Using System Dynamics Models to Enhance the Visualization of Stochastic Price Processes

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Abstract

The market prices of many financial assets and commodities can be described by stochastic processes. For example, the famous Black-Scholes formula for valuing options on common stocks is based on the assumption that stock prices move according to a geometric Brownian motion. This paper describes some models for stochastic price processes and shows how they can be formulated using the methodology of system dynamics. System dynamics lends itself to visualization of both the structure of the models and of the resultant price dynamics. For this reason, it is suggested that students' understanding of stochastic price processes can be enhanced by using such models as teaching aids. Another advantage of system dynamics as a modeling environment is that feedback loops and time delays can be easily incorporated into the models. This should facilitate the integration of stochastic price models with supply chain models and provide richer insights into the dynamics of financial and commodity markets.

Keywords

System dynamics; Simulation; Black-Scholes; Stock prices; Commodity prices

Author Biography

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1. Introduction

"We take risks to generate profit, but we never bet the farm."

Rick Buy, Executive Vice President and Chief Risk Officer, Enron Corporation

Many companies are exposed to risks created by volatility in financial markets and commodity prices. For example, a resource company will certainly experience cash flow volatility as commodity prices fluctuate. This cash flow volatility may lead to higher taxes if the firm's tax function is convex, and earnings volatility may mask the results of earnings improvement initiatives. Furthermore, if a company is highly leveraged then an unexpected downturn in commodity prices can easily lead to financial distress and possibly bankruptcy, as was the case for Dome Petroleum in 1982 following a collapse in world energy prices.

Commodity price risk also plays an important role in the evaluation of a firm's investment opportunities. Baker et al. (1998) point out that a model of commodity prices is the "engine around which any valuation methodology for commodity production projects is built." Indeed, much of the literature on commodity price modeling seems to have been motivated by the desire to improve the quality of investment evaluation under conditions of price uncertainty.

For many companies, commodity price risk presents itself as a timing risk. A firm may purchase a shipment of nickel at today's market price, but delivery of the physical product may not take place until a month from now. Then, the nickel shipment may sit in raw materials inventory, work-in-progress, and finished goods inventory for another two months before the finished product is sold for cash. For some companies, this "cash flow cycle" time can be as long as 90-120 days. Volatility in the price of nickel during this time period may constitute a significant timing risk for the company. Not surprisingly, mitigation of this timing risk is a major reason why firms engage in price risk hedging.

Firms can manage commodity price risk by hedging using options or futures, by diversifying into downstream businesses or other businesses that create a "natural hedge," or by adopting a conservative capital structure and allowing the risk to pass through to shareholders. Bodnar et al. (1998) reported that among primary product firms, commodity price risk is the most commonly managed risk with 79% of the firms in the survey indicating derivative usage.

Commodity price risk is complicated by the fact that it operates in conjunction with demand risk. Demand risk is the possibility that demand for the firm's product will be much lower than expected, and so sales will fall short of projections. Consider the market for a petrochemical product in which a few firms dominate the market. We would expect each firm to adopt a strategy to maximize its profit and, since no single firm can control the market price, each firm's strategy translates into maximization of sales volume and minimization of production costs. In a situation of falling demand for the market as a whole, each producer tries to maintain sales volume and may even try to increase sales to compensate for loss of revenue as price falls. Anticipating this, consumers of the commodity respond to producers' requests for more volume by asking for a lower price. Since many consumers purchase from more than one producer, the producer that concedes to a lower price will gain volume at the expense of the other producer. A similar scenario plays out at each and every consumer of the commodity, with each supplier winning volume at some customers and losing volume at others. But, since aggregate demand is falling, the net effect is one of falling volumes and falling prices. When the commodity cycle turns, this mechanism may operate in reverse with rising prices and rising volumes. While an economic relationship between supply and demand may well operate on average over time, a stable equilibrium will rarely occur because of price volatility.

Although it is easy enough to measure price volatility, this volatility by itself tells us nothing about the risk to the firm. To answer the question of how to measure risk to the firm, one needs a model that includes both price risk and demand risk and translates this risk into a measurable impact on the firm. Elliott et al. (2002) have described a generalized form of risk measure, which may provide some guidance:

$$\rho_{\phi,D_0}\left(R_a^{t,t_2} \mid F_t\right) = \sup\left\{\phi^{-1}\left(E_P\left[\phi\left(R_a^{t,t_2}\right) \mid F_t\right]\right) \mid p \in D_0\right\} \qquad \ldots 1\text{-}1$$

Where the terms are defined as follows:

R_a^{t,t_2}	The exposure to uncertainty associated with the risky activity *a* over the time horizon from *t* to t_2
F_t	The information available about the risky activity up to time *t*
$\rho_{\phi,D_0}(.)$	The measure of risk
$\phi(.)$	A function of risk exposure that describes the consumer of risk's attitude towards the risk, or value placed upon the risk. It may be a utility function or a simple identity function
$E_P[.]$	The expected value operator, which is a function of the probability *P*
p	The probability of a possible outcome *w*
D_0	The set of "generalized scenarios" or "probability measures"

For example, if a firm is involved in production and sale of commodity products, then the uncertainties that the firm is exposed to are the price of commodity inputs, the price of commodity outputs and the quantity of production sold. In its simplest formulation for a one-product/one feedstock firm, one would expect that the risk exposure of the firm towards commodity price risk would also depend on the size of the firm's fixed cost obligations (e.g. labour, lease costs, interest costs etc.) relative to its revenues. These uncertainties combine to create uncertainty in net cash flow, and so one may write:

$$R_a^{t,t_2} = \int_t^{t_2} \left[(P_t^O - \eta_t P_t^I)Q_t - B_t\right] dt \qquad \ldots 1\text{-}2$$

Where R_a^{t,t_2} measures the uncertainty in net cash flow for the time period t to t_2, and the other terms are defined as follows:

P_t^I , P_t^O	Commodity raw material (input) price and commodity product (output) price respectively
Q_t	Commodity product sales quantity
η_t	Quantity of inputs needed to make one unit of output
B_t	Fixed cost cash outflow at time t, which may itself be uncertain

The information filter F_t in equation 1-1 would represent price history, historical demand patterns, and other market information available to the firm.

For commodities, D_0 in equation 1-1 is a rich set of possible scenarios, which may include war in the Middle East, regulatory action in response to environmental concerns, acts of terrorism, or any number of economic "boom and bust" scenarios.

To translate the risk exposure into its impact on the firm, we are interested in the low probability "negative tail" events defined by the greatest lower bound of R_a^{t,t_2} at a user-defined small probability level α. Thus, we could define the risk function as:

$$\rho_{\phi,D_0}\left(R_a^{t,t_2}\right)=\inf\left\{x \mid p\left((w \mid R_a^{t,t_2}(w)\leq x)\mid F_t\right)>\alpha\right\} \qquad \ldots 1\text{-}3$$

This is similar in form to Elliott et al. equation 1-1 where, in this case, $\rho_{\phi,D_0}\left(R_a^{t,t_2}\right)$ measures the cash flow at risk resulting from exposure R_a^{t,t_2} at small probability level α. Note that Elliott *et al* show how equation 1-3 could be transformed into the form of equation 1-1 by the appropriate specification of the utility function $\phi(.)$.

Because the risk measurement defined by equations 1-2 and 1-3 involves a complex function of stochastic variables, and a typical firm may have more than one product and one feedstock, it is likely that no analytical solution will be possible. The most practical method of solution will most likely be through numerical methods or simulation. The method of system dynamics is a promising way to approach this problem. System dynamics is often used as a tool to model changes in demand across supply chains. By incorporating models of price processes into such supply chain models, it may be possible to construct a system dynamics model that adequately represents the complex interactions of price and quantity described above. The purpose of this paper is to present some of these stochastic price processes and show how they can be described by system dynamics models. However, because of the importance of the Black-Scholes option pricing formula to finance, and its usefulness as a tool for validating the system dynamics approach, a major portion of the paper will be devoted to modeling the Black-Scholes dynamics. The remainder of the paper will give an example of a system dynamics model of more complex stochastic differential equations used to model commodity prices.

2. An Introduction to Stochastic Price Processes

2.1 Derivation from First Principles

Taylor and Karlin (1998) formally define a stochastic process as a family of random variables X_t where t is a parameter running over a suitable index set T. In this paper we are only concerned with stochastic processes in which t represents discrete units of time. Stochastic processes are characterized by their "state space" (the range of possible values for the random variables X_t), by the index set T, and by the relationships between the random variables X_t.

While Taylor and Karlin's text is mainly aimed at stochastic modeling in the physical sciences, there are many literature sources that apply stochastic modeling in finance. The most famous of these applications is that of Black and Scholes (1973) who employ stochastic calculus to derive their famous equation for option pricing. Textbook sources for this derivation include Dixit and Pindyk (1994), Trigeorgis (1996), and Mikosch (1998). Unfortunately, few sources give a simple, clear and succinct explanation of the fundamental stochastic processes. In this section, I will endeavour to provide an intuitive explanation and derivation of the basic stochastic process that is relevant to price risk management.

Consider a stochastic process for a price P, which has a random fractional price change having a variance of σ^2 per unit time. The diagram in Figure 1 illustrates this process over a small time period Δt:

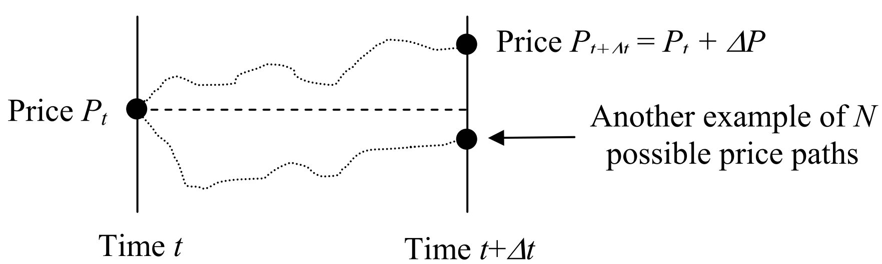


Figure 1: Motion of Price over a Small Time Period

The diagram shows just two of an infinite number of possible price paths over the time interval Δt. Let's say that there are N price paths, and therefore N different possible values of ΔP (ΔP_1, ΔP_2, …ΔP_n). The mean price and variance of the price process over this time interval are:

$$\text{Mean price } \overline{P} = \lim_{\Delta P \to 0} \left(P_t + \frac{\sum_1^N \Delta P_n}{N} \right) = P_t$$

$$\text{Variance } = \frac{\sum_{1}^{N}\left[\overline{P}-(P_t+\Delta P_n)\right]^2}{N} = \Delta P^2$$

And variance of the price change = $\dfrac{\Delta P^2}{P^2}$

From the definition of σ^2, the variance of the price change over the time interval Δt is $\sigma^2 \Delta t$ and therefore

$$\sigma^2 \Delta t = \frac{\Delta P^2}{P^2}$$

Or $$\sigma = \frac{\Delta P}{P\sqrt{\Delta t}}$$

Letting $z = \dfrac{P-\overline{P}}{P\sigma}$ be the standard normal variate, with mean of 0 and variance of 1, and remembering that volatility σ is expressed in terms of a fraction of the price, then

$$z_{t+\Delta t} - z_t = \Delta z = \frac{P_{t+\Delta t}-\overline{P}}{P_{t+\Delta t}\sigma} - \frac{P_t-\overline{P}}{P_t\sigma} = \frac{\Delta P}{P\sigma} \qquad \text{as } \Delta t \to 0$$

Therefore $$\Delta z = \frac{\Delta P}{P\sigma} = \sqrt{\Delta t} \qquad \text{…2-1}$$

From the definition of the process:

$$P_{t+\Delta t} - P_t = \Delta P = \sigma P_t \Delta z \qquad \text{…2-2}$$

And as $\Delta t \to 0$, equation 2-2 becomes:

$$\frac{dP}{P_t} = \sigma dz \qquad \text{…2-3}$$

And equation 2-1 becomes:

$$dz^2 = dt \qquad \text{…2-4}$$

Equation 2-3 describes a geometric "standard Wiener process" or geometric "standard Brownian motion" for the price variable P having price volatility σ. The process is "geometric" because we have defined the random change as being a fraction of the current state of the variable P. An interesting property of simple Brownian motion, as described by equation 2-4, is that variance of the process increases linearly with time.

If we now expand our definition of the process to include an expected fractional growth rate of α of P per unit time, the expected growth in P over time interval Δt is $\alpha P \Delta t$ and equation 2-2 now becomes:

$$P_{t+\Delta t} - P_t = \alpha P_t \Delta t + \Delta P = \alpha P_t \Delta t + \sigma P_t \Delta z$$

And as $\Delta t \to 0$

$$\frac{dP}{P_t} = \alpha dt + \sigma dz \qquad \ldots 2\text{-}5$$

To obtain an analytical solution to equation 2-5, it is possible to transform equation 2-4 by substituting $X = \ln P$ and then use Ito's Lemma of stochastic calculus (see Mikosch (1998) p. 118) to give the result:

$$dX = \left(\alpha - \frac{\sigma^2}{2}\right) dt + \sigma dz \qquad \ldots 2\text{-}5$$

Equation 2-5 can now be solved using the methods of normal calculus to give the result:

$$X_t - X_0 = \ln\left(\frac{P_t}{P_0}\right) = \left(\alpha - \frac{\sigma^2}{2}\right) t + \sigma B_t \qquad \ldots 2\text{-}6$$

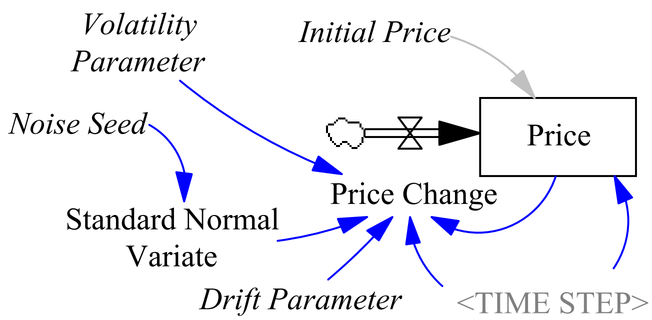
Where $B_t = \int_0^t dz$ has a normal $N(0, \sqrt{t}\,)$ distribution, and is called standard Brownian motion.

Equation 2-5 is called a "stochastic differential equation." It describes a geometric Brownian motion process with drift parameter α and volatility parameter σ. This process is commonly used to describe the motion of stock prices. For example, if we examine the long-run behavior of a stock with a history of earnings growth, then α will characterize the long-run growth in the stock price P and the volatility parameter σ will characterize the variation about this long-run trend line.

It is not always easy to solve stochastic differential equations in order to produce an analytical solution such as equation 2-6. As I shall show in the next section, an advantage of system dynamics modeling is that it allows the differential form of the price process to be modeled, from which good solutions can be found by numerical methods.

2.2 A System Dynamics Model for Geometric Brownian Motion

Geometric Brownian motion with drift, described by equation 2-4 can be represented by the system dynamics model shown in Figure 2 (Note: the sketch and equations were generated using Vensim software). The model parameters have been arbitrarily defined by a drift parameter of zero, a volatility parameter of 0.3 and an initial price of $25. The noise seed variable is required for the purpose of sensitivity simulation. To provide an accurate simulation, the TIMESTEP variable should be set at a small fraction of the model time scale. Note that the system dynamics model is set up so that Price is calculated by integrating the value of Price Change / TIMESTEP. This has the effect of summing the price change increments.



Initial Price = 25
Volatility Parameter = 0.3
Drift Parameter = 0
Noise Seed = 1
Standard Normal Variate = RANDOM NORMAL(-10, 10, 0, 1, Noise Seed)
Price Change = Drift Parameter * Price * TIME STEP + Volatility Parameter * Standard Normal Variate * Price * SQRT(TIME STEP)
Price = INTEG (Price Change / TIME STEP, Initial Price)

Figure 2: System Dynamics Model for Geometric Brownian Motion

Figure 3 shows the output from a typical single one-year run of this model with TIMESTEP = 0.00274 year (1 day).

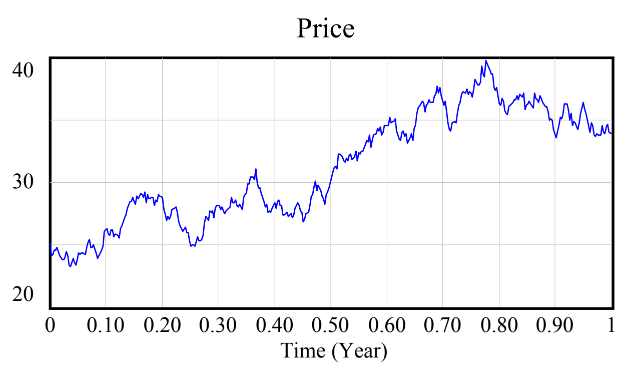


Figure 3: Typical Price Path for Geometric Brownian Motion

Since the price process is stochastic, the price path shown in Figure 3 is just one of many possible price paths. This can be illustrated by means of sensitivity analysis, in which the random noise seed is changed with each run. A sensitivity analysis for 500 runs of the model is shown in Figure 4.

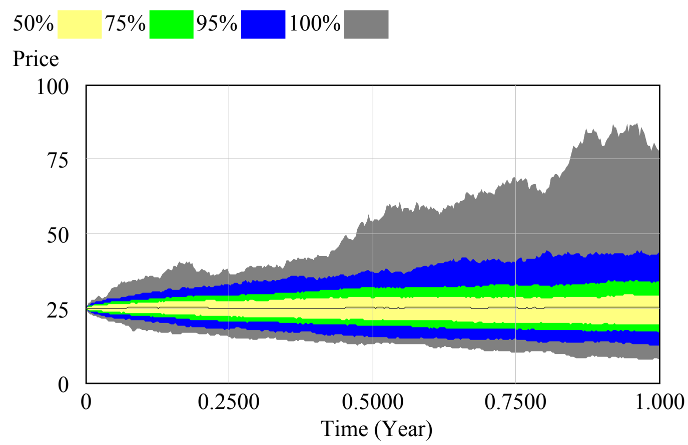


Figure 4: Sensitivity Analysis for Geometric Brownian Motion

Figure 4 illustrates some important features of geometric Brownian motion:

- If the drift parameter is zero, the mean stays constant. In the case of this example the mean remains constant at $25. This feature can be observed in many commodities. For example, the inflation-adjusted price of crude oil has remained constant at about US$21/bbl for over a 50 year period.
- The price cannot drop below zero. Again, this property is true of most financial assets and commodities.
- The upper price bound is not constrained. In fact it can be shown that the variance of the logarithm of the price increases linearly with time. This latter property can be visualized more clearly in the 100-year sensitivity analysis for the natural logarithm of Price shown in Figure 5.

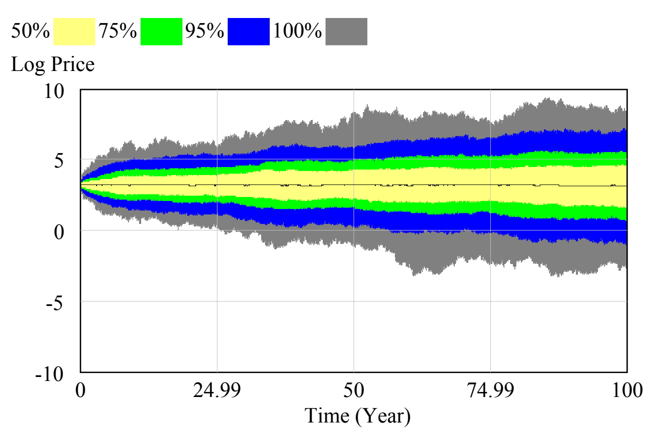


Figure 5: Sensitivity Analysis for Log Price over 100 Years

3. A Model of the Black-Scholes Dynamics

3.1 Model Description

The previous section illustrated the behavior of the geometric Brownian motion model. This model has been found to be a good model for stock prices, for which the drift parameter characterizes the growth or decay of the stock price. Black and Scholes (1973) used this model as the basis for their derivation of their famous equation for valuing call options on common stocks. The Black-Scholes equation for a call option on a common stock which pays no dividend may be written as follows:

$$C = SN(d_1) - Xe^{-rT} N(d_2)$$

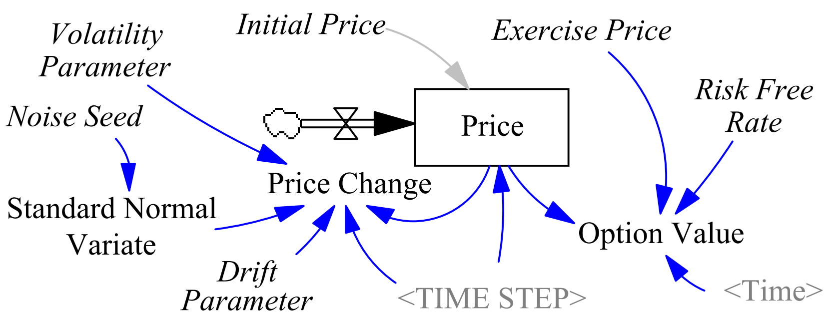
Where $d_1 = \dfrac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

And the notation is defined as follows:

C	Price of the call option
S	Price of the underlying stock
X	Exercise price of the call option
r	The "risk-free" rate of interest (usually interpreted to be US Treasury securities having a similar duration to that of the call option)
T	Time until the call option expires
$N(x)$	Standard normal distribution function
σ	Volatility parameter for the underlying stock

The reader is referred to the Finance textbooks referenced earlier for an analytical derivation of the Black-Scholes equation. Suffice it to say that the derivation of the Black-Scholes equation is quite complicated and would be covered only in Master's and Doctoral-level courses in continuous time finance. However, the intuition behind the Black-Scholes equation can easily be explained to lower-level finance students by means of a system dynamics model.

It is relatively straight-forward to extend the system dynamics model shown previously in Figure 2 in order to represent the dynamics underlying the Black-Scholes equation for option valuation. This extended model is shown in Figure 6. The option is valued by discounting the positive part of (Price – Exercise Price) back to time zero at the risk-free rate. As before, one run of the system dynamics model yields just one possible outcome or price path for the underlying stock. Therefore the model must be run many times, say 1000 or more, using the "sensitivity analysis" feature of the system dynamics software. The Black-Scholes value of a call option is then the average value of the terminal option value from 1000 simulation runs. Arnold and Henry (2003) use a similar approach to option price simulation in an Excel spreadsheet environment.



To create the Black-Scholes model, we add the following two parameters and one variable to the geometric Brownian motion model:

Exercise Price = 25
Risk Free Rate = 0.05
Option Value = max(0, (Price - Exercise Price) * exp(- Risk Free Rate * Time))

Figure 6: System Dynamics Model for the Black-Scholes Dynamics

3.2 Model Validation

The validity of the system dynamics model shown in Figure 6 can be tested by comparing the simulation results from the model with the analytical results from the Black-Scholes equation. Suppose we have the following data for a call option on a stock:

Current stock price	S = \$25
Exercise price of option	X = \$25
Volatility parameter	$\sigma = 0.3$
Drift rate	$\alpha = 0.05$
Risk free rate	$r = 0.05$
Option duration	$T = 1$ year

In other words, the stock has a volatility of 30%, is expected to grow at the same 5% per annum rate as risk-free securities, and is currently priced at the exercise value of the one-year call option. What is the value of the call option?

From the Black-Scholes equation, we calculate the value to be \$3.56. To estimate the call option value by simulation, we run the system dynamics model many times and average the result. The results for a 1 day (0.00274 year) time-step are shown in Table 1. The first row in Table 1 states the analytical result from the Black-Scholes equation. Note that the stock price at the end of the simulation is expected to be $Se^{rT} = 25e^{0.05} =$ \$26.28. The results in Table 1 raise an interesting question: Why do the results not follow a general trend towards higher precision with a higher number of simulation runs?

The explanation for the randomness in the results is likely to be related to either non-randomness in Vensim's random number generator or computational rounding errors resulting from the choice of TIMESTEP. Each sensitivity simulation used to generate the results in Table 1 was carried out using the following parameter:

Noise Seed = RANDOM_UNIFORM (0,10E6)

Conducting the same sensitivity analysis using 2000 simulations and a (-10E6, 10E6) range for the random noise seed gives a result very close to the Black-Scholes analytical result (Average price = 26.19, Average option value = 3.57).

Case: N = Number of simulation runs	Average stock price at end of simulation	Average option value at end of simulation
Analytical result	**26.28**	**3.56**
N = 100	25.80	3.01
N = 500	26.48	3.72
N = 1000	26.19	3.57
N = 2000	26.05	3.36
N = 5000	26.19	3.54

Table 1: Black-Scholes Simulation Results Showing Effect of Number of Runs

The results in Table 2 show the effect of using different values of TIMESTEP. The model is the same as before, with N = 1000 and Noise Seed = RANDOM_UNIFORM (0,10E6). The software maker recommends that TIMESTEP be a power of 0.5, with the smallest default value being 0.5 to the power 7, which equals 0.0078125. It can be seen from Table 2 that there is still randomness in the results, but the choice of a 1-day time-step appears to be reasonable for a 1-year simulation.

Case: TIMESTEP = Length of time step for Euler Integration	Average stock price at end of simulation	Average option value at end of simulation
Analytical result	**26.28**	**3.56**
TIMESTEP = 0.000114 (1 hour)	25.47	2.19
TIMESTEP = 0.00274 (1 day)	26.19	3.57
TIMESTEP = 0.005	26.13	3.42
TIMESTEP = 0.0078125 (default)	26.22	3.49
TIMESTEP = 0.01	26.24	3.49
TIMESTEP = 0.0192 (1 week)	26.37	3.63

Table 2: Black-Scholes Simulation Results Showing Effect of TIMESTEP

From the experiments reported above, and from other experiments involving a range of simulation parameters, I conclude that the system dynamics model of the Black-Scholes dynamics is valid, but the precision of the calculation often falls short of the analytical result. However, since the value of a system dynamics model is to facilitate visualization of the stochastic price process, this slight loss of accuracy may not be a problem.

3.3 Applying the Simulation Model in the "Risk-Neutral World"

So far so good, but what if the growth rate in the stock price is expected to be higher than the risk free rate? For example, suppose the growth rate of the stock in the previous example could be characterized by a drift parameter of 0.2 instead of 0.05. If the growth rate in the stock price is 20% instead of 5%, should the value of the call option not be higher?

Interestingly, the drift parameter α that characterizes the stock's growth rate does not appear in the analytical version of the Black-Scholes equation. Why not?

The answer lies in one of the assumptions underlying the derivation of the Black-Scholes equation, namely that of the "risk-neutral world." In simple terms, the assumption behind risk neutrality is that an investor can only earn a return greater than the risk-free rate by assuming more risk. Conversely, if risky investments are hedged to eliminate risk, then the hedged investment would earn no more than the risk-free rate. Therefore, to obtain the Black-Scholes equation result with the system dynamics simulation model, the drift parameter must be set to be equal to the risk-free rate.

Another way to explain the logic behind the assumption of a "risk-neutral world" is through the axiom of "no arbitrage." This axiom implies that risk-free profits cannot be made from a trading strategy. This is illustrated through the following example.

At time zero, borrow $1 at the risk free rate *r*, and buy 1/ S_0 shares of stock at price S_0. At time *t*, repay e^{rt} to the bank and sell 1/ S_0 shares of stock at price S_t. The profit on this transaction is (S_t/ S_0) - e^{rt}, which must average out to be zero in all states of the world under the "no arbitrage" axiom. Therefore, averaged across all states of the world, we have a mean stock price $\overline{S}_t = S_0 e^{rt}$.

Table 3 illustrates this point for the simple example we have been discussing so far. In fact, for all values of (α, *r*), the Black-Scholes system dynamics simulation model only gives the same result as the Black-Scholes equation when $\alpha = r$. The simulation results in Table 3 were based on 1000 runs of the Figure 6 simulation model with TIMESTEP = 0.00274 year and Noise Seed = RANDOM_UNIFORM (0,10E6).

Case: α = drift parameter *r* = risk-free rate	**Average stock price at end of simulation**	**Average option value at end of simulation**
Analytical result (*r* = 0.2)	**30.54**	**5.55**
Analytical result (*r* = 0.05)	**26.28**	**3.56**
$\alpha = 0.05$ $r = 0.05$	26.19	3.57
$\alpha = 0.2$ $r = 0.2$	30.42	5.55
$\alpha = 0.2$ $r = 0.05$	30.42	6.44

Table 3: Black-Scholes Assumes a Risk-Free World in Which $\alpha = r$

In Table 3 we can see that for $\alpha = 0.2$ in all states of the world $\overline{S}_t = 30.42$, which is close to the expected value of $S_0e^{\alpha t}$, where $S_0 = 25$, $\alpha = 0.2$ and $t = 1$ year. Could we interpret the scenario in which $\alpha = 0.2$ as being one in which the company has a strategic competitive advantage or a monopolistic market position, for example, a Microsoft? The answer is no. Finance theory argues that past price trends are not predictive of future trends and so there is no "free lunch." Expected future growth in an individual stock cannot be predicted with any degree of certainty. If it could, one would simply borrow at the risk-free rate and make a certain profit by investing in the stock. While it is possible to estimate the drift parameter from historical data, the result obtained represents just one outcome of many possible price paths. Thus, to reinforce the point made earlier, the drift rate in the Black-Scholes simulation model should always be set at the risk-free rate.

4. Multi-Factor Mean Reversion Models for Commodity Prices

Although geometric Brownian motion with drift has been shown to be a good model for stock prices, it is less suitable for commodity prices, which have been shown to exhibit a behavior known as "mean reversion" (see Baker et al. (1998) and Pindyck (1999)).

The intuitive explanation for mean reversion in commodity prices is that unlike stock prices, which may exhibit a long term growth trend driven by earnings growth, commodity prices will not exhibit long term growth unless underlying production costs drive price upwards and demand is relatively inelastic to price. However, in the absence of sustained cost-push pressure, we expect that high commodity prices would attract producers into the market until supply exceeds demand, causing prices to fall. If prices fall to levels that are "too low," then high cost producers will cut back production or shut in capacity until the reduction in supply causes upward pressure on prices.

Mean reversion models for commodity prices employing more than one stochastic variable (multi-factor models) have been considered by Gibson and Schwartz (1990), Schwartz (1997), and Schwartz and Smith (2000).. System dynamics can be used to help visualize the performance of these more complicated stochastic price processes. By way of example, I will describe a system dynamics model for the two-factor model proposed by Schwartz and Smith (2000).

Schwartz and Smith (2000) assume that the spot price of the commodity S_t is made up of two stochastic factors, one being the equilibrium price level ξ_t and the other being the short-term deviation in price χ_t:

$$\ln S_t = \chi_t + \xi_t \qquad \text{…4-1}$$

The short-term deviations are assumed to revert towards zero:

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \qquad \text{...4-2}$$

The equilibrium level is assumed to follow a Brownian motion process:

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \qquad \text{...4-3}$$

And the correlation between the Brownian motions of the two stochastic parameters is given by

$$dz_\chi dz_\xi = \rho_{\chi\xi} dt \qquad \text{...4-4}$$

To provide the reader with visualizations of this two-factor stochastic process, Figure 7 shows the probability distribution of the short-term deviation, the equilibrium price, and the spot price as described by the Schwartz-Smith model in equations 4-1 to 4-4.

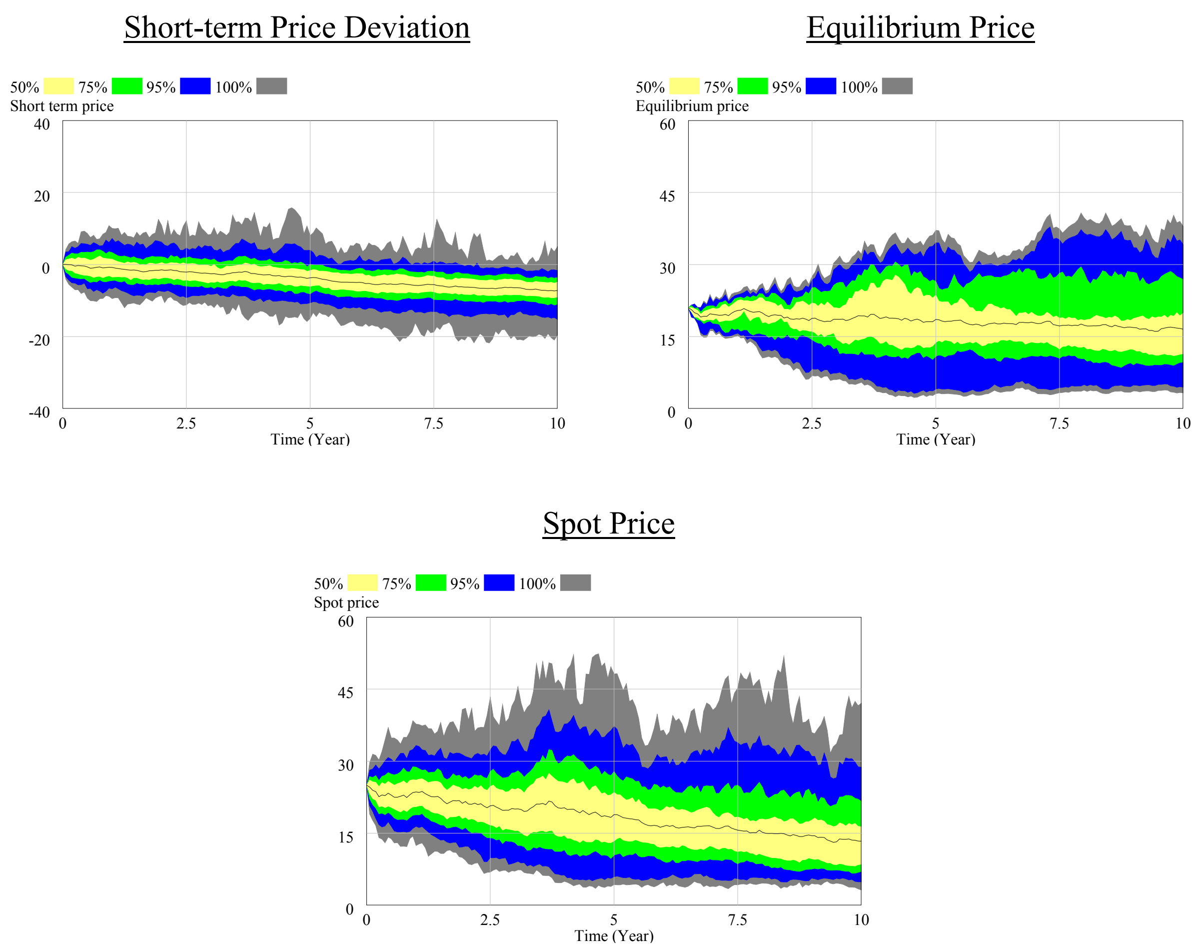


Figure 7: Schwartz-Smith Model for Commodity Prices

In modeling these equations we note that equation 4-1 can be differentiated and substitutions can be made from 4-2 and 4-3 to get:

$$\frac{dS_t}{S_t} = d\chi_t + d\xi_t = (\mu_\xi - \kappa\chi_t)dt + \sigma_\chi dz_\chi + \sigma_\xi dz_\xi \qquad \ldots 4\text{-}5$$

In Figure 7 notice that, although the mean of the short-term deviation from the equilibrium price is zero, the short-term price deviation follows the downward trend in the equilibrium price. Since the equilibrium price is not mean-reverting, the variance of the process increases linearly with time. The net result is that the upper 95+% band of the probability distribution stays roughly in the same range over the ten-year period. Figure 8 shows the system dynamics model used to generate the results in Figure 7 and the equations for this model are listed in the Appendix.

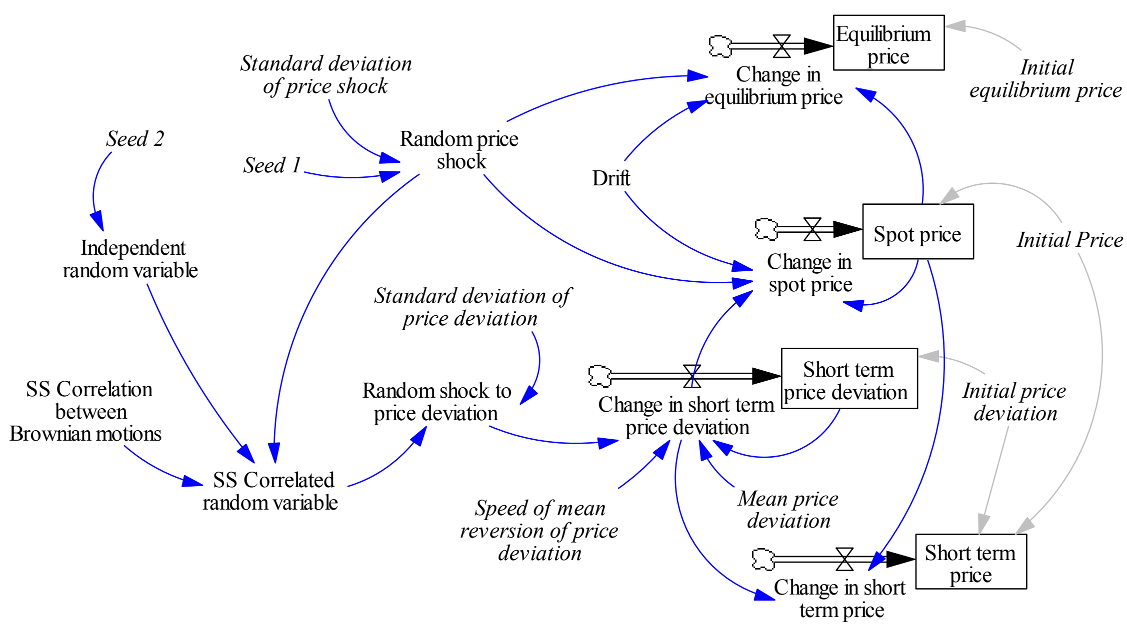


Figure 8: System Dynamics Model for Schwartz-Smith (2000)

5. Conclusions

This paper has introduced the reader to stochastic price processes and shown how these can be modeled using system dynamics. I have explained the derivation of a price process described by geometric Brownian motion, which is an important process for modeling stock prices. I have shown how this model can be easily extended to create a system dynamics model of the dynamics underlying the Black-Scholes option pricing formula. The system dynamics model was validated by showing that average values from Monte Carlo simulations yield the same result as the Black-Scholes equation when the drift parameter is set equal to the risk-free rate. Finally, I showed that more sophisticated multi-factor stochastic processes for modeling commodity prices can also be modeled using system dynamics. In particular, I presented a system dynamics model for the two-factor commodity price process proposed by Schwartz and Smith (2000), and showed how this model can be used to help visualize this price process.

The main contribution of this research is to show how many of the complex price processes that have been researched by scholars in Finance can easily be translated into system dynamics models. These models can be used both for teaching and research purposes. System dynamics software lends itself to visualization of both the structure of the models and of the resultant price dynamics. For this reason, it is suggested that using system dynamics models as teaching aids will enhance students' understanding of stochastic price processes. For example, using system dynamics models in the classroom may help students to gain a deeper insight into the behavior of the Black-Scholes model.

System dynamics models have been widely used to model supply chain processes. For example, see chapters 17-20 and the associated references in Sterman (2000). In many supply chain models, for example Berends and Romme (2001), the focus is on material flows and inventories, with prices being input as an exogenous variable. It may be possible to extend and enhance Berends and Romme's paper industry model by incorporating some of the stochastic price processes presented in this paper. The integration of stochastic price models with supply chain models may provide richer insights into the dynamics of financial markets and commodity cycles.

Appendix: Schwartz-Smith (2000) Commodity Price Model Equation Listing

Initial equilibrium price = 21

Independent random variable = RANDOM NORMAL(-5, 5, 0, 1, Seed 2)

Initial Price = 25

Seed 1 = 1

Seed 2 = 99

Change in equilibrium price = Random price shock * Spot price * SQRT(TIME STEP) + Drift * Spot price * TIME STEP

Change in short term price = Change in short term price deviation * Spot price

Change in short term price deviation = Random shock to price deviation * SQRT(TIME STEP) + Speed of mean reversion of price deviation * (Mean price deviation - Short term price deviation) * TIME STEP

Change in spot price = Random price shock * Spot price * SQRT(TIME STEP) + Drift * Spot price * TIME STEP + Change in short term price deviation * Spot price

Drift = -0.0125

Equilibrium price = INTEG (Change in equilibrium price / TIME STEP, Initial equilibrium price)

Initial price deviation = 0

Mean price deviation = 0

Random price shock = Standard deviation of price shock * RANDOM NORMAL(-5, 5, 0, 1, Seed 1)

Random shock to price deviation = Standard deviation of price deviation* SS Correlated random variable

Short term price = INTEG (Change in short term price / TIME STEP, Initial price deviation * Initial Price)

Short term price deviation = INTEG (Change in short term price deviation / TIME STEP, Initial price deviation)

Speed of mean reversion of price deviation = 1.49

Spot price = INTEG (Change in spot price / TIME STEP, Initial Price)

SS Correlated random variable = SS Correlation between Brownian motions * Random price shock + Independent random variable *SQRT(1-SS Correlation between Brownian motions^2)

SS Correlation between Brownian motions = 0.3

Standard deviation of price deviation = 0.286

Standard deviation of price shock = 0.145

References

Arnold T and Henry SC. 2003. Visualizing the Stochastic Calculus of Option Pricing With Excel and VBA. *Journal of Applied Finance* **13**(1): 56-65.

Baker MP, Mayfield ES and Parsons JE. 1998. Alternative Models of Uncertain Commodity Prices for Use With Modern Asset Pricing Methods. *Energy Journal* **19**(1): 115-148.

Berends PAL and Romme AGL. 2001. Cyclicality of Capital Intensive Industries: a System Dynamics Simulation Study of the Paper Industry. *Omega* **29** 543-552.

Black F and Scholes M. 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* **81**(May/June): 637-659.

Bodnar GM, Hayt GS and Marston RC. 1998. 1998 Wharton Survey of Financial Risk Management by US Non- Financial Firms. *Financial Management* **27**(4):

Dixit AK and Pindyk RS. 1994. *Investment Under Uncertainty*. Princeton University Press: Princeton, NJ.

Elliott RJ, Siu TK and Yang H. 2002. On a Generalized Form of Risk Measure. *International Journal of Theoretical and Applied Finance* (Submitted)

Gibson R and Schwartz ES. 1990. Stochastic Convenience Yield and the Pricing of Oil Contingent Claims. *Journal of Finance* **45** 959-976.

Mikosch T. 1998. *Elementary Stochastic Calculus With Finance in View*. Word Scientific Publishing Co. Pte. Ltd.: Singapore.

Pindyck RS . 1999. The Long-Run Evolution of Energy Prices. *Energy Journal* **20**(2): 1-27.

Schwartz ES . 1997. The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *Journal of Finance* **52**(3): 923-973.

Schwartz ES and Smith JE. 2000. Short-Term Variations and Long Term Dynamics in Commodity Prices. *Management Science* **46**(7): 893-911.

Sterman, J.D. (2000) *Business Dynamics - Systems Thinking and Modeling for a Complex World*, McGraw Hill.

Taylor HM and Karlin S. 1998. *An Introduction to Stochastic Process Modeling*. 3rd ed. Academic Press: San Diego.

Trigeorgis L. 1996. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. MIT Press: Cambridge, MA.

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