ANALYSIS of STOCK MANAGEMENT GAMING EXPERIMENTS and ALTERNATIVE ORDERING FORMULATIONS[1]

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Abstract

This paper investigates two different yet related research questions about stock management in feedback environments: The first one is to analyze the effects of selected experimental factors on the performances of subjects (players) in a stock management simulation game. In light of these results, our second objective is to evaluate the adequacy of standard decision rules typically used in dynamic stock management models and to seek improvement formulations. To carry out the research, the generic stock management problem is chosen as the interactive gaming platform. In the first part, gaming experiments are designed to test the effects of three factors on decision making behavior: different patterns of customer demand, minimum possible order decision ('review') interval and finally the type of the receiving delay. ANOVA results of these 3-factor, 2-level experiments show which factors have significant effects on ten different measures of behavior (such as max-min range of orders, inventory amplitudes, periods of oscillations and backlog durations). In the second phase of research, the performances of subjects are compared against some selected ordering heuristics (formulations). First, the patterns of ordering behavior of subjects are classified into three basic types. Comparing these three patterns with the stand-alone simulation results, we observe that the common linear "Anchoring and Adjustment Rule." can mimic well the smooth and gradually damping type of behavior, but can not replicate the non-linear and/or discrete ordering dynamics. Thus, several alternative non-linear rules are formulated and tested against subjects' behaviors. Some standard discrete inventory control rules (such as (s, Q)) common in the inventory management literature are also formulated and tested. These non-linear and/or discrete rules, compared to the linear stock adjustment rule, are found to be more representative of the subjects' ordering behavior in many cases, in the sense that these rules can generate nonlinear and/or discrete ordering dynamics. Another major finding is the fact that the well-documented oscillatory dynamic behavior of the inventory is a quite general result, not just an artifact of the linear anchor and adjust rule. When the supply line is ignored or underestimated, large inventory oscillations result, not just with the linear anchor-and-adjust rule but also with the non-linear rules, as well as the standard inventory management rules. Furthermore, depending on parameter values, nonlinear ordering rules are more prone to yield unstable oscillations -even if the supply line is taken into account.

Keywords: stock management, anchor and adjust heuristic, experimental testing of decision rules, non-linear decision heuristics, inventory control rules

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1. Stock Management Game

For the experimental testing purpose, the generic stock management problem, one of the most common dynamic decision problems, is chosen as the interactive gaming environment. The objective of the game is stated as "keeping the inventory level as low as possible while avoiding any backorders." If there is not enough goods in the inventory at any time, customer orders are entered as backorders to be supplied later. "Order decisions" are the only means of controlling the inventory level. The general structure of the stock management problem is illustrated in Figure 1. (See Sterman 2000). The three empty boxes: Expectation Formation, Goal Formation and Decision Rule are deliberately left blank, as they are unknown, since they take place in the "minds" of the players. (Later, in the simulation version of the game, these three boxes will have to be specified. For instance, the expectation formation will be formulated by exponential smoothing; inventory goal will be set to inventory coverage times expected demand and supply line goal will be order delay times expected demand. As for the decision rule, different formulations will be tried: linear stock adjustment rule, three different non-linear adjustment rules and finally various standard discrete inventory control rules.) This notion of "gaming experimentation" to analyze and test subjects' decision heuristics has been successfully used in system dynamics literature (Sterman 1987, 1989), as well as in experimental psychology (Brehmer 1989).

While playing the game, subjects can monitor the system from information displays/graphs showing their inventory, supply line levels and customer demand (Appendix A). Neither the costs associated with high inventories nor costs resulting from backorders are accounted for *explicitly* in the simulation game. However, the relation between keeping these costs as low as possible and the objective of the game is stated in the instruction given to subjects. In other words, subjects are instructed that keeping large safety stocks would result in high holding costs but backlogs must also be avoided, as they would incur large costs due to lost demand. Before beginning the game, all subjects are given a written instruction presenting the problem and their task (Appendix B). Time available to accomplish the task is not limited. No explicit, tangible reward is used to motivate the subjects.

2. Gaming Experiments

The first set of gaming experiments are designed to test the effects of three factors on decision making behavior of subjects:
- (a) Length of order decision (review) interval;
- (b) Type of the receiving delay;
- (c) Different patterns of customer demand

2.1. Length of Decision Interval

Subjects were allowed to order at "each time unit" in the first group of experiments (Short Game), whereas they were allowed to order "once every five time units" in the second group of experiments (Long Game). Short Games are simulated for 100 time units whereas Long Games are simulated for 250 time units and the receiving delays are also shorter (4 time units) in the short game, longer (10 time units) in the long game. Thus, the 'length' effect is really a package (involving longer receiving delays and longer game length as well), but this package is summarized by the term *Length of Decision Interval effect*, because this particular component will be the focus in interpreting the experimental results, as will be seen below. It is hypothesized that

subjects being free to make decisions at any point in time versus decisions allowed only every five time units would influence the difficulty of the game, hence cause differences in the performances.
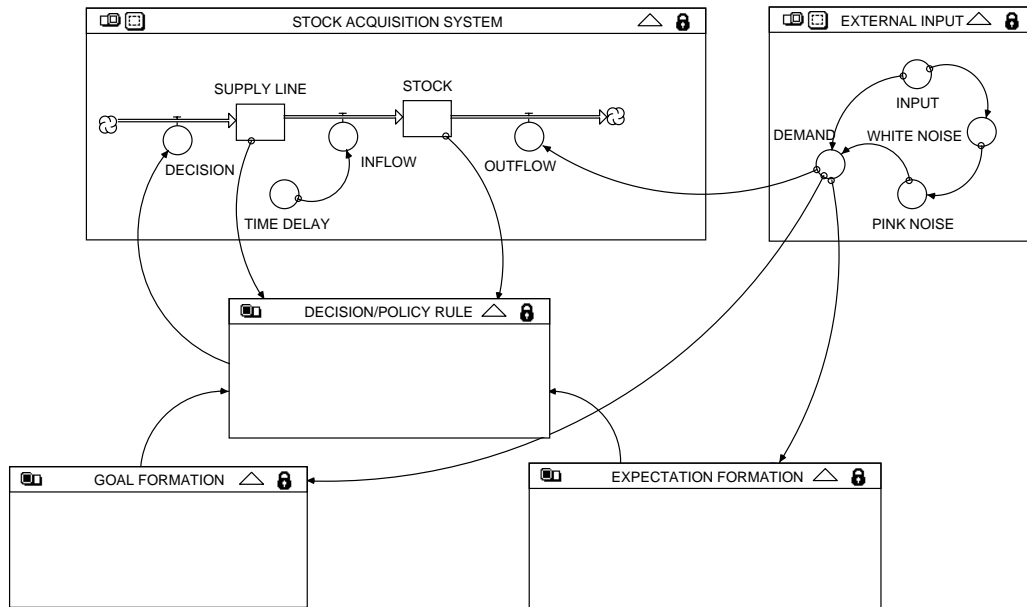

FIGURE 1. The Stock Management Problem

2.2. Type of Receiving Delay

The second independent experimental factor is the *type* of the delay. (The *length* of the delay is *not* an independent factor; since it is changed as an integral part of the *Length of Decision Interval* effect described above. Note that if one were to change the decision interval from one to five days but keep the receiving delay at four, then the nature of the game would change in an implicit and problematic way, in the sense that the receiving delay would be four times *longer* than the order interval in the short game but *shorter* than the order interval in the long one). We focus on the type of delay representation, as different delay types may be appropriate for different inventory acquisition systems, like *continuous exponential* delay or *discrete* delay representations. Since "Receiving" is the inflow to inventory, its transient behavior may influence decision-maker's interpretation of the results of his/her own order decisions. The two extremes of the exponential delay family, namely the infinite-order *discrete delay* and first order *exponential delay* are chosen as the two levels of the delay factor in the experimental design. (This is further motivated by a common criticism system dynamics games that states that continuous delays are not realistic/intuitive, so such games pose an artificial difficulty for players with no expertise in modeling. So, it would be interesting to see if subjects' performances would actually deteriorate in cases involving continuous delays).

2.3. Patterns of Customer Demand

Until the fifth decision interval, average customer demand remains constant at 20. At the beginning of the fifth decision interval (at time five in the Short Game and at time 25 in Long Game) an unannounced, one-time increase of 20 units occurs in the customer demand patterns used in the experiments. Subjects react to the disequilibrium created by this change. After the step up, demand remains constant at 40 in the first type of customer demand pattern which we call "*step up in customer demand*". (Figure 2).

In the second type of demand pattern, called "*step up and down in customer demand*," a second disturbance, a one-time decrease in demand follows the first increase, after some time interval, restoring the demand back to its original level of 20 (see Figure 3, where the step-down is set to occur at time 20). The time interval between the step up and down in customer demand is chosen as roughly half of the natural periodicity of the model (about 25 days in Short Game and about 60 in Long Game). A common perception in stock management circles is that 'poor ordering performance is caused by complex demand patterns'. The purpose of this particular experimental factor is to test this claim/hypothesis in a simplified context.

Before the demand patterns described above are used in games, "Pink noise" (auto-correlated noise) is added to the average patterns to obtain more realistic demand dynamics. The standard deviation of the white noise is set to 15 percent of average customer demand. The delay constant of the exponential smoothing (the correlation time used to create pink noise) is taken as two time units. (See Appendix C for equations).

To summarize, there are eight combinations of the above three factors across the two levels of each. So we have a 2^3 factorial design. Each condition is played six times (random replications), yielding a total of 48 experiments (see Table 1). Since the demand pattern is discovered by the subjects once the game is played and because they can improve their performance by practice, to obtain unbiased results, the same subject never played two Short Games or two Long Games. However, due to limited number of subjects, some of the subjects were allowed to play one Short Game and one Long Game, since transferring experience in-between Short and Long Games is not easy.

TABLE 1. Design of Experiments. (X) indicates the selected level of factors for the corresponding runs.

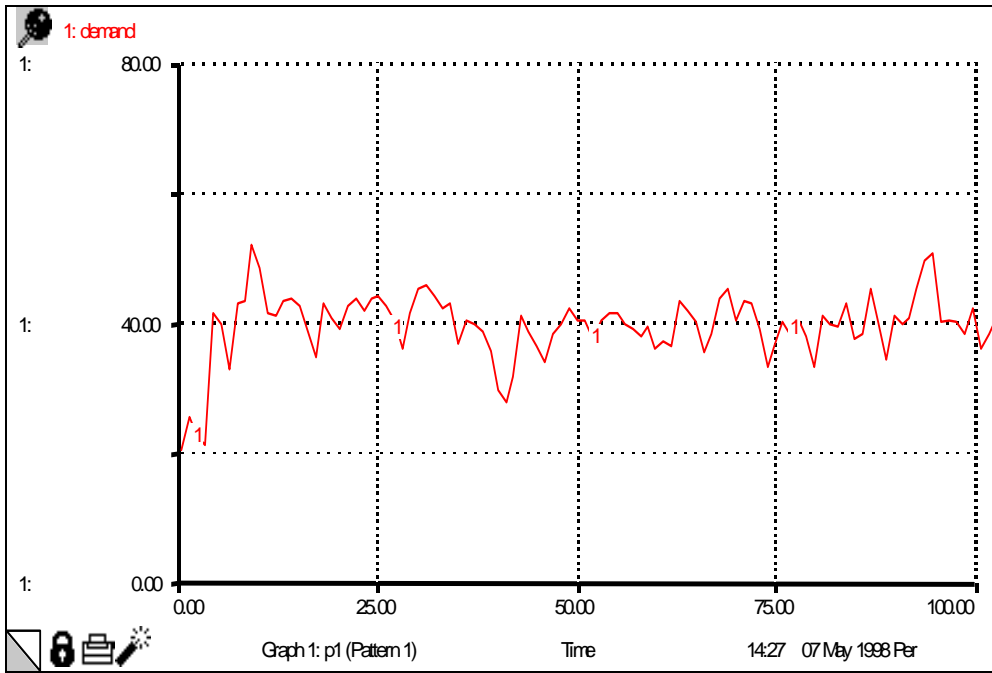| Runs	Length of Ordering Interval		Type of Receiving Delay		Pattern of Customer Demand	
	Every Time Unit (1 day)	Every Five Time Units (5 days)	Exponential	Discrete	Step-Up Customer Demand	Step-Up-and-Down Customer Demand
1	X		X		X	
2	X		X			X
3	X			X	X	
4	X			X		X
5		X	X		X	
6		X	X			X
7		X		X	X	
8		X		X		X

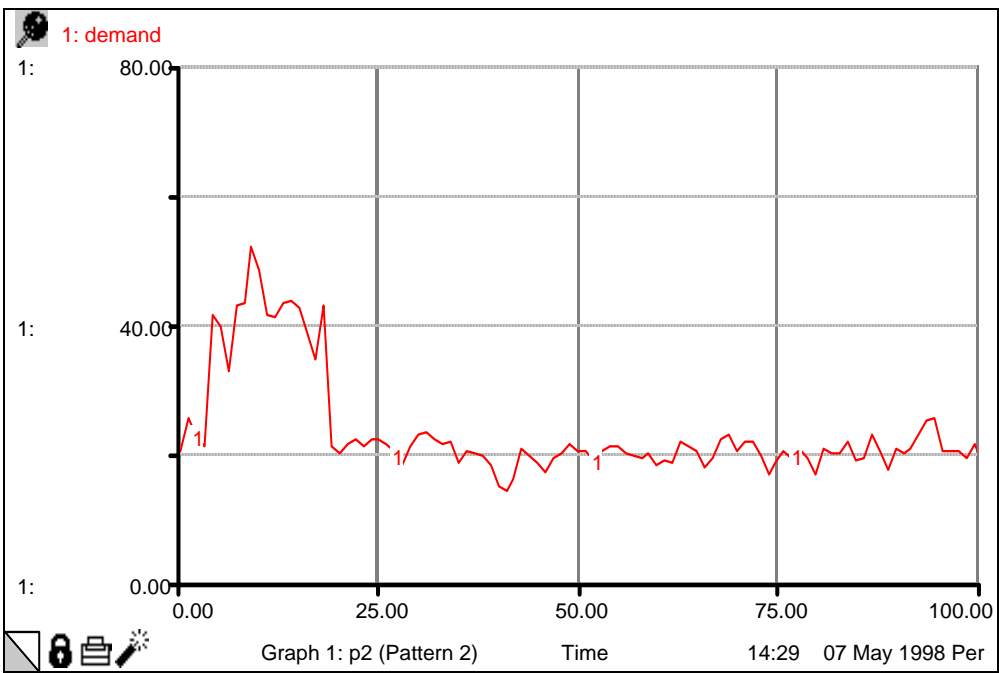FIGURE 2. Step Up Customer Demand for Short Games

FIGURE 3. Step Up and Down Customer Demand for Short Games

2.4. Initial Conditions

Games start at equilibrium. Supply line level is initially set at 80 in Short Games and 200 in Long Games such that no backordering occurs even when the decision-maker does not order goods during the first four decision intervals (at the end of which the disturbance in customer demand creates disequilibrium). Inventory levels are initially set arbitrarily (at 40 and 200 respectively) so as to satisfy the average initial customer demand for the first two decision intervals (for two days in Short Games and ten days in Long Games).

3.   Analysis of Experiments

The general, broad behavior pattern of inventory in majority of games is one of oscillations. (See Figures 4, 5 and most other games illustrated in the Figures through the article). This finding is consistent with overwhelming evidence on oscillating inventories in system dynamics literature and elsewhere (for instance, Forrester 1961, Sterman 1989, 2000, Lee et al 1997 and Tvede 1996). We will return later to this main 'qualitative' result. But first, we take a more quantitative look at the effects of the three factors.

Representative summary measures of orders and inventory levels computed from the experimental results are summarized in Table 2. The ten characteristics are tabulated for each of the 48 games. The averages of these ten measures for each of the eight experiments are also displayed. From these averages, it is possible to have some idea about the effects the three factors on each of these measures. For instance, as one moves from Run 1 to Run 2, the only input factor that changes is 'demand pattern' (from step-up to step-up-and-down). In this case, observe for example that the average Max. Order measure changes from 143.3 to 146.7, a minor change, whereas the average Max. Inventory changes from 98.3 to 167.5, clearly a more significant effect, at least at this base level of the other factors, ignoring any interactions. (A simple t-test between Run 1 and 2, ignoring other levels of the other two factors and possible interactions, would yield the same conclusion). But a more complete and definitve conclusion about the significance of each of the three factors on each of the ten output measures can be obtained by a full analysis of variance (ANOVA), considering the effects of each factor at all levels of other factors (and any possible interactions between them). A summary table derived from full ANOVA (using SPSS software) is shown in Table 3. These results are obtained from a full factorial ANOVA model involving seven effects (three main effects, three 2-way interactions and one 3-way interaction term). Since we have a total of 48 data, the degrees of freedom for residuals (errors) is 48-7-1= 40 and hence the F statistic (=Mean Squared Explained by Regression/Mean Squared Error) for each effect has (1, 40) degrees of freedom for numerator and denominator respectively. So, if the F value computed for any effect is 'large enough', we reject the hypothesis that the corresponding effect coefficient is zero, i.e. significant effect is discovered. Typical significance levels used are alpha=0.01 (99% confidence)., alpha=0.05 (95% confidence) or alpha=0.10 (90% confidence). In Table 3, we provide the F values computed for each effect (for each output measure) and the 'P value' at which the F value would be found significant. To conclude, if a P value is ≤ the chosen alpha level, we decide that the given effect has a significant effect on the selected output measure, at (1-alpha)% confidence level. Although the ANOVA results comes from a full factorial model, in Table 3, we show the main effects only for readability, because analyzing each interaction term individually is beyond our research scope.
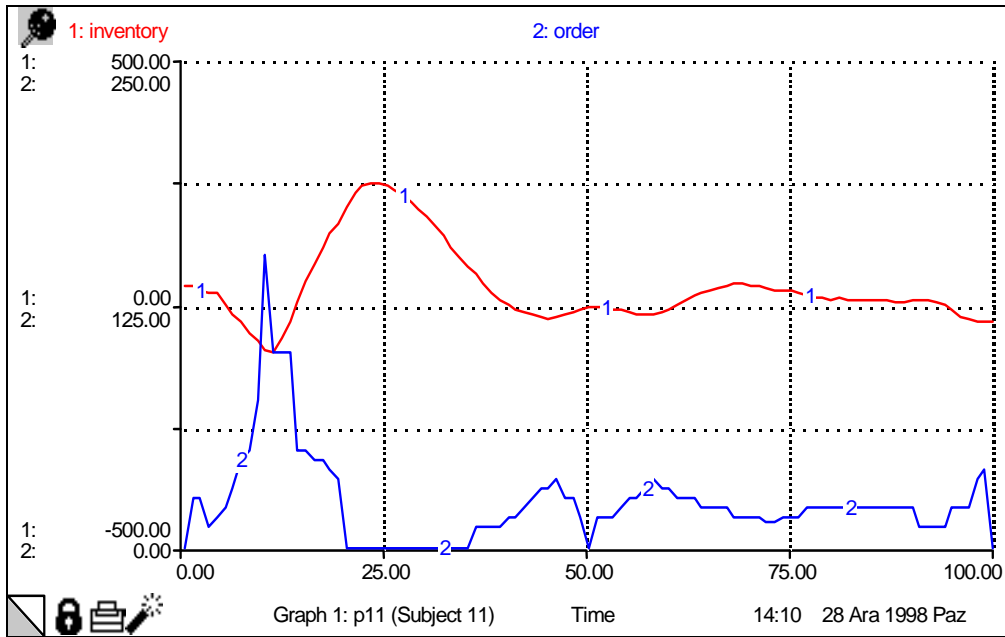
FIGURE 4. Performance of the Player in Game 11 (Short Game with Orders Each Period, Step Up and Down in Customer Demand, Exponential Delay)
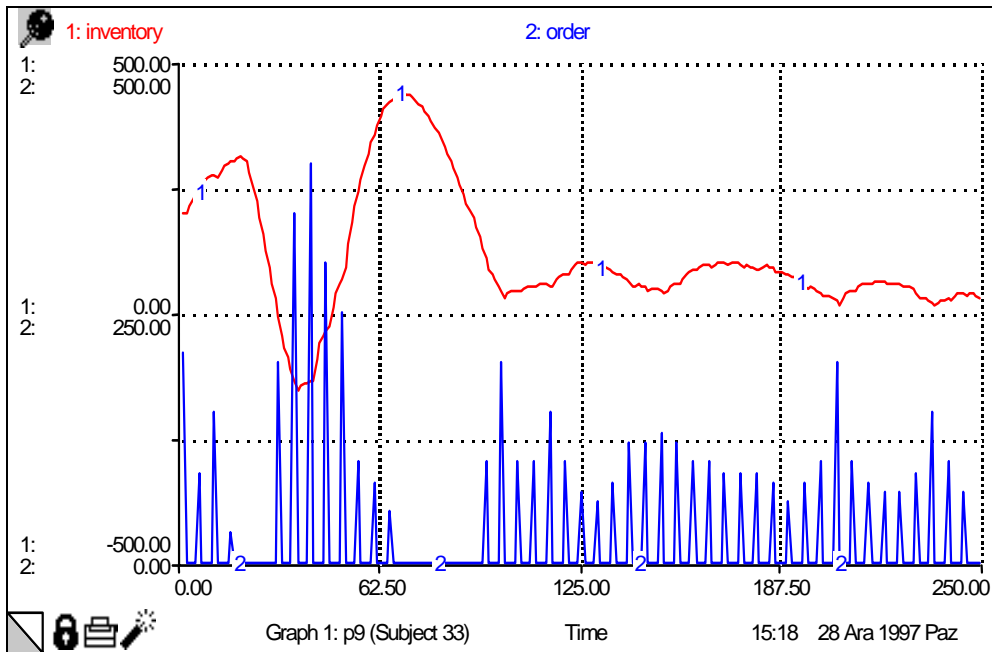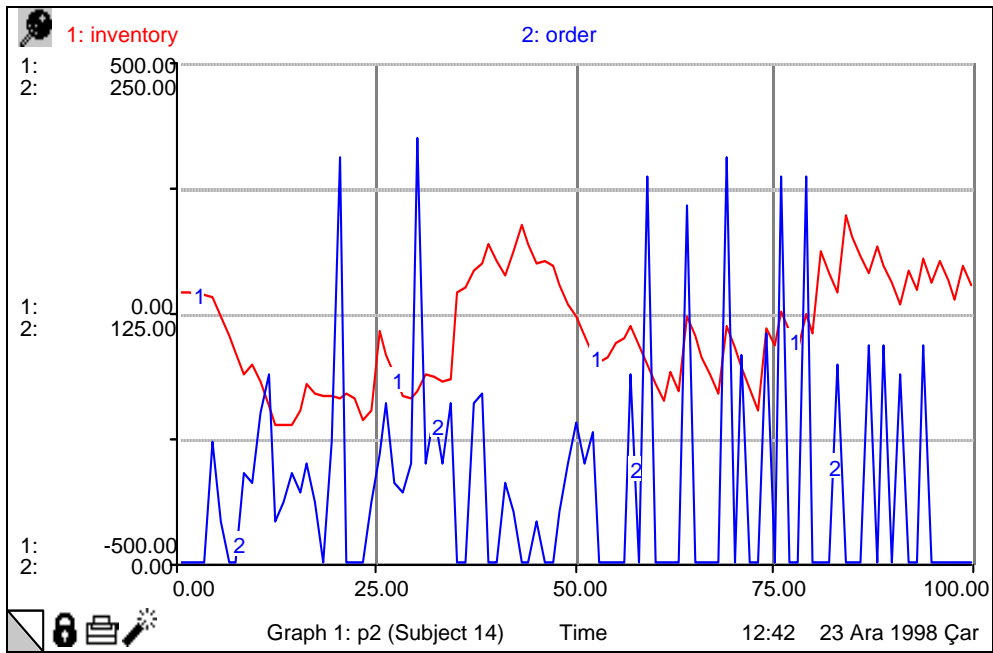

FIGURE 5. Performance of the Player in Game 33 (Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Exponential Delay)

TABLE 2. Selected Measures of Game Performances

Experiment	Min. Order	Max. Order	Range Of Orders	Min. Inven-tory	Max. Inven-tory	Range Of Inven-tory	Initial Back-order Time	Final Back-order Time	Duration Of Back-orders	Inv. Oscil-lation Period
1	0	100	100	-125	50	175	4	38	34	N/A
2	0	100	100	-125	150	275	4	33	29	31
3	0	100	100	-175	150	325	4	18	14	29
4	0	200	200	-75	100	175	6	16	10	25
5	0	300	300	-225	100	325	5	43	38	N/A
6	20	60	40	-60	40	100	6	37	31	N/A
Avg. Of Run 1	**3.3**	**143.3**	**140.0**	**-130.8**	**98.3**	**229.2**	**4.8**	**30.8**	**26.0**	**28.3**
7	0	100	100	-60	115	175	10	21	11	30
8	0	100	100	-60	250	310	5	13	8	21
9	0	60	60	-60	60	120	7	20	13	N/A
10	0	400	400	-80	180	260	6	21	15	39
11	0	150	150	-100	250	350	6	14	8	28
12	40	70	30	-60	150	210	6	18	12	N/A
Avg. Of Run 2	**6.7**	**146.7**	**140.0**	**-70.0**	**167.5**	**237.5**	**6.7**	**17.8**	**11.2**	**29.5**
13	0	100	100	-225	300	525	5	34	29	43
14	0	220	220	-225	225	450	5	35	30	45
15	0	150	150	-75	160	235	14	25	11	N/A
16	0	60	60	-15	190	215	N/A	N/A	N/A	25
17	0	100	100	-125	250	375	5	38	33	N/A
18	0	300	300	-260	180	440	5	18	13	27
Avg. Of Run 3	**0.0**	**155.0**	**155.0**	**-154.2**	**217.5**	**373.3**	**6.8**	**30.0**	**23.2**	**35.0**
19	0	150	150	-25	150	175	N/A	N/A	N/A	N/A
20	0	150	150	-100	290	390	6	21	15	N/A
21	0	80	80	-145	80	225	5	24	19	30
22	0	100	100	-40	350	390	12	18	4	32
23	0	100	100	-250	210	460	6	27	21	45
24	0	200	200	-150	250	400	5	22	17	N/A
Avg. Of Run 4	**0.0**	**130.0**	**130.0**	**-118.3**	**221.7**	**340.0**	**6.8**	**22.4**	**15.2**	**35.7**
25	0	600	600	-260	425	685	20	30	10	44
26	0	400	400	-350	200	550	25	85	60	N/A
27	0	300	300	-280	200	480	20	85	65	N/A
28	0	400	400	-300	200	500	25	70	45	63
29	0	560	560	-150	370	520	20	40	15	39
30	0	250	250	-20	425	445	35	45	10	73
Avg. Of Run 5	**0.0**	**418.3**	**418.3**	**-226.7**	**303.3**	**530.0**	**24.2**	**59.2**	**34.2**	**54.8**
31	40	375	335	-200	250	450	20	40	20	N/A
32	0	250	250	-100	200	300	25	55	30	N/A
33	0	400	400	-150	475	625	30	45	15	73
34	50	250	200	-60	200	260	30	55	25	66
35	50	300	250	0	275	275	N/A	N/A	N/A	N/A
36	100	250	150	-160	250	410	25	55	30	73
Avg. Of Run 6	**40.0**	**304.2**	**264.2**	**-111.7**	**275.0**	**386.7**	**26.0**	**50.0**	**24.0**	**70.7**
37	100	270	170	-100	500	600	20	40	20	85
38	20	500	480	-500	300	800	25	90	65	102
39	80	250	170	-40	450	490	N/A	N/A	N/A	N/A
40	50	250	200	-200	425	625	30	40	10	75
41	0	1000	1000	-1000	2000	3000	20	100	80	133
42	0	450	450	-375	420	795	35	60	25	52
Avg. Of Run 7	**41.7**	**453.3**	**411.7**	**-369.2**	**682.5**	**1051.7**	**26.0**	**66.0**	**40.0**	**89.4**
43	0	250	250	-125	400	525	N/A	N/A	N/A	75
44	0	450	450	-500	200	700	20	60	40	N/A
45	0	375	375	-750	200	950	20	70	50	N/A
46	20	300	280	-330	330	660	25	55	30	55
47	50	250	200	-180	300	480	30	45	15	114
48	0	300	300	-375	350	725	30	60	30	N/A
Avg. Of Run 8	**11.7**	**320.8**	**309.2**	**-376.7**	**296.7**	**673.3**	**25.0**	**58.0**	**33.0**	**81.3**

TABLE 3. ANOVA Results (F values) on the Significance of the Effects of the Three Experimental Factors

Factors		Minimum Order	Maximum Order	Range Of Orders	Minimum Inventory	Maximum Inventory	Range Of Inventory	Initial Backorder Time	Final Backorder Time	Duration Of Backorders	Inventory Oscillation Period
Length Of Ordering Interval	**Fo**	9.977	33.758	23.385	10.092	8.942	12.126	215,316	53.754	7.662	39.007
	P Value $(\nu_1=1,\nu_2=40)$	0.003	0	0	0.003	0.005	0.001	0	0	0.009	0
	Result	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$
Type Of Receiving Delay	**Fo**	0.013	0.137	0.103	6.61	4.383	6.782	0.346	1.048	0.637	5.626
	P Value $(\nu_1=1,\nu_2=40)$	0.91	0.713	0.749	0.014	0.043	0.013	0.560	0.313	0.430	0.027
	Result	insignificant	insignificant	insignificant	significant $\alpha=0.05$	significant $\alpha=0.05$	significant $\alpha=0.05$	insignificant	insignificant	insignificant	significant $\alpha=0.05$
Pattern Of Customer Demand	**Fo**	0.084	3.003	2.667	1.125	1.125	1.596	0.325	4.539	4.228	0.144
	P Value $(\nu_1=1,\nu_2=40)$	0.774	0.091	0.11	0.295	0.262	0.214	0.572	0.040	0.047	0.708
	Result	insignificant	significant $\alpha=0.10$	insignificant	insignificant	insignificant	insignificant	insignificant	significant $\alpha=0.05$	significant $\alpha=0.05$	insignificant

FIGURE 6. Performance of the Player in Game 14 (Short Game with Orders Each Period, Step Up in Customer Demand, Discrete Delay)
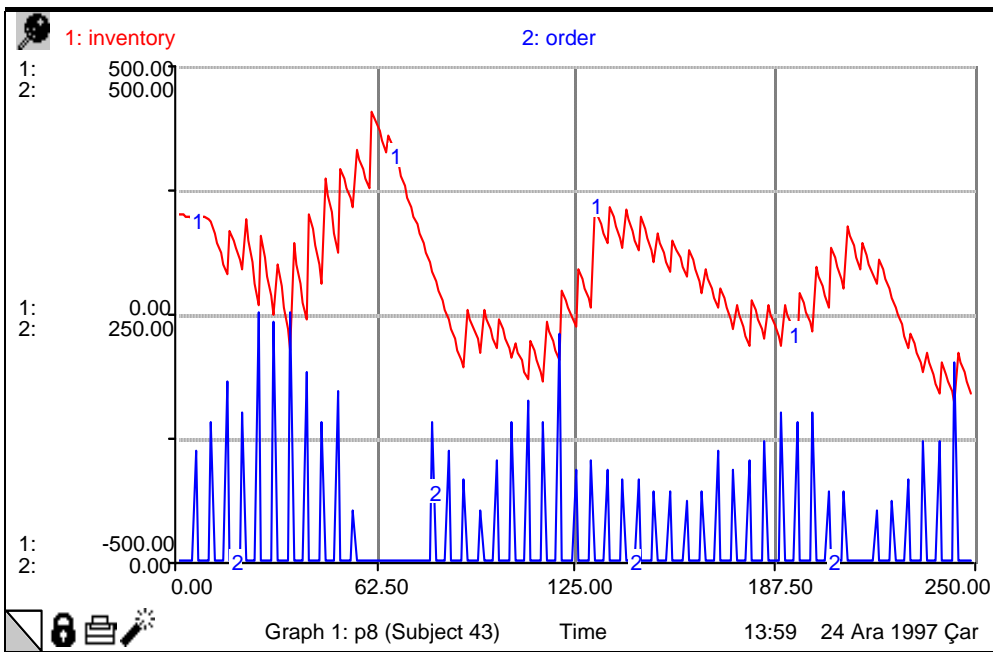

FIGURE 7. Performance of the Player in Game 43 (Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Discrete Delay)

Before we examine the ANOVA results of Table 3, note that the last four mesures in Table 2 (Initial Backorder Time, Final Backorder Time, Duration of Backorders nad Oscillation Period) have the symbol N/A in some of the cells. The meaning is that the corresponding measure was simply undefined insome particular game results. In a few instances, there are no backorders at all, so all three related measures are marked N/A. A more problematic and frequent situation is when the dynamics of inventory has no clearly identifiable, computable, constant period. This arises when the inventory is non-oscillatory, or when it exhibits very complex, highly noisy oscillations. In about two or three out of six runs in each cell we have this situation, so it is quite frequent. Since ANOVA requires 'balanced' input data, these N/A entries require some pre-treatment. Prior to ANOVA, SPSS uses a mixture of two methods, depending on the suitability: 'Estimating the missing data from neigboring points' or 'discarding data from other cells so as to balance'. In any case, it should be noted that the results about the last output measure, *period of inventory oscillations*, should be taken with caution.

Although there are ten output measures in Table 2 and 3, some of these are intermediate mesures used to compute related end-measures. Minimum and maximum orders are measured to ultimately obtain a measure of 'range of order fluctuation' (or amplitude) and the same is true for minimum and maximum inventories. Lastly, initial backorder time and final backorder time are measured to compute a the 'duration of backorders.' So to conserve space, we focus on four end-measures only and leave more detailed examination to the interested reader.

3.1. Effect of Different Patterns of Customer Demand

ANOVA results in the bottom row block of Table 3 show the effects of "demand pattern" (step-up only or step-up-and-down) on the output measures. As illustrative game dynamics, see Figures 4, 5, 7 for step-up-and-down and Figures 6, 8B and 10B for step-up demand. (For full results, compare pair wise the results of experiments 1 and 2; 3 and 4; 5 and 6; 7 and 8 in Table 2. Also see Özevin 1999).   F-values in Table 3 show that the demand pattern does not have a strong enough effect on 'Range of Orders' measure (although it *does* have some effect, since the significance is just missed at 90%). Similarly, demand pattern has no significant effect on the amplitude or period of inventory oscillations. Lastly, we observe that the demand pattern *does* have a significant effect on the 'duration of backorders'. This last finding is interesting because the *direction* of the effect is that step-up-and-down demand, compared to step-up only, causes the backlog durations to become *shorter*. So a seemingly more complex demand pattern actually happens to compensate for the players' ordering weaknesses in terms of backlog durations.

3.2. Effect of Different Representations of Receiving Delays

Illustrative game dynamics with 1st order exponential delay and with discrete delay are shown in Figures 4, 5 and Figures 6, 7 respectively. In these and many other examples, with continuous exponential delay subjects are able to manage the inventories in a relatively more stable way. It seems that when the goods ordered arrive gradually over some period of time, it prevents the players from over-ordering or under-ordering excessively. In contrast, discrete delay representation affects their performance negatively by causing large fluctuations. In these experiments, subjects yield large magnitude, long period oscillations in inventories and fail to bring these oscillations

under control in most cases. (For complete results, compare pair wise the results of experiments 1 and 3; 2 and 4; 5 and 7; 6 and 8 in Table 2. Also see Özevin 1999). Subjects seem to have difficulty in accounting for the effects of sudden receiving. The above results are consistent with research evidence on the effect of delays on dynamic decision making performance, in system dynamics literature (Sterman 1989), as well as in experimental psychology research (Brehmer 1989). The results are also consistent with the mathematical stability conditions of discrete-delay dynamical systems, much more difficult to obtain compared to continuous-delay systems. (See Driver 1977).

For a more statistical analysis, ANOVA results about the "type of receiving delay effect" are given in the middle row block of Table 3. Observe that this factor has very high significant effect on two critical output measures: *Range* of Inventory fluctuations and its *Period*. This finding statistically confirms our qualitative assessment above: that the type of receiving delay has a significant effect on the stability of inventory fluctuations. (Also observe in Table 2 that the *direction* of the effect is that discrete delay, compared to the continuous one, causes the *range* and *period* of oscillations to be larger). In a nutshell, discrete delay representation makes the system more oscillatory, less stable, and thus harder to manage for the subjects.

3.3. Effect of the Length of Decision Intervals
As illustrative game dynamics with 'order every time step' and 'order every five time steps' experiments; see Figures 4, 6 and Figures 5, 7 respectively. (For full results, compare pair wise the results of experiments 1 and 5; 2 and 6; 3 and 7; 4 and 8 in Table 2. Also see Özevin 1999). These comparisons reveal that *all* output measures change significantly, when the ordering interval is changed from 'every time step' to every five time steps'. ANOVA results given in the first row block of Table 3 are also consistent with this observation: Effects on *all* measures are *highly* significant. There may be two different sources of this high level and comprehensive significance: First, when the decision period is made longer (noting that the receiving delay is also made longer to be consistent), since some time-constants of the system are larger, time-related output measures (like *Period of Oscillations, Backorder Times and Backorder Duration)* all naturally become larger in value – a natural technical result of dynamics of the system. Secondly –and more interestingly- amplitude measures (like *Range of Orders* and *Range of Inventory*) also become statistically larger. This is more behavioral/decision-related result: When decisions can be made less frequently, orders must be larger in magnitude, feedback is less frequent, controlling of the inventory becomes harder and so inventory fluctuations are larger in magnitude as well. So again in sum, 'order every five time steps'' makes the system more oscillatory, less stable, thus harder to manage for the subjects.

4. Testing of the Alternative Decision Formulations
Section 3 above, completes the relatively more quantitative/statistical research objective of the paper. In light of the above results, the second objective is to evaluate the adequacy of standard decision rules typically used in dynamic stock management models and to seek improvement formulations. To this end, the performance *patterns* of subjects will be (qualitatively) compared with the dynamic patterns obtained using different simulated ordering formulations: As a preliminary step, the patterns of ordering behavior of subjects are observed to fall in three basic classes: i- smooth, continuous (oscillatory or non-oscillatory) damping orders (for example Figure 4), ii-

alternating large discrete and then zero orders, like a high frequency signal (for example Figure 6), iii- long periods of constant orders punctuated by a few sudden large ones (for example Figure 10-B). Using these three types of observed ordering patterns, the common linear "Anchoring and Adjustment" rule, several "nonlinear" adjustment rules and some standard discrete inventory control rules (such as (s, Q)) found in the inventory management literature will be evaluated.

4.1. Linear Anchoring and Adjustment Rule

Linear Anchoring and Adjustment Rule is frequently used to model decision-making behavior in System Dynamics models (Sterman, 2000, 1987). In making such decisions, one starts from an initial point, called the anchor, and then makes some adjustments to come up with the final decision. In the context of inventory management, a plausible anchor point for order decisions is the expected customer demand. (If the inventory manager can order only once every five periods, the anchor should be the total of expected customer demand for five periods between subsequent decisions.) When there are discrepancies between desired and actual inventory levels and/or between desired and actual supply line, adjustments are made so as to bring the inventory and the supply line back to desired levels. Thus, the order equation based on linear Anchoring and Adjustment heuristic is formulated as:

$$O_t = E_t + \alpha * (I_t^* - I_t) + \beta * (SL_t^* - SL_t) \qquad (4.1)$$

When orders can be given once every five periods, then the anchor of the rule is modified as follows (the adjustment terms being as before):

$$O_t = 5 * E_t + \alpha * (I_t^* - I_t) + \beta * (SL_t^* - SL_t) \qquad (4.2)$$

where E_t represents expected customer demand, I_t^* represents the desired inventory, I_t the inventory, SL_t^* the desired supply line and SL_t the supply line. α and β are the adjustment fractions.

In real life, 'safety stocks' are determined by balancing the inventory holding and backordering costs. Although, an optimum inventory level minimizing these costs may be found mathematically, more often safety stocks are set approximately. The desired inventory I_t^* is thus modeled as proportional to customer demand to allow adjustments in safety stocks when changes in customer demand occur.

$$I_t^* = k * E_t \qquad (4.3)$$

To maintain a receiving rate consistent with receiving delay τ and customer demand, SL_t^* is formulated as a function of τ and the expected customer demand E_t.

$$SL_t^* = \tau * E_t \qquad (4.4)$$

The linear anchor and adjust rule can mimic the subjects' performances adequately in experiments where they tend to place smooth and continuously damping orders (See Figure 8 as an example and Özevin 1999 for more). However, in certain game conditions, most subjects tend to order non-continuously (for example Figure 6 and 10B), especially when the receiving delay representation is discrete and/or the order interval is five. Such order patterns typically fall in class ii (alternating large discrete and then zero orders, like a high frequency signal) or class iii (long periods of constant orders punctuated by a few sudden large ones) described above. The linear "Anchoring and Adjustment Rule" can *not* yield such discrete-looking intermittent or occasional-large-then-constant orders. Non-linear rules may be able to represent better the subjects' performances in such situations. Different approaches will be discussed below in two separate sections: Non-linear adjustment rules and standard inventory control rules.
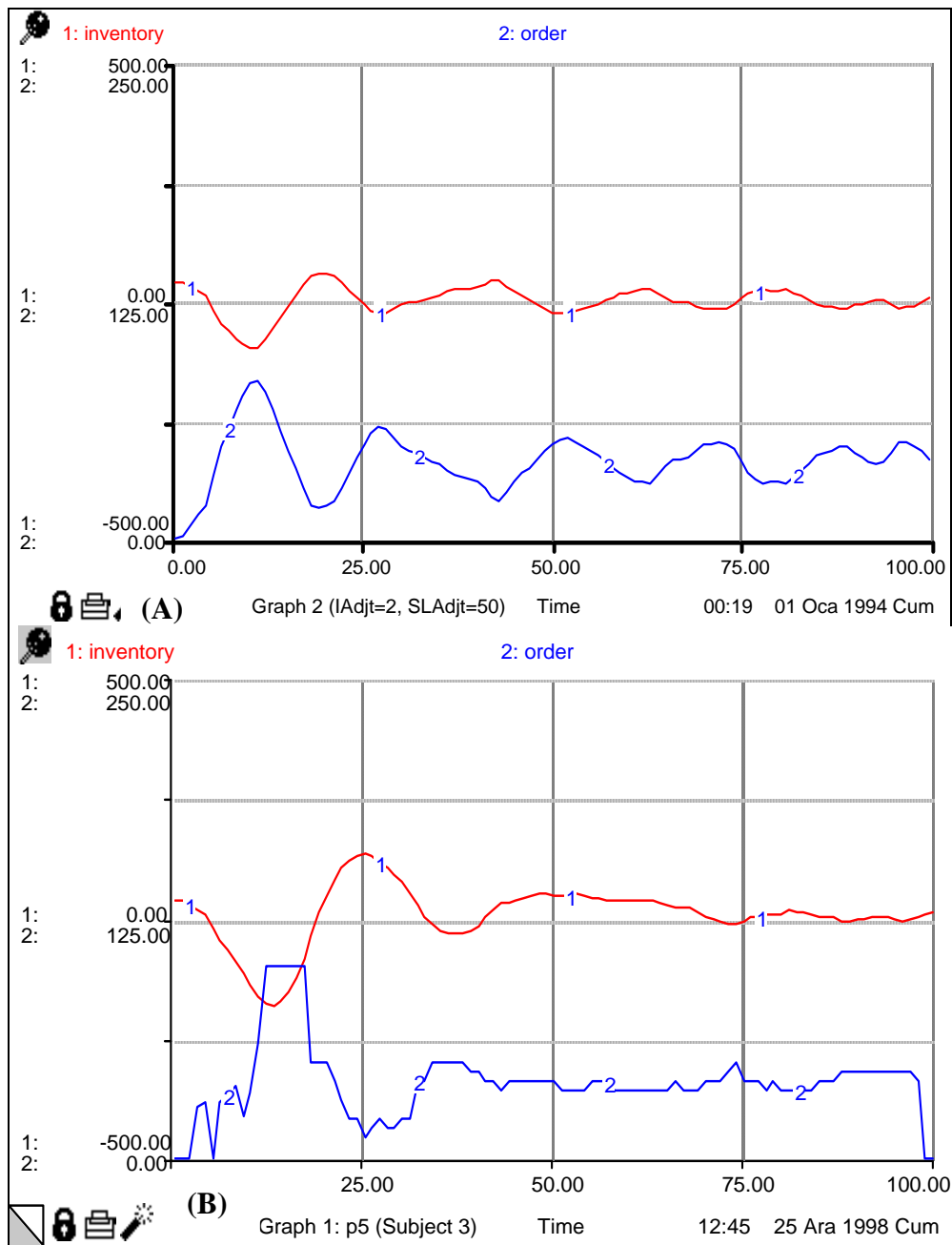
FIGURE 8. Comparison of (A) the Anchoring and Adjustment rule ($\alpha = 0.5$, $\beta = 0.02$), with (B) the Performance of the Player in Game 3 (Short Game with Orders Each Period, Step Up in Customer Demand, Exponential Delay)

4.2. Rules with Nonlinear Adjustments

The linear adjustment rule is based on 'adjustments' to orders proportional to the discrepancy between the desired and observed stock levels. Orders are placed regularly, in proportion to this discrepancy. However, some players do not place such smooth orders. They completely cease ordering when the inventory seems to be 'around a satisfactory' level and place rather large orders as the discrepancy between the desired and actual inventory becomes larger. The resulting ordering behavior is in class iii (long periods of constant orders punctuated by a few sudden large ones) described above. In this section, three alternative nonlinear decision rules will be developed and tested to address this particular class of nonlinear ordering behavior.

4.2.1. Cubic Adjustment Rules

Similar to the Linear Anchoring and Adjustment rule, Cubic Adjustment rules also start with expected customer demand as an anchor point, but the adjustments are formulated as non-linear. One or both of the adjustment terms may be cubic in discrepancies (in inventory and/or in supply line. Alternative order equations can be mathematically expressed as:

$$O_t = E_t + \alpha*(I_t^* - I_t)^3 \tag{4.5}$$

if only the inventory adjustment is taken into account, and:

$$O_t = E_t + \alpha*(I_t^* - I_t)^3 + \beta*(SL_t^* - SL_t) \tag{4.6}$$

$$O_t = E_t + \alpha*(I_t^* - I_t) + \beta*(SL_t^* - SL_t)^3 \tag{4.7}$$

where one of the adjustments is made cubic and the other is linear and finally,

$$O_t = E_t + \alpha*(I_t^* - I_t)^3 + \beta*(SL_t^* - SL_t)^3 \tag{4.8}$$

where both of the adjustments are formulated as cubic. In the equations above, E_t represents the expected customer demand; I_t^* and SL_t^* the desired inventory and supply line levels; I_t and SL_t the actual inventory and supply line and. α and β are the fraction of the discrepancy corrected by the decision-maker at each period. The internal consistency of the rules can be shown mathematically (See Özevin 1999). When orders can be given once every five periods, the adjustments terms are as above; however, the anchor term is increased to five times the expected customer demand.

The general stability properties of the cubic adjustment rules are similar to the well established linear adjustment rule. In other words, firstly the supply line must be taken into account (i.e. adjustment fraction β must be non-zero and preferably equal to α for optimum stability) and secondly, the larger the values of α and β the less stable the system tends to be. For example, Figure 9 compares two behaviors of the cubic rule with non-zero and zero β values. Observe that the behavior becomes quite unstable when the supply line adjustment term is zero. Although the behavior in Figure 9 (A) illustrates a very stable case, we should note that choosing stable values for the adjustment fractions is not easy with Cubic Adjustment rules (whereas the linear rule guarantees stability for any $\alpha=\beta < 1$). In other words, the performance of the cubic rule is too sensitive to the adjustment parameter values. In particular, when orders are given once every five periods, the cubic rule most often fails to generate stable dynamics. On the other hand, the primary advantage of the cubic rule is that it is possible to generate ordering patterns that can mimic the nonlinear ordering behavior of subjects characterized by long periods of constant orders punctuated by a few sudden large ones (described as class iii, above). Figure 10 provides comparison for such a case. Although cubic adjustment formulations can potentially yield such nonlinear order patterns with discrete delays as well as continuous ones, the range in which they can

yield stable dynamics is too narrow to be useful. (See Özevin 1999). Hence, the motivation for other nonlinear rules is to be discussed in the following sections.
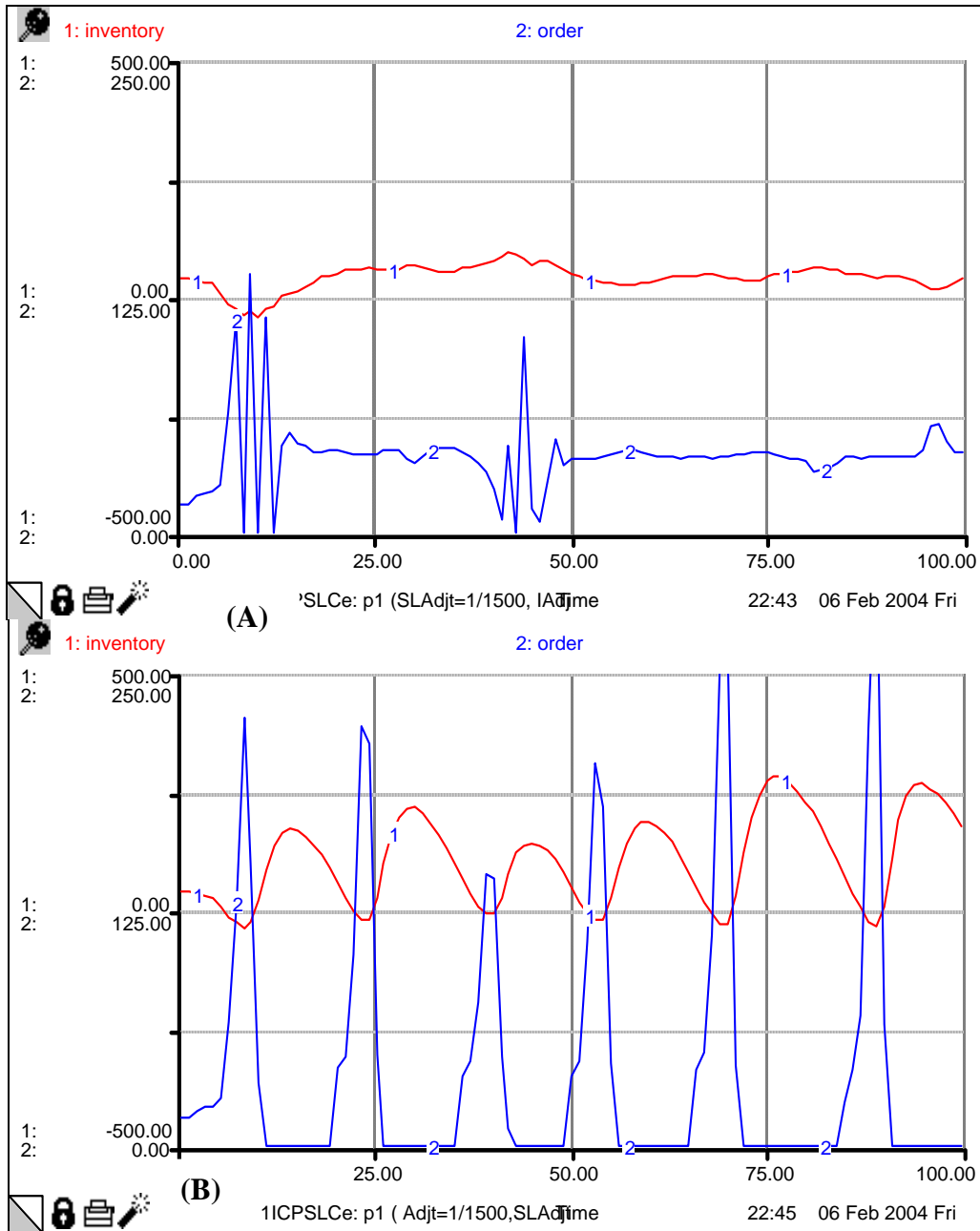

FIGURE 9. Two Different Performances of the "Cubic Supply Line and Cubic Inventory Adjustment rule" with two different parameters: (A) (with $\alpha=1/500$, $\beta=1/1500$) and (B) (with $\alpha=1/500$, $\beta=0$).
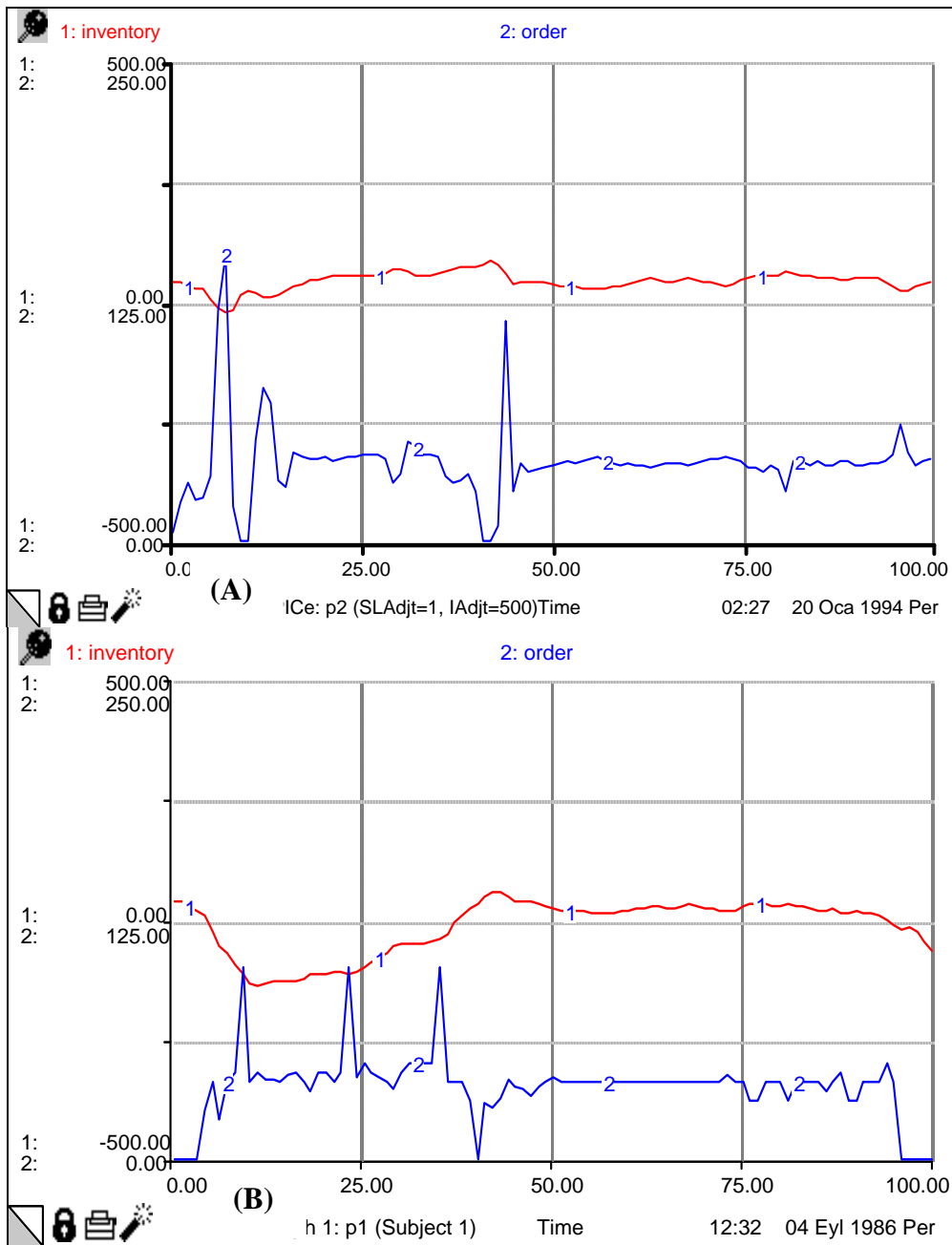
FIGURE 10. Comparison of (A) Performance of "Linear Supply Line and Cubic Inventory Adjustment rule" ($\alpha=1/500$, $\beta=1$) with (B) the Performance of the Player in Game 1 (Short Game with Orders Each Period, Step Up in Customer Demand, Exponential Delay)

4.2.2. Variable Adjustment Fraction Rule

Analogous to the Anchoring and Adjustment Rule and the Cubic Adjustment Rules, we define a 'Variable Adjustment Fraction Rule' that anchors at expectations about customer demand. However, the adjustments are increased sharply (nonlinearly) when the discrepancy in inventory increases. The simplest version of the order equation of the rule can be mathematically expressed as

$$O_t = E_t + \alpha*(I_t^*-I_t) \tag{4.9}$$
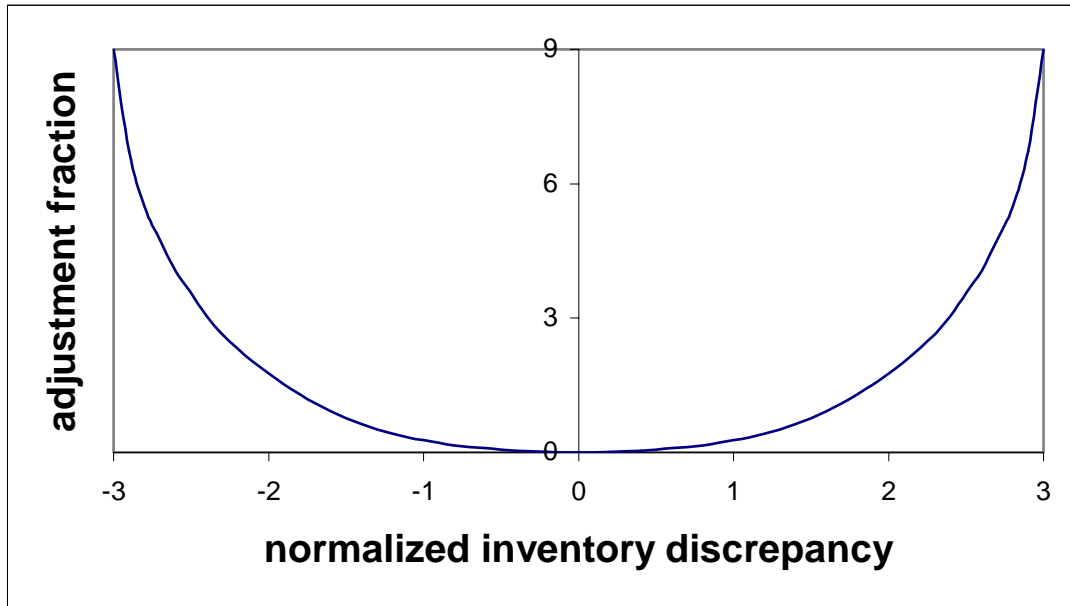
FIGURE 11. Graphical Adjustment Fraction Function

where the variable fraction α is a function of the discrepancy in inventory, normalized by the desired inventory. The shape of the function yields increased adjustments when the discrepancy in inventory is increased. Normalized inventory discrepancy δ is defined as

$$\delta = \frac{I^*-I}{I^*} \tag{4.10}$$

According to the function in Figure 11, the rule is not mathematically unbiased in the ideal known demand case; there will be some small, deliberate steady state discrepancy between the inventory and its desired level. But this may well be a "realistic" bias in order to be able to obtain a non-linear ordering behavior similar to some subjects. The rule performs quite realistically in the "noisy" demand case, where the steady state bias is negligible anyway and may be irrelevant in real life. The more general (and more stable) version of the Variable Adjustment Fraction Rule will be of the form $E_t + \alpha*(I_t^*-I_t) + \beta*(SL_t^*- SL_t)$, where β is defined by a function exactly equivalent to the one in Figure 11, except that the input would be 'normalized supply line discrepancy.' Alternatively, the supply line adjustment may be linear. (We omit this discussion further in this article to conserve space). In any case, inclusion of the supply line term will increase the stability of the system.

Variable Adjustment Fraction rule typically generates long periods of constant orders punctuated by a few sudden large ones. Therefore they may be used to represent subjects' behavior when orders are characterized by this type of nonlinearity, where linear adjustment rules would fail (See Figure 12 and Özevin 1999 for more illustrations).
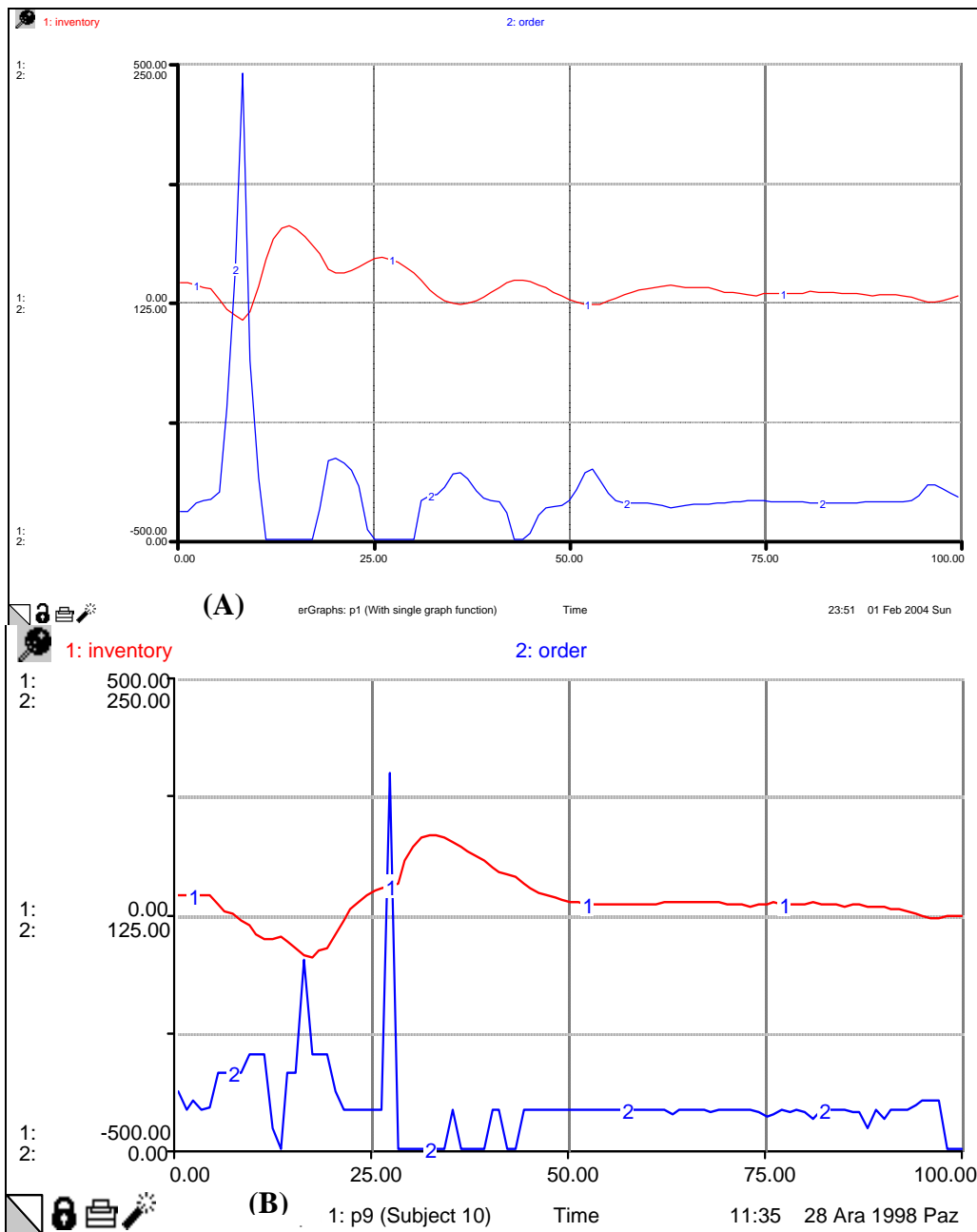

FIGURE 12. Comparison of (A) Variable Adjustment Fraction rule with (B) the Performance of the Player in Game 10 (Short Game with Orders Each Period, Step Up and Down in Customer Demand, Exponential Delay)

4.2.3. Nonlinear Expectation Adjustment Rule

The order equation of another nonlinear rule that we call 'expectation adjustment rule' can be mathematically expressed by

$$O_t = \alpha * E_t \qquad\qquad (4.11)$$

where the variable adjustment coefficient α is a function of discrepancy in inventory normalized by desired inventory and E_t represents the expected customer demand. α is equal to one when the inventory is at the desired level, since adjustments are not necessary when the system is in equilibrium (See Figure 13). The shape of the α function causes increasing upward adjustments in orders when the inventory level is below the desired level and it causes reductions in orders when the inventory is above its desired level.
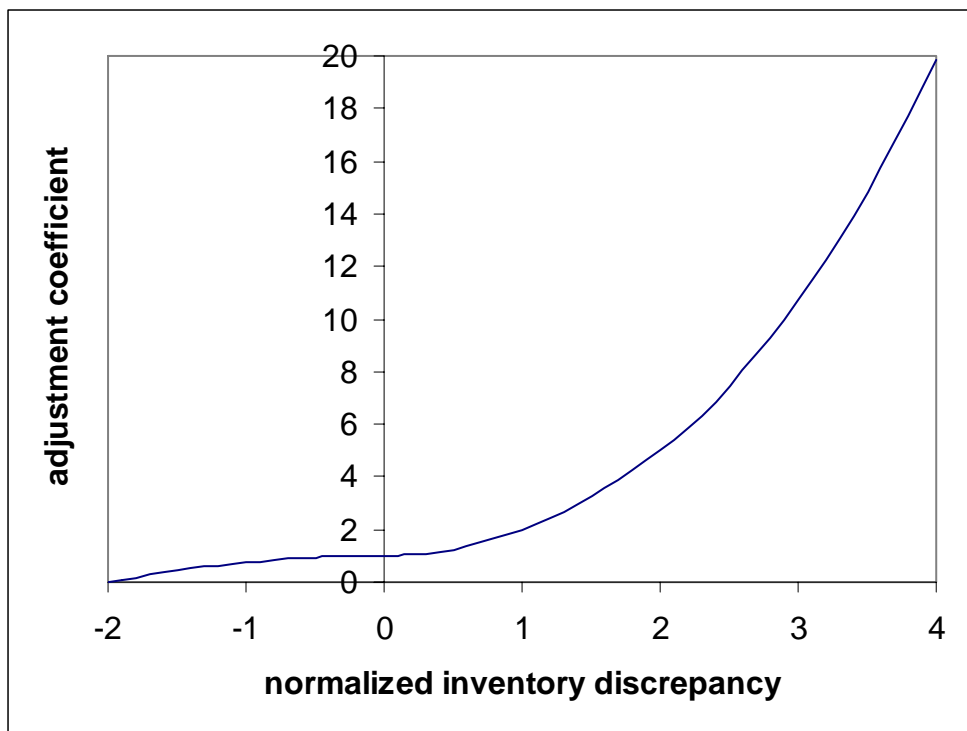
FIGURE 13. Adjustment Coefficient Function for the Non-linear Expectations Adjustment Rule

Like the previous Variable Adjustment rule, Nonlinear Expectations Rule can yield infrequent large orders. Therefore, it may provide an alternative to the linear anchoring and adjustment rule when subjects' behaviors exhibit such patterns (See Figure 14 for example).

Finally, this section yields another general result: the well-documented oscillatory dynamic behavior of the inventory when supply line is underestimated is true not only for the linear anchor-and-adjust rule but also for the non-linear rules seen above. Furthermore, in non-linear rules, stability is achieved for a rather narrow range of parameter/function values.
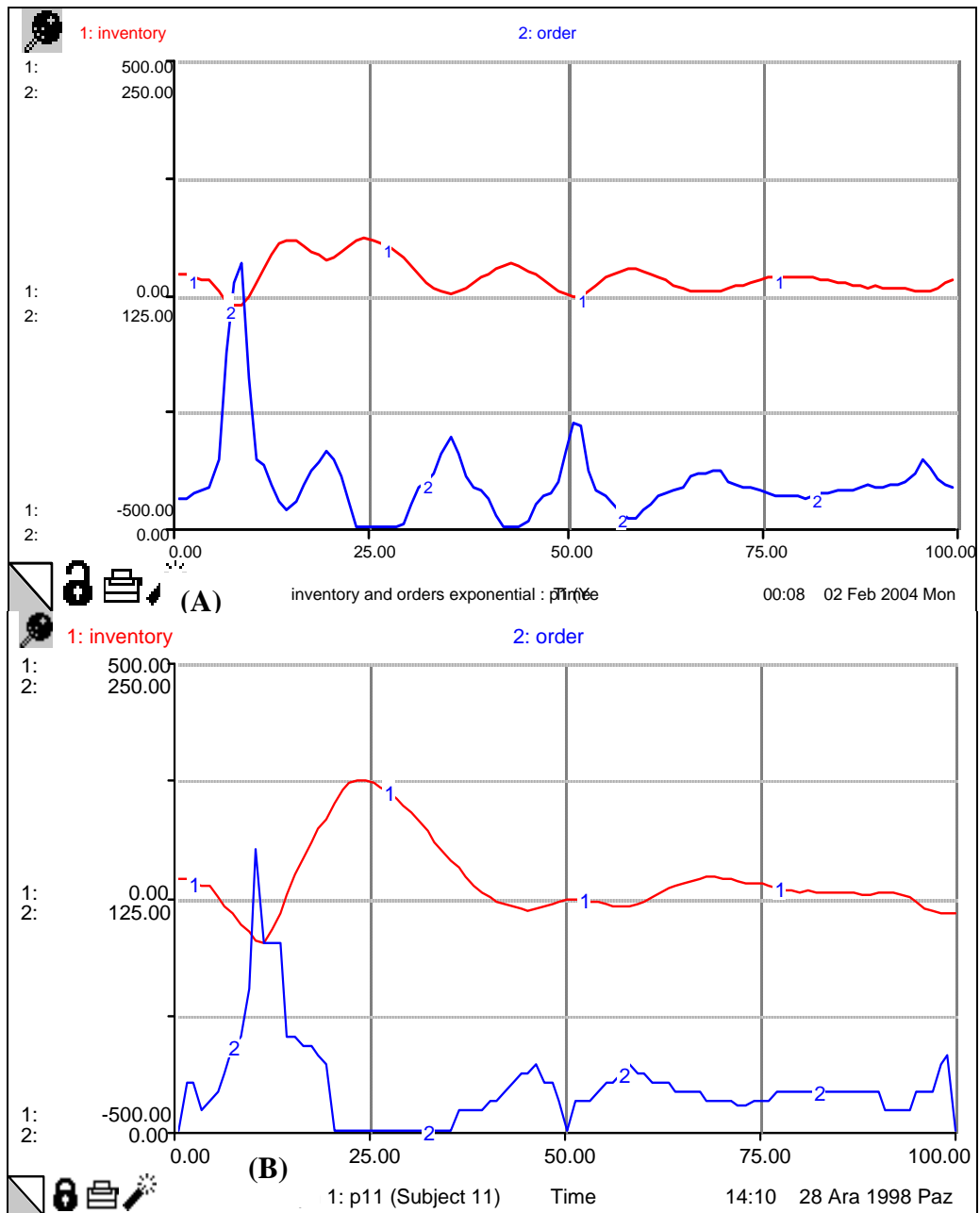
FIGURE 14. Comparison of (A) Nonlinear Expectation Rule with (B) the Performance of the Player in Game 11 (Short Game with Orders Each Period, Step Up and Down in Customer Demand, Exponential Delay).

5.   Standard Inventory Control Rules

The third type of non-linear behavior, alternating large discrete and then zero orders of players and the resulting zigzagging inventory patterns, suggests that discrete inventory control rules used in inventory management may be suitable. The following four policies are most frequently used in inventory management literature:

Order Point, Order Quantity (s, Q) Rule;
Order Point, Order Up to Level (s, S) Rule;
Review Period, Order Up to Level (R, S) Rule;
(R, s, S) Rule.

Two fundamental questions to be answered by any inventory control system are "how many" and "when" (or "how often") to order. "Order-Point" systems determine how many to order, whereas "Periodic-Review" systems determine how often to order as well (Silver and Peterson, 1985), (Tersine, 1994). When subjects can order every time unit, they are free to order at any time they desire. Therefore, order-point systems, rather than periodic review systems are more appropriate to represent the ordering behavior in these situations. In contrast, when subjects can order only, say, once every five periods, periodic-review systems with five as review period may be more appropriate as decision rules. These inventory control rules assume that time flow and changes are discrete. Therefore, these rules will be tested only with discrete delays. As will be seen, these rules are non-linear in the sense that they consist of piecewise, discontinuous functions.

5.1. Order Point-Order Quantity (s, Q) Rule:

Order Point-Order Quantity (s, Q) rule can be mathematically expressed as

$$O_t = Q, \quad \text{if } EI_t \leq s$$
$$0, \quad \text{otherwise} \tag{5.1}$$

where EI_t represents the effective inventory and s the "order point". Effective inventory and order point are calculated as follows:

$$EI_t = I_t + SL_t \tag{5.2}$$
$$s = DAVGSL_t + DMINI_t \tag{5.3}$$

I_t and SL_t represent goods in inventory and in supply line respectively. $DAVGSL_t$ refers to the desired average supply line and $DMINI_t$ to desired minimum inventory. To be consistent with the previous continuous adjustment rules, desired average supply line and desired minimum inventory can be defined as

$$DAVGSL_t = \tau * E_t \tag{5.4}$$
$$DMINI_t = E_t + SS \tag{5.5}$$

in terms of receiving delay τ, expected demand E_t and safety stocks SS. Order quantity Q and safety stock SS are fixed arbitrarily as constants in this research. Desired minimum inventory $DMINI_t$ is defined as the sum of a constant safety stock and demand expectation. With such a definition, desired minimum inventory can be adapted to variations in customer demand. The performance of this rule with deterministic demand is seen in Figure 15. Orders exhibit alternating zeros and Q's and inventory

zigzags around a constant level. Observe that the (s, Q) rule can not prevent the inventory from falling below the desired minimum inventory, even when no noise exists, primarily because the order quantity Q is constant. This particular rule is therefore not suitable for our purpose (i.e. for comparative evaluation against the continuous stock adjustment rules). (s, Q) rule will not be further evaluated; it is defined only as a preliminary for the other more realistic rules to follow.
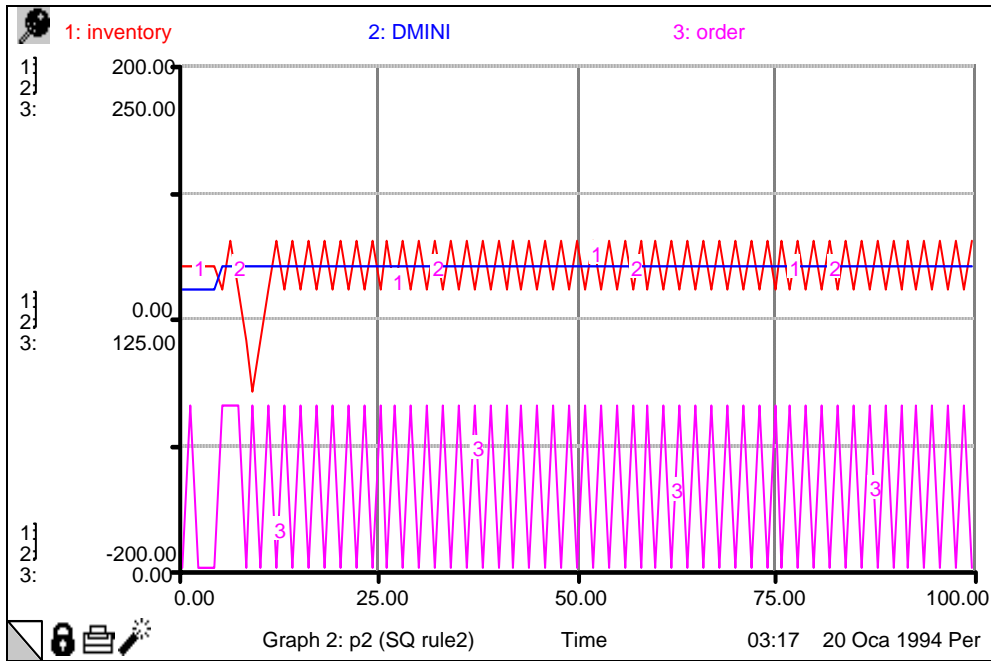

FIGURE 15. Performance of Order Point-Order Quantity Rule (Q=4*D_{t0}, DAVGSL$_t$ = 4*D_t, DMINI = 0) in Short Game with Orders Each Time Unit, Known Step Up in Customer Demand, Discrete Delay.

5.2. Order Point-Order Up to Level (s, S) Rule

Order Point-Order Up to Level (s, S) rule can be mathematically expressed as

$$O_t = S - EI_t, \quad \text{if } EI_t \leq s$$
$$0, \quad \text{otherwise} \quad (5.6)$$

where EI_t represents the effective inventory, s the order point and S the upper level of inventory. Effective inventory, the order point and the upper level S of inventory can be defined as follows:

$$EI_t = I_t + SL_t \quad (5.7)$$
$$s = DAVGSL_t + DMINI_t \quad (5.8)$$
$$S = s + Q \quad (5.9)$$

I_t and SL_t represent goods in inventory and in supply line respectively. DAVGSL$_t$ refers to the desired average supply line and DMINI$_t$ to desired minimum inventory. Desired average supply line and desired minimum inventory are defined as before:

$$DAVGSL_t = \tau * E_t \quad (5.10)$$
$$DMINI_t = E_t + SS \quad (5.11)$$

in terms of receiving delay τ, expected demand E_t and safety stocks SS. Order size Q and safety stock SS are initially set as constants. But note that the actual order quantity O_t (eq. 5.6.) is a variable in this rule. Desired minimum inventory $DMINI_t$ is defined as the sum of expectations and safety stocks. As such, desired minimum inventory $DMINI_t$ may be adapted to variations in customer demand. This rule can be shown to be unbiased and mathematically consistent in the deterministic case. (See Özevin 1999). The inventory reaches equilibrium at the desired minimum inventory level, but in the "noisy" case it may move up and down around the desired minimum, due to differences between the "expected" and "actual" customer demands (Figure 16).

      The (s, S) rule takes fully into account the supply line in the sense that the decision is based on the effective inventory EI = I + SL. Thus, in accordance with the fundamental result for the linear anchor and adjust rule, the resulting inventory dynamics is non-oscillatory. The zigzagging behavior of the inventory seen in Figure 16 is caused by the time-discrete, piecewise ordering rule yielding discrete, alternating zero and non-zero orders. (There is also some minor wavelike dynamics caused simply by the autocorrelated random demand). To explore this point further, a modified version the (s, S) rule is run by redefining EI = I + k*SL, where k is the supply line inclusion coefficient. Normally, for full inclusion of supply line k is one. To demonstrate partial inclusion of supply line, the dynamics of (s, S) model is illustrated with k=0.70 in Figure 17. Observe that the inventory now does exhibit an oscillatory pattern. So the hypothesis: 'players that generate oscillatory inventories ignore/underestimate the supply line' is compelling in general, whether the ordering heuristic is linear, non-linear or piecewise/discontinuous.
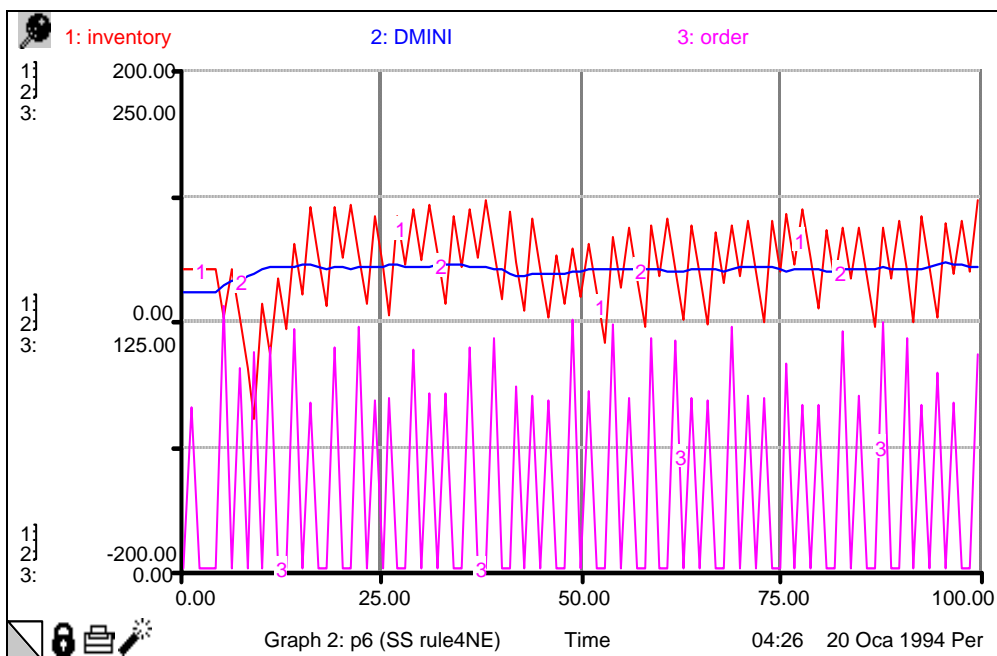

FIGURE 16. Performance of Order Point-Order Up to Level (s, S) Rule (with $DAVGSL_t = 4*E_t$, $DMINI = E_t$, $EI_t = I_t + SL_t$) in Short Game with Orders Each Time Unit, Step Up in Customer Demand, Discrete Delay.
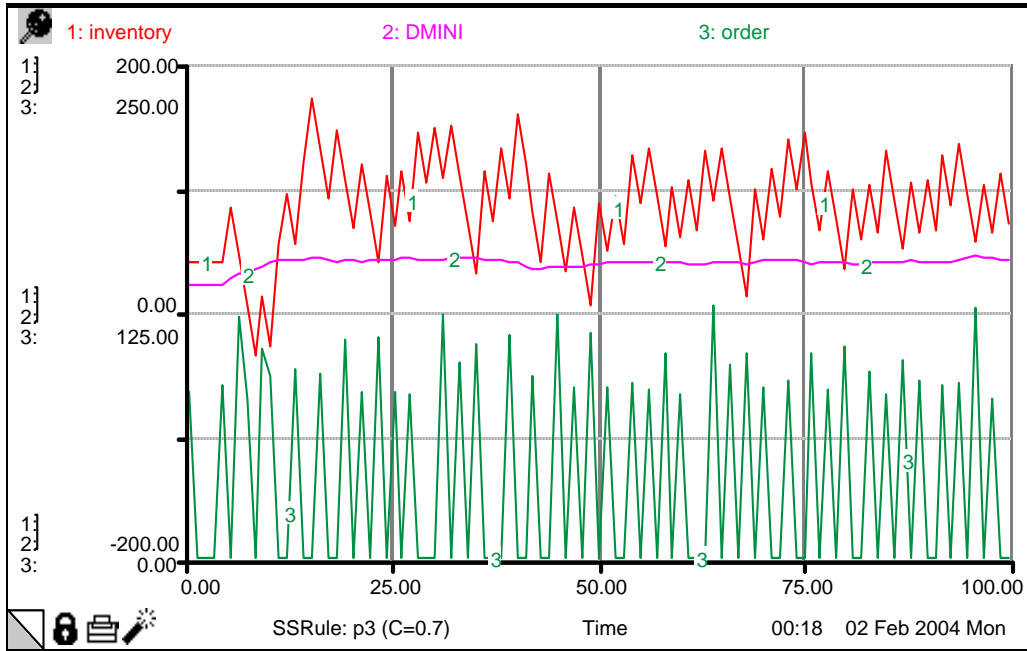
FIGURE 17. Oscillatory Performance of modified Order Point-Order Up to Level (s, S) rule (DAVGSL$_t$ = 4*E$_t$, DMINI = E$_t$ and EI$_t$ = I$_t$ +k* SL$_t$ where k=0.70) in Short Game with Orders Each Time Unit, Step Up in Customer Demand, Discrete Delay.

5.3. Review Period, Order Up to Level (R, S) Rule

Order Point-Order Up to Level rule can be mathematically expressed as:

$$O_t = S\text{-}EI_t \qquad \text{if } t = R*k$$
$$0 \qquad \text{otherwise} \qquad (5.12)$$

where EI$_t$ represents the effective inventory, t the time, S the upper level of inventory. k is an integer and R is the review period. Effective inventory, the order point and the upper level S of inventory are defined as follows

$$EI_t = I_t + SL_t \qquad (5.13)$$
$$S = DAVGSL_t + DMINI_t + R*E_t \qquad (5.14)$$

I$_t$ and SL$_t$ represent goods in inventory and in supply line respectively. Review period R is five. DAVGSL$_t$ refers to the desired average supply line and DMINI$_t$ to desired minimum inventory. Desired average supply line is defined as

$$DAVGSL_t = \tau*E_t \qquad (5.15)$$

in terms of receiving delay τ, expected demand E$_t$. Desired minimum inventory DMINI$_t$ corresponds to the safety stock. DMINI$_t$ is arbitrarily fixed as constant.

This rule can be shown to be unbiased and mathematically consistent in the deterministic case. (See Özevin 1999). The inventory reaches an equilibrium point consistent with the desired minimum inventory level, but in the "noisy" case it may fall below the desired minimum due to noise effects. (Figures 18 and 19 provide two illustrations). The basic behaviors are again alternating large discrete and then zero orders and zigzagging inventory patterns.
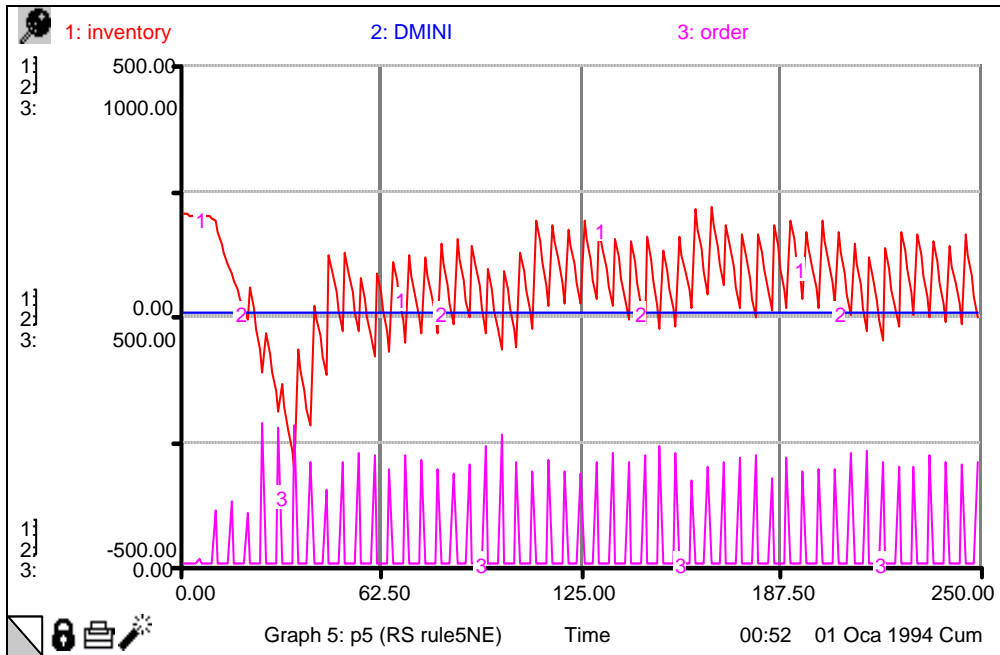
FIGURE 18. Performance of Review Period-Order Up to Level (R,S) rule (with $DAVGSL_t = 10*E_t$, DMINI = 0) in Long Game with Orders Every Five Time Units, Step Up in Customer Demand, Discrete Delay.
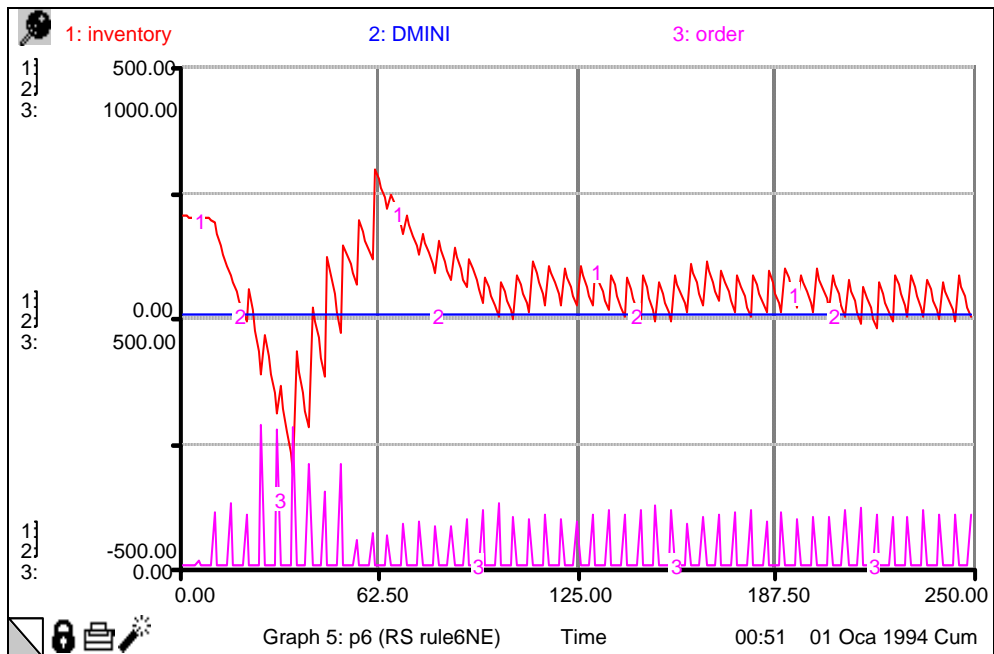

FIGURE 19. Performance of Review Period-Order Up to Level (R,S) rule (with $DAVGSL_t = 10*E_t$, DMINI = 0) in Long Game with Orders Every Five Time Units, Step Up and Down in Customer Demand, Discrete Delay.

5.4. (R, s, S) Rule:

Finally, (R, s, S) rule can be mathematically expressed as

$$O_t = S - EI_t \quad \text{if } t = k*R \text{ and } EI \le s$$
$$0 \qquad \text{otherwise} \qquad\qquad (5.16)$$

where S represents the upper level of inventory, EI_t the effective inventory, R the review period, t the time and s the order point. Review period R is five. Effective inventory, the order point, the upper level S of inventory and safety stock SS are:

$$EI_t = I_t + SL_t \qquad\qquad (5.17)$$
$$SS = R*E_t + DMINI_t \qquad\qquad (5.18)$$
$$s = DAVGSL_t + SS_t \qquad\qquad (5.19)$$
$$S = s + R*E_t \qquad\qquad (5.20)$$

I_t and SL_t represent goods in inventory and in supply line respectively. $DAVGSL_t$ refers to the desired average supply line, $DMINI_t$ to desired minimum inventory, SS to safety stock, E_t to expected demand. Desired average supply line is defined as:

$$DAVGSL_t = \tau*E_t \qquad\qquad (5.21)$$

in terms of receiving delay τ, expected demand E_t. $DMINI_t$ is determined as a constant. This rule can be shown to be unbiased and mathematically consistent in the deterministic case (See Özevin 1999). Due to the difference between the expected and actual customer demand in the noisy case, (R, s, S) rule may not result in ordering each time the effective inventory falls to or below the order point; orders are sometimes delayed until the following period. The inventory reaches an equilibrium consistent with the desired minimum inventory level, but in the "noisy" case it may occasionally fall below the desired minimum. (See Figure 20). The fundamental patterns of behavior are similar to those seen with the previous discrete inventory rules.
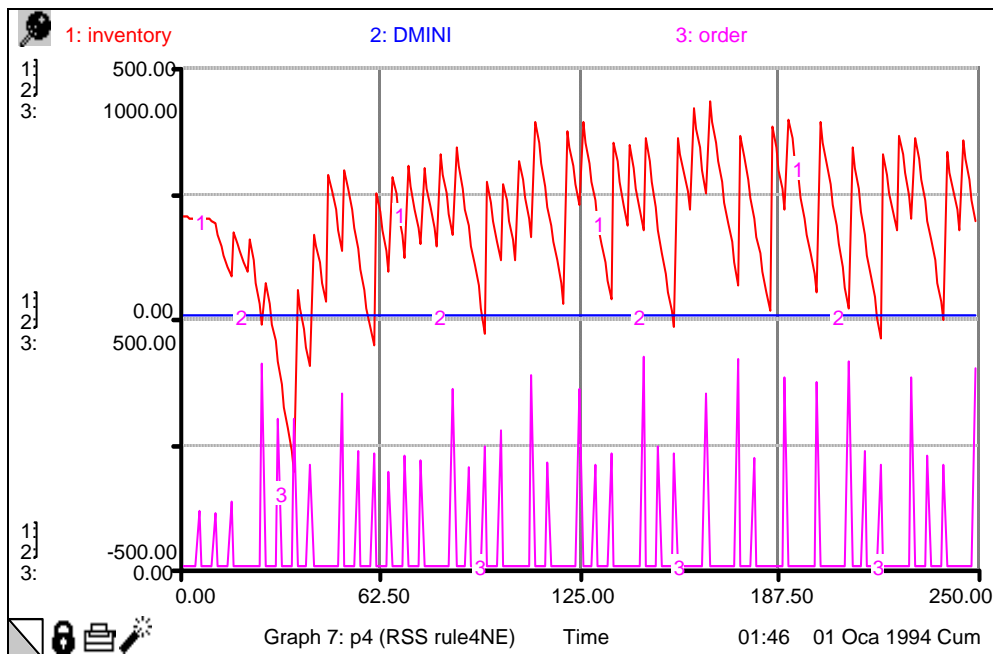

FIGURE 20. Performance of (R, s, S) Rule ($DAVGSL_t = 10*E_t$, DMINI = 0) in Long Game with Orders Once Every Five Time Units, Step Up in Customer Demand, Discrete Delay.

5.5. Comparison of the Standard Inventory Rules with Game Results

Order point-Order Quantity (s, Q) rule is not a plausible decision rule formulation when demand is not constant, as discussed above. Order Point-Order Up to Level (s, S) rule on the other hand may provide an adequate representation of subjects' performance in some cases where the continuous Anchoring and Adjustment Rule is inadequate. One such case is depicted in Figures 21 and 22. Observe that the ordering behavior of subjects is discontinuous, consisting of alternating large discrete and then zero orders, well represented by the (s, S) orders of Figure 21. The zigzagging inventory patterns of Figure 21 and 22 are also consistent. The linear or nonlinear continuous adjustment rules on the other hand can not produce such behavior patterns, especially if orders are allowed in each time step.

The ordering patterns generated by the Review Period, Order Up to Level (R, S) rule on the other hand are generally quite similar to the ones produced by the anchoring and adjustment rules when the value of R is matched with minimum ordering interval (5 days in our experiments). Therefore, (R, S) rule does not provide novel behavior patterns that anchoring and adjustment rules fail to represent.

But (R, s, S) rule can represent subjects' decisions where anchoring and adjustment rules fail, especially in cases where intervals between subjects' non-zero orders are not constant. One such comparison is depicted in Figures 23 and 24. Note that continuous anchor and adjust rules, when applied under 'orders are given every five time steps' condition, would fail to generate variable time intervals in between non-zero orders.
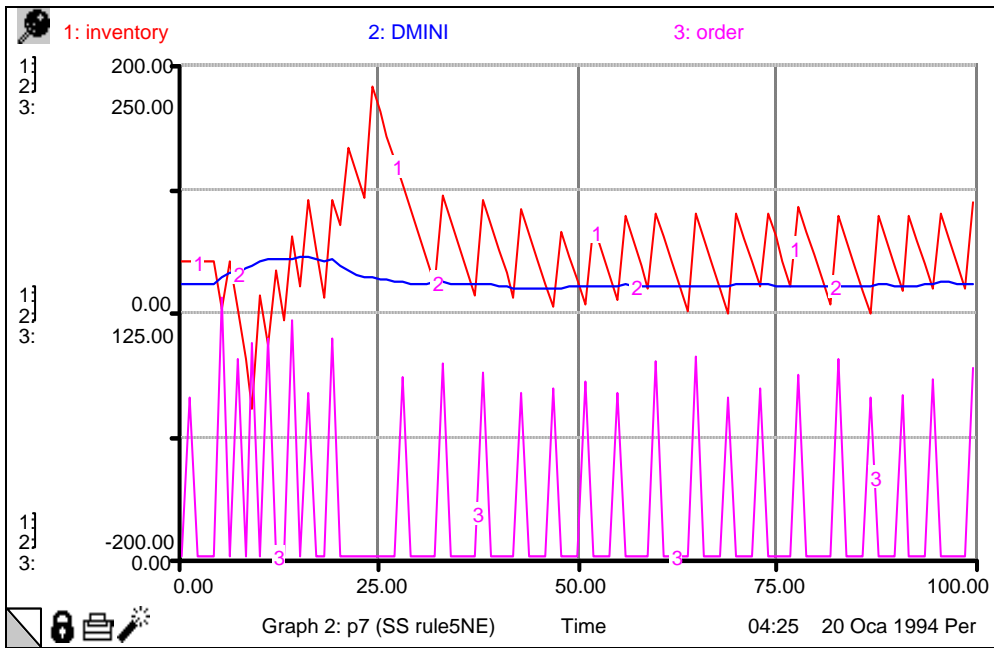
FIGURE 21. Performance of Order Point-Order Up to Level (s, S) Rule ($DAVGSL_t = 4*E_t$, $DMINI = E_t$) in Short Game with Orders Each Time Unit, Step Up and Down in Customer Demand, Discrete Delay.
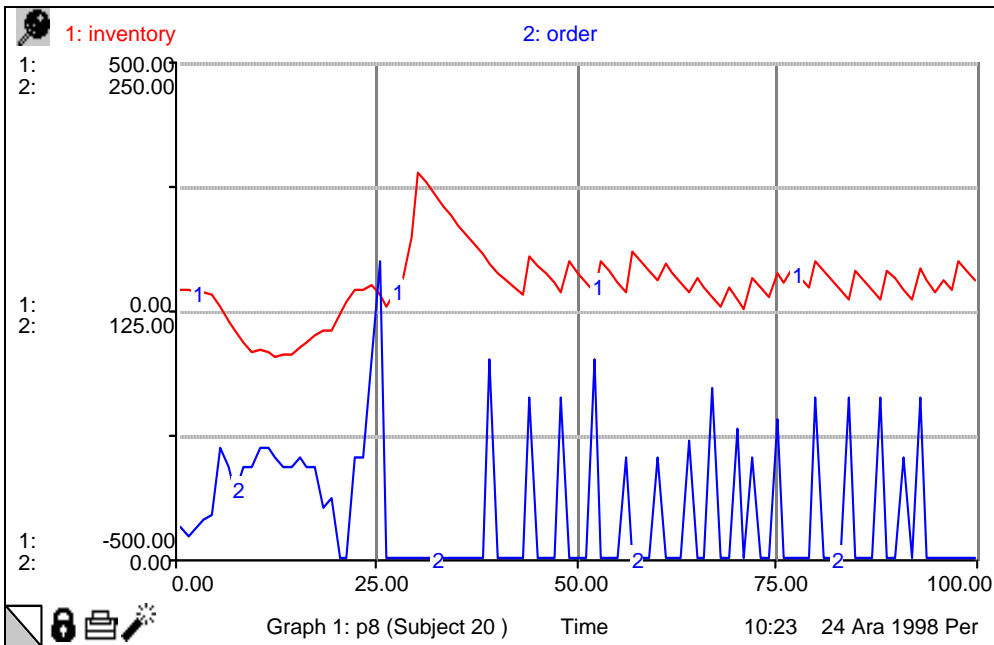

FIGURE 22. Performance of the Player in Game 20 (Short Game with Orders Each Time Unit, Step Up and Down in Customer Demand, Discrete Delay).
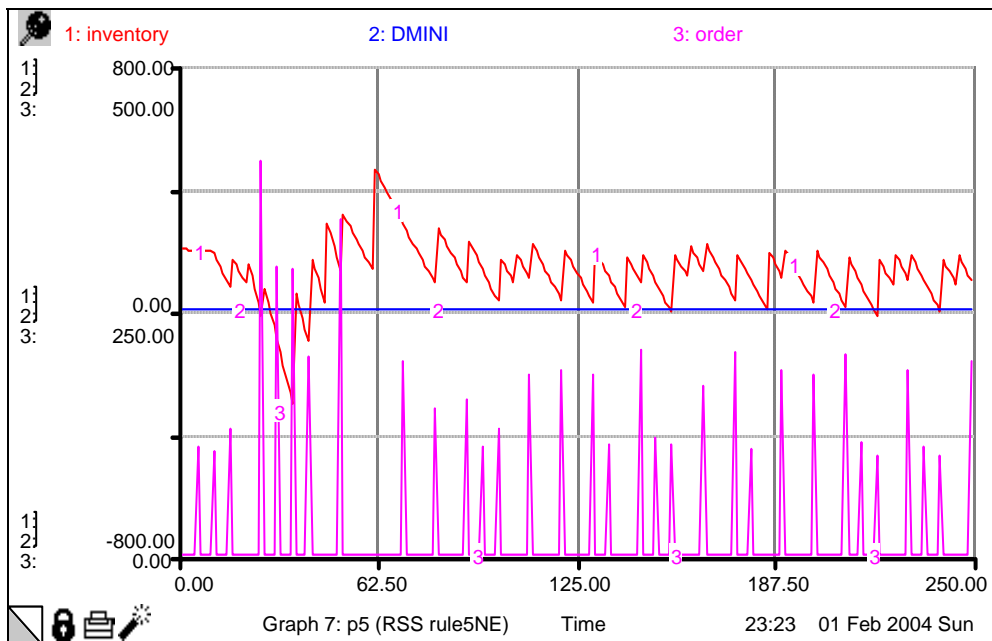
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FIGURE 23. Performance of (R, s, S) Rule ($DAVGSL_t = 10*E_t$, DMINI = 0) in Long Game with Orders Once Every Five Time Units, Step Up and Down in Customer Demand, Discrete Delay.
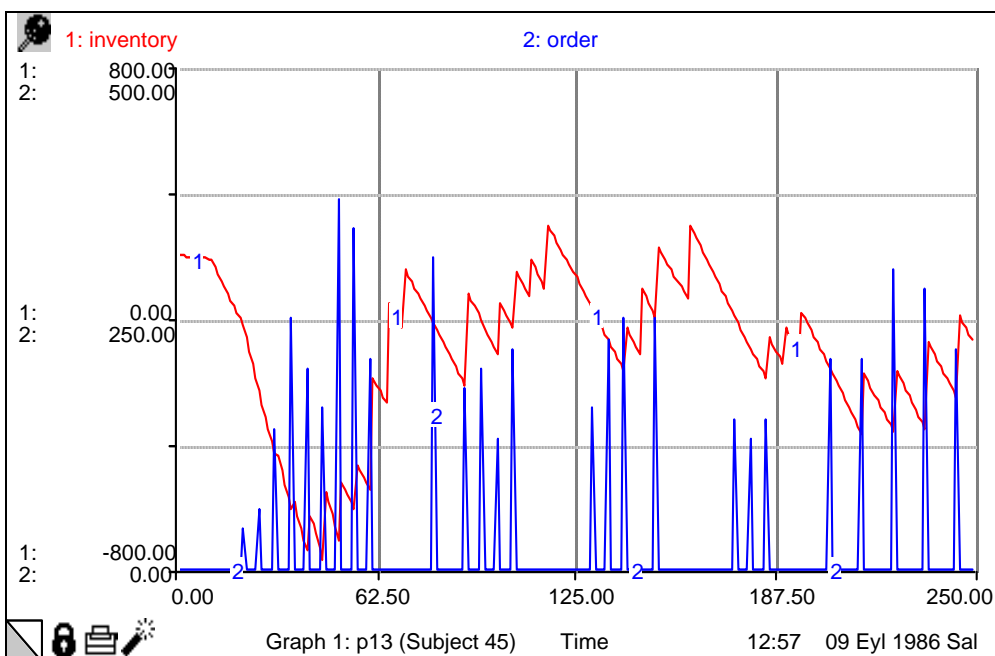

FIGURE 24. Performance of the Player in Game 45 (Long Game with Orders Once Every Five Time Units, Step Up and Down in Customer Demand, Discrete Delay).

6. Conclusion

This paper has two different research objectives: The first one is to analyze the effects of selected experimental factors on the performances of subjects (players) in a stock management simulation game. To this end, the generic stock management problem is chosen as the interactive gaming platform. Gaming experiments are designed to test the effects of three factors on decision making behavior: different patterns of customer demand, minimum possible order decision ('review') interval and finally the type of the receiving delay. ANOVA results of these 3-factor, 2-level experiments show which factors have significant effects on ten different measures of behavior (such as max-min range of orders, inventory amplitudes, periods of oscillations and backlog durations).

The second research objective is to evaluate the adequacy of standard decision rules typically used in dynamic stock management models and to seek improvement formulations. The performances of subjects are compared against some selected ordering heuristics (formulations). First, a classification of the patterns of ordering behavior of subjects reveals three basic types: i- smooth oscillatory (or non-oscillatory) damping orders, ii- alternating large and zero orders, like a high frequency discrete signal, iii- long periods of constant orders punctuated by a few sudden large ones. Comparing these behaviors with the stand-alone simulation results, we observe that the common linear "Anchoring and Adjustment Rule." can mimic well the first (i) type of behavior, but can not, due to its linear nature, replicate the next two types. Several alternative non-linear rules are formulated and tested against subjects' behaviors. Some standard discrete inventory control rules (such as (s, Q)) common in the inventory management literature are also formulated and tested. The non-linear adjustment rules are found to be more representative of subjects' decisions in cases where subjects' ordering patterns fall in class (iii) above. The standard discrete inventory rules are more representative of the subjects' ordering behavior in cases where decision patterns consist of alternating discrete large and zero orders. Another finding is the fact that the well-documented oscillatory dynamic behavior of the inventory is a quite general result, not just an artifact of the linear anchor and adjust rule. When the supply line is ignored or underestimated, large inventory oscillations result, not just with the linear anchor-and-adjust rule but also with the non-linear rules, as well as the standard inventory management rules. Furthermore, depending on parameter values, nonlinear ordering rules are more prone to instability -even if the supply line is taken into account.

More research is needed to formulate and test other non-linear formulations. There is also need to test these rules in more complex and realistic game environments (such as more stocks, delays and multi-player supply chains). Currently, we are examining the effects of information delays in the order decisions (in addition to supply line delays). Initial results indicate that the oscillations become more unstable and the basic anchor and adjust heuristic needs to be modified properly in order to reduce the instability. Finally, a long term research question is: if individuals tend to order intermittently and discrete delays further exacerbate the situation, are mostly continuous system dynamics decision structures invalid? Or is it true that even if individuals decide in discrete and intermittent fashion, the *macro* decision making behavior of the *aggregate* system (that we are typically interested in) can be/should be modeled continuously? There is a fundamental research question here that should bridge the micro behavior of actors (using perhaps agent-based modeling) to the macro behavior of the aggregate system.

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**Appendix A:** Interactive Inventory Management Game Interface
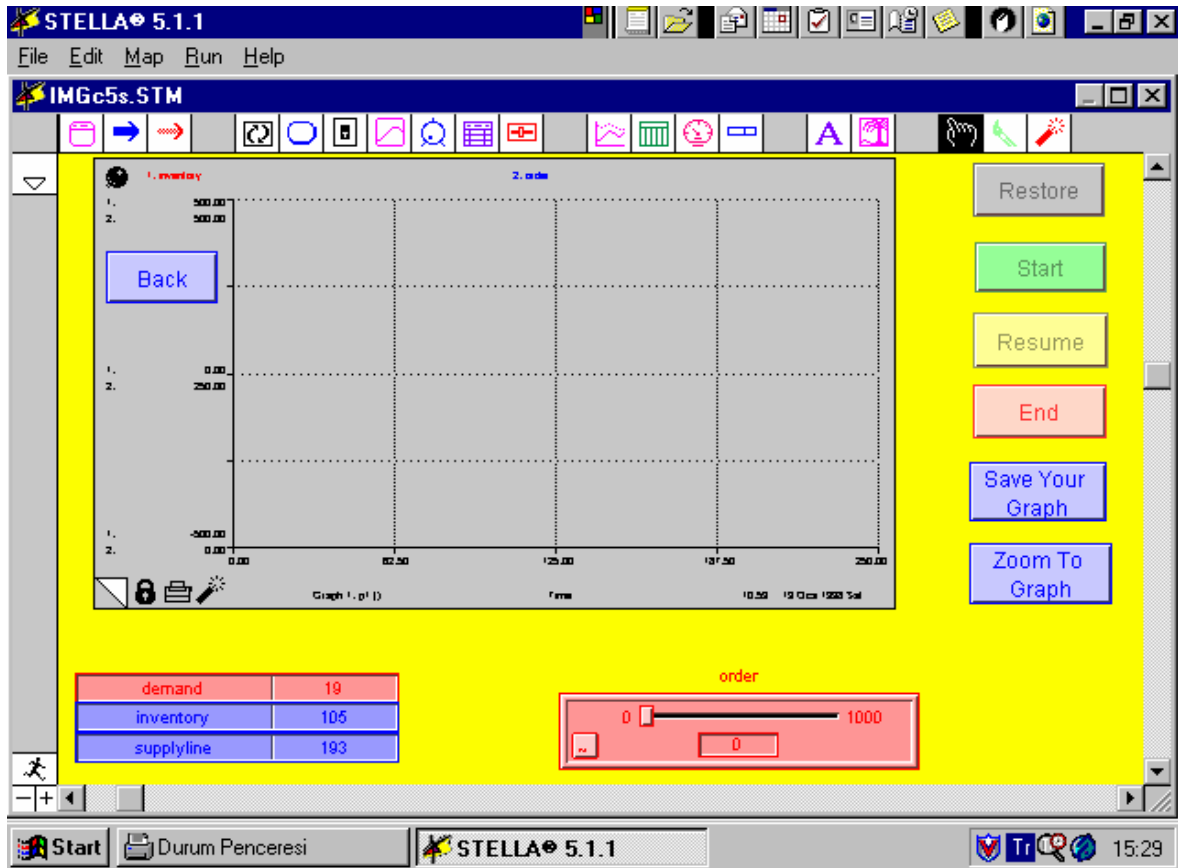
Appendix B: Game Instruction Sheet

<div style="border:1px solid black; padding:10px;">

Inventory Management Game (Short Version)

1. Objective

In this game, as an inventory manager, you will control the inventory level of a certain good such that your company must not backlog too many orders from your customers. You can achieve this by keeping a large safety stock. However, large safety stocks result in high inventory costs. Therefore, you should keep your inventory level as low as possible while trying not to backlog.
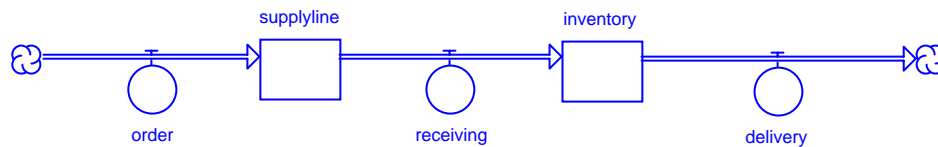
2. How the inventory is controlled?

You will control your inventory by ordering new goods. You can order new goods once every day. While ordering new goods you should consider the following three variables: *inventory, demand* and *supply line. Inventory* is the quantity of goods you have in hand. *Demand* is the quantity of goods requested by within a time unit (a day). If there are no enough goods in your inventory at any time, you will take the order as a backlog and supply the goods later. In this case, your inventory will be negative until you receive enough goods from your suppliers. *Supply line* corresponds to the goods you have ordered previously but you have not received yet. Remember that there are time delays between your placing of orders and receiving them.

3. Remarks

- There is no cost associated with ordering. Therefore you may order as frequently as every time period.
- By inspecting the graph of inventory and your orders over time, you may have some idea about the supply line delay and the future demand.
- You can enter your order either by typing it or by sliding the input device. After entering your order, press the play button.
- The game will last 100 days. At the end, please save your game under C: \STELLA5\ as yourname.stm

</div>

Appendix C: Model Equations (Illustrated with linear anchor and adjust rule)

Demand Sector
demand = step_input +noise
noise = SMTH1(whitenoise,2,0)
step_input = 20+STEP(20,4)-STEP(20,19)
whitenoise = NORMAL(0,0.15*step_input,67779)

Goal Sector
desired_inventory = inventory_coefficient*step_input
desired_onorder = step_input *receiving_delay
inventory_coefficient = 2

Order Sector
adjtime1 = 5
adjtime2 = 3
decision_rule = demand+inventory_adjustment+supplyline_adjustment
inventory_adjustment = inventory_discrepancy/adjtime2
supplyline_adjustment = supplyline_discrepancy/adjtime1

Production Line Sector
inventory(t) = inventory(t - dt) + (receiving - delivery) * dt
INIT inventory = inventory_coefficient*step_input

INFLOWS:
receiving = supplyline/receiving_delay
OUTFLOWS:
delivery = demand
supplyline(t) = supplyline(t - dt) + (order - receiving) * dt
INIT supplyline = 80

INFLOWS:
order = decision_rule
OUTFLOWS:
receiving = supplyline/receiving_delay
inventory_discrepancy = desired_inventory-inventory
receiving_delay = 4
supplyline_discrepancy = desired_onorder-supplyline