A FORMAL METHOD FOR ANALYZING AND ASSESSING OPERATIONAL RISK IN SUPPLY CHAINS

T. Rossi, C. Noé, F. Dallari
Cattaneo University, Italy
Corso Matteotti 22, 21054, Castellanza (VA), Italy
++39.0331.572388
trossi@liuc.it, cnoe@liuc.it, fdallari@liuc.it

Abstract
This paper is aimed at formalizing an objective method to analyze and assess operational risk in supply chains. The proposed approach consists of exploiting the analogy among logistic networks and dynamical systems; in particular, it proposes to identify the risky events characterizing a generic supply chain by studying its attributed Petri net and the corresponding coverability graph, whereas it suggests to assess the risky events effects by building the logistic network simulation model, experimenting on it and applying ANOVA to analyze the results and, then, define the order of importance among the risky events previously figured out. Finally, the method has been applied to a single-item, 3-stages supply chain to show how it can be practically used.

Keywords: Risk, supply chain, Petri nets, simulation

1. INTRODUCTION

In recent years, the problem of supply chain risk has become a major research subject as the several conference proceedings, the special journal issues and the books focused on it can testify.

Such a problem has been pushed to the fore by numerous events [1], e.g. BSE outbreak, millennium bug, terrorist attacks of 11th September 2001, tensions and war in the Middle East, etc., which have urged both academicians and practitioners to consider and investigate the issue of supply chains vulnerability, as well as how risk in logistic networks can be assessed and faced.

In particular, according to some authors [2], the vulnerability of modern commercial supply chains is intrinsically due to their nature. As a matter of fact, these dynamic networks of interconnected firms and industries, which have been driven towards efficiency during last years, are more and more reliant on efficient and reliable transport and communications. As a consequence, not only terrorist attacks, natural disasters or industrial disputes, but also day-to-day risks to the business from more routine supply chain failures (i.e. internal and external disturbances concerning with both materials and information flows) could result in disruptions of the logistic network operations.

Notwithstanding the level of relevance reached by the supply chain risk issue, some writers emphasize that the relationship between day-to-day risk and the implications for supply chain management is poorly understood and is in need of further exploration [3], [4].

For this reason the paper is aimed at developing a formal approach to analyze and assess the operational risk (see paragraph 2.1) within a generic logistic network. In particular, the proposed method is based on the idea of supply chain as dynamical system and exploits attributed Petri nets both to represent the considered logistic network and to identify the risk it deals with. According to the proposed approach, instead, risk evaluation is performed by means of simulation techniques and statistical analyses.

In more detail, the paper is arranged as follows: section 2 is devoted to present the background the work refers to; section 3 illustrates the proposed approach and applies it to a single-item, 3-stages supply chain supposed to belong to the fast moving consumer goods (FMCG) sector; whereas, in section 4 some concluding remarks and future research paths are given.

2. BACKGROUND

The topic addressed by this study involves three main areas. The first relates to risk and supply chains, the second refers to the traditional risk management methodology, while the third deals with supply chains as dynamical systems.

2.1 Risk and supply chains

According to Tapiero (2004), risks in logistic networks can be grouped into four categories:

- Operational risks, which are due to the daily disturbances that material and information flows characterizing the supply chain can suffer from.
- External risks, the elements this class is composed of range from technological to political risks, as well as from financial to market structure risks.
- Strategic risks, which arise when supply chains are typified by information and power asymmetries.
- Risk externalities. Both positive and negative externalities create a risk because divergences between private and social costs can result in damages for the firms the supply chain is composed of.

As mentioned in the Introduction, the present paper refers to the first category of risks (i.e. operational risks) only. They result from many reasons, which can be internal, i.e. failures in operations and service management, as well as external, i.e. uncontrollable events the supply chain is not ready for, and essentially lie in the inability of the supply chain to meet the customers requirements (e.g. in terms of quality, delivery lead time, etc.).

With reference to the measure of such kind of risks, either the consequences of deviating from customers expectations (measured and estimated by money) or the number of times the customers requirements are not met are quite commonly used in literature [3], [4], [5].

With reference to the way of facing operational risks in logistic networks, instead, the traditional risk management process is often applied.

2.2 Risk management process

It is characterized by a precise structure (see figure 1), which can be divided into 3 main phases (the output of each phase is the input for the following one) [5]:

- Risk analysis, after the definition of the bounds of the system under study and the problems it can suffer from, risky events are identified.
- Risk assessment, where risk is assessed by evaluating both the frequency of each event and the severity of its consequences.
- Risk control: in this phase the appropriate measures on how to manage risk are chosen.

Concerning with the steps the present work refers to, i.e. risk analysis and risk assessment, they are traditionally conducted on by means of relatively subjective methods [5]. As a matter of fact, risky events are typically identified through simple fault tree analyses or cause-effect diagrams, whereas quite often the severity of their consequences is given by qualitative judgments of academicians or practitioners. As a result, develop an objective approach for analyzing and assessing operational risk in supply chains exploiting formal methods can be really helpful. In particular, the methodologies originally studied for modeling and controlling dynamical systems seem the most suitable for this purpose, due to the analogy between dynamical systems and logistic networks.

2.3 Supply chains as dynamical systems

A dynamical system can be defined by the 8-tuple S = <T, U, Ω, X, Y, Γ, φ, η> [6], [7], where:

T is a time set ($T \in \Re$).

U is the collection of all the system inputs.

Ω represents the collection of all the admissible input functions of the system.

X is the states set.

Y is the collection of all the system outputs.

Γ represents the output functions set.

φ is the transition state function; it allows the system state at the instant $t \in T$ to be determined starting from the initial state $x \in X$ at the instant $\tau \in T$ and applying the input function $u(\cdot) \in \Omega$, i.e. $x(t) = \varphi(t, \tau, x, u(\cdot))$.

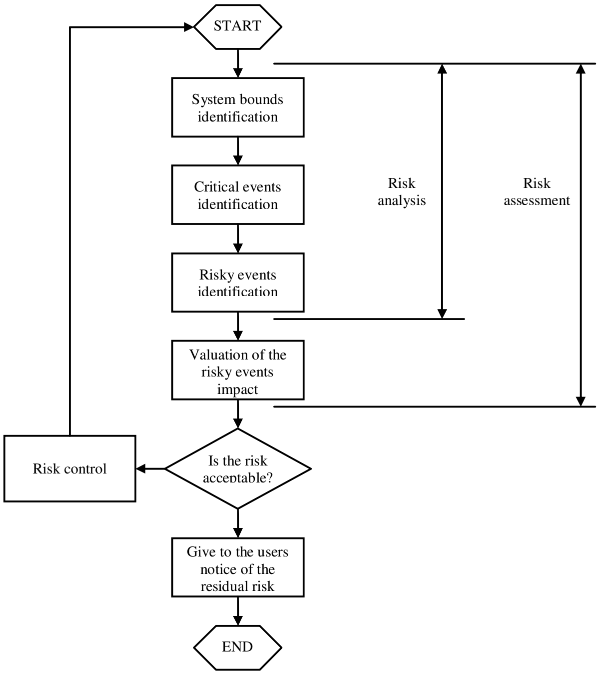


Figure 1: Structure of the risk management process.

η is the function that defines the output $y \in Y$ of the system, i.e. $y(t) = \eta(t, x(t))$.

As well known, supply chains can be described according to the above reported representation [8], [9]. In particular, to contain the degree of complexity, let's consider a single-item logistic network constituted of 1 retailer and 1 manufacturer only. Let's suppose that the former faces a constant daily demand equal to D and manages its stocks according to the EOQ model, where the economic order quantity and the re-order point are indicated as Q and S respectively (to simplify, S is equal to 0 and safety stock is not present). Let's suppose, instead, that the latter produces one lot of L units of the item at hand every K-lt days (where lt is the production lead time) and is characterized by a null delivery lead time. Then, the supply chain in question can be described by the 8-tuple S = <T, U, Ω, X, Y, Γ, φ, η> where:

$T \in \aleph$; as a matter of fact the time axis has been implicitly divided into daily time buckets. In any case, $\aleph \in \Re$.

U = {u}; the only input of the simplified supply chain introduced above is given by the customer demand.

$\Omega = \{u(t)\}$, where $u(t) = D \; \forall \; t \in T$.

$X = \{\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}\}$, i.e. the states set is composed of n state vectors. The generic state vector is: $\mathbf{x_i} = [x_{i1}, x_{i2}, x_{i3}]$.

$x_{i1} = h_i \cdot D$, with $h_i \in \aleph$ and $0 \leq h_i \leq Q/D$, is the item stock level at the retailer.

x_{i2} is the item stock level at the manufacturer (its expression depends on the ratios among the values of L, Q, K and D).

x_{i3} is the Boolean variable that indicates if the manufacturer production system is in operation or not. In particular: $x_{i3} = 1$ if $t \in [j \cdot (K\text{-}lt), j \cdot K]$, with $j \in \aleph$; $x_{i3} = 0$ if $t \in (j \cdot K, (j+1) \cdot (K\text{-}lt))$, with $j \in \aleph$.

$Y = \{y\}$; the only supply chain output is given by the number of items daily sold to the customers.

Γ is composed of the output function $y(t)$, which is defined by $\eta(t, x(t)) = D$ if $x_{i1}(t) \geq D$, $\eta(t, x(t)) = 0$ otherwise.

Then, knowing the state vector at the instant $\tau \in T$ and the input function $u(t)$ the state vector at the instant $t \in T$ can be determined. Therefore, it is possible to affirm that a sort of transition state function, which allows the next supply chain state to be found, exists, i.e. $\mathbf{x}(t) = \varphi(t, \tau, \mathbf{x}, u(t))$.

As a consequence of the above depicted analogy, supply chains can be opportunely described, modeled and studied by means of Petri nets, which are a quite common formalism for representing dynamical systems [10], [11]. In particular, within the framework of Petri nets, the attributed ones seem to be the most useful tool for the supply chains logical modeling. As a matter of fact, the attributes the tokens are characterized by can be suitable for taking into account several logistic network parameters: the economic order quantity of the retailer, the lot size at the manufacturer stage, etc.

3. THE PROPOSED APPROACH

The proposed methodology for identifying risky events in supply chains and for estimating them consists of applying attributed Petri nets and simulation techniques/statistical analyses to the risk analysis and risk assessment phases respectively (for a brief overview on Petri nets see Appendix).

In particular, concerning with the first phase, after the disruptions the considered logistic network can suffer from have been defined (as made explicit in paragraph 2.1 disruptions are represented by not met customers requirements), the attributed Petri net corresponding to the supply chain under study is built. At this stage it is important to put in the net also places which do not correspond to any physical elements of the logistic network but record, through the tokens they contain, the number of times each kind of disruptions is occurred (as a matter of fact, the proposed approach measures the operational risk in terms of number of times a disruption occurs, see paragraph 2.1). Therefore, by means of the coverability graph of the Petri net at hand, sequences of fired transitions which enable marks characterized by tokens in the above mentioned places can be determined, i.e. risky events can be identified. Obviously, computing the coverability graph of a Petri net which models a real-life logistic network can be highly expensive. For this reason, developing *ad hoc* algorithms and software applications to automatically perform such activity or using the ones already available [12], [13], [14], is recommended.

Once risk analysis has been done, the second phase can be faced. It is conducted by building, from the previously obtained attributed Petri net, the supply chain physical model and by experimenting on it (it is worth to note that the physical model is derived by manually putting into a simulation language the logical one, i.e. the objective representation of the system in terms of elements it is composed of and relations among them, according to the scheme for a sound simulation study proposed by Law and Kelton [15]). In more detail, since time in supply chains is considered as a discrete variable (see paragraph 2.3), the physical model is suggested to be based on an event-driven simulation language. With reference to the experimental campaigns, instead, if 'n' are the risky events the logistic network can suffer from, ‘n+1’ experiments have to be conducted. In the first one the supply chain is studied when no risky events happen, whereas in the other ‘n’ each event is separately taken into account. Results of the experimental campaigns are the number of times customers requirements have not been satisfied, i.e. disruptions have been occurred. Applying ANOVA to these results and, in particular, comparing the main effects plots for the number of occurred disruptions of the different risky events allow the impact of each of them to be valuated [16] and, as a consequence, the risk assessment phase to be carried out.

Finally, before the proposed approach is thoroughly explained by applying it to the simplified logistic network depicted in the next paragraph, a couple of clarifications can be done on such an approach (which table 1 synthesizes in terms of both phases it is composed of and used techniques and formalisms/tools).

Table 1: The proposed approach.

PHASE	*TECHNIQUE*	*FORMALISM / TOOL*
Risk analysis	Dynamical systems logical modeling and control	Attributed Petri nets Petri nets coverability graph
Risk assessment	Simulation	Event-driven simulation language
	Statistical analysis	ANOVA

First of all, even if Petri nets are absolutely able to represent the dynamics of the systems they describe and, as a consequence, their coverability trees do not only depend on the static structures of the same systems, graphs may include, from a theoretical point of view, cases that could in fact not occur, i.e. may reveal non-trivial insights into the supply chain. However, this is coherent with the risk analysis phase which is aimed at identifying all the possible risky events the logistic network under study can suffer from, whereas the valuation of their relevance is up to the risk assessment phase (in other words, experimenting on the simulation model allows the importance of the risky events identified by means of the supply chain Petri net and the corresponding coverability tree to be figured out). Secondly, also at the rate of the above reported observation, it could be not clear why not directly building the logistic network simulation model, and proceeding with the risk identification straight on this model. Nevertheless, in this way, that is without a previous and clear definition of the risky events, it could be quite difficult to design an effective experimental campaign, i.e. an experimental campaign able to return the inputs requested by the ANOVA analysis (moreover, here it is opportune to recall that for a sound simulation study it is necessary to build the logical model of the system and, then, derive from it the physical one [15]).

3.1 The context

To contain the level of complexity, a single-item, 3-stages supply chain is used for exemplifying the proposed approach. In particular, the considered logistic network, which is supposed to belong to the FMCG sector, is composed of: 1 retailer, 1 distributor and 1 manufacturer.

Concerning with the first, it is worth to note that no promotions are managed by it and the replenishment policy it refers to is the EOQ model modified according to the forecast system logics. In more detail, the elements characterizing the retailer are: daily demand, which is given by the probability distributions of the customers inter-arrival time and of the number of items bought by the single customer; economic order quantity, re-order point, safety stock (each of them depends on the sales forecasts estimated from time to time by the retailer), standard and increased supplier delivery lead times (represented by the probability distributions of the time the retailer has to wait for receiving goods in normal conditions and when some problems occur respectively); forecasting horizon and forecast accuracy.

Table 2: Synthetic view of the considered supply chain

	ELEMENT	*EXPRESSION/ VALUE*
Retailer	Customers inter-arrival time	exp(0.0069) [days]
	Items bought by the single customer	disc(0.5,1,0.8,2,0.95,3,1,10) [units]
	Initial inventory (R)	6500 [units]
	Economic order quantity (EOQ_r)	$(2*100*Dr/3.5*0.005)^{1/2}$
	Re-order point (ROP_r)	(Dr/15)*2+SS
	Safety stock (SS_r)	$1.96*[(Dr/15)^2*0.5^2+2*(1.25*MAD)^2]^{1/2}$
	Supplier delivery lead time (standard)	norm(2,0.5) [days]
	Supplier delivery lead time (no-standard)	norm(2.5,0.6) [days]
	Forecasting horizon	15 [days]
	Forecast accuracy	0.9
Distributor	Initial inventory (S)	11000 [units]
	Economic order quantity (EOQ_d)	$(2*100*Dd/2.5*0.015)^{1/2}$
	Re-order point (ROP_d)	(Dd/3)*0.13+SS
	Safety stock (SS_d)	$1.96*[(Dd/3)^2*0.03^2+0.13*(1.25*MAD)^2]^{1/2}$
	Transportation resource availability	0.8
	Supplier delivery lead time (standard)	norm(0.13,0.03) [15days]
	Supplier delivery lead time (no-standard)	norm(0.17,0.04) [15days]
	Forecasting horizon	45 [days]
	Forecast accuracy	0.9
Manufacturer	Initial inventory (U)	20000 [units]
	Lot size (LS)	30000 [units]
	Time interval among 2 campaigns (IC)	40 [days]
	Production lead time	5 [days]
	Production resource availability	0.8
	Transportation resource availability	0.8

Even the distributor manages its inventories according to the modified EOQ model. Therefore, it is characterized by the same elements that have been defined with reference to the retailer (the only exceptions are represented by the availability of the distribution transportation resource, not present at the retailer stage, and by the daily demand; as a matter of fact, for the distributor, the demand is given by the retailer orders).

Concerning with the manufacturer, it realizes items through predefined production campaigns. As a consequence, the elements it is important to consider are: availability of the manufacturer transportation and production resources; lot sizing policy; time interval among 2 subsequent campaigns; production lead time.

A synthetic view of the above described logistic network is given in table 2, where expressions and values of each element the supply chain nodes are characterized by are shown (obviously, some of them will result in risky events) and where:

- Dr and Dd are the sales forecast at the retailer and the distributor stages respectively.

- MAD is the mean absolute deviation and is calculated with reference, for instance, to the retailer stage as: (Dr-0.9*Dr)/15.

Finally, it is worth to specify that, the only disruption the considered supply chain is supposed to refer to is given by the impossibility to meet the customers demand, i.e. the stock-outs occurrence.

3.2 Risk analysis

To perform the risk analysis phase in accordance with the proposed approach, it is necessary to define the attributed Petri net which represents the logistic network introduced in the previous paragraph. Figure 2 shows such a net.

Supply chain logical model

Place P1 and transition T1 allow the customers arrival at the retailer to be modeled. As a matter of fact, every time a token is in P1, T1 starts to fire; after a duration drawn from the probability distribution of the customers inter-arrival times, the transition creates 1 token in P2 and assigns to its attribute, here indicated with 'd', a value drawn from the empirical distribution of the number of items bought by the single customer. If the number of tokens in P6 is lower than 'd', that is if the quantity of items at the retailer is not sufficient to satisfy the customer request, T2 becomes active.

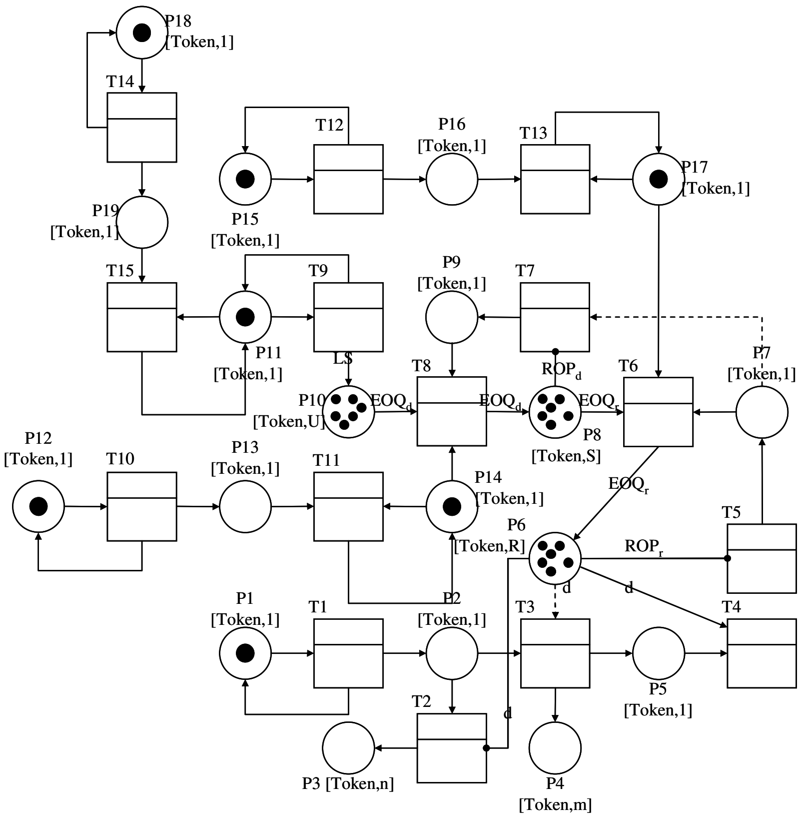


Figure 2: Attributed Petri net of the considered supply chain.

It cancels the token in P2 and creates 1 token in P3 (where the number of occurred stock-outs is recorded). Otherwise, T3 is activated; it creates 1 token in P4 (where the number of satisfied customers is recorded) and in P5 respectively. In particular, the latter allow transition T4 to fire, i.e. 'd' tokens to be canceled from P6 (which represents the retailer warehouse). When the number of tokens in P6 is equal or lower than the retailer re-order quantity, T5 fires causing the creation of 1 token in P7. At this point, if at least the tokens necessary to meet the retailer order are in P8, i.e. in the distributor warehouse, and if 1 token is in P17 (that is, the transportation resource of the distributor is available), transition T6 becomes active. It cancels 1 token from both P7 and P17 and EOQ_r tokens from P8; then, after a duration drawn from the distribution of the distributor delivery lead times, it creates EOQ_r tokens in P6. Instead, if the number of tokens in P8 is lower than the distributor re-order point, transition T7 fires creating 1 token in P9. This token, which represents the order issued to the manufacturer by the distributor, allows transition T8 to be activated if both the number of items in the manufacturer warehouse is higher than the requested quantity (i.e. the number of tokens in P10 is greater than EOQ_d) and the manufacturer transportation resource is available (i.e. 1 token is in P14). In particular, transition T8 cancels 1 token from P9 and P14 respectively and EOQ_d tokens from P10. After a delay drawn from the distribution of the manufacturer delivery lead times, it creates EOQ_d tokens in P8 and 1 token in P14. Finally, place P11 and transition T9 model the manufacturer production campaigns. As a matter of fact, they allow LS tokens to be created in P10 every IC days (the groups of places and transitions P12, T10, P13, T11; P15, T12, P16, T13 and P18, T14, P19, T15, instead, permit the not availability of the manufacturer transportation resource, the distributor transportation resource and the manufacturer production resource respectively to be represented).

Risk identification

To identify the risky events the supply chain is characterized by, the coverability graph (or tree) of the Petri net in question is used. In particular, the coverability tree, which is depicted in table 3, allows the transitions sequences enabling marks with tokens in P3 to be figured out. Then, from these sequences and from the logistic network logical model risky events can be identified (it has been possible to parameterize the coverability graph depicted in table 3 and, as a consequence, to make agiler the proposed approach description, because the same graph has been manually obtained. Obviously, if the coverability tree of the supply chain under study was computed by means of algorithms and software applications, it should contain numeric values only).

Concerning with table 3, the first column refers to the transition which, starting from the current mark, becomes active; the second makes explicit the transition status (i.e. firing – fg, or fired – fd. As a matter of fact the supply chain logical model is a timed attributed Petri net); whereas the third refers to the new mark enabled by the transition in question. Here it is worth to specify that, with reference to the generic transition occupying the *i*th raw of the table, the current mark is in the raw '*i*-1', whereas the new one is in the same *i*th raw. Moreover, concerning with the stock level at the retailer, distributor and manufacturer warehouses, it is indicated, after the *j*th picking, by the letter which identifies the initial inventory level followed by the index 'j', e.g. R_j.

Table 4: Risky events.

MARK	*RISKY EVENT*
$[10\mathbf{1}n0R_n0S0U110110110]$	Forecast inaccuracy at the retailer
$[10\mathbf{1}k0R_k1S_m0U110110010]$	Distributor delivery lead time higher than the standard value
$[10\mathbf{1}k0R_k1S0U110110110]$	Transportation resource of the distributor not available
$[10\mathbf{1}z0(R_z+m)0\ S_{m+1}0\ U_1110110110]$	Forecast inaccuracy at the distributor
$[10\mathbf{1}z0(R_z+m)0S_m0U110010110]$	Transportation resource of the manufacturer not available
$[10\mathbf{1}w0(R_w+v)0(S_v+y)0U_y110010110]$	Production delivery lead time higher than the standard value
$[10\mathbf{1}h0(R_h+p)0(S_p+q)0U_q010010110]$	Production resource of the manufacturer not available

Tables 3a and 3b: Coverability tree of the considered supply chain.

T	*TS*	*MARK*
		[10000R0S0U110110110]
T1	fd	[11000R0S0U110110110]
T3	fd	[10011R0S0U110110110]
T4	fd	[10010$R_1$0S0U110110110]
T1	fd	[11000$R_1$0S0U110110110]
T3	fd	[10021$R_1$0S0U110110110]
T4	fd	[10020$R_2$0S0U110110110]
…	…	…
T4	fd	[100n0R_n0S0U110110110]
T1	fd	[110n0R_n0S0U110110110]
T2	fd	[10**1**n0R_n0S0U110110110]
T1	fd	[110n0R_n0S0U110110110]
T3	fd	[100(n+1)1R_n0S0U110110110]
T4	fd	[100(n+1)0R_{n+1}0S0U110110110]
T5	fd	[100(n+1)0R_{n+1}1S0U110110110]
T6	fg	[100(n+1)0R_{n+1}0$S_1$0U110110010]
T1	fd	[110(n+1)0R_{n+1}0$S_1$0U110110010]
T3	fd	[100(n+2)1R_{n+1}1$S_1$0U110110010]
T4	fd	[100(n+2)0R_{n+2}1$S_1$0U110110010]
…	…	…
T4	fd	[100k0R_k1S_m0U110110010]
T1	fd	[110k0R_k1S_m0U110110010]
T2	fd	[10**1**k0 R_k1S_m0U110110010]
T1	fg	[000n0R_n0S0U110110110]
T12	fd	[000n0R_n0S0U110111110]
T18	fg	[000n0R_n0S0U110111010]
T1	fd	[110n0R_n0S0U110110110]
T3	fd	[100(n+1)1R_n0S0U110110110]
T4	fd	[100(n+1)0R_{n+1}0S0U110110110]
T5	fd	[100(n+1)0R_{n+1}1S0U110110110]
T1	fd	[110(n+1)0R_{n+1}1S0U110110110]
T3	fd	[100(n+2)1R_{n+1}1S0U110110110]
T4	fd	[100(n+2)0R_{n+2}1S0U110110110]
…	…	…
T4	fd	[100k0R_k1S0U110110110]
T1	fd	[110k0R_k1S0U110110110]
T2	fd	[10**1**k0 R_k1S0U110110110]

T	*TS*	*MARK*
T1	fd	[110n0R_n0S0U110110110]
T3	fd	[100(n+1)1R_n0S0U110110110]
T4	fd	[100(n+1)0R_{n+1}0S0U110110110]
T5	fd	[100(n+1)0R_{n+1}1S0U110110110]
T6	fd	[100(n+1)0(R_{n+1}+1)0$S_1$0U110110110]
T1	fd	[110(n+1)0(R_{n+1}+1)0$S_1$0U110110110]
T3	fd	[100(n+2)1(R_{n+1}+1)0$S_1$0U110110110]
T4	fd	[100(n+2)0(R_{n+2}+1)0$S_1$0U110110110]
…	…	…
T4	fd	[100k0(R_k+1)0$S_1$0U110110110]
T5	fd	[100k0(R_k+1)1$S_1$0U110110110]
T6	fd	[100k0(R_k+2)0$S_2$0U110110110]
…	…	…
T6	fd	[100x0(R_x+m)0S_m0U110110110]
T1	fd	[110x0(R_x+m)0S_m0U110110110]
T3	fd	[100(x+1)1(R_{x+1}+m)0S_m0U110110110]
T4	fd	[100(x+1)0(R_{x+1}+m)0S_m0U110110110]
…	…	…
T4	fd	[100z0(R_z+m)0S_m0U110110110]
T5	fd	[100z0(R_z+m)1S_m0U110110110]
T7	fd	[100z0(R_z+m)1S_m1U110110110]
T8	fg	[100z0(R_z+m)1S_m0$U_1$110010110]
T1	fd	[110z0(R_z+m)0S_{m+1}0$U_1$110110110]
T3	fd	[100(z+1)1(R_{z+1}+m)0 S_{m+1}0$U_1$110110110]
T4	fd	[100(z+1)0(R_{z+1}+m)0 S_{m+1}0$U_1$110110110]
…	…	…
T4	fd	[100w0(R_w+m)0 S_{m+1}0 $U_1$110110110]
T1	fd	[110w0(R_w+m)0 S_{m+1}0 $U_1$110110110]
T2	fd	[10**1**z0(R_z+m)0 S_{m+1}0 $U_1$110110110]
T1	fd	[110x0(R_x+m)0S_m0U110010110]
T3	fd	[100(x+1)1(R_{x+1}+m)0S_m0U110010110]
T4	fd	[100(x+1)0(R_{x+1}+m)0S_m0U110010110]
…	…	…
T4	fd	[100z0(R_z+m)0S_m0U110010110]
T5	fd	[100z0(R_z+m)1S_m0U110010110]
T7	fd	[100z0(R_z+m)1S_m1U110010110]
T8	fg	[100z0(R_z+m)1S_m0$U_1$010010110]
T1	fd	[110z0(R_z+m)0S_m0$U_1$110010110]
T2	fd	[10**1**(z+1)1(R_z+m)0S_m 0$U_1$110010110]

On the other hand, the stock level after the *h*th storing is given by the expression of the previous inventory level and the notation “+h”, e.g. R_j+h.

Finally, as mentioned before, interpreting the below coverability tree according to the verbal description of the logical model depicted in figure 2, the risky events the supply chain refers to can be outlined. In particular, they are shown in table 4.

Tables 3c and 3d: Coverability tree of the considered supply chain.

T	*TS*	*MARK*
T1	fg	$[000x0(R_x+m)0S_m0U110110110]$
T10	fd	$[000x0(R_x+m)0S_m0U111110110]$
T11	fg	$[000x0(R_x+m)0S_m0U110010110]$
T1	fd	$[110x0(R_x+m)0S_m0U110010110]$
T3	fd	$[100(x+1)1(R_{x+1}+m)0S_m0U110010110]$
T4	fd	$[100(x+1)0(R_{x+1}+m)0S_m0U110010110]$
…	…	…
T4	fd	$[100z0(R_z+m)0S_m0U110010110]$
T5	fd	$[100z0(R_z+m)1S_m0U110010110]$
T7	fd	$[100z0(R_z+m)1S_m1U110010110]$
T1	fd	$[110z0(R_z+m)0S_m0U110010110]$
T3	fd	$[100(z+1)1(R_{z+1}+m)0S_m0U110010110]$
T4	fd	$[100(z+1)0(R_{z+1}+m)0S_m0U110010110]$
…	…	…
T4	fd	$[100w0(R_w+m)0S_m0U110010110]$
T1	fd	$[110w0(R_w+m)0S_m0U110010110]$
T2	fd	$[10\mathbf{1}z0(R_z+m)0S_m0U110010110]$

T	*TS*	*MARK*
T1	fg	$[000x0(R_x+m)0S_m0U110010110]$
T14	fd	$[110x0(R_x+m)0S_m0U110010111]$
T15	fg	$[110x0(R_x+m)0S_m0U010010110]$
T1	fd	$[110x0(R_x+m)0S_m0U010010110]$
T3	fd	$[100(x+1)1(R_{x+1}+m)0S_m0U010010110]$
T4	fd	$[100(x+1)0(R_{x+1}+m)0S_m0U010010110]$
…	…	…
T4	d	$[100z0(R_z+m)0S_m0U010010110]$
T5	d	$[100z0(R_z+m)1S_m0U010010110]$
T7	fd	$[100z0(R_z+m)1S_m1U010010110]$
T8	fd	$[100z0(R_z+m)1(S_m+1)0U_1010010110]$
T6	Fd	$[100z0(R_z+m+1)0(S_{m+1}+1)0U_1010010110]$
T1	fd	$[110z0(R_z+m+1)0(S_{m+1}+1)0U_1010010110]$
T3	fd	$[100(z+1)1(R_z+m+1)0(S_{m+1}+1)0U_1010010110]$
T4	fd	$[100(z+1)0(R_{z+1}+m+1)0(S_{m+1}+1)0U_1010010110]$
…	…	…
T4	fd	$[100h0(R_h+p)0(S_p+q)0U_q010010110]$
T5	fd	$[100h0(R_h+p)1(S_p+q)0U_q010010110]$
T7	fd	$[100h0(R_h+p)1(S_p+q)1U_q010010110]$
T1	fd	$[110h0(R_h+p)0(S_p+q)0U_q010010110]$
T2	fd	$[10\mathbf{1}h0(R_h+p)0(S_p+q)0U_q010010110]$

3.3 Risk assessment

According to the proposed approach, the valuation of the risky events identified in the previous paragraph is performed through simulation techniques and statistical analyses.

For this reason, the supply chain physical model is built (ARENA is the applied simulation meta-language [17]) and 8 experimental campaigns are conducted on it (every campaign consists of 100 replication runs and the single run is characterized by a 180-days length). The first campaign refers to the logistic network where no problems occur, whereas in the others a different risky event is separately considered. Then, since the only supply chain disruption taken into account is to not satisfy final customers demand (see paragraph 3.1), the variable chosen as benchmark for the comparison is the number of stock-outs. The relative importance of the single risky event is determined, i.e. the risk assessment phase is completed, by applying ANOVA to the outputs of all the experimental campaigns and, in particular, by comparing among them the main effects plots for the number of stock-outs of the different events.

In figure 3 such a comparison is shown. From such a figure it is possible to claim that the most relevant risky event is given by 'distributor delivery lead time higher than the standard value'. As a matter of fact, when this event is activated the mean of the stock-outs number changes significantly. The risky event represented by 'distributor transportation resource not available' follows in order of importance the above mentioned one. Even in this case the benchmark variable mean is appreciably influenced by the activation of the event in question. With reference to the other risky events, instead, their significance in explaining the stock-outs number variance is quite low. In particular, the events dealing with the manufacturer stage are substantially equivalent, whereas ‘forecast inaccuracy at the retailer’ is a more relevant risky event than ‘forecast inaccuracy at the distributor’.

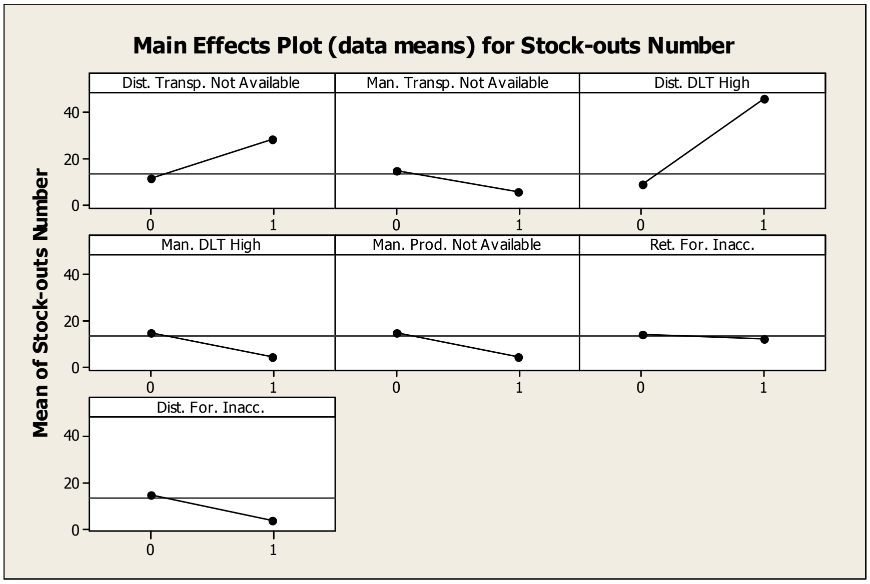


Figure 3: Effects of the different risky events on the number of stock-outs.

Exactly this latter evidence testifies, at a first glance, the goodness of the proposed risk assessment procedure; as a matter of fact, it is quite evident that the influence on the stock-outs number of an inaccurate forecasting process of the retailer, which has a negative impact also on the upstream stage, is greater than the influence due to the inaccurate forecasting process of the distributor only.

Finally, it is worth to note that the order of importance of the above mentioned risky events, which is shown in table 5, is not general but depends on both the supply chain structure and the values assumed by the elements the logistic network is characterized by.

Table 5: Order of importance of risky events.

ORDER OF IMPORTANCE	*RISKY EVENT*
1	Distributor delivery lead time higher than the standard value
2	Transportation resource of the distributor not available
3	Forecast inaccuracy at the retailer
4	Forecast inaccuracy at the distributor
5	Manufacturer delivery lead time higher than the standard value
5	Transportation resource of the manufacturer not available
5	Production resource of the manufacturer not available

4. CONCLUDING REMARKS

The traditional risk management process has been applied in the last years to face supply chain operational risks. However, concerning with the first 2 phases of the process, i.e. risk analysis and risk assessment, often the lack of an objective methodology to perform them has not allowed the day-to-day logistic network risks to be opportunely managed. For this reason, the main aim of the study presented in this paper consisted of identifying a formal method for conducting the phases of risk analysis and assessment. The suggested approach is based on the idea of supply chain as dynamical system and, in more detail, exploits, on one hand, attributed Petri nets and coverability trees to identify the risky events a generic supply chain can suffer from and, on the other hand, simulation techniques and ANOVA to evaluate the effects of each risky event previously defined. The proposed approach has been exemplified by applying it to a single-item, 3-stages logistic network. If the use of simulation and ANOVA has proved to be quite effective for an objective risk assessment, the risk analysis phase supported by attributed Petri nets and coverability trees has allowed for identifying the same risky events a traditional cause-effect diagram would have figured out. However, this is probably due to the very simple supply chain taken into account; as a matter of fact, the advantages in using formal methods for the risk assessment phase seem to increase with the complexity and the peculiarity of the system, in this case of the supply chain, under study. Of course, this last statement should be more deeply verified: for this reason real-life industrial cases concerning with the proposed approach application have been already planned. In addition to it, future research steps deal with the development of a formal procedure based on simulation to control operational risk in supply chains.

5. APPENDIX

Broadly speaking, Petri nets are a graphical and mathematical formalism for the logical modeling of dynamical systems. Such nets, which allow not only a particular status but also the evolution of the system under study to be represented, are characterized by 2 types of elements, i.e. places and transitions, interacting with each other by means of arcs (the graphical representation of the above mentioned elements is given in figure 4). Places and transitions manage tokens that allow the instantaneous status of the system to be described and the transitions to be activated or deactivated.

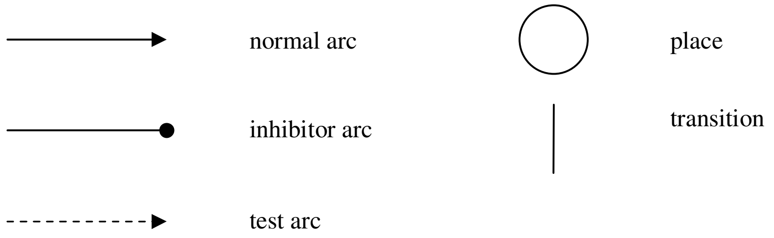


Figure 4 – Petri nets elements

In more formal terms a Petri net can be defined as a tuple $R = <P, T, Pre, Post, M_0>$, where P is the finite set of places (with $|P|= n$); T is the finite set of transitions (with $|T|= m$); $P \cap T = \varnothing$ and $P \cup T \neq \varnothing$; $Pre(P_i, T_j)$ is a function that defines arcs from a place to a transition; $Post(P_i, T_j)$ is a function that defines arcs from a transition to a place; M_0 is the initial mark of the net (for possible combination among Petri nets elements see figure 5).

A special type of Petri nets is represented by attributed Petri nets. They are, substantially, Petri nets with attributed tokens. In figure 6 an overview of attributed Petri nets elements is given:

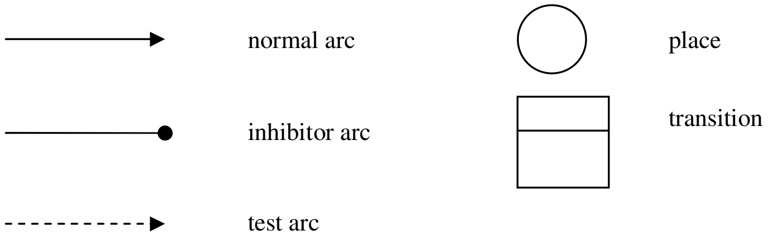


Figure 6 – Attributed Petri nets elements

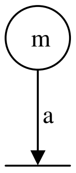
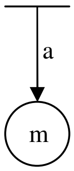
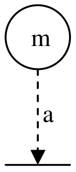
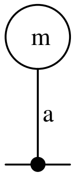
Input arcs	Output arcs
 classical relation among place and transition; transition fires if m ≥ a	 classical relation among place and transition; token created in the discrete place are equal to a
 control relation; transition fires if m ≥ a (no token flow)	
 control relation; transition fires if m < a (no token flow)	

Figure 5 – Possible combination among Petri nets elements

6. REFERENCES

[1] Peck H., 2004, Interesting Times, Freight Transport Review, 2,1-3.

[2] Chapman P., Christopher M., Juttner U., Peck H., Wilding R., 2003, Identifying and managing supply-chain vulnerability, Logistics and Transport Focus, 4, 1-4.

[3] Tapiero C. S., 2004, Risk measurement and supply chains, International Workshop on Performance and Risk Measurement, Milan, Italy, 113-129.

[4] Proth J. M., 2004, Supply chains: goal and specific risks, International Workshop on Performance and Risk Measurement, Milan, Italy, 131-136.

[5] Culp C. L., 2001, The risk management process: business strategy and tactics, Wiley, New York.

[6] Rinaldi S., 1993, System Theory, CLUP, Milan, Italy (in Italian).

[7] Girod B., Rabenstein R., Stenger A., 2001, Signals and Systems, Wiley, Chichester, UK.

[8] Bunimovich L., 2001, Dynamical systems and operations research: a basic model, Discrete and Continuous Dynamical Systems-Series B, 1, 209-218.

[9] Radons G., Neugebauer R., 2004, Nonlinear dynamics of production systems, Wiley-VCH, Weinheim, Germany.

[10] David R., Alla H., 1994, Petri nets for modeling of dynamical systems: a survey, Automatica, Vol. 30, No. 2, 175-202.

[11] Murata T., 1989, Petri nets: properties, analysis and applications, Proceedings of IEEE, Vol. 77, 541-580.

[12] Giua A., Seatzu C., 2001, The Observer Coverability Graph for the Analysis of Observability Properties of Place/Transitions Nets, Proceedings of the 6th European Control Conference, Porto, Portugal, 1339-1344.

[13] Basile F., 2002, Lecture notes on Petri nets, Federico II University, Naples, Italy (in Italian).

[14] Crestani D., Jean-Marie A., Covès C., 2005, Petri nets analysis: complexity and finite coverability graph in modular design, Studies in Informatics and Control, Vo. 14, No. 2, 55-64.

[15] Law A. M., Kelton W. D. (2000) “Simulation modelling and analysis”, McGraw-Hill, Boston, Massachusetts.

[16] Greene W., 2000, Econometric Analysis, Prentice Hall, Upper Saddle River, New Jersey.

[17] Kelton W. D., Sadowski R. P., Sadowski D. A., 2002, Simulation with ARENA, McGraw-Hill, Boston, Massachusetts.