Spatial Dynamics of Social Network Evolution

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Abstract

This paper explores the problem of fragmenting social networks enabled by spatial distancing between distinct socioeconomic classes. Such fragmentation is evidenced by the experience of urban sprawl without population growth. We develop a prototype model to examine the spatial dynamics of social network evolution in the face of neighborhood migration. This model draws upon the small world analogy by using an initial template of connections that are "rewired" over time. Spatially, connections are established for neighborhood proximity. Socially, connections are added based upon similarity of economic class. Migration patterns thus affect the probability of rewiring social connections. In effect, the probability of rewiring becomes endogenous as the network evolves over time. Analyses are conducted to explore the relative cohesiveness of the emergent community networks, and the income differentials between neighborhoods. The development of this abstract model is discussed in relation to further application and calibration to a real-world case community.

Introduction

Even in the absence of population growth, many communities continue to experience urban sprawl, or low-density fringe development. Since one's ability to move to a new home usually requires an income well above the poverty level, new development serves to separate those who can afford to choose from those who cannot. In this way, sprawl dynamics exacerbate spatial disparity between socioeconomic classes. But underlying the aggregate tendency of sprawl is the individual choice of where to live. For a current resident, this is a choice of whether to stay or leave a neighborhood in favor of another neighborhood or another community altogether. For a newcomer, this is a choice of where to settle upon arrival. In aggregate these choices shape the spatial and temporal dimensions of urban dynamics such as sprawl and its accomplice, spatial disparity. As evidenced by the presence of urban sprawl in the absence of population growth, such social fragmentation becomes a concern for community efficacy in establishing new economic opportunities and for potential sources of conflict between sub-communities.

The recursive effect of social networks on neighborhood choices becomes difficult to test due to the inherent impracticality of experimenting with policies on a real community. Virtual experimentation via computer models is therefore a logical if not essential choice for exploring the effects of social networks on neighborhood choice over time. While a computer model will not predict the future of a real community, it is the only practical means of testing alternative assumptions and policies in an internally consistent framework. And it is this process of modeling and testing, this virtual experimentation, which offers opportunities for insight or surprise relative to expectations.

Background

A social network refers to a set of interpersonal relationships. In this case, the relationships are considered to be communication links that evolve in the face of neighborhood and community migration. Although networks are powerful, they are difficult to define and measure, let alone simulate over time. Research on social networks has tended to be ethnographic (Rowe and Wolch 1990, Gilbert 1998), empirical as in much of social network analysis (Wasserman and Faust 1994), or abstract as for emerging simulation techniques (Watts and Strogatz 1998, Barabasi and Albert 1999). Emerging indirect estimation techniques (Conley and Topa 2003) enable calibration of abstract models from spatial socioeconomic data.

The study of social networks is rich with methods of structural analysis that are based upon graph theory as invigorated in large part by Erdos and Renyi (1960). Social networks enable a purely relational perspective, where the biggest challenge is in identifying relationships to be analyzed. For the proposed research, the relationship is one of communication. Two areas of emerging research involve *spatial* and *dynamic* social networks.

Social Networks are Dynamic

The *dynamics* of social networks may be considered in two ways – the dynamics of behavior within a network structure, or the evolution of the network itself over time. To connect structure with dynamics, White (2004) presents a synthesis of social network theory in relation to social dynamics, including a detailed conception of how statics and dynamics operate in balance through many patterns witnessed in social network theory. In a related work, White et al (2004) focus on cohesive network topologies in both organizational contexts and emergent fields, and the ways in which these interact. Applying an epidemiological notion of diffusion to the social context, Granovetter (1978) demonstrates the utility of threshold models in understanding collective behavior. Social thresholds refer to the minimum fraction of one's peers who have made a decision before the individual in question does. In a similar manner, Crane (1991) utilizes a contagion model to examine the nonlinear social effects of neighborhood dynamics as behaviors transmit among members.

Epstein and Axtell (1996) introduce ways in which simulated agents can represent human connections and interactions over time. Indeed, they emphasize the importance of transients and dynamics more than the quest for equilibrium conditions. With a similar approach to modeling, Young (1998) focuses on theory underlying such individual-based conceptions and the institutions that result from such interaction. The extent to which individual preferences are selfish or altruistic has also been examined in recent simulation studies (Bowles 2001, Lazar et al

2002) of the simultaneity and reflexivity of network evolution and individual preference evolution. These studies reveal not only that trends toward conformity within groups can sustain difference between groups (see also Young 2001), but also that within-group cohesion may result from socially influenced altruism that runs counter to selfish motives. Moreover, Lazar et al (2002) note the importance of being able to detect others' type that enables homophily in the first place. In a related study, Macy et al (2003) examine how polarization or segregated clustering can occur in the absence of resource competition. They encode an attractor network in their model, such that agents are attracted to others of similar states and are also influenced by others. While a novel approach, their work is consistent with prior findings of polarization under the principles of structural balance.

Watts (1999a, 1999b) and Watts and Strogatz (1998) explore self-organizing dynamics of network formation, emphasizing that where self-organization occurs, the resulting structure lies between randomness and order. The prime example of such a mix of randomness and order is the small world network inspired by Milgram (1967), in which local clusters are dense but are connected globally through a few cross-cutting links between hubs. Accordingly, Watts (1999a) developed algorithms for evolving small worlds in which interpersonal connections are locally dense (e.g., most of my friends are also friends) but globally sparse (e.g., everybody is not directly connected to everybody else). This small world lies in interesting region between complete subgroup isolation and complete network connectivity. While the suitability of the small world structure to describe real-world social networks is still under evaluation, it is one of the most promising quantifiable theories of social structure. A critical element of Watts' (1999a, 1999b) small world formulation is a probability of "rewiring" social connections from an initially ordered structure (usually a ring lattice of one-dimensional connectivity).

The study of network dynamics adds complexity to social network analysis, a field that is already full of techniques to test social relationships. And yet, as evidenced by the above literature, it offers the prospect of additional insight in understanding the simultaneity of how the network influences the individual, and how the individual influences the network. By exploring relationships at the level of individual interaction, we can learn about behavior over time at both micro and macro social network scales of analysis.

Social Networks are Spatial

Stanley Milgram (1967) conducted an inherently *spatial* social network experiment that came to undergird the small world theory as extended above by Watts (1999a, 1999b). Milgram gave research participants in Kansas and Nebraska a letter describing a target person in Massachusetts. If the participant knew the target on a personal basis, he/she was asked to send the letter directly to that person. Otherwise, the participants were to give the letter to a personal acquaintance who was more likely to know the person. Of the letters that were returned, the median number of intermediate links was 5.5, rounding up to the cliche "six degrees of separation." What is striking about Milgram's research is that it was inherently geographical in nature, intended to measure social distance spanning geographic locations between arbitrarily chosen individuals. And yet the subsequent research in social networks has tended to involve purely "relational" space, without consideration of geographic distance.

Spatial simulations of neighborhood networks date back to the cellular automata simulations of Hagerstrand (1965) and Schelling (1971, 1978). Cellular automata are discrete cells located on a grid that update their state based on their previous state and the state of their neighbors (Shalizi 2003). Cellular automata are a sort of precursor to the more richly structured decision rules of mobile interactive agents. Hagerstrand (1965) utilized empirical data on telephone network density to explore stochastic simulations of spatial diffusion of farming subsidies in Sweden. Schelling (1971, 1978) employed an abstract cellular framework for examining the emergence of segregation from low thresholds of preference for similar neighbors. While such simulation utilizes abstract space in the form of a uniform grid of household locations, it enables the development of intuition about the conditions under which spatial clustering emerge and the degree of contingency in its patterns.

In a related abstract spatial simulation, Arthur (1988) explores the uniqueness that can play out as a result of historical path dependence from industrial location decisions. While Arthur (1988) illustrates the potency of a simplified abstract model, geographers with expertise in Geographic Information Systems (GIS) are ready to add realism to spatial simulations. Dibble and Feldman (2004) present a computational laboratory in which simulated agents may interact in networks across abstract or empirical space using GIS templates.

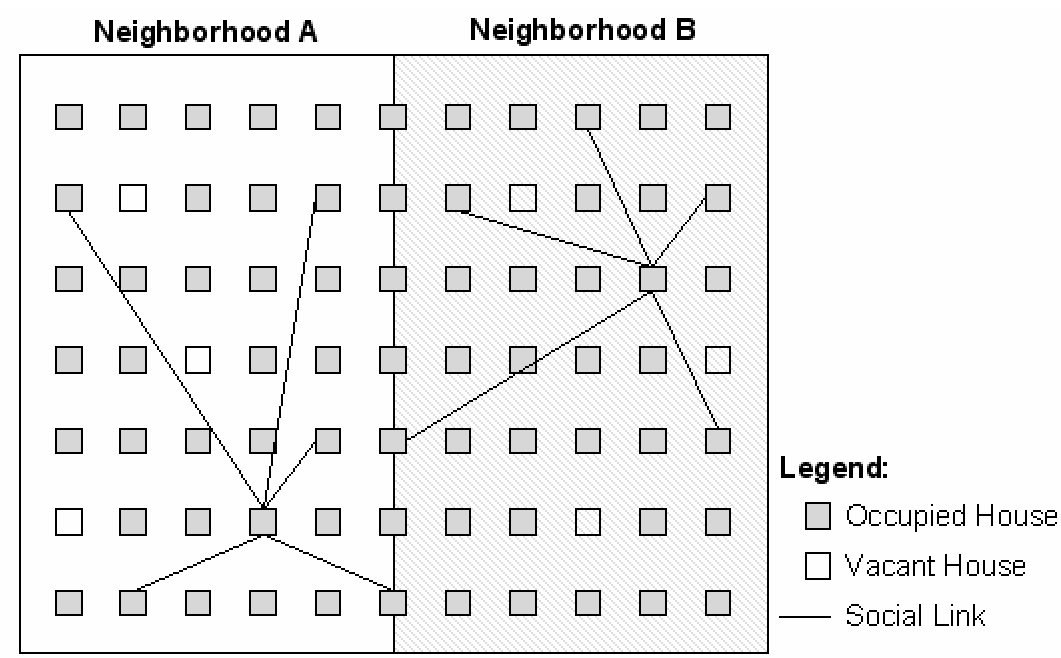

Figure 1. Example Template for a Spatial Social Network.
This template demonstrates how a social network may be embedded in 2-dimensional space. In this case, houses are distributed uniformly across a grid between two neighborhoods. The social links of one house from each neighborhood are shown. While links are predominantly contained within each house's neighborhood, they expand beyond direct neighbors and may ultimately connect across neighborhoods through boundary households.

The template in Figure 1 above is an abstraction that simply represents location in space as residential location. Social links are strong in broad domains ("neighborhoods"), though not restricted to adjacent neighbors (as in the Schelling 1971, 1978 case). This is a juxtaposition of the relational space of the social network as visualized in 2-dimensional space. The argument that social networks are spatial is at once obvious and yet superficial to sociologists who have resisted it with a pure focus on relational space instead. And as geographers recognize, life space is far from Euclidean (Adams 1995). Yet abstractions of space in just two dimensions can prove pragmatic for navigating our world, much as abstractions of relational space in social networks are relevant to human understanding. The combination of these spaces offers a way to make the small world (Watts 1999a) rewiring probability endogenous in a spatially explicit environment that incorporates individual choices about whether to leave neighborhoods. A major contribution of this research to the field of social network analysis is this dynamic evolution of networks in a spatially explicit environment.

Methods

To prototype and test a variety of virtual experiments, we utilize the AnyLogic[1] modeling software. AnyLogic has several features that make it attractive for use in simulating spatial social dynamics such as this. While it is Java-based and therefore compatible with existing models built in Java environments such as RePast[2], AnyLogic has a more intuitive user interface for nonexpert programmers. Although it is commercial software, simulations are easily exported as Java applets for broader dissemination. Functionally, it incorporates the hybrid approach with building blocks for both discrete and continuous functions. Sensitivity testing and optimization are facilitated through an experiment setup, and agent parameters may be imported from a broad suite of probability distributions. Although it contains many simplifications for novice users, the model's underlying code exports easily for expert examination.

Two choice formulations are central to the model structure: friendship and neighborhood choice. Discrete choice probabilities may be represented mathematically by the convenient logit formulation (Ben-Akiva and Lerman 1985). Equation 1 utilizes a binary logit formulation to represent the choice of whether or not to gain a friend (if the switch s is negative), or whether or not to lose a friend (if the switch s is positive).

Equation 1. Binary Logit Formulation for Friendship Choice

$$P_{ij} = \frac{P_{base} \cdot \exp\left(s \cdot \frac{|I_i - I_j|}{I_{\max} - I_{\min}} \right)}{1 - P_{base} + P_{base} \cdot \exp\left(s \cdot \frac{|I_i - I_j|}{I_{\max} - I_{\min}} \right)}$$

where

P_{ij}	= Probability that person i will choose person j as a friend
P_{base}	= Baseline probability (neighborhood dependent)
s	= Switch (-1 if making a friend, +1 if losing a friend)
I_i, I_j	= Individual annual income of person i and of person j
I_{max}	= Maximum possible annual income, \$60,000/year
I_{min}	= Minimum possible annual income, \$20,000/year

The friendship choice is socioeconomic due to the relative income portion of Equation 1. At the start of the simulation, each agent is assigned an income between \$20,000 and \$60,000 per year. This income does not change over time, and is simply a continuous representation of socioeconomic differences. A large difference between the incomes of two agents increases the likelihood of the friendship ending, and decreases the likelihood of a new friendship beginning.

[1] http://www.xjtek.com
[2] http://repast.sourceforge.net

The spatial element of friendship choice derives from the baseline probability, which is neighborhood dependent. We define a parameter F_{out} to represent the preference for friendships outside one's current neighborhood. A balanced preference in the two-neighborhood model would invoke F_{out} of 0.5. In contrast, an unbalanced preference favoring one's current neighborhood would invoke $F_{out} < 0.5$. The results presented in the next section compare an unbalanced F_{out} of 0.3 with the balanced case. The balanced case relies solely on socioeconomic preferences, while the unbalanced case involves both socioeconomic and spatial preferences. Both cases have spatial implications, however.

The network of friendships developed through the friendship choices in Equation 1 become part of the neighborhood choice formulation expressed in Equation 2. The probability that an agent will choose a different neighborhood is a function of affordability and utility. Affordability is expressed as one's income relative to the neighborhood average income. This value could vary between 1/3 (least affordable) and 3 (most affordable) due to the minimum and maximum incomes of \$20,000 and \$60,000 respectively. The affordability of one's current neighborhood is "grandfathered" as unity, however.

Equation 2. Neighborhood Choice Formulation

$$P_{ni} = \frac{a_{ni} \cdot \exp(U_{ni})}{\sum_n a_{ni} \cdot \exp(U_{ni})}$$

$$a_{ni} = \frac{I_i}{I_n}$$

$$U_{ni} = \left(1 - \left(\frac{I_{\max} - I_n}{I_{\max} - I_{\min}}\right)\right) + \frac{F_{ni}}{\sum_n F_{ni}}$$

where

P_{ni} = Probability that person *i* will choose neighborhood *n*
a_{ni} = Affordability of neighborhood *n* to person *i* (1 if current neighborhood)
U_{ni} = Utility of neighborhood *n* to person *i*
I_n = Average annual income in neighborhood *n*
F_{ni} = Number of friends that person *i* has in neighborhood *n*

As expressed in Equation 2, neighborhood utility derives from income-based attractiveness and the individual's fraction of friends in the target neighborhood. The income-based attractiveness (the first component of the utility equation) is neighborhood-specific and designed to vary between zero and unity. If the neighborhood average income is \$20,000 (I_{min}), attractiveness is zero; if neighborhood average income is \$60,000 (I_{max}), attractiveness is unity. While neighborhood attractiveness is universal, the friendship fraction is unique to each individual. Recall from Equation 1 that friendships reflect spatial and socioeconomic preferences.

The evaluation frequency is critical for understanding the dynamics of neighborhood and friendship choice. In our baseline case, we define the rate of evaluating neighborhoods to be exponentially distributed around once in 100 months (8.3 years). The stochastic distribution of times enables asynchronous evaluation by individual agents. Whether or not a move is made depends upon whether the probability in Equation 2 is greater than a randomly generated number between 0 and 1, and whether houses are available in the target neighborhood. If a move is made, the agent moving updates their friendship network by making one friend and losing a friend according to Equation 1. In addition to the neighborhood evaluation, agents update their network at a rate that is ten times more frequent (once in 10 months in the baseline case), and is also stochastic under an exponential frequency distribution. The friendship network is adjusted with a 50% probability at each evaluation point. Friendships are made or broken stochastically to adjust toward a target number of five friendships per person.

Results

The simulation results that follow are based upon the two-neighborhood template illustrated in Figure 1, with 200 total houses and agents of varying incomes assigned randomly to houses at a 90% occupancy rate. Agents are connected using a random network (Erdos and Renyi 1960) designed to have five average friendships per agent. The random initial network, layout, and stochastic dynamics are fixed using a random seed for comparative purposes.

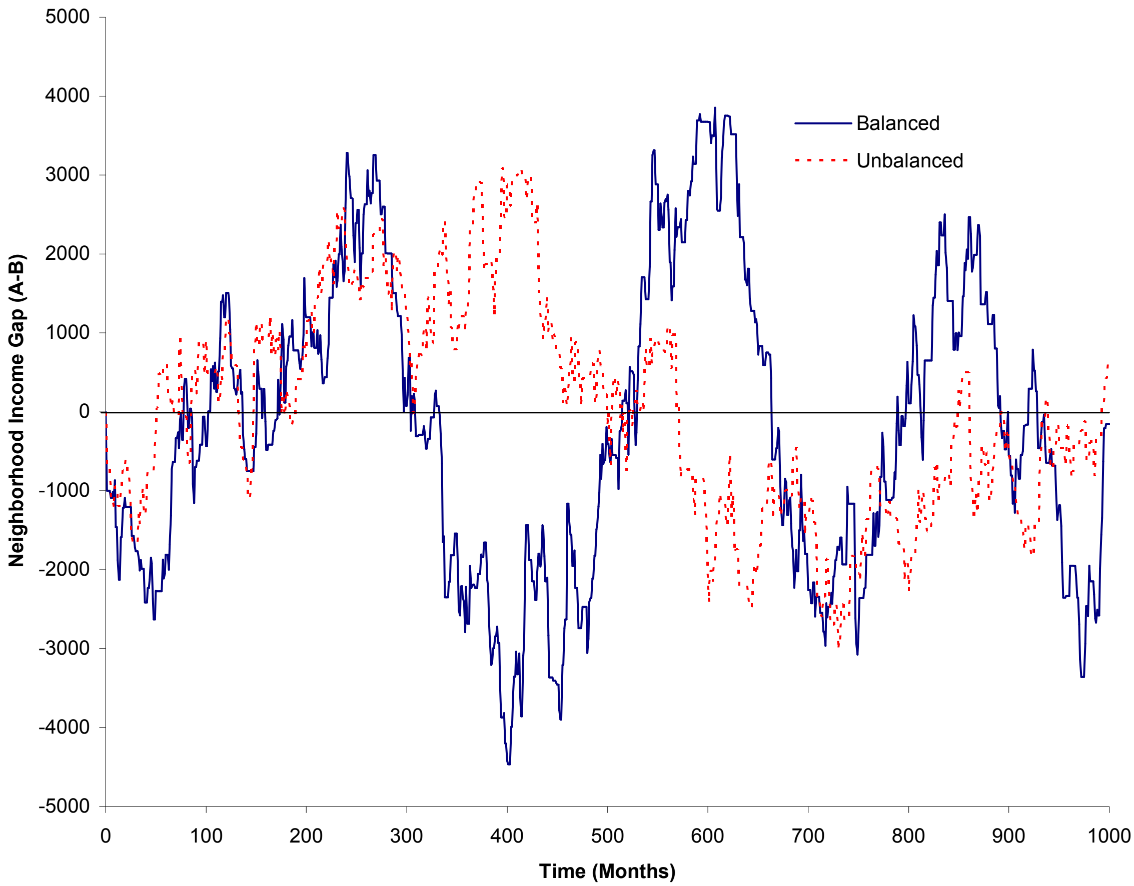

Figure 2. Effect of Balanced and Unbalanced Neighborhood Preferences on Income Gap. Income gap is the difference in average income between the two neighborhoods. The raw value of the income gap is positive or negative depending on which neighborhood is more affluent on average. The balanced neighborhood preference is based upon F_{out} of 0.5, such that the baseline probability is 50%. In contrast, the unbalanced neighborhood preference is based upon F_{out} of 0.3, such that the baseline probability of choosing friends outside one's neighborhood is 30%, versus 70% within one's own neighborhood. Although the balanced case exhibits more oscillation between neighborhoods over the simulation, the intensity (absolute value) of the income gap is greater than the unbalanced case on average. These simulations utilize the baseline (low) evaluation frequency of 100 months for moving and 10 months for network updates.

A measure of spatial disparity is reflected in neighborhood income gap (Figure 2). Neighborhood income gap simply compares the average neighborhood incomes at each point in time. The dynamic simulation results for neighborhood income gap are illustrated in Figure 2 for the balanced and unbalanced cases with preferences for the fraction of friends outside one's neighborhood, F_{out}, set to 0.5 and 0.3 respectively. The balanced case therefore only invokes socioeconomic preferences in the friendship choice. However, both cases illustrate strong spatial dynamics as measured by the income gap between neighborhoods. Over the simulation horizon of 1000 months (83 years), the balanced case involves strong income gaps that oscillate as the concentration of affluent individuals shifts from one neighborhood to the other. The unbalanced case retains a single neighborhood bias for longer, but with weaker income gaps. Invoking only the element of socioeconomic preference therefore has stronger spatial implications for socioeconomic disparity.

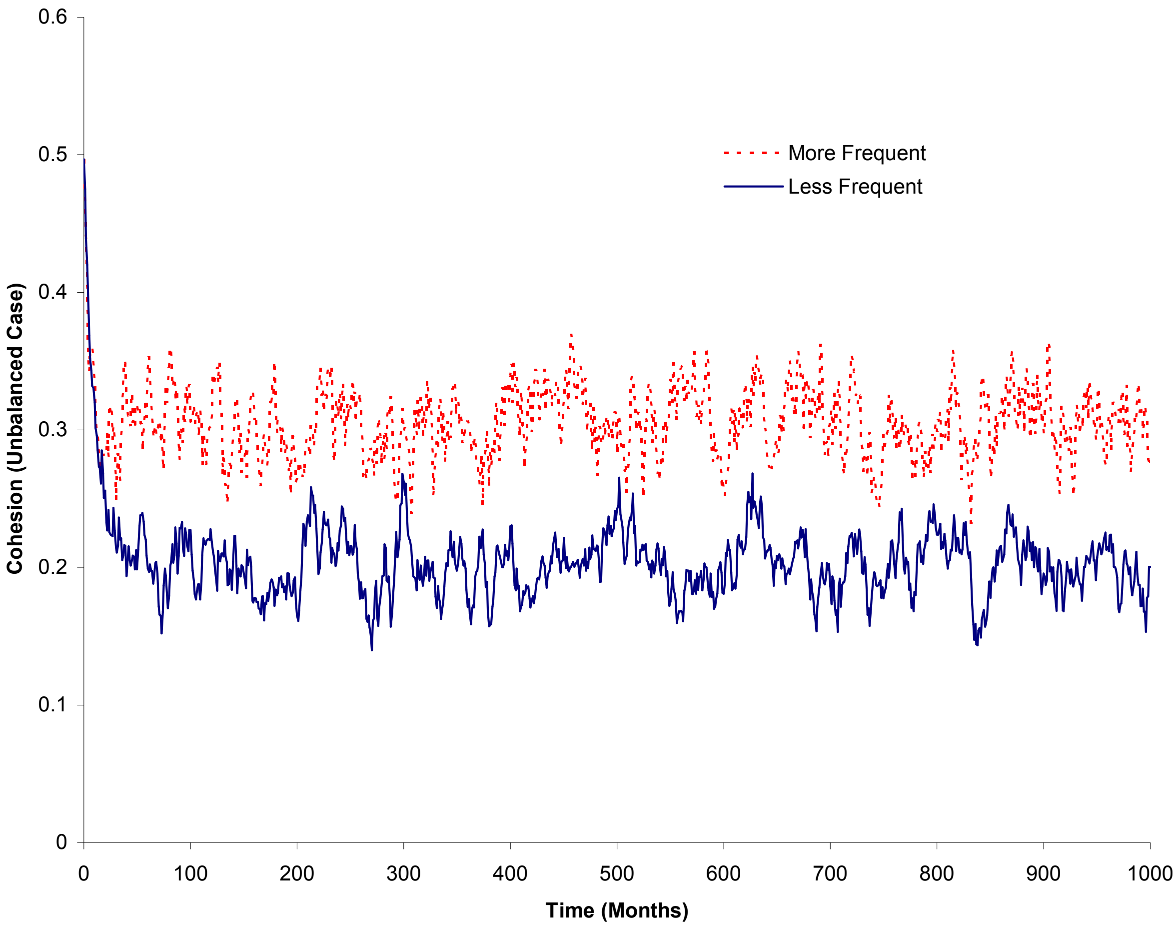

Figure 3. Effect of Evaluation Frequency on Cohesion under Unbalanced Preference. Cohesion is measured as the fraction of social links that span across neighborhoods. The evaluation frequency has an effect on the average cohesion over time. For the less frequent evaluation of 100 months (~8.3 years) for moving and 10 months for network updates, the average cohesion is 0.205. In contrast, the more frequent evaluation of 10 months for moving and 1 month for network updates exhibits a higher average cohesion of 0.304. These simulations are conducted for the unbalanced neighborhood preference based upon F_{out} of 0.3, such that the baseline probability of choosing friends outside one's neighborhood is 30%, versus 70% within one's own neighborhood.

A measure for spatial fragmentation is reflected by the cohesion measure (Figure 3), where cohesion is the cumulative fraction of social links that span neighborhood boundaries. Cohesion is the opposite of fragmentation, such that when cohesion is low, fragmentation is high. Figure 3 compares the effect of evaluation frequency on the overall level of cohesion under the case unbalanced neighborhood preferences (F_{out} = 0.3). Network cohesion initializes near 50%, but declines over time as it approaches a dynamic equilibrium. In the case of more frequent neighborhood and network evaluation, the average cohesion is approximately 0.3. In the less frequent (baseline) case, average cohesion is approximately 0.2. Therefore, evaluation frequency has an impact on the overall cohesion level.

Further sensitivity testing is necessary to assess the extent to which these results are generalizable under a variety of random settings. Specific results such as the approximate equivalence of cohesion to F_{out} in the higher frequency case may be related to questions of path dependence. Path dependence, the extent to which a result depends upon initial conditions, is expected be less significant under high frequency of evaluation, and under conditions of balanced preferences.

Table 1. Effects of Evaluation Frequency and Neighborhood Preference on Cohesion and Income Gap.

	Less Frequent	**More Frequent**
Balanced Preference (F_{out} = 0.5)	**Higher:** Avg. Cohesion (0.495) Avg. Income Gap ($1570)	**Lower:** Avg. Cohesion (0.466) Avg. Income Gap ($1277)
Unbalanced Preference (F_{out} = 0.3)	**Lower:** Avg. Cohesion (0.205) Avg. Income Gap ($1135)	**Higher:** Avg. Cohesion (0.304) Avg. Income Gap ($1334)

Table 1 above summarizes the effects of evaluation frequency and neighborhood preference on cohesion and income gap. These results reflect parameter variations under the identical random seed conditions. In contrast to the income gap displayed in Figure 2, the income gap averaged in Table 1 is based on the absolute difference at each point in time, not raw values. These results reveal that our model formulation prevents high spatial disparity (income gap) from coexisting with high fragmentation (low cohesion), as average values for both cohesion and income gap vary in the same direction for each case. The highest cohesion and income gap occurs in the less frequent, balanced preference case. The lowest cohesion and income gap occurs in the less frequent, unbalanced preference case. The effect of frequency depends upon whether neighborhood preference is balanced.

Discussion

We conclude that the income-based attractiveness is stronger in the utility equation when neighborhood preferences are balanced. This occurs because the second term of the utility equation (the friendship fraction) averages to 0.5 in the balanced case, versus 0.7 in the unbalanced case. Those who can afford to move to the more attractive neighborhood do so, producing an agglomeration of higher income individuals in the attractive neighborhood. At the same time, balanced preferences result in a higher cohesion level, so that spatial disparity and fragmentation do not coexist at their extremes under this formulation.

Extensions

Alternative constructions of this model will include variants of small worlds and scale free networks that evolve with neighborhood and community migration. These networks will be tested for tipping points of fragmentation in the face of alternative migration probabilities. Sensitivity testing will be conducted with varying migration parameters under this endogenous rewiring to explore the relative cohesiveness of the emergent community networks. The development of this abstract model will then be extended to the real-world case community of Danville, Illinois.

The socioeconomic and spatial assumptions underlying network connectivity may be derived from census data as integrated into the model with Geographic Information Systems (GIS). Additional GIS data include zoning information and business analysis (if possible, a master plan of zoning changes will be obtained from the community). While GIS census data are available for areas such as census tracts or blocks, such data will be disaggregated to correspond to individual households. Two methods of disaggregation will be considered: the first is with parcel data, which may be available in GIS form by the city of Danville. However, such parcel data are not uniformly available nationwide at this time (though a national parcel database is under construction). The second method is with programming algorithms to assign household locations within the tract or block area in either a random or arranged manner. Such algorithms are transparent and transferable to other study sites. Moreover, accuracy of household location is not of primary concern, as the social networks that emerge will be based more broadly on neighborhood proximity, rather than immediate adjacency. This is not to say that immediate adjacency is not a factor in real-world networks, just that for approximating an entire community using census area-based data, accuracy is relegated to broader resolution.

To demonstrate the viability of locating households in space with an algorithm, we will first attempt to obtain household location via parcel data or as derived from aerial photography. Simulation results from this exact location method may then be compared with simulation results from the assignment algorithm to demonstrate whether or not location accuracy arises as an issue in this model. In any case, converting the census counts into specific locations in space enables readier translation into the AnyLogic modeling software described earlier.

As individual household objects are located in space, they retain identification with census tracts or block groups. Such identification enables heterogeneous parameterization of attributes based upon distribution of counts recorded in the census (e.g., 10 households with income less than $20,000, 5 houses unoccupied, etc.). Regardless of exact point location, accuracy of assignment is achieved only at the broader level of census area. But such accuracy is not of primary concern – rather the range of heterogeneity is of greater importance in structuring social networks. The translation into networks will be guided by techniques of indirect estimation using socioeconomic data as developed by Conley and Topa (2003). In analyzing unemployment, Conley and Topa (2003) use broad spatial proximity to structure the social network of local interactions, with the majority occurring within and adjacent to the tract in question. While this strictly spatial algorithm is highly simplified, they demonstrate that it works better for anticipating shifts in unemployment than an aggregate black box approach would. In addition to the spatial considerations of social network structure, we incorporate social connections also based upon socioeconomic status. The specific algorithms for such connections may be varied for scenario analysis, along with migration parameters and their effect on social network structure.

Closing Comments

The goal of this research is not to produce one defining model but rather to provide an integrative simulation tool for considering alternative assumptions and theories about how social networks relate to neighborhood choice. The importance of virtual experimentation with computer simulation models is central to the research design. Figure 4 highlights the role that such experimentation plays in influencing researchers' own mental maps of the problem at hand. The model described herein is the first of several that will be constructed and tested for insight. It is through the process of iteration in model design that we learn about the question at hand. Virtual experimentation enables studies of human behavior to be tested in ways that are not feasible or ethical in real communities. As a corollary, virtual experimentation offers a range of simulated realities that may or may not emerge in the "real world." In Figure 4, this learning from virtual experiments is illustrated as embedded within learning from the real world.

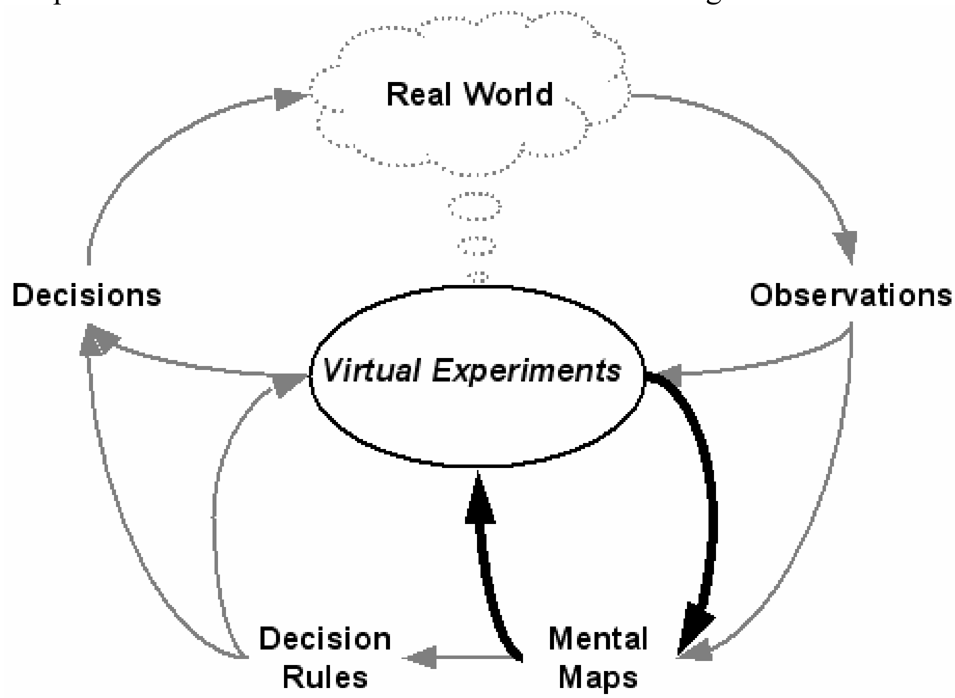

Figure 4. Learning from Virtual Experiments.
This diagram illustrates how the process of computer modeling as virtual experimentation is embedded in the "real world." Observations of this perceived reality help to shape our mental maps, in turn affecting the decision rules by which we operate, thereby influencing decisions themselves that feed back to actions in our reality. With virtual experimentation, observations and decision rules may be input as data and assumptions into our models, and our output may inform policies for making decisions. But the most critical aspect of virtual experimentation is learning from the reflexive relationship between such experimentation and our own mental maps. Adapted from Sterman (2000:88).

Whether through statistical measures or direct experience, real world observations shape the design of virtual experiments and our own mental maps that interpret them. This research involves not only the development of abstract models, but also the inclusion of primary and secondary data for the case community of Danville, Illinois. In the context of virtual experimentation (Figure 4), such real world observations will serve to guide model development not only through parameterization, but also through researchers' own mental maps.

References

Adams, P. 1995. "A Reconsideration of Personal Boundaries in Space-Time." *Annals of the Association of American Geographers* 85(2): 267-285.

Arthur, W. B. 1988. "Urban Systems and Historical Path Dependence." In J. H. Ausubel and R. Herman (eds), *Cities and their Vital Systems: Infrastructure, Past, Present, and Future*. Washington, DC: National Academy Press, 85-97.

Ben-Akiva, M. and S. R. Lerman. 1985. *Discrete Choice Analysis*. Cambridge, MA: MIT Press.

Bowles, S. 2001. "Individual Interactions, Group Conflicts, and the Evolution of Preferences." In S. N. Durlauf and H. P. Young (eds.) *Social Dynamics*. Washington, DC: Brookings Institution Press. pp. 155-190.

Burt, R. S. 1987. "Social Contagion and Innovation: Cohesion Versus Structural Equivalence." *American Journal of Sociology* 92(6): 1287-1335.

Conley, T. and G. Topa. 2003. "Identification of Local Interaction Models with Imperfect Location Data." *Journal of Applied Econometrics* 18(5): 605.

Crane, J. 1991. "The Epidemic Theory of Ghettos and Neighborhood Effects on Dropping Out and Teenage Childbearing." *American Journal of Sociology* 96(5): 1226-1259.

Dibble, C. and P. G. Feldman. 2004. "The GeoGraph 3D Computational Laboratory: Network Terrain Landscapes for RePast." *Journal of Artificial Societies and Social Simulation* 7(1). http://jasss.soc.surrey.ac.uk/7/1/7.html

Epstein, J. M. and R. Axtell. 1996. *Growing Artificial Societies: Social Science from the Bottom Up*. Washington, DC: Brookings Institution Press.

Erdos, P. and A. Renyi. 1960. "On the Evolution of Random Graphs." *Publications of the Mathematical Institute of the Hungarian Academy of Sciences* 5: 17-61.

Gilbert, M. 1998. "Race, Space, and Power: The Survival Strategies of Working Poor Women." *Annals of the American Association of Geographers* 88(4): 595-621.

Granovetter, M. 1978. "Threshold Models of Collective Behavior." *American Journal of Sociology* 83(6): 1420-1443.

Hagerstrand, T. 1965. "A Monte Carlo Approach to Diffusion." *Archives Europeenes de Sociologie* 6: 43-67.

Lazar, A., D. Chavalarias, and T. K. Ahn. 2002. "Endogenous Network Formation and the Evolution of Preferences." Presented at Agent 2002, University of Chicago, October 11-12. http://www.econ.iastate.edu/tesfatsi/LazarChavAhn.EndNet.pdf

Macy, M. W., J. A. Kitts, and A. Flache. 2003. "Polarization in Dynamic Networks: A Hopfield Model of Emergent Structure." In R. Breiger, K. Carley, and P. Pattison (eds.) *Dynamic Social Network Modeling and Analysis: workshop summary and papers*. Washington, DC: National Academy Press.

Milgram, S. 1967. "The Small World Problem." *Physiology Today* 2: 60-67.

Rowe, S. and J. Wolch. 1990. "Social networks in time and space: homeless women in Skid Row, Los Angeles." *Annals of the Association of American Geographers* 80(2): 184-204.

Schelling, T. C. 1971. "Dynamic Models of Segregation." *Journal of Mathematical Sociology* 1:143.

Schelling, T. C. 1978. *Micromotives and Macrobehavior*. New York, NY: W.W. Norton & Company.

Shalizi, C. R. 2005. "Methods and Techniques of Complex Systems Science: An Overview." In T. S. Deisboeck and J. Y. Kresh (eds.), *Complex Systems Science in Biomedicine*. Dordrecht, The Netherlands: Kluwer Academic Press. http://arxiv.org/PS_cache/nlin/pdf/0307/0307015.pdf

Sterman, J. D. 2000. *Business Dynamics: Systems Thinking and Modeling in a Complex World*. Boston, MA: McGraw-Hill.

Wasserman, S. and K. Faust. 1994. *Social Network Analysis: Methods and Applications*. Cambridge, UK: Cambridge University Press.

Watts, D. J. 1999a. *Small Worlds: The Dynamics of Networks Between Order and Randomness*. Princeton, NJ: Princeton University Press.

Watts, D. J. 1999b. "Networks, Dynamics and the Small World Phenomenon." *American Journal of Sociology* 105(2):493-527.

Watts, D. J. and S. H. Strogatz. 1998. "Collective Dynamics of 'Small-World' Networks." *Nature* 393: 440-442.

White, D. R. 2004. "Network Analysis and Social Dynamics." *Cybernetics and Systems* 35(2-3): 173-192.

White, D. R., J. Owen-Smith, J. Moody, and W. W. Powell. 2004. "Networks, Fields and Organizations: Micro-Dynamics, Scale and Cohesive Embeddings." *Computational and Mathematical Organization Theory* 10(1):95-117.

Young, H. P. 1998. *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*. Princeton, NJ: Princeton University Press.

Young, H. P. 2001. "The Dynamics of Conformity." In S. N. Durlauf and H. P. Young (eds.) *Social Dynamics*. Washington, DC: Brookings Institution Press. pp. 133-153.