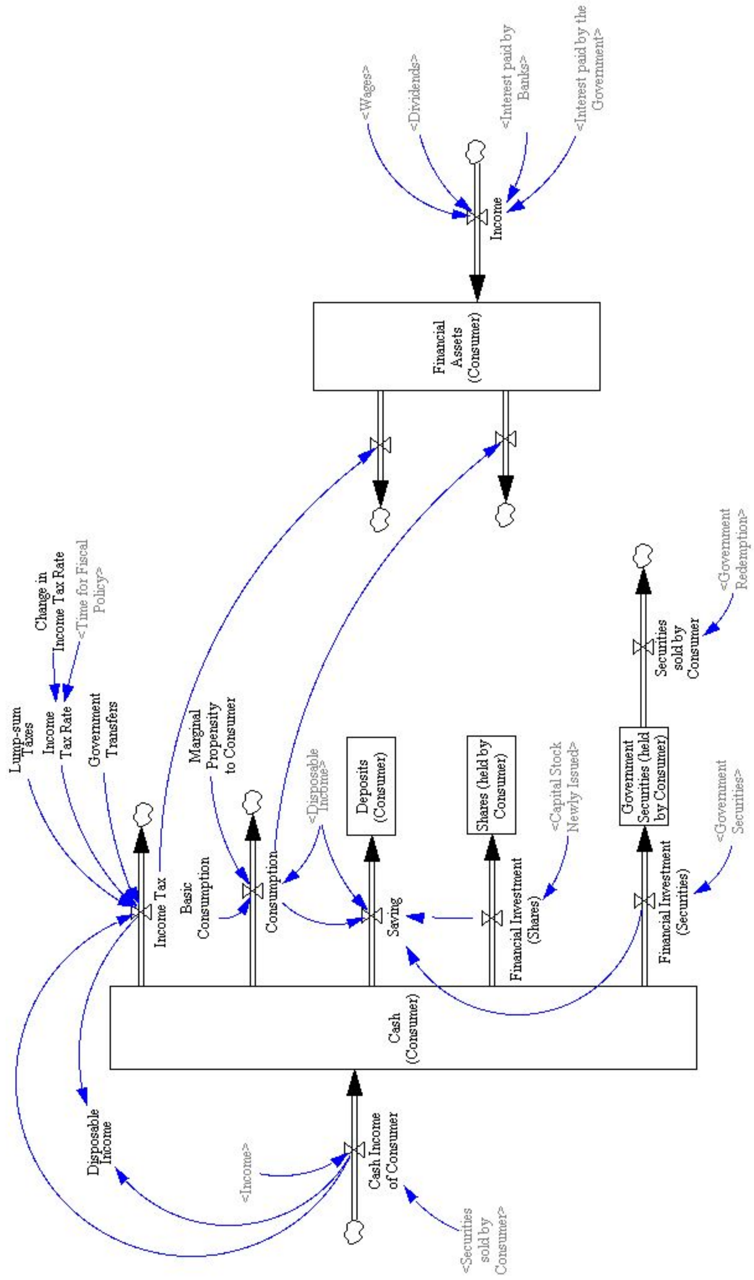

Figure 10: Transactions of Consumer

choice. Consumption decision is assumed to be made by a standard consumption function (3). (It could also be made dependent on their financial assets). As to the portfolio decision we simply assume that consumers first buy government securities and save the remaining amount to the banks as deposits.

Transactions of consumers are illustrated in Figure 10, some of which are summarized as follows.

- Consumers receive wages and dividends.
- Financial assets of consumers consist of bank deposits and government securities, against which they receive financial income of interests from banks and government. (In this paper, no corporate shares are assumed to be held by consumers).
- In addition to the income such as wages, interests, and dividends, consumers receive cash whenever previous securities are partly redeemed annually by the government.
- Out of these cash income as a whole, consumers pay income taxes, and the remaining income becomes their disposal income.
- Out of their disposal income, they spend on consumption. The remaining amount are either spent to purchase government securities or saved. Accordingly, no cash is assumed to be withheld by the consumers.

Banks

In our model, banks are assumed to play a very passive role; that is, they only make loans to producers by the amount asked by them. In other words, they don't purchase government securities and accordingly need to make no portfolio decisions between loans and securities. Transactions of banks are illustrated in Figure 11, some of which are summarized as follows.

- Banks receive deposits from consumers, against which they pay interests.
- They make loans to producers and receive interests.
- Their retained earnings thus become interest receipts from producers less interest payment to consumers. Since the same interest rate is applied for the calculation of the amount of interests, banks cannot make any profits and their retained earnings become zero.

Government

Government faces decisions such as how much taxes to levy as revenues and how much to spend as expenditures. Tax revenues are assumed to be collected according to the standard formula in (20), while expenditures are determined

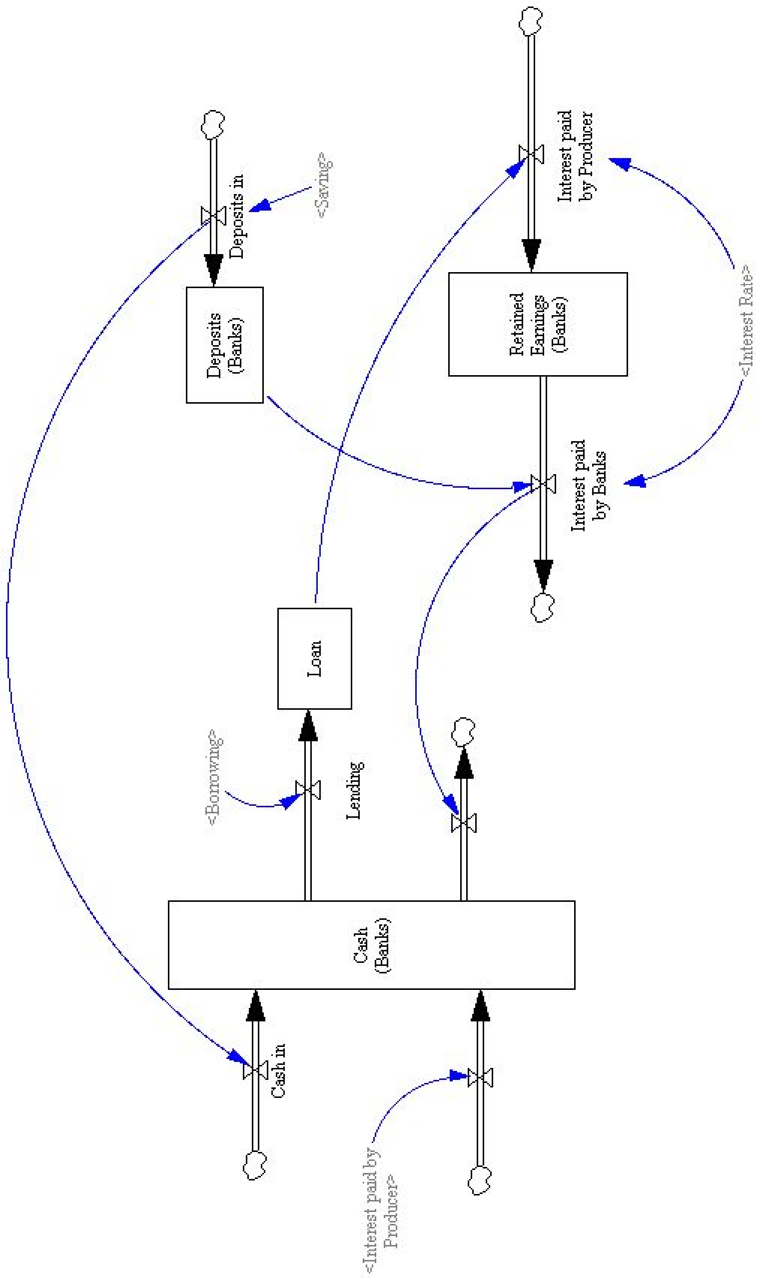

Figure 11: Transactions of Banks

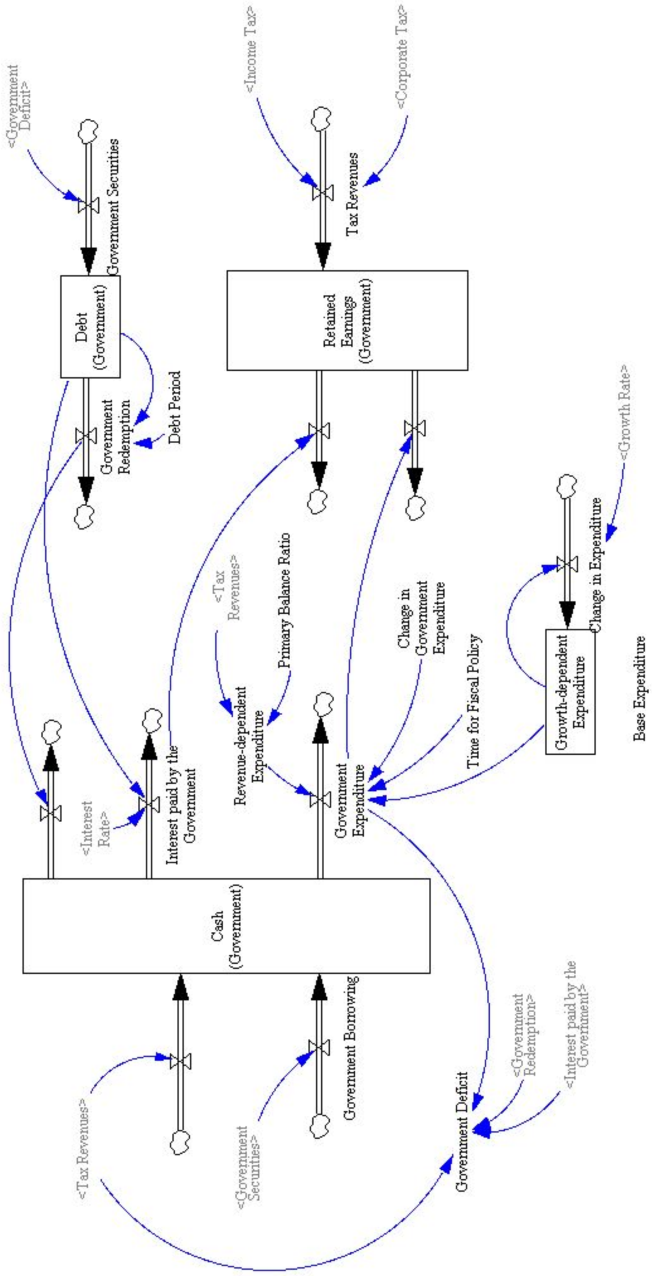

Figure 12: Transactions of Government

either by growth-dependent amount (26) or revenue-dependent amount (27). In the model, expenditures are easily switched to either one.

Transactions of the government are illustrated in Figure 12, some of which are summarized as follows.

- Government receives, as tax revenues, income taxes from consumers and corporate taxes from producers.
- Government spending consists of government expenditures and payments to the consumers for its partial debt redemption and interests against its securities.
- Government expenditures are assumed to be endogenously determined by either the growth-dependent expenditures or tax revenue-dependent expenditures.
- If spending exceeds tax revenues, government has to borrow cash from consumers by newly issuing government securities.

5 Behaviors of Aggregate Demand Equilibria

We now see how aggregate demand equilibrium of (Y^*, i^*) is attained in our SD model constructed above. This model is built by deleting the equation (10) and in this sense, as already discussed above, Y^* needs not be equal to a production level of full capacity, Y_{full}. Surely, the full production level is a maximum level of output in the economy beyond which no physical output is possible. To introduce this upper bound of production level, the equation (19) has to be revised as follows.

$$Y = \mathrm{Min}(Y_{full},\ Y^D) \tag{33}$$

Moreover, the full capacity output level in equation (14) is specified as follows:

$$Y_{full} = \frac{1}{\theta}K \tag{34}$$

where θ is a capital-output ratio. The determination process of GDP in our SD model is illustrated in Figure 13.

Left-hand diagram in Figure 14 shows that initial discrepancy between production and aggregate demand is brought to an equilibrium around $Y^* = 450$ at period 10, while full capacity output is around $Y_{full} = 507$ at the same period. Moreover, aggregate demand equilibrium of GDP is shown to be constantly lower than the full production level. This implies that the economy is in a state of disequilibria or recession.

Right-hand diagram is loci of aggregate demand equilibrium (Y^*, i^*) such as an intersection between IS-LM curves as illustrated in Figure 8. Our model can capture these dynamic movements of the aggregate demand equilibria in contrast with comparative static ones in standard textbooks. This may be another contribution of SD macroeconomic modeling.

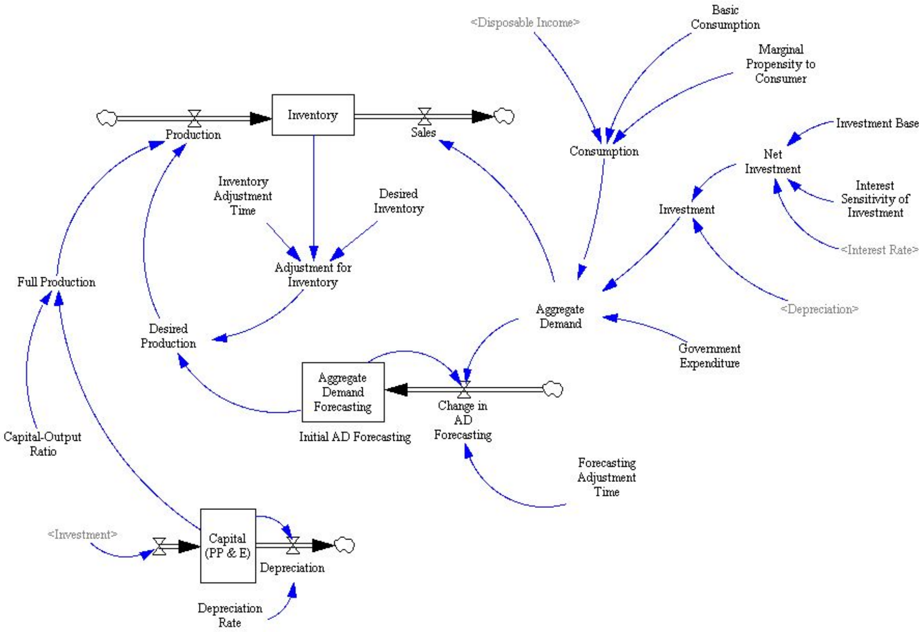

Figure 13: Full Capacity Production

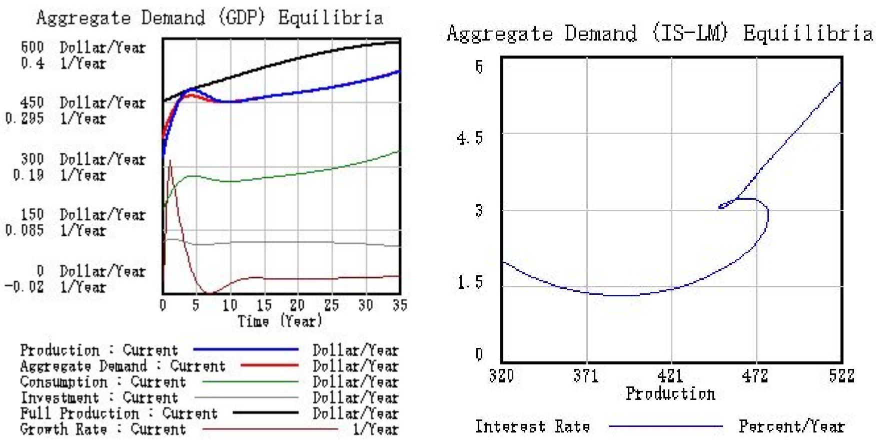

Figure 14: Aggregate Demand Equilibria

Monetary Policy

How can we, then, attain a true Y_{full} equilibrium in which full capacity output is completely sold out? As already illustrated in Figure 3, it could be done by an increase in aggregate demand. The essence of the Keynesian theory is that aggregate demand can be stimulated by monetary and fiscal policies; that is, macro-economy is manageable!

Let us examine monetary policy first by increasing the amount of money supply by 27 at period 10. According to textbook explanation, this increase in money supply shifts the LM curve to the right, and accordingly i^* is lowered while Y^* increases. In the left-hand diagram of Figure 15 a full capacity equilibrium is shown to be attained around $Y_{full} = Y^* = 550$ at period 15. Unfortunately, however, this equilibrium cannot be sustained, because capital continues to accumulate and accordingly production capacity also continues to increase, eventually exceeding aggregate demand.

Right-hand diagram shows how interest rate suddenly begins to decline due to the increase in money supply. However, it eventually begins to increase as aggregate demand fails to sustain a full capacity equilibrium.

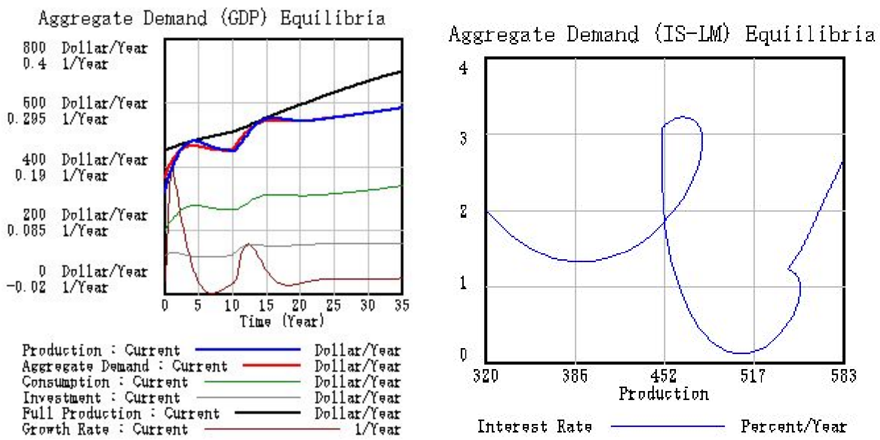

Figure 15: Effect of Monetary Policy

Fiscal Policy

Now we try to attain a Y_{full} equilibrium through fiscal policy. Specifically we try to increase government expenditures by 20 at period 10. According to textbook explanation, this increase shifts the IS curve to the right and stimulates the economy by increasing Y^*. However, as illustrated in the right-hand diagram of Figure 16, it also pushes up i^*, and eventually discourages investment, which is a well known crowding-out effect of government expenditures.

To confirm this, let's take a look at the left-hand diagram, in which a full capacity equilibrium is shown to be attained around $Y_{full} = Y^* = 530$ at period 21. Beyond this point, however, only aggregate demand continues to grow, while production as well as full capacity output begin to decline due to a continued

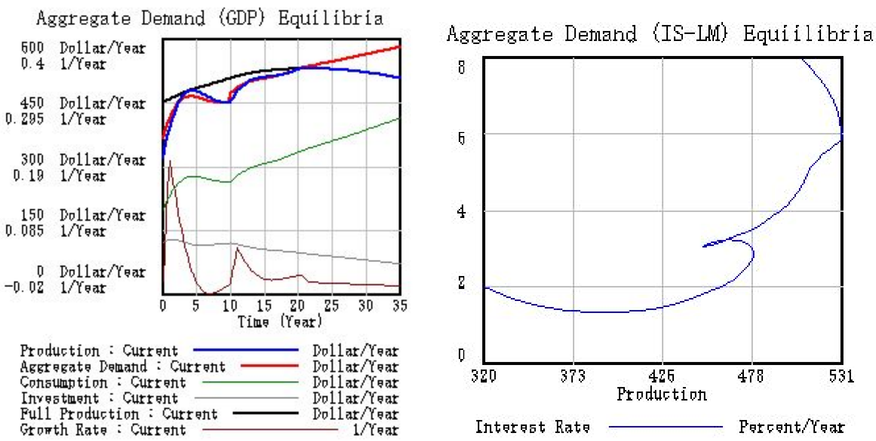

Figure 16: Effect of Fiscal Policy

increase in interest rate and discouraged investment following it. This movement suggests the existence of crowding-out effect caused by fiscal policy.

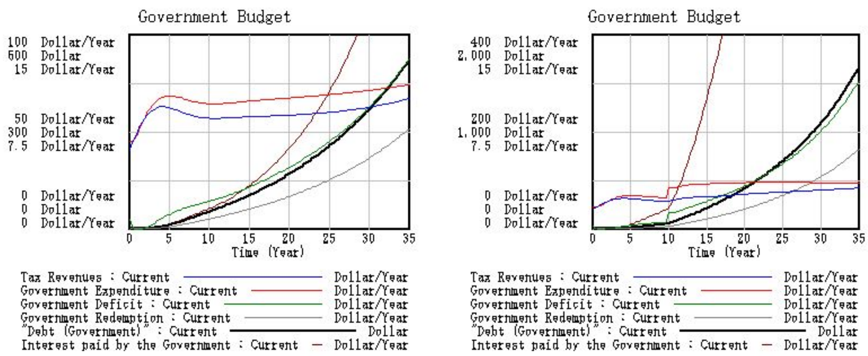

Figure 17: Skyrocketing Government Debt

This state of continued increase in aggregate demand above full capacity output may also be regarded as a state of hyper-inflation, even though price is fixed in this model. Can such a hyper-inflationary situation be sustainable? Figure 17 shows how government debt has accumulated by this fiscal policy from 517 (in the left-hand diagram) to 1,644 (in the right-hand diagram) at the period 35, more than three times (Notice a change in scale between two diagrams). In reality the increasing government debt will lower the price of government securities, and further increase the interest rate. This will not only cause a loss of government credibility, but also bring investment activities simultaneously to a complete standstill; in short, a total breakdown of national economy eventually.

In standard textbooks fiscal policy is usually introduced as a very effective policy to stimulate the economy, while the skyrocketing effect of government debt, the other side of the coin, is left out from the picture, giving an impression to the students that fiscal policy works well without any problem. Our system dynamics analysis is able to successfully capture the other side of the coin. Hence, this could be another contribution of our SD macroeconomic modeling.

In this Keynesian aggregate demand analysis, no feedback structure is built in to reverse the situation of hyper-inflation and a possible collapse of the economy, simply because price is assumed to be fixed, which will be dropped in the next section.

6 Price Flexibility

It is now clear that the Keynesian theory of aggregate demand equilibria is imperfect from a SD model-building point of view, because price level is assumed to be sticky and there exists no built-in mechanism to restore a full capacity production equilibrium unless monetary and fiscal policies are carried out.

In fact, let us rewrite the aggregate demand equilibrium of GDP obtained in equation (25) as a function of price:

$$Y^*(P) = A + B\frac{M^s}{P} \tag{35}$$

Then it is becomes clear that this equation only provides a relation between Y^* and P. Hence, Y^* is called an aggregate demand function. It is now obvious that, unless price is flexible, there exists no mechanism to attain a true equilibrium such that

$$Y_{full} = Y^*(P) \tag{36}$$

It is shown in the previous section that, even though monetary and fiscal policies can attain a full capacity production equilibrium, central bank and government need to constantly fine-tune their policies to sustain such equilibrium. Can they really perform the task under a sticky price in the short run? If so, how short is a short run, practically speaking, to apply such policies?

From a dynamic point of view, it's very hard to specify how short is a short run. Is this moment in the short run or in the long run? It depends on when to specify an initial point. This moment could be in the short run to justify current policies. Or it could be already in the long run, and a long-run price adjustment mechanism, to be discussed below, may be under way. If so, current policy applications might worsen economic situations.

Accordingly, a better way of modeling a macroeconomic system has to allow price flexibility in the model from the beginning and let the price adjust disequilibria, including a fixed price as its special case. To do this formally, we have to bring a previously neglected equation (10). To avoid a redundancy of equation by doing so, we need to introduce another variable of price, and let it adjust discrepancies between full capacity output and aggregate demand equilibrium as in the equation of price adjustment mechanism (11). Such an adjustment equation is here specified as

$$\frac{dP}{dt} = \Psi(Y^D - Y_{full}). \tag{37}$$

Additionally, to make investment more sensitive to changes in the interest rate, the investment function of equation (21) is redefined as follows:

$$I(i) = \frac{\alpha I_0}{i} \tag{38}$$

This completes our construction of SD macroeconomic modeling. Figure 18 illustrates adjustment processes of price and interest rate. With the introduction of flexible price, behaviors of the model turns out to be surprisingly different from the previous model under a fixed price as Figure 19 illustrates. First, aggregate demand equilibrium, Y^*, can no longer be attained as in the previous fixed price case, but they fluctuates alternatively, which we call aggregate demand alternations.

Second, this alternation moves along a full capacity output level, and occasionally approaches to a full capacity equilibrium such that $Y_{full} = Y^*$ as if butterflies moves around flowers and occasionally rest on them. This vividly contrast with the previous fixed price case in which aggregate demand equilibrium can be attained through monetary and fiscal policies, but it will eventually diverge from a full capacity output level. Therefore, under a flexible price, monetary and fiscal policies might not be effective to attain full capacity equilibrium.

Third, economic growth rates turn out to fluctuate periodically as illustrated in Figure 19, in which the business cycle of growth rates, produced by the inner forces fo the system structure itself, can be observed to have a period of about eight years. This is an entirely unexpected behavior to us. Can we avoid this business cycle by practising monetary and fiscal policies? These will be open questions to be challenged in our later research with more integrated model.

Left-hand diagram of Figure 20 illustrates combined movements of (i, P) against Y. (i, Y) is no longer considered as aggregate demand equilibria, and (P, Y) does no longer represent downward sloping aggregate demand curve, simply because they are constantly off equilibrium under price flexibility

In the right-hand diagram, money supply is shown to be a horizontal line, while real money supply (which is defined as money supply divided by price) continues to increase after period 8 because price keeps going down around the same time. According to the standard textbook explanation, this constantly increasing real money supply shifts the LM curve to the right and lowers the interest rate but stimulates GDP. Figure 20 confirms similar movements, though they are constantly off equilibria under price flexibility.

Behind the background of these business cycle movements, government budget becomes deficit around period 8, though it started with surplus, and begins to accumulate as the left-hand diagram of Figure 21 illustrates. This run-away accumulation of government debt can be reversed by raising an income tax rate

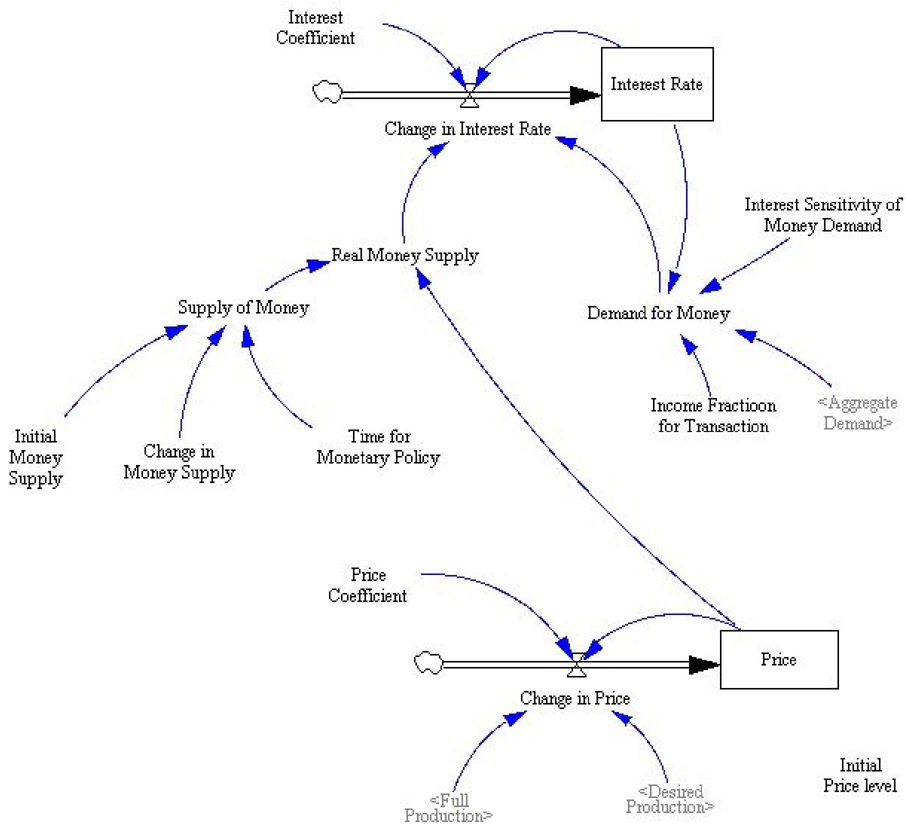

Figure 18: Interest Rate and Price Adjustment Process

by 2 % at the period 10 as shown in the right-hand diagram. To our surprise, this fiscal policy of raising income tax rate does not affect the overall macroeconomic behaviors illustrated in Figure 19 as if the effect of the fiscal policy is totally absorbed by the flexible price.

Growth-dependent Money Supply

The above behaviors are observed under a fixed money supply. It will be interesting to see how a change in money supply affects the behaviors of overall economy. To do so, however, our macroeconomic model here must be first of all integrated with the money supply model developed in my previous paper. Otherwise, it will be misleading to merely change money supply without examining its feedback relations within the system.

Even so, just for our curiosity, let us change money supply proportionally to an economic growth rate. Monetarist argue that money is neutral so that a

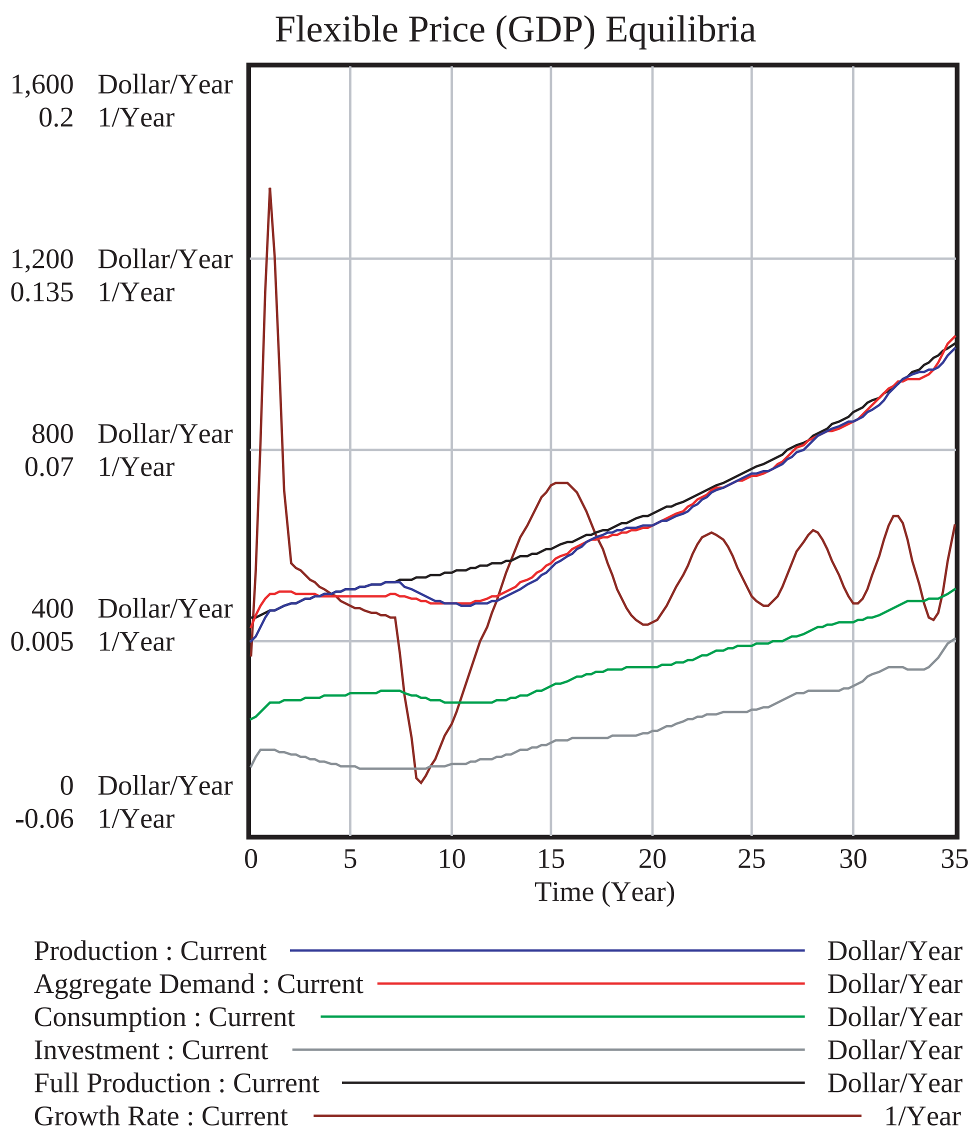

Figure 19: Flexible Price Equilibria of GDP

change in money supply along with the economic growth does not affect true behaviors of its real part.

To observe the effect, let us introduce growth-dependent money supply in the same fashion as we introduced growth-dependent government expenditures in equation (26):

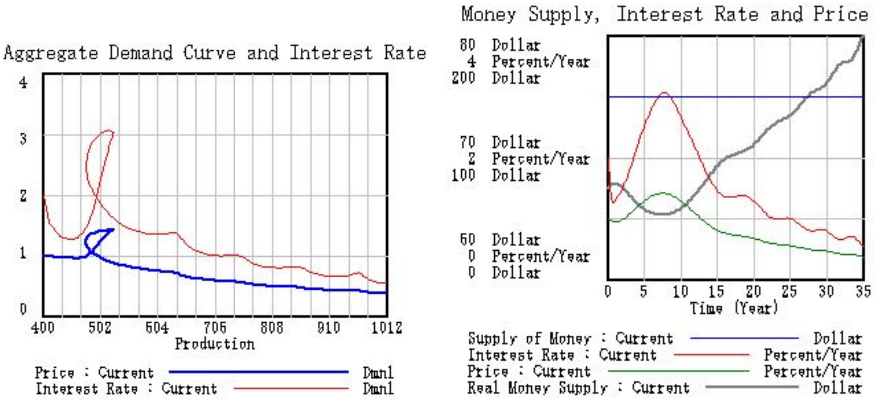

Figure 20: Money Supply, Interest Rate, and Price

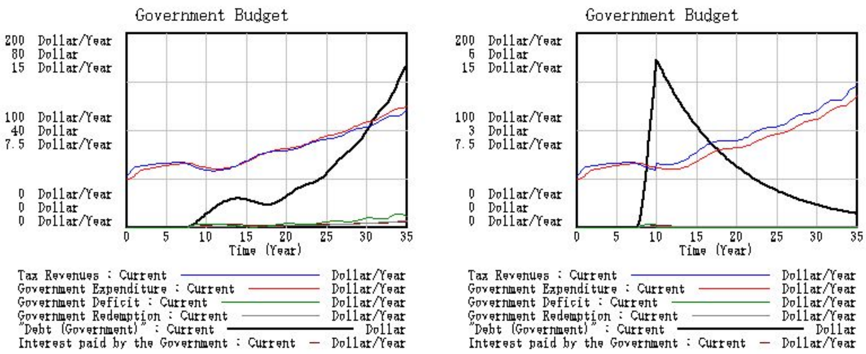

Figure 21: Government Debt under Price Flexibility

$$\frac{dM^s}{dt} = g(t)M^s. \tag{39}$$

Left-hand diagram of Figure 22 illustrates that, compared with a fixed money supply, business cycles scales up. Right-hand diagram shows that price fluctuates rather than continues to decline in the previous case. Accordingly, real money supply also fluctuates and moves oppositely to price level and interest rate as standard textbooks explain. But as pointed out above, true interpretations of these behaviors have to be postponed until the current macroeconomic model is integrated with the previous money supply model

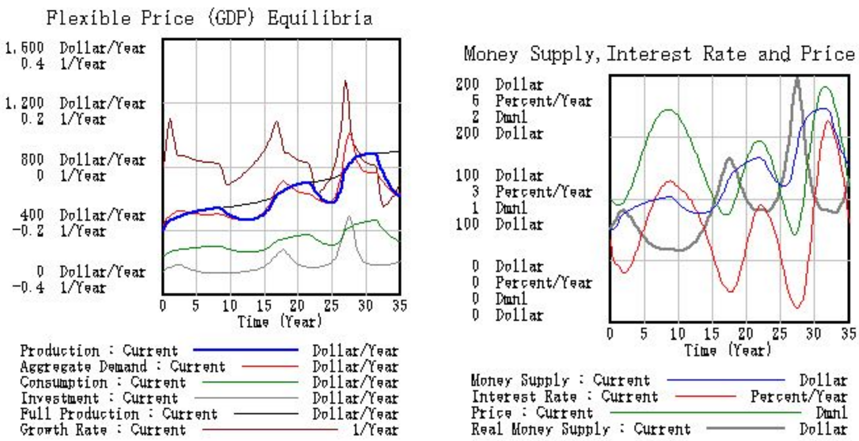

Figure 22: Behaviors under Growth-dependent Money Supply

7 Conclusion

In this second paper of a series of macroeconomic modeling, we have successfully built a real part of macroeconomic system, based on our analytical tool of double entry accounting system. Our model comprises dynamic processes of determining GDP, interest rate and price level. It integrates both Keynesian and neoclassical frameworks, starting first with a standard Keynesian model, then expanding it as an aggregate demand equilibrium model of IS-LM curves, and finally introducing neoclassical long-run feature of price flexibility.

From the analysis of our SD macroeconomic modeling, some of the main features we have obtained are as follows.

- A standard Keynesian macroeconomic adjustment process overshoots an equilibrium GDP when SD inventory adjustment process is introduced.
- Under a balanced budget, a reduction in ramp-sum taxes does not stimulate the aggregate demand.
- Under a Keynesian sticky price, full capacity equilibrium cannot be sustained by monetary and fiscal policies unless they are constantly fine-tuned.
- Fiscal policy to attain full capacity equilibrium will skyrocket the government debt.
- Keynesian aggregate demand equilibria can be presented as loci of the intersections of IS-LM curves.
- Specifically, fiscal policy crowds out the investment opportunities by increasing interest rate. At the same time government debt continues to

accumulate, which may eventually leads to an incredibility of government securities and a total collapse of the economy with worsening production capacity caused by the decrease in investment.

- Under a flexible price, aggregate demand equilibria can no longer be attained, instead production and aggregate demand alternates. Moreover, they fluctuates around a full capacity output level.
- Under such circumstances, monetary and fiscal policies might be no longer effective as a tool to attain full capacity equilibrium.
- On the other hand, accumulated debt of the government can be reduced by increasing tax rates without affecting the overall behaviors of the economy.
- When money supply is fixed under a flexible price, price and interest rate continue to plummet.
- When money supply is changed proportionately to an economic growth rate, price and interest rate begin to fluctuate, while real money supply also fluctuates but oppositely to them.

It may be too early to confirm some of the above features until we will integrate the model presented in this paper with the model of money supply in my first paper, because a change in money supply may trigger an overall feedback reactions among all macroeconomic sectors, and the mechanistic change in money supply introduced here might be misleading.

Apparently, this leads to our next target of research toward the integration of two models mentioned above. Another missing features in our current research such as labor markets and an open economy with foreign sectors will also have to be challenged eventually.

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Appendix: SD Macroeconomic Model

Core part of our SD macroeconomic model is described as follows.

$$
\begin{aligned}
Y_{full} &= \frac{1}{\theta}K \quad \text{(Full Capacity Output)} && (40)\\
Y &= Min(Y_{full},\ Y^D) \quad \text{(Production Decision)} && (41)\\
Y^D &= (I^*_{nv} - I_{nv}) + AD \quad \text{(Desired Production)} && (42)\\
\frac{dI_{nv}}{dt} &= Y - AD \quad \text{(Inventory Adjustment)} && (43)\\
AD &= C + I + G \quad \text{(Aggregate Demand)} && (44)\\
C &= C_0 + cY_d \quad \text{(Consumption Decisions)} && (45)\\
Y_d &= Y - T - \delta K \quad \text{(Disposable Income)} && (46)\\
T &= T_0 + tY - T_r \quad \text{(Tax Revenues)} && (47)\\
I &= \frac{\alpha I_0}{i} \text{ (or } = I_0 - \alpha i)(\quad \text{(Investment Decisions)} && (48)\\
\frac{dK}{dt} &= I - \delta K \quad \text{(Net Capital Accumulation)} && (49)\\
\frac{dG}{dt} &= gG \text{ (or } G = \beta T) \quad \text{(Government Expenditures)} && (50)\\
\frac{dP}{dt} &= \Psi(Y^D - Y_{full}) \quad \text{(Price Adjustment)} && (51)\\
m^s &= \frac{M^s}{P} \quad \text{(Real Money Supply)} && (52)\\
m^d &= aY - bi \quad \text{(Demand for Money)} && (53)\\
\frac{d\,i}{dt} &= \Phi(m^d - m^s) \quad \text{(Interest Adjustment)} && (54)
\end{aligned}
$$

This macroeconomic model consists of 15 equations with 15 unknown variables; that is, $Y_{full}, K, Y, Y^D, I_{nv}, AD, C, I, G, Y_d, T, I, P, MS, MD)$.