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Expectation Formation and Parameter Estimation in Uncertain Dynamical Systems: The System Dynamics Approach to Post Keynesian-Institutional Economics

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> Our social systems are 'broadband,' by which we mean that they do not have the structural character that will cause present conditions to determine the exact state of the system far into the future.…the exact path followed by an industrial or economic system depends on two separate considerations. First is the orderly system structure for which we can hypothesize approximate rules of behavior. The second is noise that occurs in all decision functions and about which we know only its existence and perhaps its approximate magnitude and statistical character. Noise is that part of the decision flow for which we have no satisfactory causal hypothesis.
>
> Jay W. Forrester (1961, p. 430)

> I hope that we shall hear no more of the 'Harrod knife-edge.'
>
> Sir Roy Harrod (1970, p. 741)

> The primary dictionary definitions of 'validation,' verification' and similar terms relate to being objectively true, and this is precisely what all models are not. Any model that refers to the real world (that is, that is not only a mathematical tautology) is, by definition, not the real world but a selected, filtered, abbreviated, constructed representation. It differs from the real world in multiple, indeed infinitely many ways. It is not correct, true, or valid. It is wrong. Get used to it. The question is whether the model is appropriate and useful for a particular purpose. Claims that models are 'valid' or have been 'validated' are usually part of a rhetorical strategy the modeler or others are using to legitimate their analysis or themselves, get their policies adopted, or gain other advantage.
>
> John D. Sterman (2004)

Introduction

Twenty years ago, Paul Davidson (1983-1984) first put forth his view that any economist practicing normal science within the Post Keynesian paradigm must produce models and theories that are, among other things, nonergodic. In a nutshell, nonergodic systems are uncertain and unpredictable. Since the notion that economic systems are uncertain, particularly in the long-run, is a crucial insight that can be traced back to (among others) Keynes himself, Davidson argued that any economic theory that utilizes the rational expectations hypothesis and

[1] I'd like to thank Robert L. Eberlein for help in setting-up the FIMLOF algorithm used in the simulations presented in this paper. Any errors, of course, are mine alone.

its reduction of the future into probabilistic outcomes, cannot be considered Post Keynesian and is most probably invalid.

In a separate stream of thought, Radzicki (1988, 1990, 2003, 2005) has suggested that, due to the striking similarities in their underlying methodologies, Post Keynesian economics, institutional economics, and system dynamics computer simulation modeling can be combined to form a superior form of heterodox economics. In an effort to extend this line of thinking this paper will lay-out, from a system dynamics perspective, three arguments that parallel Davidson's views and that should thus be of interest to heterodox economists. The first is that dynamic, nonlinear, feedback systems (that is socioeconomic systems) are indeed uncertain and unpredictable, except perhaps in the very short-run. This will be accomplished by replicating a well-known simulation experiment that was devised in the earliest days of the field of system dynamics. The second argument is that, in direct contrast to rational expectations models, it is possible to produce formal models that mimic the formation of actual human expectations and can thus be used when modeling uncertain dynamical systems. This will be accomplished by presenting a well-known bounded rational system dynamics expectations structure and showing how it can closely mimic actual human expectations data. The third argument is that, just as in the case of the pattern modeling approach used in institutional economics, the issue of model validity is complicated and goes far beyond the ability of a model to mimic historical data. This will be accomplished by fitting a version of the Harrod economic growth model tightly to U.S. macroeconomic data, and discussing this result in light of system dynamics criteria for judging a model's validity.

Dynamic, Nonlinear, Feedback Systems are Uncertain and Unpredictable

In 1961, Jay W. Forrester (1961, Appendix K) published a very clever simulation experiment that supported his argument that dynamic, nonlinear, feedback systems are unpredictable in principle, except perhaps in the very short-run when any actions taken by decision makers on the basis of their "correct" predictions cannot influence system behavior. More specifically, Forrester presented a system dynamics model that exhibited an oscillatory behavior mode when perturbed by a small amount of noise and designated it as the “real world.” He then made an exact copy of the model and called it a perfectly specified “model” of the “real world.” Since the “model” was an exact replica of the “real world” its structure, initial conditions, and parameters were all perfectly estimated – conditions that are impossible to achieve in a real modeling situation.

To conduct the experiment, Forrester made a single change to the “model” of the “real world” system: he altered the seed value to the noise input that perturbed the “model.” This meant that the specific sequence of random numbers that perturbed the “model” differed from the sequence perturbing the “real world” system, even though both sequences came from the same probability distribution with the same mean and variance.[2]

A replication of Forrester’s experiment is shown in Figures 1 and 2. The model in Figure 1 is a simplified version of a portion of the mega-corp sector of the Post Keynesian-Institutionalist-System Dynamics model that is currently under construction at Worcester Polytechnic Institute.[3] It represents the “real world” portion of Forrester’s experiment.[4]

Place Figures 1 & 2 About Here

The logic of the model is straightforward. Orders for widgets are received by the sector and accumulated in its backlog. Discrepancies between the sector’s desired backlog, which is based on the normal time it takes the sector to fill an order as well as managerial perceptions of the incoming order rate, and its actual backlog, along with discrepancies between the sector’s desired level of inventory and its actual level of inventory, send signals to the sector to hire/fire workers and produce more/less widgets. Widgets are shipped to customers at a normal rate, consistent with the sector’s normal delivery delay, unless the sector is experiencing a shortage of inventory due to an insufficient production rate. When the sector ships its widgets, it simultaneously clears its backlog of orders.

Figure 2 presents two time series plots, one for labor and one for inventory, from a simulation of the model shown in Figure 1. Clearly, the perfectly specified “model” of the “real world” system is of no use in predicting the future value of either variable.

Recently, Forrester (2003) has refined his arguments about the unpredictability of nonlinear, dynamic feedback systems. He notes that it may be possible to predict the future of such systems in the very short-run, when their momentum overwhelmingly determines their behavior, but it is not possible to do so in the longer-run when the random factors that influence their behavior are amplified. Moreover, any decisions that are made on the basis of accurate short-run predictions will not influence the behavior of these systems until time periods

[2] For a full discussion of noise and its use in system dynamics modeling see Sterman (2000, Appendix B). Changing the seed value for the noise function effectively meant that Forrester was introducing a small amount of uncertainty into the structure of the "perfectly specified model."
[3] For an overview of the nomenclature of system dynamics modeling see Radzicki (2003) or Sterman (2000). For additional information about the Post Keynesian-Institutionalist-System Dynamics model see Radzicki (2005). In contrast to the model shown in Figure 1, Forrester used a production-distribution or supply chain model for his original experiment.
[4] To save space, the exact copy of this model that serves as the “model” in the experiment is not shown. The equations for this model are contained in the Appendix, as are the equations for all of the other models presented in this paper.

further into the future, when accurate prediction is impossible. Quoting Forrester (2003, p. 10): “one can forecast in the time zone in which one cannot act, and can act in the time zone in which one cannot forecast.”[5]

Parameter Estimation and Fitting Models to Historical Data

Parameter estimation and fitting models to historical data are somewhat contentious issues within the field of system dynamics. Generally speaking, there are two opposing schools of thought related to the issues. The first has been termed the “classical” school by David Peterson (2003) and the second has been termed the “statistically inclined" school by George Richardson (1981). Practitioners within the classical school believe that it is unnecessary to estimate the parameters of a system dynamics model with formal statistical techniques or to tightly fit its behavior to a set of historical time series data, while practitioners within the statistically inclined school take the opposite position. A possible third school of thought that offers something of a compromise between the classical and statistically inclined schools might be termed the “hand calibration school” [see Lyneis and Pugh (1996)]. System dynamicists who practice hand calibration obtain initial parameter estimates for a model the same way that classical system dynamicists do, but then "tweak" them by hand until the model's behavior closely mimics a set of historical time series data.

The Classical School. Practitioners within the classical school believe that the outcome of a properly conducted system dynamics modeling study is a robust feedback policy or rule[6] that can be followed by policy makers to keep an actual system behaving in a desired manner, without the system dynamics model that created it and without the need to forecast the future value of any variable. Moreover, the system dynamics model from which the robust feedback policy is derived can be created and tested without fitting it tightly to any particular set of historical time series data.[7] Instead, the model is built to mimic the basic time shapes defined by the system's "reference mode." A reference mode is a picture of the time paths of various variables deemed important to the problem under study.[8] It is thus a dynamic "picture" of the problem to which the modeling effort is directed.

According to members of the classical school, the parameters of a system dynamics model need not be estimated econometrically (i.e., by teasing them out of time series and/or cross sectional data) but instead can be derived outside of the model, where possible, "at or below the level of aggregation in the model."[9] This means that a system dynamics modeler must seek-out information at the level of the individual decision maker or the

[5] See Forrester (2003, pp. 9-10), especially Figure 4.
[6] Actually, the "rule" that is created is often an institution. See, for example, Saeed and Fukuda's (2003) system dynamics model of a mitigation banking system.
[7] And, in fact, the appropriate time series data often do not exist!
[8] These time paths can be derived quantitatively from time series data that have been properly "sliced" [see Saeed (1992)] and/or qualitatively from the "mental models" of system experts.
[9] See Graham (1980) for a discussion.

individual physical process to parameterize his or her model. This process has been likened to the case study or pattern modeling approach of institutional economists[10] and involves such techniques as interviewing people who make decisions in the actual system, measuring delays in physical processes, soliciting expert opinion, participant observation by the modeler, and the use of focus groups and archival material.[11]

Although the process of estimating parameters at or below the level of model aggregation does not involve formal statistical procedures, classical system dynamicists emphasize that it is neither a sloppy nor unscientific endeavor. Moreover they point out that since socioeconomic systems are characterized by rich nonlinear stock-flow-feedback loop structures, the models they create are remarkably robust and contain parameter values that can typically be varied 10% or more in either direction without significantly altering model dynamics or the fundamental policy insights the models are generating.[12] The need for pin-point accuracy in the estimation of a model's parameters is thus not deemed a high priority in classical system dynamics modeling.

In sum, the classical approach to system dynamics places relatively little emphasis on knowing the precise values of all of a model's parameters and/or on fitting a model tightly to historical data. Moreover, the robust feedback policies generated by the classical approach are not at all dependent on a model or person being able to forecast the future value of anything, which is fortuitous in light of the belief that socioeconomic systems are unpredictable in principle. As a consequence, in addition to being controversial within the field of system dynamics, the classical approach to parameter estimation in system dynamics modeling is typically viewed with skepticism by economists and other social scientists because it differs so significantly from the statistical and econometric approaches they have been trained to follow.[13]

Combining System Dynamics and Econometrics. During the first decade and a half of its existence, system dynamics went largely unnoticed by the economics profession. This was most probably due to the focus of the early system dynamicists on management-oriented, rather than public sector-oriented, problems. With the publication and popularity of the Urban Dynamics study in 1969, the World Dynamics study in 1970, and The Limits to Growth study in 1972, however, this began to change. Many economists were severely critical of the urban and world models and one of the most common complaints was that the parameters of these models had

[10] See Radzicki (1988, 1990, 2003) for more a more detailed discussion of the system dynamics modeling approach and its relationship to the institutionalist pattern modeling approach.
[11] For a discussion of how these techniques are used in system dynamics modeling see Sterman (2000, p. 859).
[12] Stated differently, it is the stock-flow-feedback loop structure of a system, and not its particular parameter values, that determine its behavior. See the discussion in Legasto and Maciariello (1980), Forrester (1969, pp. 227-240; 1980a; 1980b), and Zellner (1980).
[13] For more on the differences between system dynamics modeling and econometric modeling see Sterman (1988b).

not been estimated econometrically.[14] Of course, this criticism was pushed by some economists beyond the urban and world models and directed toward system dynamics modeling in general.[15]

In response to the attacks from economists, during the mid-1970s and early 1980s several system dynamicists undertook studies aimed at determining whether or not econometric techniques could be used to accurately estimate the parameters of a system dynamics model. These studies generally took the form of synthetic data experiments in which a system dynamics model with known parameter values was assumed to be the "real world" and the time series data it generated when simulated had various econometric techniques applied to it. The results of these studies showed overwhelmingly that under almost all but perfect circumstances, the econometric techniques were unable to accurately recover the model's parameter values. In particular, any time a small amount of measurement error (noise) was added to the data being generated by the model, or a small amount of uncertainty (noise) was added to the model's structural equations, accurate and consistent parameter recovery was shown to be next to impossible.[16] In light of such studies as Oscar Morgenstern's <u>On the Accuracy of Economic Observations</u>, as well as the synthetic data experiments described above and Forrester's Appendix K results,[17] the conclusion within the system dynamics community was that it was both impossible and unnecessary to econometrically estimate the parameters of a system dynamics model.[18]

The Statistically Inclined School. At the same time that some system dynamicists were exploring the possibility of utilizing econometric techniques to estimate the parameters of system dynamics models, a system dynamicst named David Peterson undertook a different approach. Peterson noticed that engineers attacked the problems of parameter estimation and historical data fitting somewhat differently than did economists. More

[14] See for example the critique of <u>World Dynamics</u> by Nordhaus (1973).
[15] Radzicki (1990) documents additional attacks by economists on system dynamics modeling in general.
[16] More specifically, Senge (1977), and Mass and Senge (1978) showed how ordinary least squares and generalized least squares generate parameter estimates that are highly sensitive to errors in data measurement and could thus lead to misleading results. Morecroft (1977) confirmed these results for ordinary least squares but showed that instrumental variable estimation could sometimes yield improved results in the face of measurement error. By contrast, Richardson (1981) assumed no measurement error and focused instead on the issue of "process noise" - i.e., the uncertainty inherent in the specification of a model's structure (equations). He found that, in most instances, ordinary least squares, generalized least squares, and nonlinear least squares produced "dramatically inaccurate" parameter estimates. Moreover, he identified the "sampling interval problem" (i.e., the problems that arise when data measured at discrete intervals are used to estimate the parameters of a continuous time system dynamics model) as being the most troublesome in his study.
[17] See also the discussion in Johnson (1980).
[18] Sommer (1984b) re-examined the studies by Senge (1977), Mass and Senge (1978), and Morecroft (1977) and drew much more optimistic conclusions about the use of econometric techniques for system dynamics modeling. He found that the underlying problem was not econometric parameter estimation in general, but rather the econometric estimation of the parameters of exponentially distributed lags in system dynamics models. He also criticized the "kind and size" of the measurement errors introduced in the Senge (1977), Mass and Senge (1978), and Morecroft (1977) studies. Sommer (1984b) concluded with a set of guidelines for the successful application of econometric techniques to system dynamics models [see also Sommer (1984a)].

specifically, he noticed that an electrical engineer named Fred Schweppe (1973) had developed an accurate method for fitting models of uncertain dynamical physical systems to data and estimating their parameter values.[19] This technique is known as **F**ull **I**nformation **M**aximum **L**ikelihood via **O**ptimal **F**iltering (FIMLOF). Schweppe had created his version of FIMLOF in the 1960s to solve the "star wars" problem of identifying and tracking nuclear missiles launched toward the United States, deciding whether or not they were decoys, and then shooting them down.[20] Unfortunately, the computers of the 1960s were so slow that, in a real situation, by the time they were able to complete Schweppe's computations, the missiles would have already struck their targets.

Peterson worked with Schweppe to adapt FIMLOF to system dynamics modeling and the special problems associated with parameter estimation in the social sciences. The result of their collaboration was an original software package called the **G**eneral **P**urpose **S**ystem **I**dentifier and **E**valuator or GPSIE.[21] Today, Schweppe's techniques are embodied in a well-known system dynamics software package called Vensim.[22]

According to Peterson [(1975, pp. 6-7), (1980, pp. 234-235)], unlike most econometric techniques FIMLOF can operate under the following conditions:

1. Nonlinearities in model dynamics
2. Nonlinear measurement functions (e.g., ratios)
3. Errors in the measurement of variables from the actual system
4. Mixed sampling intervals (e.g., FIMLOF can estimate a weekly model using monthly and/or yearly data)
5. Models with unmeasured endogenous variables (i.e., FIMLOF does not require historical data on all of a system's variables)
6. Mixed cross sectional and time series data
7. Unknown characteristics of equation errors (i.e., when the model's structure differs from the real world structure)
8. Unknown characteristics of measurement noise
9. Missing data
10. Corrupt data (FIMLOF can even <u>detect</u> corrupt data)

[19] Schweppe's techniques are themselves based on the original work in engineering filtering by Kalman (1960) and Kalman and Busy (1961). Schweppe (1965a, 1973) was the first to link Kalman filtering to maximum likelihood estimation, while Anstrom (1970) contributed greatly to the overall framework for hypothesis testing and parameter identification in noisy dynamical systems with missing and corrupted data. See the discussion in Graham (2002) for more details.
[20] See Schweppe (1965b) for some of the details.
[21] See Peterson and Schweppe (1974), Peterson (1975), and Peterson (1980) for the details.
[22] Vensim is available from Ventana Systems, Inc. http://www.vensim.com/

11. Indirect measurements (e.g., yearly summations, averages, and functions of multiple stocks)

Although the mathematics of FIMLOF are a bit difficult to master, the basic ideas behind it are fairly simple to understand. An excellent intuitive description is provided by Graham (2002, Appendix):

> The heart of the FIMLOF algorithm is the “predict-correct” cycle: starting from estimated values of the [stock] variables, use the model equations to simulate forward to the next time for which real data are available. The “predict” part is then using [the] model equations to predict what the observed data “should” be *a priori* for that time, i.e., the estimated observation given the estimated [stocks] from the previous time. Of course, the real data will differ from the estimate, because of random noise driving the dynamics and random noise corrupting the data. Those differences are called the *residuals*. Standard Bayesian estimation can use [the] model equations and the residuals for that time to calculate an *a posteriori* estimate of the [stocks] for that point in time. Those estimates are the starting point for the next predict-correct cycle. So the algorithm described thus far takes a stream of observations, and produces a stream of estimated [stocks], and a stream of residuals. This is the Optimal Filtering (OF) part of FIMLOF.
>
> Full-information maximum likelihood estimation backs into parameter values by calculating, for a given sample set of real observations and their residuals, the parameters that maximize the probability density of that aggregate sample. It turns out that the logarithm of the probability density, the likelihood, is a function of a quadratic weighting of the residuals. Therefore, the Full-Information Maximum Likelihood (FIML) estimate of the parameter values is a particular Weighted Least Squares (WLS) parameter estimate, which can be found by standard search methods. In essence, then, FIMLOF estimates parameters by finding a best WLS fit for a predict-correct cycle.

FIMLOF has shown great promise for use in estimating the parameters of system dynamics models and for fitting system dynamics models tightly to historical data, especially in circumstances in which traditional econometric techniques fail.[23] Indeed, when Peterson [(1975, pp. 58-62), (1980, pp. 233-243)] used FIMLOF to re-examine the results obtained by Senge (1977) and Mass and Senge (1978), he found that it did an excellent job of recovering the original parameters specified in the synthetic data experiments.

Intuitively, the main reason that FIMLOF works very well with system dynamics models is that the predict-correct cycle described above by Graham provides an estimate of the noise sequences that perturb the actual system.[24] In fact, in principle, if FIMLOF were properly applied to the "model" of the "real world system" presented in Figure 1, the original parameters of the "real world system" could be recovered and the model tightly fit to the "actual" data. This is because FIMLOF would provide an estimate of the particular noise sequence that perturbs the "real world system."

Two Related Views within the Statistically Inclined School. For completeness, two sub-schools of thought within the statistically inclined school of system dynamics modeling need to be mentioned. The first is the "model reference optimization" (MRO) sub-school and the second is the "optimal stochastic system dynamics" (OSSD) sub-school. The MRO sub-school refers to a group of system dynamicists who utilize a set of formal statistical techniques to estimate the parameters of a system dynamics model and find its best fit to a set of historical data *without the use of engineering filtering and the limitations it imposes* (e.g., the need to linearize a nonlinear system, the need to know the location(s) and characteristics of a system's noise, and computational intensity).[25] OSSD refers to a small group of system dynamicists who utilize system dynamics models to develop contingent forecasts for use in optimal decision making.

OSSD goes beyond the use of FIMLOF to "simply" estimate a model's parameters and fit some of its variables tightly to historical data. In contrast to classical system dynamicists, who use a model to derive an robust feedback policy and then discard the model when it comes time for day-to-day decision making, OSSD practitioners formally integrate a parameterized and historically-fit system dynamics model into the day-to-day decision making processes within the socioeconomic system for which it was created. The step-by-step process for doing this has been defined by Peterson (2003):

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[23] See Eberlein and Wang (1985) and Eberlien (1986) for more technical overviews of the strengths and weaknesses of various statistical methods relative to FIMLOF.

[24] That is, it provides estimates of the errors committed in measuring the variables in the real system and of the errors committed in estimating the proper structure of the system.

[25] Vensim uses a modified Powell algorithm for model reference optimization while DYNAMO [now replaced by Jitia -- see Eubanks and Yeager (2001)] uses a variant of the Broyden-Fletcher-Goldfarb-Shanno variable metric method. For more information on the use of these techniques see the discussion in Lyneis and Pugh (1996, pp 3-4). Within Vensim, FIMLOF requires the use of <u>both</u> the modified Powell algorithm and Kalman filtering <u>together</u>, while model reference optimization is undertaken by simply turning off the Kalman filter and using the modified Powell algorithm by itself.

1. Use a robust system dynamics model to estimate a system's parameters and its level of uncertainty, using all available data.
2. Use [the] data to estimate the current state of the system.
3. Starting from the current state of the system, compute the full range of possible futures.
4. Repeat step three for all possible future decision sequences.
5. Implement the first step of the decision with the best long-term payoff and risk profile.
6. When new data arrives, repeat the above five steps.

According to Peterson, the computationally intensive six-step OSSD process provides policy makers with "contingent forecasts" that can be used to make optimal decisions. These forecasts are bounded by confidence bands that get larger and larger (i.e., the forecasts get less and less accurate) as the forecast horizon gets longer and longer. This result, of course, is consistent with Forrester's argument about the unpredictability of systems, as OSSD could presumably be used to predict the future state of a system fairly accurately in the short-term (where actions based on the prediction have no effect on the system), but not in the long-term [where actions based on the (now uncertain) prediction can influence the system's behavior].

Hand Calibration School. A possible third school of thought that offers something of a compromise between the classical and statistically inclined schools might be termed the “hand calibration school.” Hand calibration involves estimating a model's parameters at or below its level of aggregation and then iteratively revising the parameters by hand until the model’s behavior closely mimics a set of historical data. Lyneis and Pugh (1996, pp. 1-2) point out that this practice has been criticized because: (1) it is a difficult process that seems to rely more on the experience and intuition of the modeler than on any particular scientific procedure (i.e., it's more of an "art" than a "science"); (2) the process and results do not seem replicable (i.e., different modelers are likely to come up with different parameterizations - especially if the calibration process involves altering a model's structure); (3) a modeler cannot be sure that his/her final calibration is the best that can be achieved; and (4) in the case where a model must be recalibrated to a revised set of parameters, the generation of sensitivity analyses and confidence bands is more cumbersome and less robust. Despite these concerns, Lyneis and Pugh (1996, p. 10) found via an experimental study that, although "automated calibration offers great promise," "[h]and calibration works, and is less of an art and more replicable than might be expected. Moreover, it produces results which are as close to the true values as automated calibration, and are typically close enough to make no significant difference to the outcome of policy interventions."

Nonergodicity and System Dynamics Models

Based on the discussion up to this point the following question can be asked: Although system dynamics models appear to be uncertain and unpredictable, at least in the long-run, are they nonergodic and thus appropriate for Post Keynesian and institutional analysis?

Much has been written during the last decade on the issue of nonergodicity and uncertainty in economic modeling. Rosser (1995, 2001) has been at the forefront of surveying economic modeling techniques and relating them to the issue of Post Keynesian uncertainty. While agreeing with Davidson that nonergodic systems are uncertain, he makes a compelling case that "one may have fundamental uncertainty without nonergodicity, as in models displaying chaotic dynamics" and that "nonlinear and complex dynamics provide a clear foundation for fundamental uncertainty that embraces cases not covered by nonergodicity."

In actuality, a particular system dynamics model can be either ergodic or nonergodic. System dynamics models are typically nonlinear and complex and can, under the right circumstances, generate chaos and nonergodic behaviors such as path dependence,[26] hysteresis, and agent-based "emergent" dynamics. Moreover, the bounded rational decision making structures that system dynamics models typically embody almost always guarantee that economic agents will be unable to make accurate predictions of the future,[27] whether the system of which they are a part is nonergodic or not. Thus, the quick and simple answer to the question posed in this section is that system dynamics models are indeed appropriate for Post Keynesian and institutional analysis.

A larger modeling issue remains, however. One of the most fundamental principles in system dynamics modeling is that a system's structure determines its behavior. If a modeler does not include structures that will/might influence the future behavior of a system because he or she is unaware of them or is unable to specify what they are, agents in the model, regardless of how they form their expectations, will not be able to take them into account when they make their crucial decisions. This is the sort of thing that Keynes [(1973, pp. 114-115; quoted in Davidson (1982-1983, p. 188)] meant when he said that:

> “By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty….The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new

[26] For examples of path dependent system dynamics economics models see Radzicki (2003) and the pricing sub-sector of the mega-corp sector of the Post Keynesian-Institutionalist-System-Dynamics model in Radzicki (2005).
[27] Except perhaps in a deterministic steady state.

> invention….About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know!"

There are two final points to make concerning the issue of model nonergodicity. First, it is important to distinguish between the inability of economic agents in a model to predict its future behavior, and the inability of a modeler to use a model to predict the precise future of an actual system. OSSD offers a prime example of why this distinction is important. An OSSD model may embody bounded rational economic agents who cannot predict the future behavior of the model of which they are a part, but the modeler can take the model and use it make on-going, contingent forecasts banded by confidence intervals that, in the short-term (for certain problems), might be fairly accurate.

The second point is that the nonergodicity debate, in a sense, misses the point when it comes to system dynamics modeling. A properly conducted system dynamics study will yield a robust feedback policy or rule, which implies the redesign of a system's structure (and sometimes means creating or redesigning an institution). This rule helps decision makers keep the system behaving in a desired manner. Robust feedback policies are termed "robust" because they enable the system to respond "well" to unexpected shocks that are, by definition, unpredictable and that will inevitably perturb the system.

A Bounded Rational System Dynamics Expectations Structure

Given that dynamic, nonlinear, feedback systems are unpredictable in principle, particularly in the long-run, and that economic agents must thus make their decisions about the future in the face of uncertainty, a logical question to ask is whether or not it is possible to model the way in which humans actually form their expectations.

John Sterman [1987, 1988a, (2000, pp. 631-660)] has been quite successful at developing system dynamics models that can mimic the actual human expectations formation process. He notes that even though contemporary decision makers have unprecedented access to sophisticated mathematical forecasting tools, more often than not they make judgmental forecasts that are based on "gut feeling," experience, and "tweaks" to forecasts generated by formal techniques. In other words, Sterman believes that forecasting is a cultural process that is typically infested with cognitive flaws:

> "Though many organizations spend considerable resources generating and purchasing forecasts, forecasting is a social, political, and bureaucratic activity, not a scientific one…[This] means [that] many judgmental heuristics and other manifestations of bounded rationality may have considerable influence on the

forecasting process and can lead to persistent, systematic forecasting errors." [Sterman (2000, p .632)]

Figure 3 presents the bounded rational system dynamics structure that Sterman has used to model and test human expectation formation. It is based on insights from behavioral decision theory,[28] investigations into the ways in which decision makers actually make forecasts, and four behavioral characteristics of human expectation formation: (1) decision makers overwhelmingly focus on the behavior of the variable they are trying to forecast,[29] (2) decision makers need time to collect data on a variable about which they are forming expectations, smooth out any high frequency noise, analyze it, and form an opinion about its value, (3) decision makers compare their perception or opinion of a variable about which they are forming expectations to a slow-moving reference condition or anchor to determine whether or not the variable is rising or falling, and (4) decision makers need time to recognize trends in a variable about which they are forming expectations.

Place Figure 3 About Here

Sterman has used variations on this structure to successfully explain (mimic) actual forecasts of inflation [Sterman (1987, 2000, pp. 645-654)], energy consumption [Sterman (1987, 1988a, 2000, pp. 638-643)], and cattle prices [Sterman (2000, pp. 643-645)]. It has been modified here to mimic the formation of expectations about future hog (barrow and gilt) prices put forth by Glen Grimes, an agricultural economist at the University of Missouri and a well-known agricultural commodity analyst.[30]

In Figure 3, the smoothing process described by the stock **Perceived Hog Price** and its associated flow **Chg Perceived Hog Price** represents the data collection, analysis, and perception formation process undertaken by Grimes, the smoothing process described by the stock **Reference Hog Price** and its associated flow **Chg Reference Hog Price** represents the formation of the slow-moving anchor used by Grimes to assess changes in hog prices, and the stock **Perceived Trend in Hog Price** and its associated flow **Chg Expected Trend in Hog Price** represents the trend recognition process used by Grimes when he forms an opinion about the rate of growth of hog prices.

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[28] For more complete discussions about behavioral decision theory and its use in economics see Tversky and Kahneman (1974), Lavoie (1992), Harvey (1998, 1999) and Mullainathan and Thaler (2000).

[29] Of course, decision makers have access to, and undoubtedly utilize, a variety of sources of information when they formulate their expectations. The structure in Figure 3, however, implicitly assumes that the impact of these influences is small relative to the impact of the variable that is being predicted. See Sterman (2000, Chapter 16) for evidence that supports this position.

[30] Sterman (2000) used the structure in Figure 3 to mimic Glen Grimes' formation of expected <u>cattle</u> prices.

The structure in Figure 3 has four key parameters: **Time to Perceive Hog Price**, **Time Horizon for Reference Hog Price**, **Time to Perceive Trend in Hog Price**, and **Hog Price Forecast Horizon**, and three initial values: **Initial Perceived Hog Price**, **Initial Reference Hog Price**, and **Initial Perceived Trend in Hog Price**. Since it will be used to mimic the one-quarter-ahead forecasts of hog prices provided by Grimes, the **Hog Price Forecast Horizon** parameter is set equal to 1 quarter. The remaining three parameters and three initial values will be estimated using FIMLOF.

Place Figure 4 About Here

Figure 4 presents the one-quarter-ahead forecast of the seven market average cash price for barrows & gilts, that was made by Glen Grimes, from the first quarter of 1972 to the second quarter of 1986, plotted against the actual seven market average cash price for barrows & gilts for the same period.[31] Inspection of the plots clearly reveals that, although Grimes is pretty good at forecasting actual hog prices, he often misses turning points in the data. As Sterman (2000, p. 644) points out, phase lag (and overshoot) are characteristic of a forecast made "by smoothing recent prices and then extrapolating the recent trend."

To estimate the parameters and initial values of the expectations structure of Figure 3, and to see if it could produce expected hog prices that are similar to those provided by Grimes, the structure was fed the data on <u>actual</u> hog prices from Figure 4[32] and asked to kick-out a one-period-ahead forecast or expectation of future hog prices. The results, using FIMLOF, are shown in Figure 5.

Place Figure 5 About Here

Figure 5 shows Grimes' forecast of hog prices from the previous figure plotted against the expected hog prices generated by the system dynamics model. A simple visual inspection is all that is needed to conclude that the model does an excellent job of mimicking Grimes' expectations.

Table 1 is a summary of the FIMLOF estimates of the model's three free parameters and three initial values along with their corresponding 95% confidence intervals. Question marks in the confidence intervals indicate

[31] These data are from Bessler and Brandt (1992).
[32] These, of course, are the same prices that Grimes used to make his forecasts.

places where FIMLOF was not able to zero-in on a particular value due to integration and round-off errors encountered during simulation.[33]

Place Table 1 About Here

All three parameter estimates are reasonable. **Time to Perceive Hog Price** was estimated to be .49 quarters, **Time Horizon for Reference Hog Price** was estimated to be 14.29 quarters, and **Time to Perceive Trend in Hog Price** was estimated to be .61 quarters.[34] Two of the three estimated initial values of the model's stocks were also reasonable. **Initial Perceived Hog Price** was estimated to be $23.38 per barrow & gilt and **Initial Perceived Trend in Hog Price** was estimated to be .004 (fraction/quarter). **Initial Reference Hog Price** was estimated to be $79.72 per barrow or gilt, which is quite high, but apparently necessary for the Kalman filter to optimally adjust to the model's stocks to the data.

Harrod's Model and the System Dynamics Expectations Structure

The evidence presented in Figure 5 and Table 1 suggests that it is possible to formally estimate the parameters of a system dynamics model and fit it tightly to historical data, and to behaviorally model the formation of actual human expectations. An interesting follow-up question is: Can the system dynamics expectations structure presented in Figure 3 be incorporated into an economics model to shed light on some issues that are of interest to Post Keynesian and Institutional economists? To answer this question, this section of the paper will show how the system dynamics expectations structure from Figure 3 can be added to the Harrod growth model, how this modified Harrod model can shed light on the issue of Harrod's "knife-edge," and how the modified Harrod model can also closely mimic historical data from the U.S. economy.

The Harrod Growth Model. Modern economic growth theory often begins with the model put forth by Sir Roy Harrod in 1939 [Harrod (1939, 1948, 1973)].[35] The standard story is that Harrod developed the model because he

[33] Which are unavoidable when simulating continuous-time dynamical systems on a digital computer.

[34] Sterman (2000, p. 644) econometrically estimated these three parameters with data on actual cattle prices along with Grimes' expectations of those prices. His results were: **Time to Perceive Cattle Price** = .60 years; **Time Horizon for Reference Cattle Price** = 6.00 years; and **Time to Perceive Trend in Cattle Price** = .56 years.

[35] For example, see Jones (1976) and Hahn and Matthews (1964, p. 783). Regardless of whether or not all of modern growth theory starts with the Harrod model, a strong argument can be made that it is the starting point for dynamic Post Keynesian theory.

was originally interested in both bringing the mathematics of dynamics formally into economic theory, and in understanding the macroeconomic interactions between the trend and cycle.[36]

As is known to virtually all serious students of economics, Harrod's model boils down to his "fundamental equation" of $G_w = (s/v)$. That is, the warranted rate of growth in his model economy, G_w, is defined to equal to the propensity to save divided by the incremental capital output ratio. If the actual rate of growth of the economy, G_a, is equal to the warranted rate, then the economy will grow on an equilibrium path such that managers have no incentive to change their investment behavior. More specifically, if $G_a = G_w$ then $(s/v) = (s/v_r)$, or $G_a{*}v = s = G_w{*}v_r$, which implies that $v = v_r$ or that the actual increase in the capital stock equals the increase required by managers so that they will be satisfied that the amount of capital in the economy is precisely that which is necessary to produce the current level of output. Stated differently, if $G_a = G_w$ then the economy's stock of capital will be at its desired level over time and managerial expectations will be satisfied. In addition, if G_a and G_w are equal to the rate of growth of the labor force, n, that is if $G_a = (s/v) = (s/v_r) = n$, the economy will be on a "golden age" growth path such that, over time, everyone who wants to work has a job.

The Two Harrod Problems. Traditionally, Harrod's model is said to possess two separate but interrelated problems. The first is an existence problem which arises from the simple observation that since s, v, v_r and n are all independently determined,[37] there is no reason to believe that they will take-on (except by chance) the values necessary for an economy to be on its golden age growth path.

The second Harrod problem is a stability problem. Harrod argued that the warranted rate of growth is unstable in that deviations of G_a from G_w will not correct themselves and, in fact, will become even greater as pressures caused by "centrifugal forces" push G_a farther and farther away from G_w. The standard textbook description of this phenomenon is Harrod's "knife-edge" or the "razor's edge." Harrod argued that the centrifugal forces that push the economy away from the warranted rate are caused by incorrect error adjustments made by managers. More precisely, managers who observe a rate of growth above the warranted rate, i.e., $G_a > G_w$, will find that the actual increase in the capital stock (investment) is less than the increase they require in order to produce the current level of output, i.e., $v < v_r$.[38] Harrod envisioned that managers would respond to this discrepancy by investing at a higher rate, which would force G_a even farther away from G_w.[39]

[36] See Kregal (1980) but also see Harrod's earlier book on the trade cycle [Harrod (1936)].
[37] Harrod originally treated these parameters as constants in the first part of his analysis and then allowed them to vary in the second part of his analysis. See Harrod (1973, Chapter 3) and Kregel (1980) for more details.
[38] Stated differently, the managers would find the economy's stock of capital to be below the level they desired it to be.
[39] Of course, the dynamics are just the opposite if $G_a < G_w$.

Harrod, however, objected to the characterization of the instability problem as the knife-edge and argued that, although the warranted rate was indeed unstable, the economy, if knocked off of its warranted path, would eventually return to its warranted path.[40] Toward the end of his life he put forth his most comprehensive thinking on the matter[41]. The warranted rate is like a ball resting on a grassy slope. It might initially require a hard kick to get it rolling (i.e., it will require a large shock to knock an economy off its warranted growth path), but once it starts rolling, due to centrifugal forces, it'll go farther than a ball lying on a flat field that is kicked with the same force. The ball will not continue to roll down the hill forever, though. If G_a deviates from G_w, households and firms will react by changing their saving and consumption behavior, which has the effect of shifting the warranted rate of growth. Harrod termed the shifted warranted rates "special warranted rates." When G_w exceeds G_a, households and firms react to the slump by reducing their propensity to save so that their consumption spending is supported. This reaction will drop the special G_w below G_a and, thereafter, the saving adjustment process will reverse. Harrod argued that this process would also work when G_a exceeds G_w, but additionally noted that the ultimate limiting factor within this scenario is full employment. The exact level of full employment may be a bit "spongy" because people who have previously never worked outside of the home may decide to enter the labor force, but a zero rate of unemployment must eventually stop the increasing divergence of G_a from G_w.

Place Figure 6 About Here

A System Dynamics Representation of the Harrod Model. Figure 6 presents a system dynamics representation of the simple Harrod model that is based on the versions presented in Allen (1967, pp. 198-201) and Jones (1976, Chapter 3). In its simplest form, the Harrod model is a second order system that possesses two minor positive feedback loops. Loop 1 controls the growth of the labor force and loop 2 is responsible for the growth of the capital stock.

Figures 7, 8, and 9 show the results of a simulation of the simple Harrod model. In this "golden age run," capital, labor, and output grow at the same rate (s/v), which is equal to the model economy's actual, warranted, and natural rates of growth. The parameters of this model can be easily changed so that it exhibits non-golden age behavior.

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[40] In other words, Harrod still viewed his model as being able to shed light on the macroeconomic interactions between the trend and the cycle. See Kregel (1980) for more details.
[41] See Harrod (1973).

Adding Managerial Expectations to the Harrod Model. As both Hahn and Matthews (1964, p. 805) and Jones (1976, p. 51) emphasize, the existence of the Harrod instability problem relies crucially on the details of the error adjustment mechanism that managers are assumed to follow when G_a deviates from G_w. Amartya Sen (1970, pp. 10-14), for example, created a mathematical version of the Harrod model that includes a specific managerial expectations formation process[42] and generates unmitigated knife-edge behavior any time the expected rate of growth G_e deviates from the warranted rate of growth. Sen (1970, p. 14) does, however, note that inflationary and deflationary pressures could stop the divergence of G_a from G_w and admits that his version of the Harrod model is thus incomplete.

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Place Figures 7-9 About Here

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In an effort to “complete” the Harrod model,[43] Figure 10 presents the simple Harrod model from Figure 6, to which the behavioral expectations structure from Figure 3 has been added. The system is now fifth order and contains a "bird's nest" of feedback loops -- <u>some</u> of which are shown in Figure 11.

Place Figures 10 & 11 About Here

The logic behind the model is straightforward. Capital accumulation is the key to economic growth. Managers use the rate of economic growth they expect to prevail to make their investment decisions. In order to keep the economy on a golden age growth path (where expectations are realized), the managers’ aggregate investment decision must correct the capital stock for the expected growth of the economy and for any discrepancy that arises between the actual capital stock and the desired capital stock. The desired stock of capital is a function of the rate of economic growth expected by the managers, and the capital output ratio they require in order to keep the capital stock at its desired level, given the rate of economic growth they expect.

The most important feedback loop in the model is the major negative loop that helps alter the desired capital stock via the required capital to output ratio. This loop is shown with "thick" arrows in Figure 11 and is primarily responsible for the model’s oscillatory mode of behavior. Generally speaking, major negative feedback loops are

[42] From a system dynamics point of view, Sen's expectations formulation is a simple first order smooth. As Sterman (2000, pp. 632-633) points out, this type of expectations structure "outperform[s] many other forecasting methods over longer term horizons," but is problematic because it yields steady state error when its input variable grows exponentially.
[43] Sen (1970, p. 14) does note that there are many possible ways to "complete" the Harrod model.

destabilizing and tend to cause systems to oscillate. Minor negative loops, on the other hand, are stabilizing and tend to damp system oscillations. Positive feedback loops are usually destabilizing and cause systems to grow or decline.[44] In Figure 11, no minor negative loops are shown (they are shown in Figure 10). The three positive loops, however, are responsible for the system's growth mode of behavior.

Figures 12 through 16 show a "golden age" simulation of the model. As in the case of the simple Harrod model shown in Figure 6, when expectations are realized the model economy operates on its golden age growth path. Its actual, expected, warranted, and natural rates of growth are the same (s/v), and the capital output ratio is always the one required by managers to maintain the economy's capital stock at its desired level over time.

Place Figures 12-16 About Here

Figure 16 also presents an overlaid plot of the model's output from an "unrealized expectations" run. When the model economy is knocked off its golden age growth path in period 5 by a one-time managerial over-estimation of the economy's growth rate, the model responds by exhibiting both a growth trend and a cycle. Because the required capital output ratio fluctuates as managerial expectations about the economy's growth rate change, the "special warranted rate" changes and the economy eventually is able to return to its golden age growth path.[45] The implications of this are that: (1) Harrod appears to be correct when he argues that macroeconomic interactions between the trend and the cycle are significantly influenced by managerial expectations and (2) managerial expectations do not cause centrifugal forces to continuously pull the model economy away from its warranted rate of growth.

Fitting the Harrod to U.S. Macroeconomic Data. The simulation run in Figure 16 is theoretically pleasing in that it provides some evidence that when realistic managerial expectations are added to the Harrod model, the knife-edge disappears and Harrod's belief about the eventual return of the system to the warranted rate is correct. Given this result, the next question to ask is: Can the modified Harrod model mimic historical time series data from the real economy?

Figures 17–19 show the results of using FIMLOF to estimate the parameters and initial values of the modified Harrod model, and to fit it tightly to some U.S. macroeconomic data. Visual inspection of the figures reveals that,

[44] From a system dynamics point of view Harrod's centrifugal forces are positive feedback loops.
[45] Actually, depending on the parameters chosen, the model can exhibit balanced or unbalanced exponential growth, growth with damped fluctuations (as in Figure 16), growth with steady fluctuations, or growth with expanding oscillations that become bounded by the ceiling of full employment.

for the years 1929 to 2002, the modified Harrod model fits U.S. real GDP, labor supply, and real investment spending data extremely well. Table 2 lists the parameter and initial value estimates for the model, along with their 95% confidence intervals. The good news is that many of the estimates are quite reasonable.

Place Figures 17-19 About Here

In terms of parameter and initial value estimation, the following estimates are of particular interest:

1. The model's estimated propensity to save is .051 (fraction) and its estimated capital-output ratio is 2.81 years, which yields an estimated warranted rate of growth of .018 (fraction/year).
2. The model's estimated natural rate of growth of the labor force is .016 (fraction/year).
3. The model's estimate of the initial growth rate expected by managers is .04 (fraction/year).
4. The model's other estimated time constants are: **Time to Perceive Output** = 20 years; **Time Horizon for Reference Output** = 1.94 years; **Time to Perceive Trend in Output** = 2.07 years; and **Output Forecast Horizon** = 48.41 years.

Place Table 2 About Here

The estimate of the propensity to save of .051 (fraction) is reasonable, especially since it represents the propensity to save for all sectors of the economy. The estimated capital-output ratio of 2.81 years is consistent with results from many empirical studies of the U.S. economy. The estimated rate of growth of the labor force of .016 (fraction/year) is consistent with results from demographic studies of the U.S. economy and is close to the estimated warranted rate of growth for the U.S. economy of .0183 (fraction/year). The estimates of the time constants in the expectations structure come from psychological variables that are unobservable in the real economy and show that managers (in the aggregate) have fairly long memories when it comes to forming an opinion about the economy's level of real GDP, but short memories when it comes to creating a psychological anchor for real GDP and forming an opinion about the rate of growth in GDP. They also show that managers (in the aggregate) base their investment decisions on a very long-term time horizon. Unfortunately, these estimates seem a bit turned around. A priori, one would expect the **Time Horizon for Reference Output** to be the largest time constant and the **Time to Perceive Trend in Output** to be the smallest. The **Output Forecast Horizon** is also unusually long.[46]

Model Validity

The results presented in Figures 17-19 and in Table 2 suggest that the modified Harrod model may be "valid" because it has some theoretically pleasing properties, can mimic some aspects of the historical behavior of the U.S. economy very well, and yields at least some parameter estimates that are reasonable and somewhat interesting. That said, the next question to ask is: How do system dynamicists judge the validity of a model and, using these criteria, how valid is the revised Harrod model?

Model validity is a subject that is of considerable interest to system dynamicists. In fact, it is quite possible that system dynamicists have devoted more time, per capita, to the issues of model validity and proper modeling practice than researchers in any other field of scholarly inquiry. System dynamicists reject the binary notion that a model is either valid or invalid and replace it with the concept of building confidence in a model along multiple dimensions. To build confidence in a model, a system dynamicist will subject it to a large battery of tests. As the number of tests the model can pass increases, the system dynamicist becomes more confident that a model is telling him or her something useful.

Peterson (1975, Appendix B), Forrester and Senge (1980), Barlas (1989, 1996), and Sterman (2000, p. 852 and pp. 858-891) all provide extensive descriptions of tests for system dynamics models. Some of these tests are judgmental in nature and some are quite rigorous and mathematical.[47] A synthesized list, presented in the form of questions a researcher should ask about a model, is presented here.

Tests of Model Purpose

1. Does the model have a clear purpose?
2. Does the model have a well-defined perspective? (i.e., has the modeler "stepped back" far enough from the actual system to view it holistically and at the proper level of aggregation?)
3. Does the model address important questions and problems?

Boundary Adequacy Tests

1. Are the important concepts for addressing the problem to which the modeling study is directed endogenous to the model?

[46] Experimentation with the model uncovered FIMLOF runs that yielded much more reasonable parameter estimates for the time constants in the expectations structure, but they corresponded to a poorer fit of the model to the real investment time series.
[47] For some interesting technical tools for building confidence in system dynamics models see Barlas (1989), Peterson and Eberlein (1994), Reichelt et al. (1996), and Sterman (2000, pp. 874-880).

2. Does the behavior of the model change when its boundary assumptions (i.e., what to include and exclude) are relaxed?
3. Do the policy recommendations stemming from the model change when the model's boundary is extended?

Structural Assessment Tests

1. Is the model's structure consistent with the relevant descriptive knowledge of the actual system?
2. Does the model conform to the basic laws of physics?
3. Do the model's decision rules accurately capture the behavior of actors in the systems?
4. Does every variable and parameter have a precise definition relating to things in the real world?
5. Are the model's parameter values consistent with the relevant descriptive and numerical knowledge of the actual system?
6. Are the model's constants really constant for the purpose of the model?
7. Are the model's exogenous variables really independent of all the other variables in the model?
8. Do all of the critical points in the model's functions and relationships have physical meanings and plausible justifications?
9. Does the model's structure include policy space?

Dimensional Consistency Test

1. Is each equation dimensionally consistent without the need to add parameters or "fudge factors" that have no real world meaning?

Extreme Conditions Tests

1. Does each equation in the model make sense even when its inputs take on extreme values?
2. Does the model respond plausibly when tested with extreme policies, shocks, and parameters?

Integration Error Test

1. Are model results sensitive to the choice of time step or numerical integration method?

Tests of Model Sub-Sector Behavior

1. Do sudden or large changes in the model's input variables cause the model to behave in a plausible manner that is consistent with past observations and historical data (where appropriate)?
2. Is the behavior of equations and sub-sectors reasonable for extreme values of input variables?

Tests of Overall Model Behavior

1. Does the model reproduce the qualitative and quantitative behaviors of interest observed in the real system?
2. Does the model endogenously generate the symptoms of difficulty that motivate the study?
3. Do the frequencies and phase relationships among the variables match the data?
4. Do abrupt changes in model behavior make sense? Do they correspond with those seen historically in the real system?

<u>Behavior Anomaly Test</u>

1. Do anomalous behaviors result when the assumptions of the model are changed or deleted?

<u>Family Member Test</u>

1. Can the model generate the behavior observed in other instances of the same system?

<u>Surprise Behavior Test</u>

1. Does the model generate previously unobserved or unrecognized behavior? Is it reasonable?
2. Does the model successfully anticipate the response of the system to novel conditions?

<u>Sensitivity Tests</u>

1. Do the model's numerical values change significantly when assumptions about its parameters, boundary, and aggregation are varied over the plausible range of uncertainty?
2. Do the modes of behavior generated by the model change significantly when assumptions about its parameters, boundary, and aggregation are varied over the plausible range of uncertainty?
3. Do the policy implications of the model change significantly when assumptions about its parameters, boundary, and aggregation are varied over the plausible range of uncertainty?

<u>Pragmatic Tests</u>

1. Did the modeling process help to change the actual system for the better?
2. Does the model correctly predict the direction of system change resulting from policy changes?
3. Does the model correctly predict the extent of system change resulting from policy changes?

A detailed discussion of each of the tests in this list is beyond the scope of this paper. However, the modified Harrod model can be subjected to several of the tests in an effort to assess some of its strengths and weaknesses with respect to the purposes of this study.

From Figures 17-19 it appears that the modified Harrod model passes all of the "tests of overall model behavior" because it nicely mimics some of the observed behavior of the U.S. economy. In addition, it is dimensionally consistent and its parameters have precise definitions related to aspects of the real world.

Moreover, its estimated parameter values are "consistent with the relevant descriptive and numerical knowledge of the actual system." Lastly, its structure and behavior are appropriate for its purpose, which is to illustrate parameter estimation and data fitting in system dynamics modeling, to show how a behavioral expectations structure can be added to a well-known economics model to produce interesting results, and to initiate a discussion about model validity within the context of parameter estimation and data fitting.

On the down-side, the modified Harrod model contains no policy space with which the dynamics of policy changes can be assessed. This is particularly regrettable since much of the research done by Harrod and the Oxbridge economists during the golden age of economics in the U.K. was very policy-oriented. It is also more sensitive to changes in its parameter values than is a typical system dynamics model because it is simple and thus contains a relatively small number of feedback loops.

More troubling is a theoretical issue related to the fifth system dynamics structural assessment test. A properly constructed system dynamics model will contain variables that have a precise definition relating to things in the actual system. Harrod's original model, and hence the modified Harrod model presented in this paper, represents a single good economy (e.g. a corn economy). This, of course, makes issues surrounding the appropriateness of an aggregate capital stock in a growth model moot. The legitimacy of fitting a single good model to aggregate macroeconomic data, however, can be questioned.

Conclusions

The results of this study have shown that system dynamics models are uncertain and unpredictable and that bounded rational expectations structures can be added to them to realistically represent the ways in which economic agents cope with uncertainty and unpredictability. System dynamics modeling is thus suitable for Post Keynesian-institutionalist analyses.

This study has also shown that, when an empirically tested system dynamics expectations structure is added to the Harrod growth model, the centrifugal forces that pull the model economy away from its warranted rate disappear and it is able to generate macroeconomic interactions between the trend and the cycle that closely mimic the historical behavior of the U.S. economy. Harrod's original intuition about these issues was thus shown to be essentially correct and the existence of the "Harrod knife-edge" is called into question.

This paper also outlined the issues related to parameter estimation and curve fitting in system dynamics modeling and illustrated how (some) system dynamicists engage in these activities using FIMLOF. Issues related to model "validation" were also discussed in an effort to shown how parameter estimation and data fitting are only a small (but important) part of the system dynamics modeling process.

Finally, when viewed holistically, this paper has laid-out another argument as to why Post Keynesian, institutional, and other heterodox economists should begin utilizing system dynamics models to help them improve the behavior of real economic systems. Indeed, improving actual socioeconomic systems should be the ultimate goal of any research endeavor within heterodox economics.

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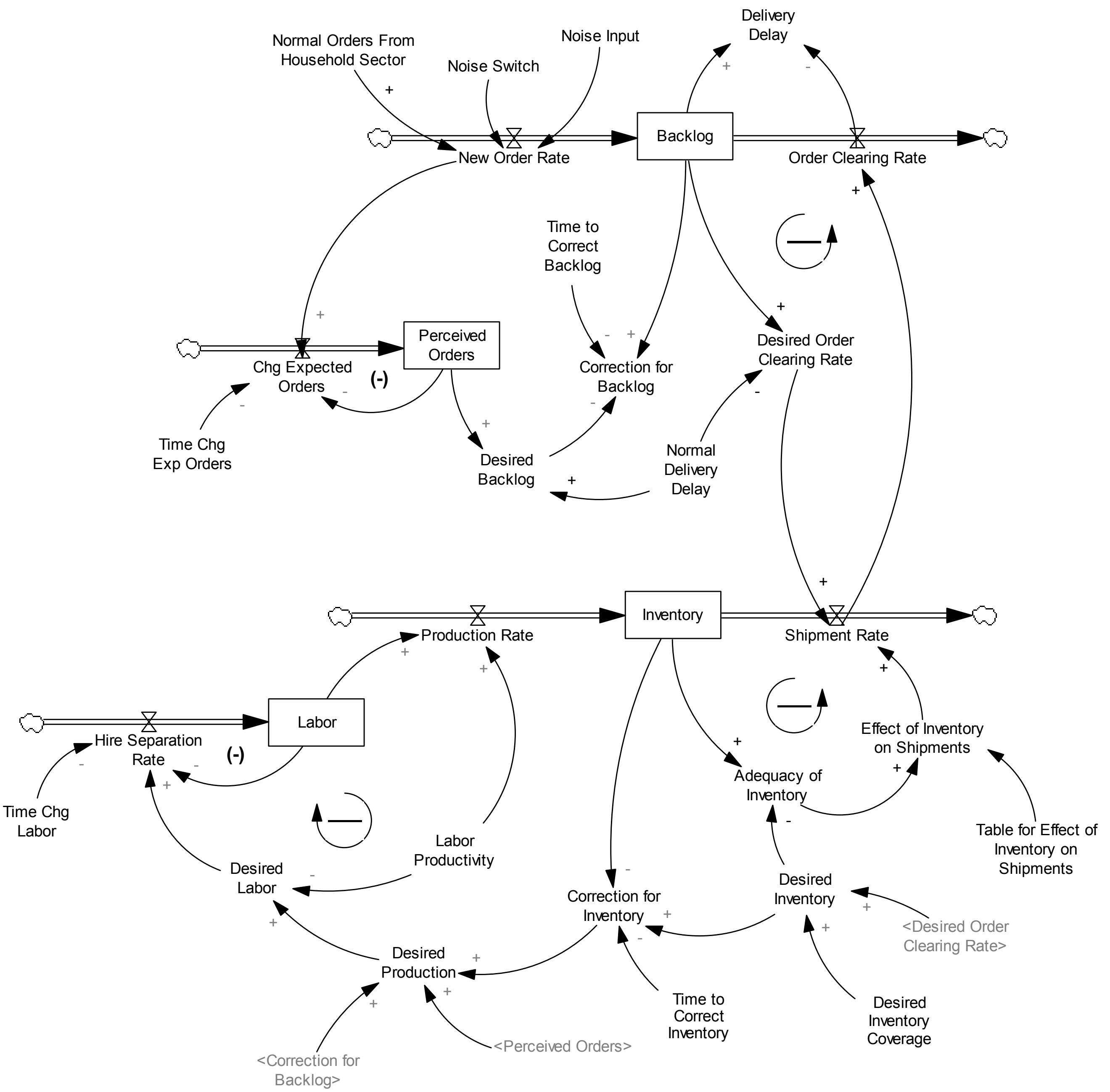
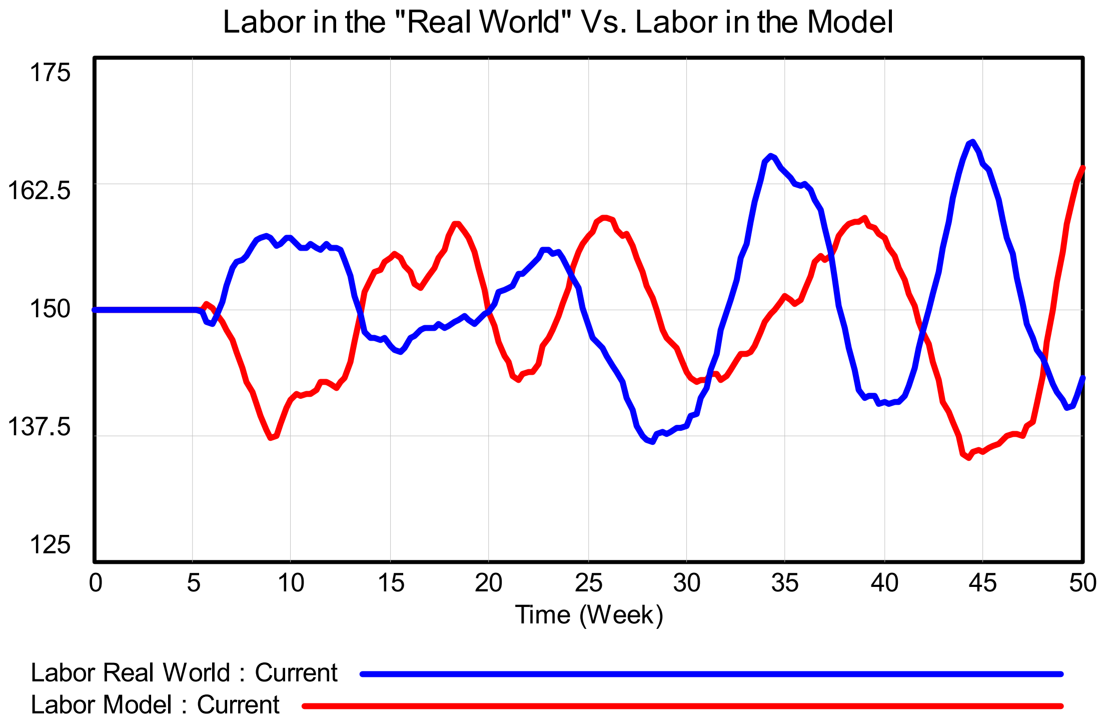


Figure 1: System Dynamics Model of the "Real World" Used to Show the Non-Predictability of Dynamic, Nonlinear, Feedback Systems



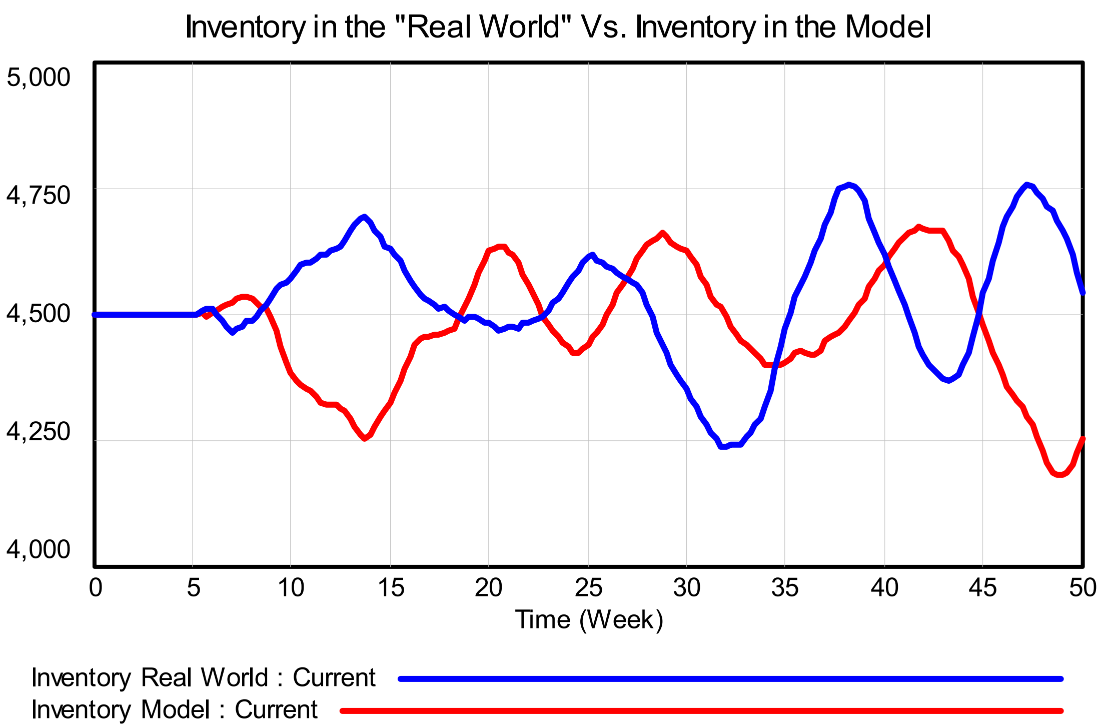


Figure 2: Simulation of the "Real World" and the Perfectly Specified Model of the "Real World"

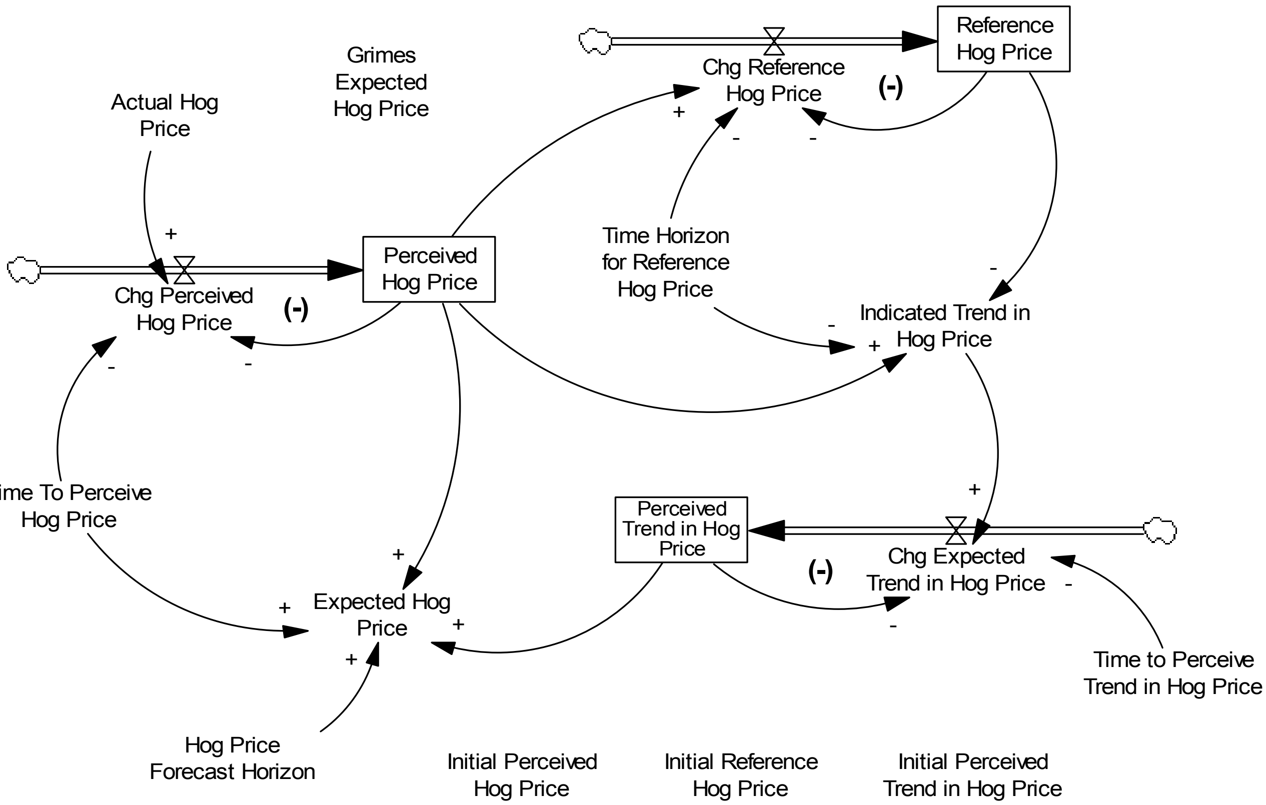


Figure 3: Bounded Rational System Dynamics Expectations Structure

Parameters & Initial Values	95% Confidence Intervals for Parameters & Initial Values
Time To Perceive Hog Price (Quarters)	???? <= 0.486111 <= ????
Time Horizon for Reference Hog Price (Quarters)	14.0982 <= 14.2922 <= ????
Time to Perceive Trend in Hog Price (Quarters)	0.403568 <= 0.613893 <= ????
Initial Perceived Hog Price ($ / Barrow or Gilt)	???? <= 23.3761 <= ????
Initial Reference Hog Price ($ / Barrow or Gilt)	???? <= 79.7191 <= 80.2868
Initial Perceived Trend in Hog Price (1/Quarter)	???? <= 0.00355728 <= ????

Table 1: Estimated Parameters, Estimated Initial Values, & 95% Confidence Intervals for the Bounded Rational System Dynamics Expectations Structure

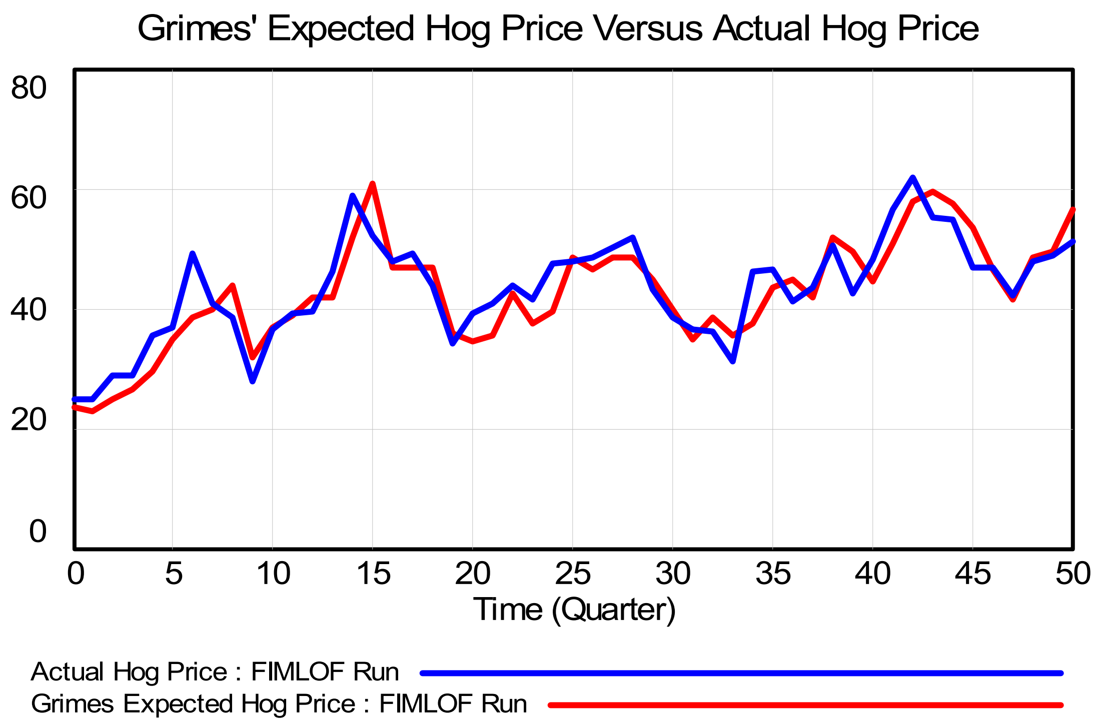


Figure 4: One Quarter Ahead Forecast of the Seven Market Average Cash Price for Barrows & Gilts by Glen Grimes, over the Period 1972 Q1 through 1986 Q2, versus the Actual Outcomes [Bessler & Brandt (1992)]

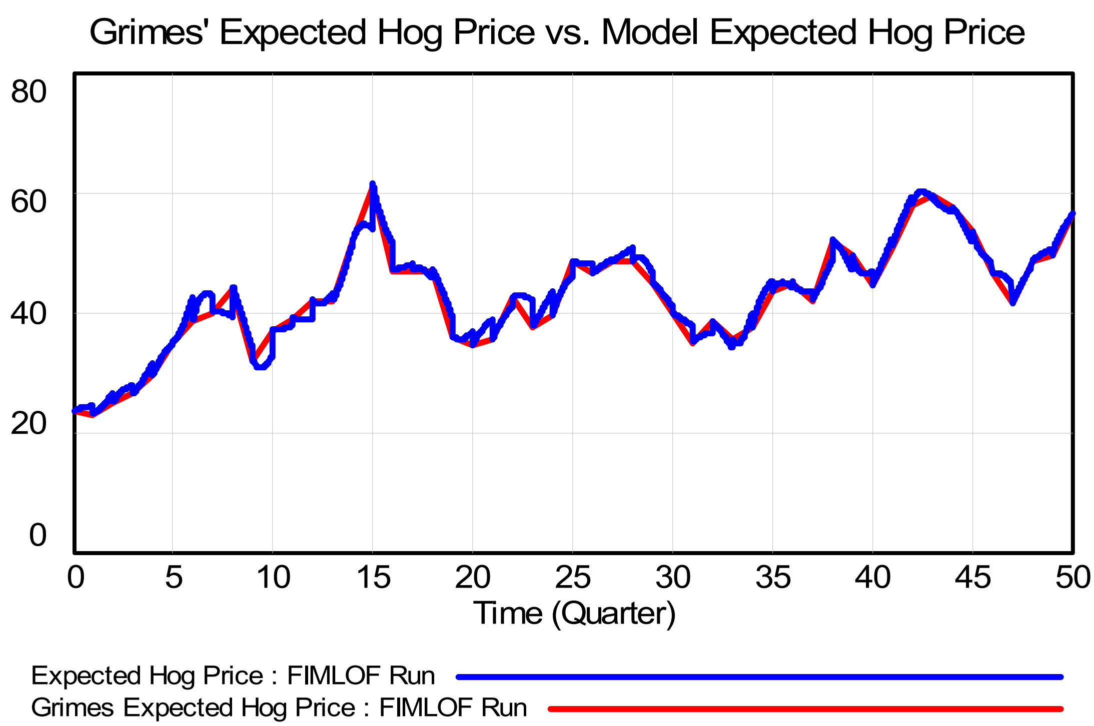


Figure 5: Actual One Quarter Ahead Forecast of the Seven Market Average Cash Price for Barrows & Gilts by Glen Grimes, over the Period 1972 Q1 through 1986 Q2, Plotted Against the Model-Generated Forecast

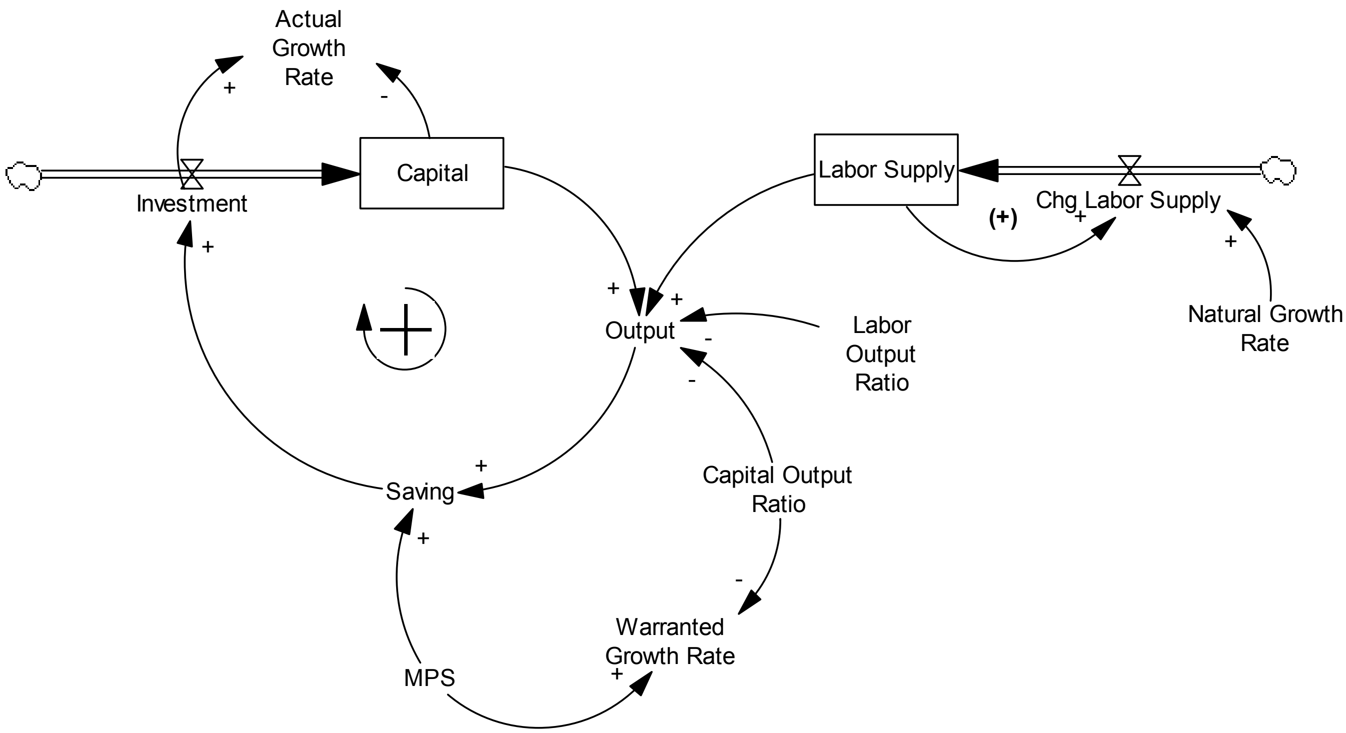


Figure 6: System Dynamics Representation of the Simple Harrod Model

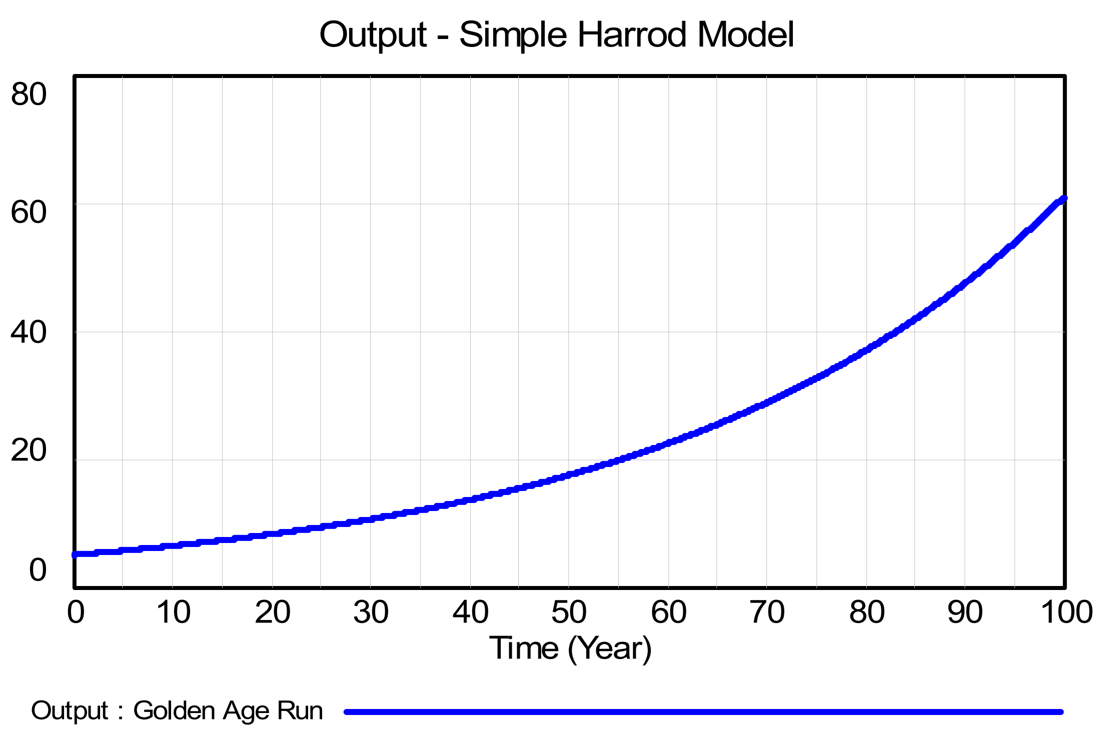


Figure 7: Time Series Plot of Output from the Simple Harrod Model

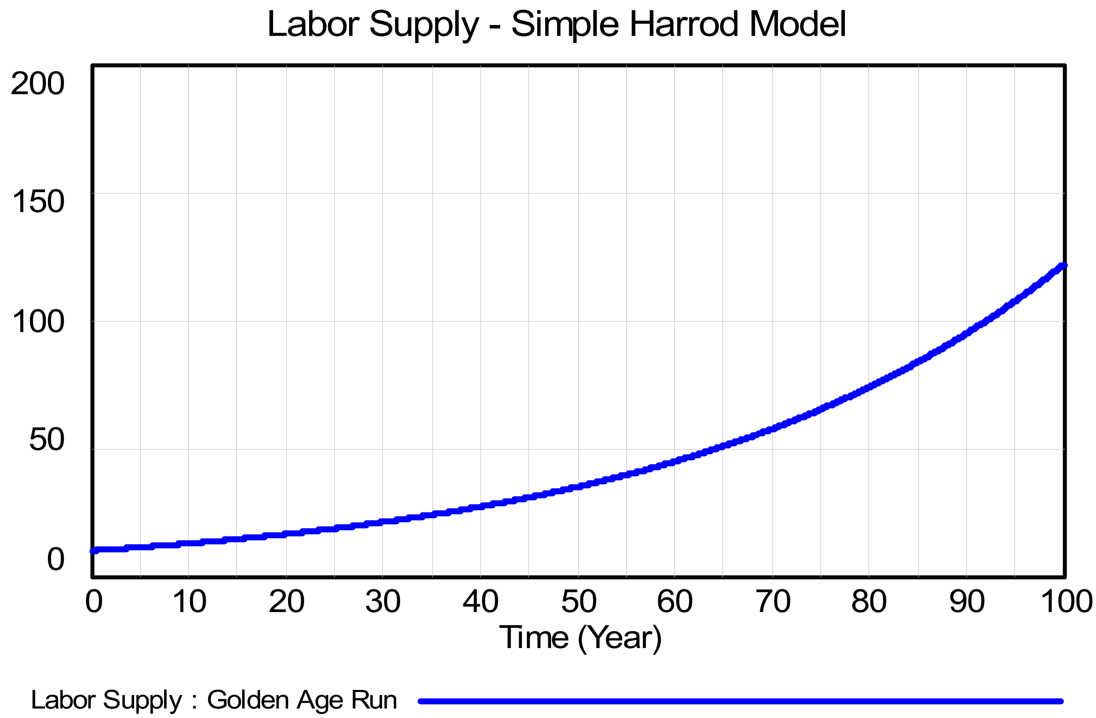


Figure 8: Time Series Plot of Labor from the Simple Harrod Model

Figure 9: Time Series Plot of the Actual, Warranted, and Natural Rates of Growth from the Simple Harrod Model

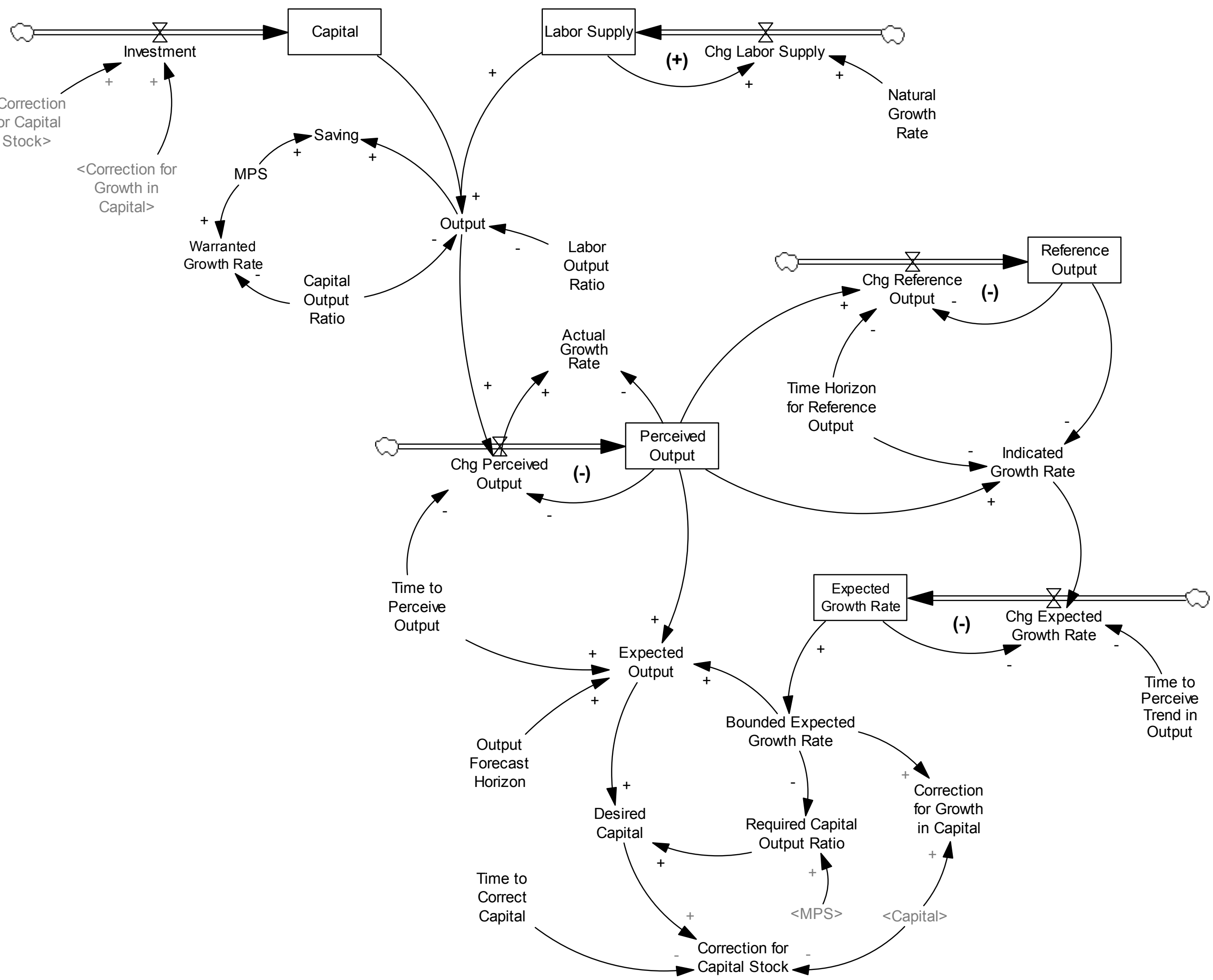


Figure 10: System Dynamics Representation of the Harrod Model with a Bounded Rational Managerial Expectations Structure

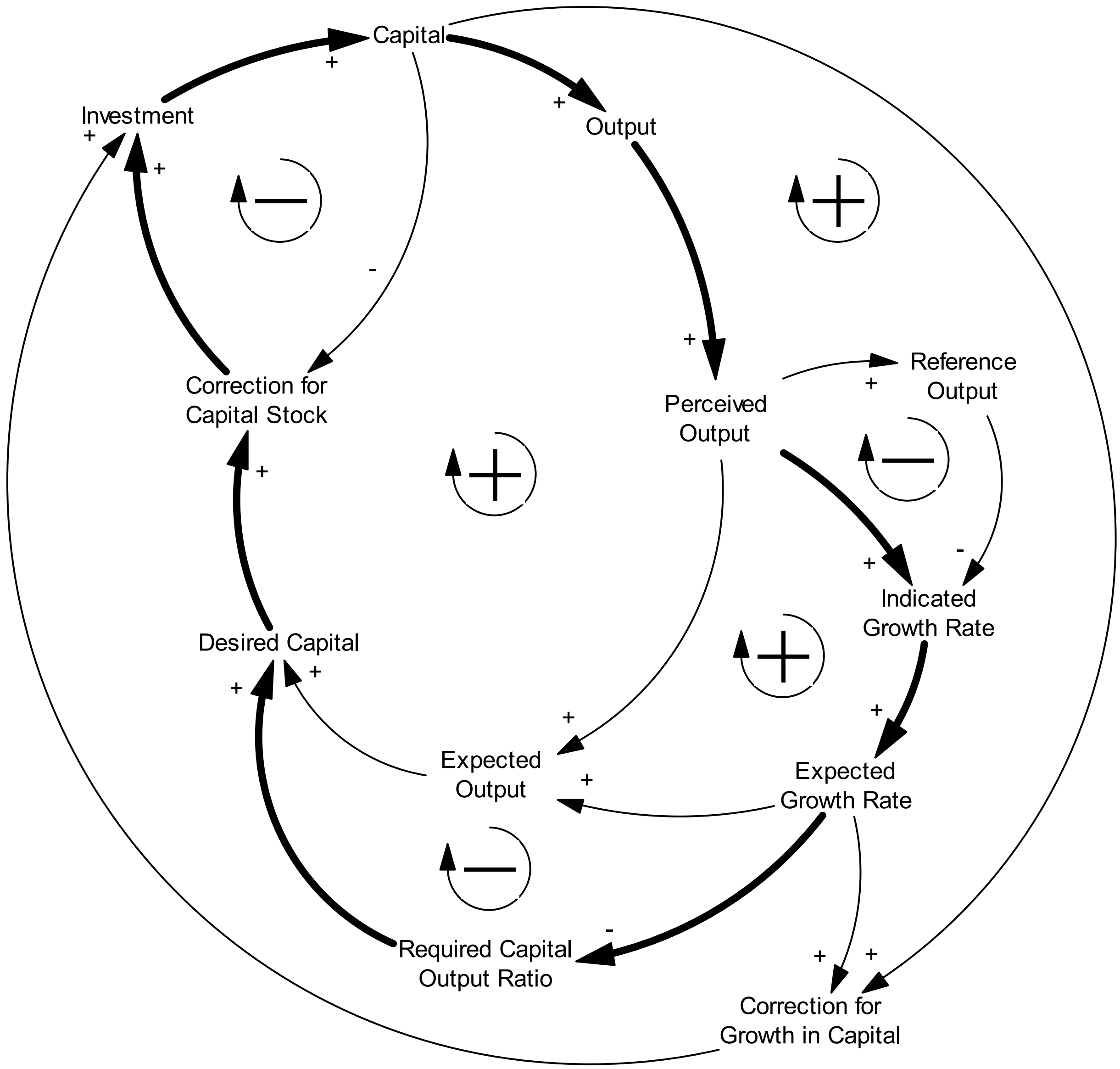


Figure 11: Causal Loop Diagram of the Major Feedback Loops in the Harrod Model with Bounded Rational Managerial Expectations

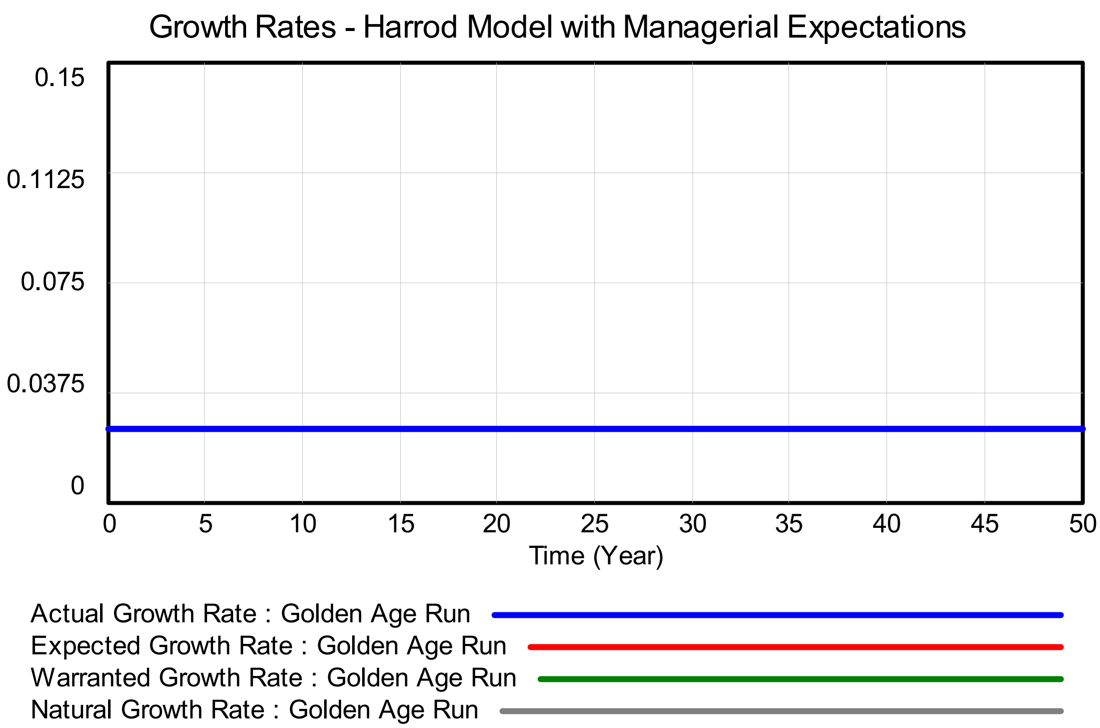


Figure 12: Growth Rates in the Harrod Model with Bounded Rational Managerial Expectations– Golden Age Case

Figure 13: Actual & Required Capital-Output Ratios in the Harrod Model with Bounded Rational Managerial Expectations– Golden Age Case

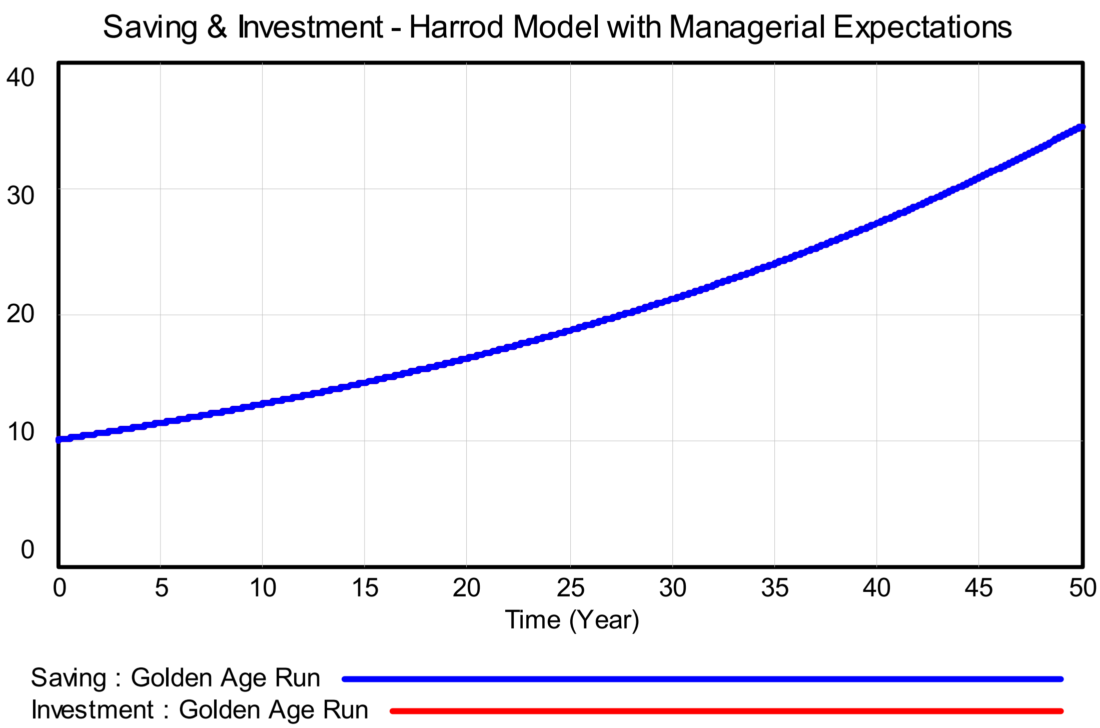


Figure 14: Saving & Investment in the Harrod Model with Bounded Rational Managerial Expectations–Golden Age Case

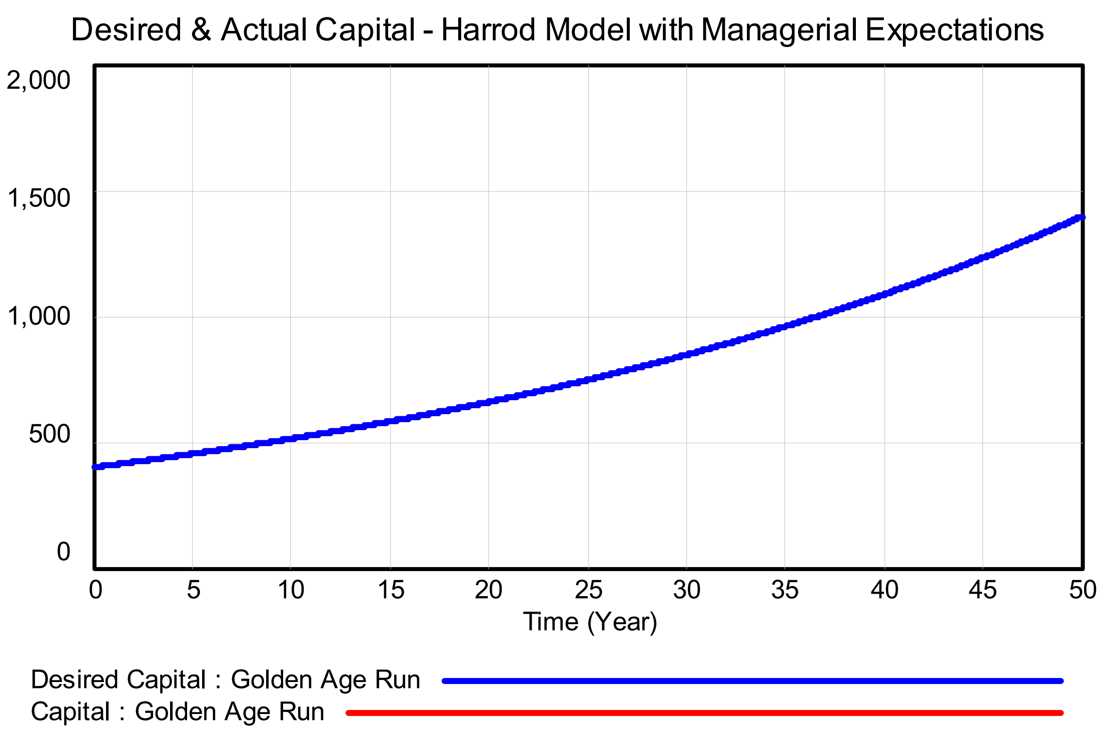


Figure 15: Desired & Actual Capital in the Harrod Model with Bounded Rational Managerial Expectations–Golden Age Case

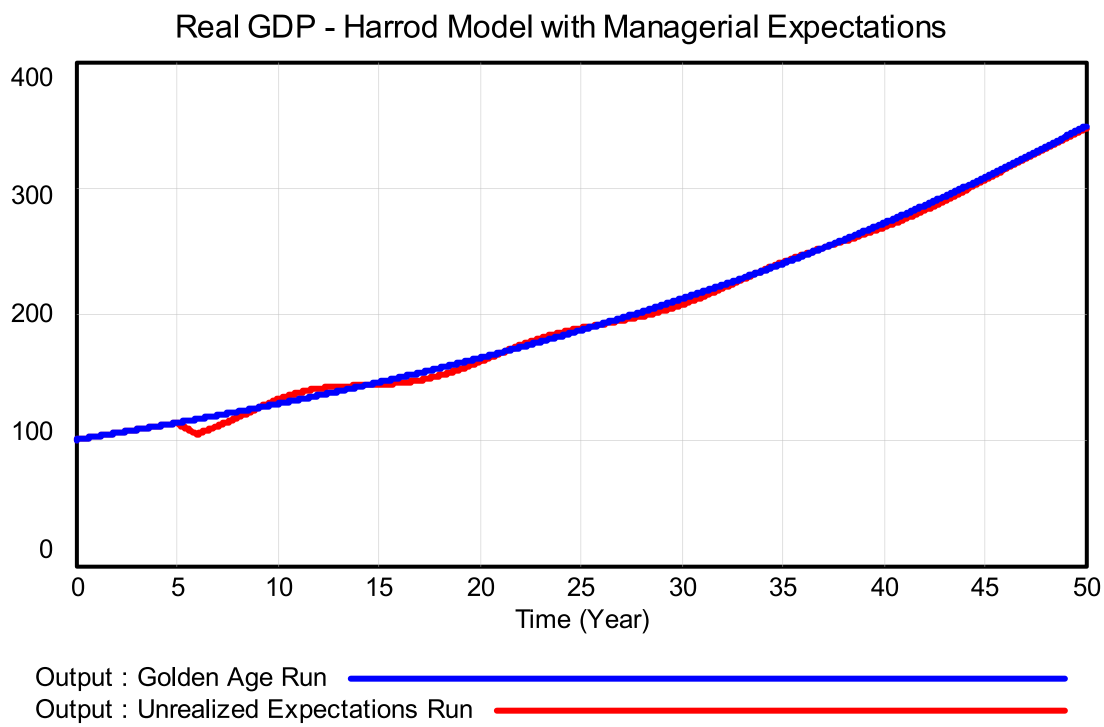


Figure 16: Output in the Harrod Model with Bounded Rational Managerial Expectations– Golden Age Case & Unrealized Expectations Case

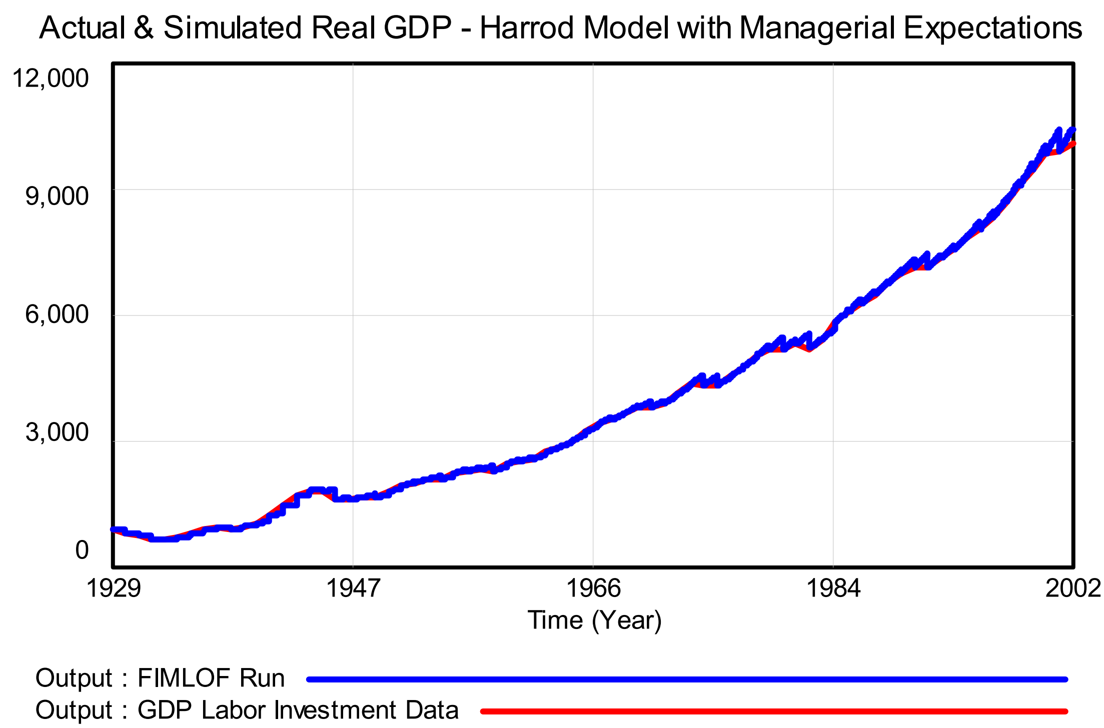


Figure 17: Actual & Simulated Real U.S. GDP in the Harrod Model with Bounded Rational Managerial Expectations

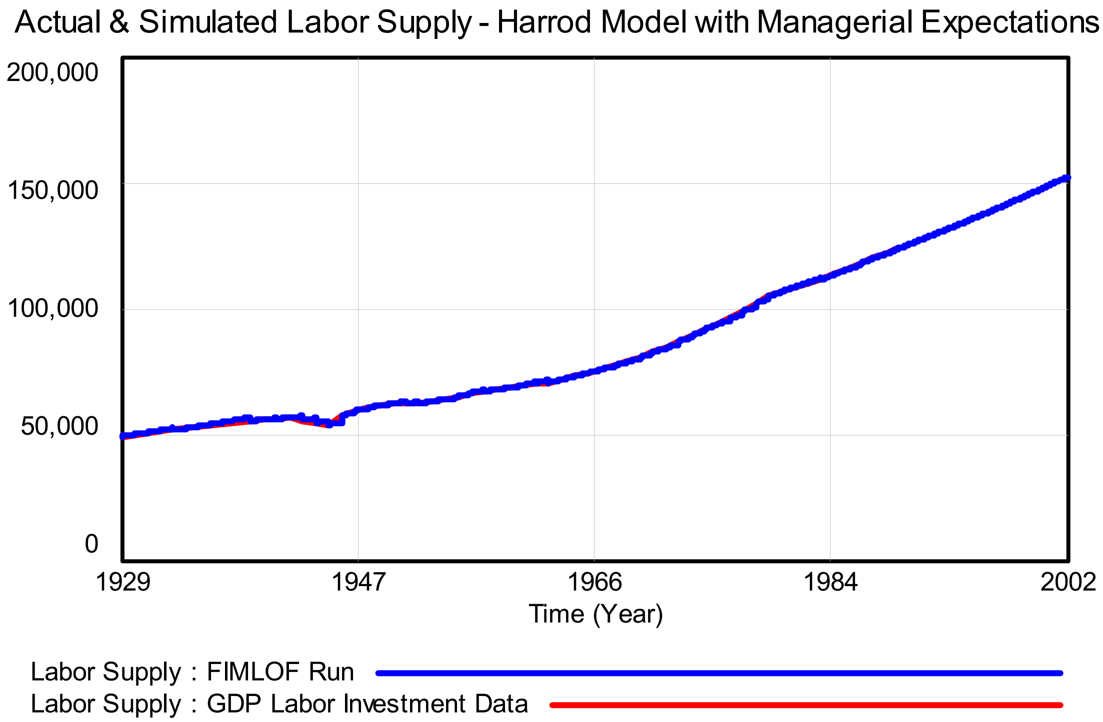


Figure 18: Actual & Simulated Labor Supply in the Harrod Model with Bounded Rational Managerial Expectations

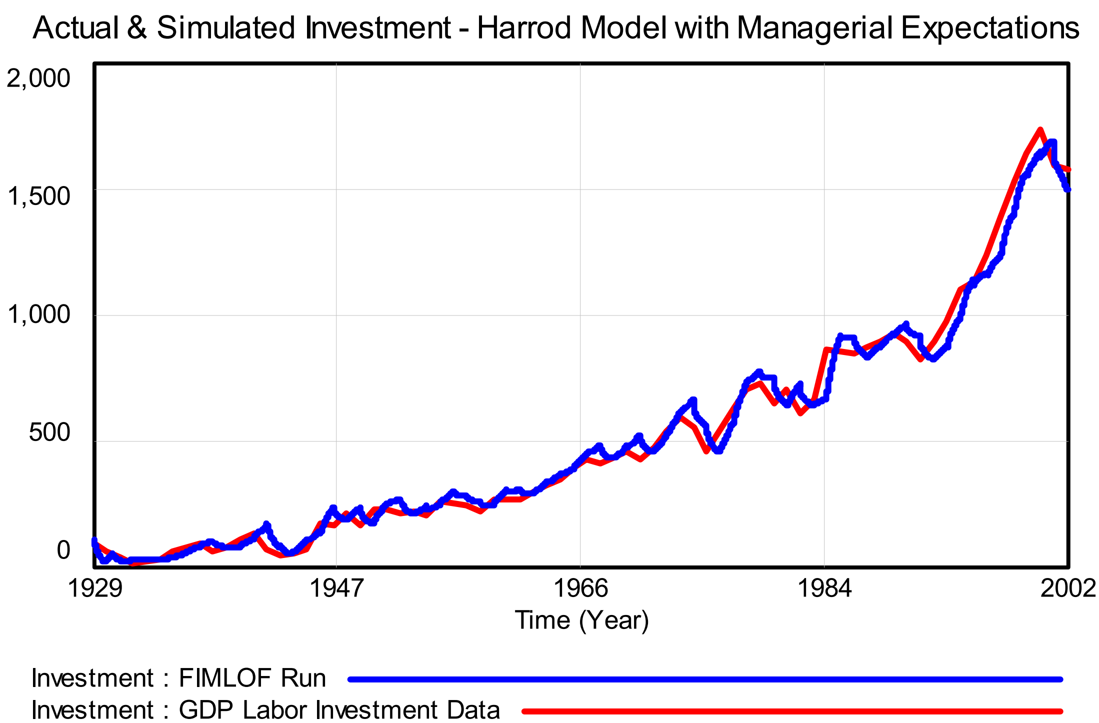


Figure 19: Actual & Simulated Investment Spending in the Harrod Model with Bounded Rational Managerial Expectations

Parameters & Initial Values	95% Confidence Intervals for Parameters & Initial Values
MPS (Fraction)	.0363202 <= 0.0508878 <= .0728591
Natural Rate of Growth (Fraction/Year)	???? <= 0.0158742 <= ????
Capital Output Ratio (Years)	2.80146 <= 2.80491 <= ????
Estimated Warranted Rate of Growth (Fraction/Year)	.0181424
Initial Expected Growth Rate (Fraction/Year)	0.0179739 <= 0.0403708 <= 0.0425465
Labor Output Ratio (People/ $ of Output/Year)	8.06716 <= 8.06958 <= ????
Time To Perceive Output (Years)	13.2384 <= 20 <= 20***
Time Horizon for Reference Output (Years)	1.3 <= 1.94271 <= 4.05091
Time to Perceive Trend in Output (Years)	???? <= 2.0661 <= 4.36797
Output Forecast Horizon (Years)	40.8537 <= 48.4051 <= ????
Initial Capital ($ of Capital Units)	???? <= 2427.87 <= 2430.85
Initial Perceived Output ($ of Output/Year)	???? <= 525.011 <= 763.387
Initial Reference Output ($ of Output/Year)	307.344 <= 457.406 <= 536.615

Table 2: Estimated Parameters, Estimated Initial Values, & 95% Confidence Intervals for the Harrod Model with Bounded Rational Managerial Expectations [* = the FIMLOF algorithm bumped-up against the parameter's estimation constraint]**

Appendix

Equations for Figure 1: System Dynamics Model of the "Real World" Used to Show the Non-Predictability of Dynamic, Nonlinear, Feedback Systems

Adequacy of Inventory = Inventory / (Desired Inventory + 1e-007) {Dmnl}

Adequacy of Inventory Model = Inventory Model / (Desired Inventory Model + 1e-007) {Dmnl}

Backlog = INTEG (New Order Rate-Order Clearing Rate,New Order Rate * Normal Delivery Delay) {Orders}

Backlog Model = INTEG (New Order Rate Model-Order Clearing Rate Model, New Order Rate Model * Normal Delivery Delay Model) {Orders}

Chg Expected Orders = (New Order Rate - Perceived Orders) / Time Chg Exp Orders {Orders/Year/Year}

Chg Expected Orders Model = (New Order Rate Model - Perceived Orders Model) / Time Chg Exp Orders Model {Orders/Year/Year}

Correction for Backlog = (Backlog - Desired Backlog) / Time to Correct Backlog {Orders/Year}

Correction for Backlog Model = (Backlog Model - Desired Backlog Model) / Time to Correct Backlog Model {Orders/Year}

Correction for Inventory = (Desired Inventory - Inventory) / Time to Correct Inventory {Widgets/Year}

Correction for Inventory Model = (Desired Inventory Model - Inventory Model) / Time to Correct Inventory Model {Widgets/Year}

Delivery Delay = Backlog / (Order Clearing Rate + 1e-007) {Years}

Delivery Delay Model = Backlog Model / (Order Clearing Rate Model + 1e-007) {Years}

Desired Backlog = Perceived Orders * Normal Delivery Delay {Orders}

Desired Backlog Model = Perceived Orders Model * Normal Delivery Delay Model {Orders}

Desired Inventory = Desired Order Clearing Rate * Desired Inventory Coverage {Widgets}

Desired Inventory Model = Desired Order Clearing Rate Model * Desired Inventory Coverage Model {Widgets}

Desired Inventory Coverage = 3 {Years}

Desired Inventory Coverage Model = 3 {Years}

Desired Labor = Desired Production / Labor Productivity {People}

Desired Labor Model = Desired Production Model / Labor Productivity Model {People}

Desired Order Clearing Rate = Backlog / Normal Delivery Delay {Orders/Year}

Desired Order Clearing Rate Model = Backlog Model / Normal Delivery Delay Model {Orders/Year}

Desired Production = Max(0, Perceived Orders + Correction for Backlog + Correction for Inventory) {Widgets/Year}

Desired Production Model = Max(0, Perceived Orders Model + Correction for Backlog Model + Correction for Inventory Model) {Widgets/Year}

Effect of Inventory on Shipments = Table for Effect of Inventory on Shipments(Adequacy of Inventory) {Dmnl}

Effect of Inventory on Shipments Model = Table for Effect of Inventory on Shipments Model(Adequacy of Inventory Model) {Dmnl}

Hire Separation Rate = (Desired Labor - Labor) / Time Chg Labor {People/Year}

Hire Separation Rate Model = (Desired Labor Model - Labor Model) / Time Chg Labor Model {People/Year}

Inventory = INTEG (Production Rate-Shipment Rate,Desired Inventory) {Widges}

Inventory Model = INTEG (Production Rate Model-Shipment Rate Model,Desired Inventory Model) {Widgets}

Labor = INTEG (Hire Separation Rate,Desired Labor) {People}

Labor Model= INTEG (Hire Separation Rate Model,Desired Labor Model) {People}

Labor Productivity = 10 {Widgets/Person/Year}

Labor Productivity Model = 10 {Widgets/Person/Year}

New Order Rate = ((1 - Noise Switch) * Normal Orders From Household Sector) + (Noise Switch * Normal Orders From Household Sector * (1 + Noise Input)) {Orders/Year}

New Order Rate Model ((1 - Noise Switch Model) * Normal Orders From Household Sector Model) + (Noise Switch Model * Normal Orders From Household Sector Model * (1 + Noise Input Model)) {Orders/Year}

Noise Input = RANDOM NORMAL(-0.1, 0.1, 0, 1, 321) {Dmnl}

Noise Input Model = RANDOM NORMAL(-0.1, 0.1, 0, 1, 456) {Dmnl}

Noise Switch = Step(1,5) {Dmnl}

Noise Switch Model = Step(1,5) {Dmnl}

Normal Delivery Delay = 1 {Years}

Normal Delivery Delay Model = 1 {Years}

Normal Orders From Household Sector = 1500 {Orders/Year}

Normal Orders From Household Sector Model = 1500 {Orders/Year}

Order Clearing Rate = Shipment Rate {Orders/Year}

Order Clearing Rate Model = Shipment Rate Model {Orders/Year}

Perceived Orders = INTEG (Chg Expected Orders,New Order Rate) {Orders/Year}

Perceived Orders Model = INTEG (Chg Expected Orders Model, New Order Rate Model) {Orders/Year}

Production Rate = Labor Productivity * Labor {Widgets/Year}

Production Rate Model = Labor Productivity Model * Labor Model {Widgets/Year}

Shipment Rate = Desired Order Clearing Rate * Effect of Inventory on Shipments {Widgets/Year}

Shipment Rate Model = Desired Order Clearing Rate Model * Effect of Inventory on Shipments Model {Widgets/Year}

Table for Effect of Inventory on Shipments([(0,0)-(1,1)],(0,0),(0.25,0.75),(0.5,0.9),(0.75,0.98),(1,1)) {Dmnl}

Table for Effect of Inventory on Shipments Model([(0,0)-(1,1)],(0,0),(0.25,0.75),(0.5,0.9),(0.75,0.98),(1,1)) {Dmnl}

Time Chg Exp Orders = 12 {Years}

Time Chg Exp Orders Model = 12 {Years}

Time Chg Labor = 4 {Years}

Time Chg Labor Model = 4 {Years}

Time to Correct Backlog = 1 {Years}

Time to Correct Backlog Model = 1 {Years}

Time to Correct Inventory = 1 {Years}

Time to Correct Inventory Model = 1 {Years}

Equations for Figure 3: Bounded Rational System Dynamics Expectations Structure

Chg Expected Trend in Hog Price = (Indicated Trend in Hog Price - Perceived Trend in Hog Price) / Time to Perceive Trend in Hog Price {Fraction/Quarter/Quarter}

Chg Perceived Hog Price = (Actual Hog Price - Perceived Hog Price) / Time To Perceive Hog Price {$/Barrow or Gilt/Quarter}

Chg Reference Hog Price = (Perceived Hog Price - Reference Hog Price) / Time Horizon for Reference Hog Price {$/Barrow or Gilt/Quarter}

Expected Hog Price = Perceived Hog Price * (1 + Perceived Trend in Hog Price * Time To Perceive Hog Price) * Exp(Hog Price Forecast Horizon * Perceived Trend in Hog Price) {$/Barrow or Gilt/Quarter}

Hog Price Forecast Horizon = 1 {Quarters}

Indicated Trend in Hog Price = (Perceived Hog Price - Reference Hog Price) / (Reference Hog Price * Time Horizon for Reference Hog Price) {Fraction/Quarter}

Initial Perceived Hog Price = 24.67 {$/Barrow or Gilt/Quarter}

Initial Perceived Trend in Hog Price = 0 {Fraction/Year}

Initial Reference Hog Price = 24.67 {$/Barrow or Gilt/Quarter}

Perceived Hog Price = INTEG (Chg Perceived Hog Price,Initial Perceived Hog Price) {$/Barrow or Gilt/Quarter}

Perceived Trend in Hog Price = INTEG (Chg Expected Trend in Hog Price,Initial Perceived Trend in Hog Price) {Fraction/Quarter}

Reference Hog Price = INTEG (Chg Reference Hog Price,Initial Reference Hog Price) {$/Barrow or Gilt/Quarter}

Time Horizon for Reference Hog Price = 4 {Quarters}

Time To Perceive Hog Price = 0.5 {Quarters}

Time to Perceive Trend in Hog Price = 0.5 {Quarters}

Equations for Figure 6: System Dynamics Representation of the Simple Harrod Model

Actual Growth Rate = Investment / Capital {Fraction/Year}

Capital = INTEG (Investment,10) {$}

Capital Output Ratio = 2 {Years}

Chg Labor Supply = Labor Supply*Natural Growth Rate {People/Year}

Investment = Saving {$/Year}

Labor Output Ratio = 1 {People/$/Year}

Labor Supply = INTEG (Chg Labor Supply,10) {People}

MPS = 0.05 + Step(0.01,5) {Fraction}

Natural Growth Rate = 0.025 {Fraction/Year}

Output = Min((Capital/Capital Output Ratio),(Labor Supply/Labor Output Ratio)) {$/Year}

Saving = MPS*Output {$/Year}

Warranted Growth Rate = MPS/Capital Output Ratio {Fraction/Year}

Equations for Figure 10: System Dynamics Representation of the Harrod Model with a Bounded Rational Managerial Expectations Structure

Actual Growth Rate = Investment / (Capital + 1e-006) {Fraction/Year}

Bounded Expected Growth Rate = Min(MAX(Expected Growth Rate,-0.05),0.05) {Fraction/Year}

Capital = INTEG (Investment,Initial Capital) {$}

Capital Output Ratio = 2.28 {Years}

Chg Expected Growth Rate = (Indicated Growth Rate - Expected Growth Rate) / Time to Perceive Trend in Output {Fraction/Year/Year}

Chg Labor Supply = Labor Supply * Natural Rate of Growth {People/Year}

Chg Perceived Output = (Output - Perceived Output) / Time to Perceive Output {$/Year/Year}

Chg Reference Output = (Perceived Output - Reference Output) / Time Horizon for Reference Output {$/Year/Year}

Correction for Capital Stock = (Desired Capital - Capital) / Time to Correct Capital {$/Year}

Correction for Growth in Capital = Capital * Bounded Expected Growth Rate {$/Year}

Desired Capital = Required Capital Output Ratio * Expected Output {$}

Expected Growth Rate = INTEG (Chg Expected Growth Rate,Init Expected Growth Rate) {Fraction/Year}

Expected Output = Perceived Output * (1 + Bounded Expected Growth Rate * Time to Perceive Output) * Exp(Output Forecast Horizon * Bounded Expected Growth Rate) {$/Year}

Indicated Growth Rate = (Perceived Output - Reference Output) / (Reference Output * Time Horizon for Reference Output + 1e-007) {Fraction/Year}

Init Expected Growth Rate = 0.045 {Fraction/Year}

Initial Capital = 1741 {$}

Initial Labor Supply = 49180 {People}

Initial Perceived Output = 273 {$/Year}

Initial Reference Output = 425 {$/Year}

Investment = Correction for Capital Stock + Correction for Growth in Capital {$/Year}

Labor Output Ratio = 4.05 {People/$/Year}

Labor Supply = INTEG (Chg Labor Supply,Initial Labor Supply) {People}

MPS = 0.05 {Fraction}

Natural Rate of Growth = 0.025 {Fraction/Year}

Output = Min((Capital / (Capital Output Ratio + 1e-007)),(Labor Supply / (Labor Output Ratio + 1e-007))) {$/Year}

Output Forecast Horizon = 1 {Years}

Perceived Output = INTEG (Chg Perceived Output,Initial Perceived Output) {$/Year}

Reference Output = INTEG (Chg Reference Output,Initial Reference Output) {$/Year}

Required Capital Output Ratio = MPS / (Bounded Expected Growth Rate +1e-008) {Years}

Saving = MPS * Output {$/Year}

Time Horizon for Reference Output = 6 {Years}

Time to Correct Capital = 5 {Years}

Time to Perceive Output = 3 {Years}

Time to Perceive Trend in Output = 3 {Years}

Warranted Rate of Growth = MPS / (Capital Output Ratio + 1e-007) {Fraction/Year}