INTRODUCING AUTOREGRESSIVE ELEMENTS IN SYSTEM DYNAMICS MODELS

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Abstract

Autoregressive, vector autoregressive and structural vector autoregressive models may be described, in general, as those models that explain, at least partially, the values of a variable or set of variables, based on the past values of this variable or set of variables. During the last decades these models have increased their presence and importance within the field of economic and econometric analysis. It has been found that this kind of simple models, with a small number of variables and parameters, can seriously compete in terms of their forecasting capabilities with the large macroeconomic models, with hundreds of variables and parameters, developed during the fifties and sixties.

This paper explains how System Dynamics models built using Vensim simulation environment may easily incorporate the main elements of autoregressive models. In order to do that we have developed a structural autoregressive model using stock and flow diagrams built with Vensim software and provided the code for the mathematical formulation in a way that this tool can be later used in System Dynamics models. This tool provides short term forecasting capabilities to System Dynamics models built using Vensim. As an illustration, we present an application to the study of the Spanish labor market.

Keywords: Autoregressive Models, System Dynamics Models, Impulse-Response Functions, Forecasting, Labor Market.

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1. Introduction

In this paper we present an approximation to autoregressive models –autoregressive (AR), vector autoregressive (VAR) and structural vector autoregressive (SVAR)- from the point of view of the usefulness that they can provide to the System Dynamics (SD) modelers. The purpose of the paper is to use the SVAR methodology to elaborate an stock and flow diagram and the corresponding formulation and code written in Vensim, in a way that this new tool (macro) can be used to simulate and forecast the behavior of a variable in the short term when the past information of this variable and other related variables is known. Moreover, the proposed model allows to build the map of contemporaneous relations among the considered variables. Therefore, the aforementioned model does not try to be a SD model in itself, but a tool to be used with Vensim when building a wider SD model. This tool will endow the wider model with the required endogenous structure to increase its short term forecasting capabilities.

VAR models allow us to analyze the dynamic relations among a set of variables and offer bigger possibilities to study and contrast theoretical models. Sims (1980) also mentioned that an additional interest of estimating VAR models is the type of information derived from the estimated set of equations. For example, it is possible to analyze the sign, the intensity, the timing and the persistence that each one of the stochastic innovations have on the variables of the model, by means of the impulse-response functions. Another basic element of the VAR analysis is the variance decomposition of the forecasting error, from which it is possible to study the relative weight of every disturbance in the variability of the model endogenous variables.

The SVAR models appear as a response to the criticism received by VAR models regarding their absence of theoretical background. In this way, a VAR model turns out to be a reduced form of a dynamic structural model -theoretical-, which can be estimated from its reduced form and from a set of restrictions on the model parameters.

The rest of the paper is organized as follows. The second section describes briefly the autoregressive models methodology, providing the appropriate references for a more detailed study. The third section describes how to implement an autoregressive stock and flow model using the Vensim simulation environment. This model is composed by two sub-models that articulate the SVAR structure. The fourth section develops an example, based on a previous work which applied the SVAR methodology to the Spanish labor market. Finally, the fifth section concludes.

2. Introduction to AR, VAR and SVAR models

AR models may be described as those in which a variable is explained, at least partially, depending on its past values. VAR models can be understood as a vector generalization of AR models[1]. During the last decades these models have increased their presence and importance within the field of economic and econometric analysis. It has been found that this kind of simple models, with a small number of variables and parameters, can seriously compete in terms of their forecasting capabilities with the large macroeconomic models, with hundreds of variables and parameters, developed during the fifties and sixties.

The VAR models relate several variables in a form such that the value that each of them takes in a period of time is related to the values that the same variable and all other variables take in previous periods. A VAR model can be formulated as follows:

$$\mathbf{y_t} = \mathbf{\Phi_1}\, \mathbf{y_{t-1}} + \mathbf{\Phi_2}\, \mathbf{y_{t-2}} + \ldots + \mathbf{\Phi_p}\, \mathbf{y_{t-p}} + \mathbf{c} + \boldsymbol{\varepsilon}_\mathbf{t} \qquad [1]$$

[1] Autoregressive models methodology (AR, VAR and SVAR) can be found in Hamilton (1994).

where $\mathbf{y_t}$, $\mathbf{y_{t-1}}$,…, $\mathbf{y_{t-p}}$ are vectors (n x 1) containing the values of the variables in periods t, t-1,…, t-p; $\mathbf{\Phi_1}$, $\mathbf{\Phi_2}$,…, $\mathbf{\Phi_p}$ are matrices (n x n) containing the model parameters that can be estimated; **c** is a vector (n x 1) containing the model constants, that can equally be estimated; and $\boldsymbol{\varepsilon_t}$ is a vector of random perturbation terms, also denominated *innovations*, as this is the only new information that enters in period *t*, with respect to what it is already available from previous periods. In this model $\mathbf{\Omega}=E(\boldsymbol{\varepsilon_t}\,\boldsymbol{\varepsilon_t}')$ is the variance-covariance matrix (n x n) of the innovations. This matrix can be a non-diagonal matrix reflecting the fact that innovations can be contemporaneously correlated among them. The model in [1] is denoted VAR(*p*), where the *order p* is the number of time lags of the model. Equation [1] can be written:

$$(\mathbf{I_n} - \mathbf{\Phi}_1 L - \mathbf{\Phi_2} L^2 - \ldots - \mathbf{\Phi_p} L^p)\, \mathbf{y_t} = \mathbf{\Phi}(L)\, \mathbf{y_t} = \mathbf{c} + \boldsymbol{\varepsilon_t} \qquad [2]$$

where $\mathbf{\Phi}(L)$ is a matrix polynomial in the lag operator[2] with (n x n) matrices $\mathbf{\Phi}_j$, and $\mathbf{I_n}$ represents the identity matrix of order *n*.

Under suitable conditions, VAR models can be transformed in moving average infinite-order vector models (*reduced moving average form*):

$$\mathbf{y_t} = \boldsymbol{\varepsilon_t} + \mathbf{\Psi_1}\boldsymbol{\varepsilon_{t-1}} + \mathbf{\Psi_2}\boldsymbol{\varepsilon_{t-2}} + \ldots + \boldsymbol{\mu} = (\mathbf{I_n} + \mathbf{\Psi_1} L + \mathbf{\Psi_2} L^2 + \ldots)\, \boldsymbol{\varepsilon_t} + \boldsymbol{\mu} = \mathbf{\Psi}(L)\, \boldsymbol{\varepsilon_t} + \boldsymbol{\mu} \qquad [3]$$

where $\boldsymbol{\varepsilon_t}, \boldsymbol{\varepsilon_{t-1}}, \boldsymbol{\varepsilon_{t-2}}, \ldots$ are the innovations vectors in t, t-1,… ; $\boldsymbol{\mu}$ is a vector of constants and $\mathbf{\Psi}(L)$ is a matrix polynomial in the lag operator with infinite (n x n) matrices $\mathbf{\Psi}_j$. Comparing [2] and [3], $\mathbf{\Psi}(L)=\mathbf{\Phi}(L)^{-1}$ should apply.

Notice that, as $\mathbf{I_n}$ is the identity matrix, in the VAR model in equation [2] each element of the vector $\mathbf{y_t}$ (*endogenous* variables determined within the system) is expressed as a function of lagged values of all the elements in the same vector (variables *predetermined* in previous periods). However, do not appear contemporaneous relations among the variables, in other words, each variable is not related to the values of the others in that same period. Thus, equation [2] can be viewed as the reduced autoregressive form that could be obtained from a SVAR model in which there would be a relation among endogenous variables for the current time period:

$$\mathbf{B}(L)\, \mathbf{y}_t = (\mathbf{B}_0 - \mathbf{B}_1 L - \mathbf{B}_2 L^2 - \ldots - \mathbf{B}_p L^p)\, \mathbf{y}_t = \mathbf{k} + \mathbf{u}_t \qquad [4]$$

where **k** is a vector (*n* x 1) with constants, $\mathbf{u}_t$ a vector (*n* x 1) of perturbations, which in this structural model are called structural shocks, and $\mathbf{B}(L)$ is a matrix polynomial in the lag operator with (n x n) matrices $\mathbf{B}_j$.

Notice how $\mathbf{B}_0$ denotes the contemporaneous relations among the endogenous variables[3] $\mathbf{y}_t$. Moreover, it is common to suppose that $\mathbf{u}_t$ are standardized structural shocks, not contemporaneously correlated to each other, so that their variance-covariance matrix is the identity ($E\,(\mathbf{u}_t\,\mathbf{u}_t') = \mathbf{I}$).

Equation [4] is the structural autoregressive form of the model. If we pre-multiply both members of the equation by $\mathbf{B}_0^{-1}$ we would obtain [2] -reduced autoregressive form- and, vice versa, known the matrix $\mathbf{B}_0$, the structural autoregressive form of the model can be obtained pre-multiplying by $\mathbf{B}_0$ in both members of [2]. However, it can be demonstrated that the information

[2] The lag operator L is defined as follows: $L\,x_t \equiv x_{t-1}$; $L^2\,x_t \equiv x_{t-2}$; …; $L^p\,x_t \equiv x_{t-p}$.

[3] The model in its structural form [4] cannot be directly estimated by OLS in a consistent way, as there are endogenous variables among the system regressors. It is therefore required to implement the estimation in its reduced form [2]. This problem in the OLS estimation of the models in its structural form is discussed in Greene (2003).

contained in $\mathbf{y}_t$ is not enough to identify the matrix $\mathbf{B}_0$, and some additional restrictions are required. These additional restrictions can be obtained from the implications that theoretical models have on the expected behavior of the variables $\mathbf{y}_t$. In this sense, it can be affirmed that whereas in the VAR model of equation [2] the theoretical requirements are minimum (the set of variables whose interaction is going to be analyzed and the number of time lags to be included), in the SVAR model a greater theoretical content can be found, given by the model from which the above mentioned additional restrictions are obtained previously.

Like VAR models in reduced form [2], VAR models in its structural form [4], under suitable conditions, can be transformed into the structural moving average form:

$$\mathbf{y}_t = \mathbf{C}(L)\, \mathbf{u}_t + \mathbf{h} = (\mathbf{C}_0 + \mathbf{C}_1 L + \mathbf{C}_2 L^2 + \ldots + \mathbf{C}_s L^s + \ldots)\, \mathbf{u}_t + \mathbf{h} \qquad [5]$$

where $\mathbf{h}$ is a vector (n x 1) of constants, and $\mathbf{C}(L) = \mathbf{B}(L)^{-1}$.

Equation [5] represents the model [1] in moving average form with orthogonal shocks. Each element ij of the matrix $\mathbf{C}_s$ of the polynomial $\mathbf{C}(L)$ -c^s_{ij}- identifies the effect of an orthogonal unitary shock in y_j (j=1,..,n) at date t -u_{jt}- on the variable y_i (i=1,..,n) at date t+s, under the assumption that there is not other kind of shock at date t or earlier. Therefore, the elements of matrix $\mathbf{C}_s$ describe the temporary effect of a unitary shock –impulse- on the model variables –response-. The matrix of dynamic multipliers $\mathbf{C}_s$ is known as the *orthogonalized* impulse-response function. To obtain the matrix polynomial $\mathbf{C}(L)$ it is required to calculate previously the matrix $\mathbf{B}_0$; and once we know $\mathbf{B}_0$, we can obtain u_t from ε_t, and $\mathbf{C}(L)$ from $\mathbf{\Psi}(L)$ according to the equation $\mathbf{C}(L) = \mathbf{\Psi}(L)\, \mathbf{B}_0^{-1}$.

Another basic element in VAR analysis, besides the impulse-response functions, is the variance decomposition of the forecasting error, from which it is possible to study the relative weight of each shock in the variability of the model endogenous variables. Thus, the weight of a shock in y_j at date t (u_{jt}) in the variability of the variable y_i at date t+s ($y_{i\,t+s}$) will be given by:

$$\frac{(c^s_{ij})^2}{\sum_{j=1}^{n} (c^s_{ij})^2} \qquad [6]$$

3. Design of a structural autoregressive stock and flow model in Vensim

We will now show how it is possible to implement a stock and flow SVAR model using Vensim (version 4.0[4]). We will explain how we have implemented this model by constructing two basic stock-flow sub-models (sub-model 1 and sub-model 2), each of which corresponds to different phases of the analysis process, as it will be showed in detail later. We want to remark here that it is not the purpose of this paper to design a SD model, but to provide a sort of "macro" that will facilitate the SD modelers the incorporation of SVAR methodology elements.

The core of both sub-models is the forecast of the variables in every period from their values in the previous periods, according to the reduced autoregressive form of equation [2]. The main difference between both sub-models is that sub-model 1 uses the real data of the variables in the previous periods, whereas sub-model 2 uses the forecasts for the previous periods given by the sub-model. For this reason, we can say that the forecast horizon is one period in the first sub-model, and multi-period in the second one.

[4] We have used Vensim 4.0, which is a Trade Mark of Ventana System Inc.

Sub-model 1 is used to estimate the parameters of the model in the reduced autoregressive form [2] from a series of real values of the variables. This sub-model analyzes the fit between the forecasts of the estimated model [2] and the series of real values of the variables, computing the differences between them (residuals or estimated values of innovations $\boldsymbol{\varepsilon}_t$) and their variance-covariance matrix.

With the values of the estimated parameters, sub-model 2 is used to calculate the $\mathbf{B}_0$ matrix, with the additional restrictions that the theoretical model imposes on the dynamics of the variables. After that, we can obtain the structural autoregressive form from the reduced autoregressive form. Sub-model 2 also allows simulating the response of the variables to different impulses, in innovations $\boldsymbol{\varepsilon}_t$ or in structural shocks $\mathbf{u}_t$ -impulse-response functions-. These functions are also related to the moving average forms of the model. In order to obtain all these results sub-model 2 is used in three versions, differing only in the magnitude of the innovations.

In the next two sections we present in detail the analysis process that both sub-models follow. In appendix A the corresponding code for both sub-models is presented, in an application to the study of the Spanish labor market.

3.1. Detailed explanation of sub-model 1

Sub-model 1 stock and flow diagram can be observed in figure 1 (the names of the variables will be indicated within quotation marks as they are explained).

First, the sub-model reads the data imported from an external file obtaining "variables in t". In order to obtain the lags structure of the model, the level variables "variables in t-i" are generated (where i means the order of the lag) and updated at the end of each period. This is done with an inflow named "incr var" that is used to store data of vector $\mathbf{y}_t$, in that period and in the p-1 past periods, as lagged data for the following period, and an outflow named "decr var" which eliminates previously stored data in the level variable.

As we said before, the core of sub-model 1 is the forecast of the variables for every period ("variables forecast in t ") from their values in the previous periods, according to the model in reduced autoregressive form [2]. The model parameters to estimate are "variable in t-i coefficients" and the "constants". The variable "Time" is used to control the periods in which the model is initialized, introducing the real values of the variables as first lags.

The estimation of the model parameters, starting from the series of the variables real values, is done using a modified Powell Method[5] included in the "calibration" option of Vensim. By doing so, the values obtained for the parameters minimize the sum of the squared residuals (real values of the variables minus forecasts of the model) for all the periods that compose the estimation interval. A joint estimation is done for all the equations that compose [2], corresponding to each variable in the vector $\mathbf{y}_t$, giving the same weight to the sums of the squares of every equation residuals in the global payoff function to minimize.

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[5] Among the numerical optimization techniques, the direct-search method that does not evaluate the gradient is most suitable for the analysis of dynamics of complex nonlinear control systems. The Powell method (Powell, 1964) is well known in order to have an ultimate fast convergence among direct-search methods. The basic idea behind the Powell method is to break the *N* dimensional minimization down into *N* separate one-dimensional (1D) minimization problems. Then, for each 1D problem a binary search is implemented to find the local minimum within a given range. Furthermore, on subsequent iterations, an estimate is made of the best directions to use for the 1D searches.

Some problems, however, are not always assured of optimal solutions because the direction vectors are not always linearly independent. To overcome this difficulty, the method was revised (Powell,1968) by introducing new criteria for the formation of linearly independent direction vectors. This revised method, which is the one used in this paper, is called "The Modified Powell Method".

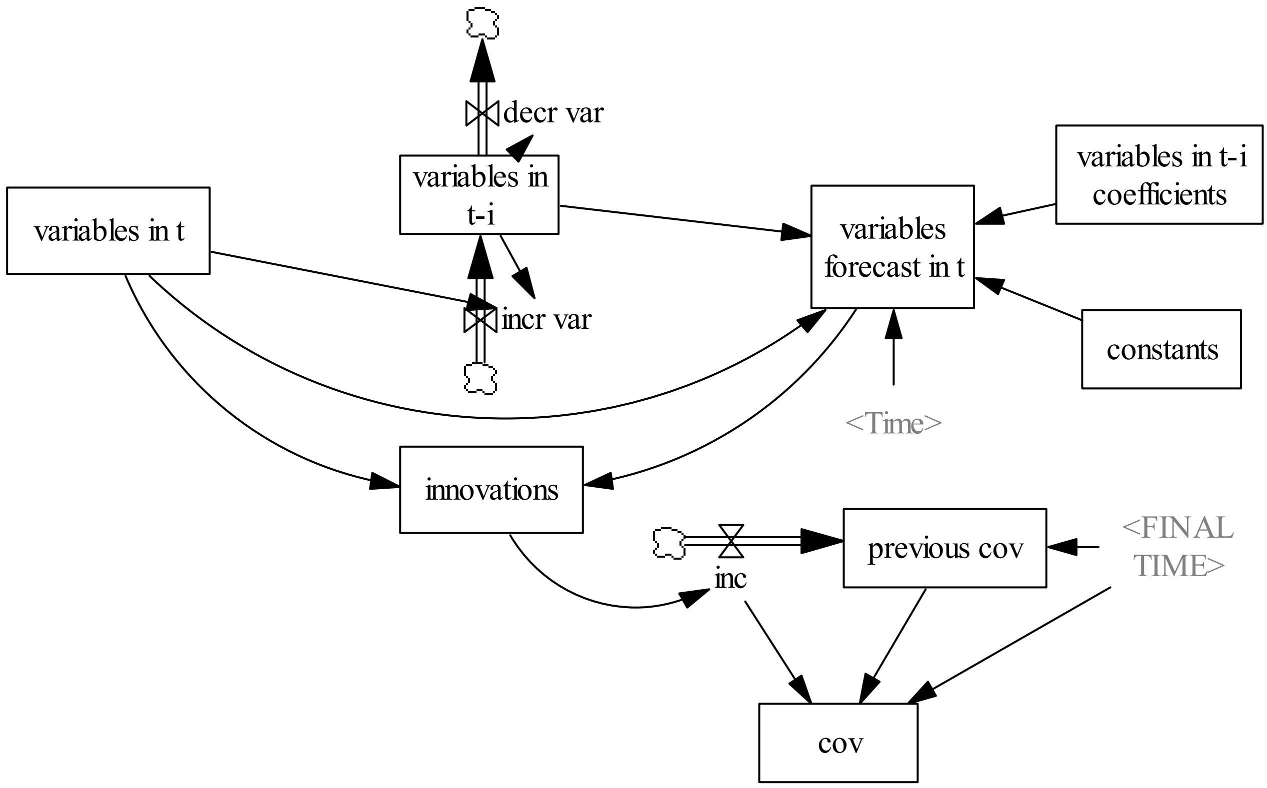


Figure 1. Stock and flow diagram of sub-model 1

With the estimated values of the parameters we obtain the estimated “innovations”, and from those values we obtain the estimate of the variance–covariance matrix "cov". The flow variable "inc" increases in every period the accumulated level "previous cov" of the sum of the residuals products for all the previous periods. Finally, the variable "FINAL TIME” provides the number of periods that it is necessary to take into account in this process.

3.2. Detailed explanation of sub-model 2

In this section, we present in detail the analysis process that sub-model 2 follows, divided in four steps. In order to obtain the results of this sub-model, three versions of the same model are developed differing only in the magnitude of the innovations. The first version corresponds to step 1), the second to step 2) and the third to steps 3) and 4). The sub-model 2 stock and flow diagram can be observed in figure 2.

1) Obtaining the polynomial matrix $\Psi(L)$ corresponding to the reduced moving average form, and the impulse-response functions (non orthogonalized)

The non orthogonalized impulse-response functions are obtained as a result of the simulation[6] of the response of the vector of variables $\mathbf{y}_t$ to impulses in the innovations $\boldsymbol{\varepsilon}_t$. The specification "non orthogonalized" refers to the fact that innovations appear contemporaneously correlated among them. The response obtained as a result of the simulation is "variables forecast in t", which corresponds to the vector $\mathbf{y}_t$, obtained by means of the model in reduced autoregressive form of equation [2], with "variables in t-i coefficients" estimated in the sub-model 1. The "variables in t-i", in this sub-model 2, are generated from the forecast in t[7], updating them at the end of every period with an input flow, "incr var", that stores as lagged data for the following period the forecast in this

[6] See Hamilton (1994).
[7] Initially, their values are made equal to 0.

period and the variables in the p-1 previous periods, and an output flow, "decr var", which eliminates the information previously stored.

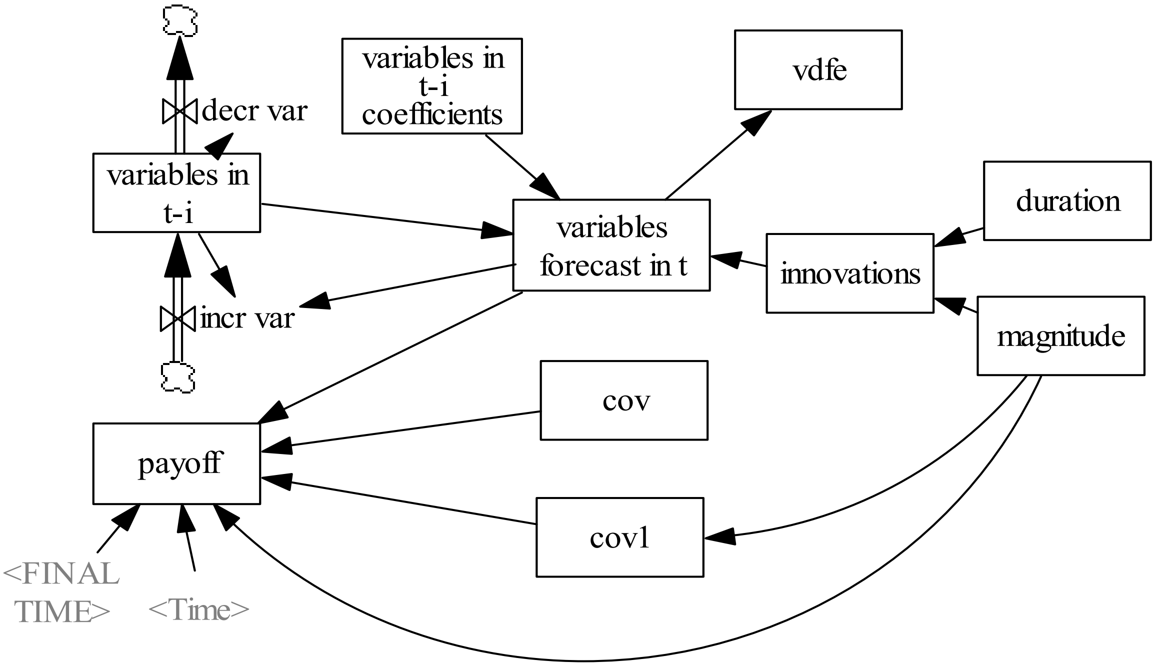


Figure 2. Stock and flow diagram of sub-model 2

The impulse are the "innovations" $\boldsymbol{\varepsilon}_t$, that are made equal to 1 in the initial period for the corresponding variable of the vector $\mathbf{y}_t$, whereas they are made equal to zero for the remaining variables in this period and for all the variables in the following periods. Therefore, n simulations will be required. Each simulation will depend on which variable of the vector **y** experiments the initial unitary impulse. Nevertheless, it is possible to use subscripts in order to carry out all the simulations at the same time. The innovations are obtained as the product of "duration", which establishes the time that the innovation lasts (in this case, an initial impulse that disappears later), by the "magnitude" of the same innovation (in this case, the first version of sub-model 2, the "magnitude" is 1 for the variable that experiences the impulse and 0 for the others).

From the impulse-response functions we can obtain the matrix $\boldsymbol{\Psi}(L)$ corresponding to the moving average reduced form. It is sufficient to note that in $\boldsymbol{\Psi}(L)$ the term corresponding to the lag s is composed by the elements of the impulse-response functions corresponding to the period s of simulation.

2) Obtaining the matrix $\boldsymbol{S} = \boldsymbol{B}_0^{-1}$, the structural autoregressive form, the structural moving average form, and the structural shocks

As it was previously exposed in section 2, pre-multiplying both members of the reduced autoregressive form [2] by $\mathbf{B}_0$, the structural autoregressive form [4] can be obtained and, vice versa, known the matrix $\mathbf{S} = \mathbf{B}_0^{-1}$, it is possible to obtain [2] from [4], pre-multiplying both members of this equation by **S**. Given that $E\,(\mathbf{u}_t\,\mathbf{u}_t') = \mathbf{I}$ and $\boldsymbol{\varepsilon}_t = \mathbf{S}\,\mathbf{u}_t$, then:

$$E\,(\boldsymbol{\varepsilon}_t\,\boldsymbol{\varepsilon}_t') = \boldsymbol{\Omega} = \mathbf{S}\,\mathbf{S}' \qquad [7]$$

where $\boldsymbol{\Omega}$ is the variance-covariance matrix of the innovations $\boldsymbol{\varepsilon}_t$ estimated in the sub-model 1. As $\boldsymbol{\Omega}$ is a n x n symmetrical matrix, the equation [7] provides $(n^2+n)/2$ conditions to identify the n^2

elements of **S**. The other $(n^2\text{-}n)/2$ conditions, as it was exposed in section 2, are obtained as implications of theoretical models.

Since the sub-model 2 corresponds to the reduced autoregressive form, we must consider that, according to the equation $\boldsymbol{\varepsilon}_t = \mathbf{S}\,\mathbf{u}_t$, an unitary value of one of the shocks $\mathbf{u}_t$ is equivalent to a vector of innovations $\boldsymbol{\varepsilon}_t$ of magnitude equal to the respective column of the matrix **S**. Therefore, the matrix **S** that we look for will be composed by the values for the variable "magnitude" in the second version of the sub-model 2.

The numerical optimization is guided by the fulfillment of the aforementioned conditions, by means of the maximization of a vector of n^2 variables "payoff", giving the same weight to all these variables. The first $(n^2\text{-}n)/2$ variables capture the theoretical restrictions required for the identification of the SVAR model, while the remaining $(n^2+n)/2$ guarantee the fulfillment of equation [7]. Regarding theoretical restrictions, sometimes they directly influence the matrix $\mathbf{S} = \mathbf{B}_0^{-1}$, while in other cases, as it happens in our application, they affect dynamic forecasts of the model acting indirectly on S. On the other hand, the additional $(n^2+n)/2$ restrictions are the square of the differences among all non identical elements of the symmetrical matrices **SS'** and **Ω,** with negative sign. Initially, in the second version of the sub-model 2, we impose initial unitary values to all the elements of "magnitude", and therefore to all the elements of **S**. After that, the process of optimization continues until the values of the above mentioned elements that approximate the payoff sufficiently to its maximum possible value are found. In this maximum value, the last $(n^2+n)/2$ variables of the payoff function will have a value equal to zero and the theoretical restrictions will be fulfilled. In the variables "cov" and "cov1" (with the corresponding subscripts) are respectively the elements of the matrix **Ω** estimated in the sub-model 1, and the elements of the product **SS'**, obtained from the values of "magnitude" forming the matrix **S**. The variables "Time" and "FINAL TIME" are used to control that the payoff is calculated in the period corresponding to the theoretical restrictions used.

Once the matrix $\mathbf{S} = \mathbf{B_0}^{-1}$ has been obtained, pre-multiplying both members of the reduced autoregressive form [2] by $\mathbf{B}_0$, the structural autoregressive form [4] and the structural shocks $\mathbf{u}_t = \mathbf{B}_0\,\boldsymbol{\varepsilon}_t$ are obtained. The structural moving average form [5] can also be obtained from the moving average reduced form [3], obtained in step 1), multiplying $\boldsymbol{\Psi}(L)$ by **S**, since, as $\boldsymbol{\varepsilon}_t = \mathbf{S}\,\mathbf{u}_t$, we get:

$$\mathbf{y}_t = \boldsymbol{\Psi}(L)\,\boldsymbol{\varepsilon}_t + \ldots = \boldsymbol{\Psi}(L)\,\mathbf{S}\,\mathbf{S}^{-1}\,\boldsymbol{\varepsilon}_t + \ldots = \mathbf{C}(L)\,\mathbf{u}_t + \ldots \qquad [8]$$

where $\mathbf{C}(L) = \boldsymbol{\Psi}(L)\mathbf{S}$ is the polynomial matrix corresponding to the structural moving average form.

3) Obtaining the orthogonalized impulse-response functions

In the third version of sub-model 2 the elements of "magnitude" are made equal to the values obtained for S in the previous step. Each of the parallel simulations thus carried out with the reduced autoregressive form corresponds to unitary values in the initial period of each one of the structural shocks. Therefore, the values obtained in the simulations of the variables in "variables forecast in t", represent the orthogonalized impulse-response functions for these variables. The specification "orthogonalized" refers to the fact that the structural shocks are not contemporaneously correlated among each other.

4) Variance decomposition of the forecasting error

From the values, in each period, of the variables forecast in the orthogonalized impulse-response functions of the previous step, we can obtain the decomposition of the variance of the forecasting error "vdfe" in the same period. This forecasting error is originated by the responses to each one of the n structural shocks. So, for each variable, the percentage that supposes the square of its value in each simulation is calculated in relation to the sum of all the squares.

4. An application to the Spanish labor market

We present now an application of the model explained in the previous section, implementing in Vensim 4.0 a SVAR model referred to the Spanish labor market[8], developed by Dolado and Gómez (1997)[9], following Blanchard and Diamond (1989).

Dolado and Gómez (1997) SVAR model focuses on the quarterly series of three variables: unemployment (U), vacancies (V), and labor force (L). As we will see, in this model the vector $\mathbf{y}_t$ is obtained from a few previous transformations, and it is composed by the variables $v1=\Delta(v-u)$, $v2=\Delta u$ and $v3=\Delta l$, where v, u and l are the logarithms of V, U and L, and where Δ indicates the first difference of the corresponding variable. These three transformed variables correspond respectively to the rates of growth of the vacancies/unemployment ratio, unemployment and labor force.

Relating each of these three transformed variables with the lagged values (up to 4 quarters) of all of them, the reduced autoregressive form [2] is derived, including also a vector of dummy quarterly variables $\mathbf{d}_t$ with its coefficients matrix $\mathbf{D}$, to control for the seasonal effects:

$$\mathbf{\Phi}(L)\, \mathbf{y}_t = \mathbf{c} + \mathbf{D}\, \mathbf{d}_t + \boldsymbol{\varepsilon}_t \qquad [9]$$

As it was mentioned in section 2, in this reduced autoregressive form, contemporaneous relations do not appear among the variables, that is, each variable is not related to the values of the others in the same period. These contemporaneous relations do appear in the structural autoregressive form [4]. The matrix $\mathbf{B}_0$, within the polynomial in the lag operator $\mathbf{B}(L)$, reflects the contemporaneous relations among the variables. As it was also exposed in section 2, the information contained in the time series $\mathbf{y}_t$ is not sufficient to identify the elements of $\mathbf{B}_0$, and therefore it is necessary to add restrictions. These restrictions can be obtained from the implications that theoretical models may have on the expected behavior of the variables $\mathbf{y}_t$.

Dolado and Gómez (1997) use a theoretical model, following a flow approach[10] to labor market, made up of four blocks: the flows of job creation and job destruction, the hiring process through a matching function between vacancies and unemployment, the wage determination as a function of the excess demand in the labor market, and the labor supply or labor force as a function of wages and unemployment. All this is used to obtain a relation among the transformed variables that compose the vector $\mathbf{y}_t$ in the structural autoregressive form [4]. At the same time, the structural shocks $\mathbf{u}_t$, are identified using three types of disturbances in the economy: *aggregate activity shocks,* due to disturbances in the different components of aggregate demand, *reallocation shocks,* due to disturbances affecting the efficiency in the matching process between vacancies and unemployed (skill mismatch, geographical mismatch …) and *labor force shocks,* due to disturbances that affect directly this variable (women participation in the labor market …). The additional restrictions for the identification of $\mathbf{B}_0$, obtained as implications of this theoretical model, are that a labor force

[8] Pioneering works in the application of the SVAR analysis to the labor market are Blanchard y Quah (1989), Bean (1992) and Galí (1992).
[9] Other work in this line, for the Andalusian labor market, is Usabiaga *et al.* (2001).
[10] A detailed analysis of the labor market, following the flow approach, can be found in Blanchard and Diamond (1992) and Pissarides (2000).

shock does not have permanent effects on unemployment and vacancies and that a reallocation shock does not have permanent effects on the vacancies/unemployment ratio.

In order to develop this application with Vensim we are going to follow the steps described in the previous section with both sub-models. Since these steps correspond faithfully to those followed in the process of econometric estimation of a SVAR model, the numerical results obtained are practically identical to those of Dolado and Gómez[11]. In this application, the sub-models 1 and 2 are renamed " labor 1 " and " labor 2 " respectively, and their code is shown in appendix A.

4.1. Sub-model labor 1

The stock and flow diagram corresponding to the sub-model labor 1 is presented in figure 3.

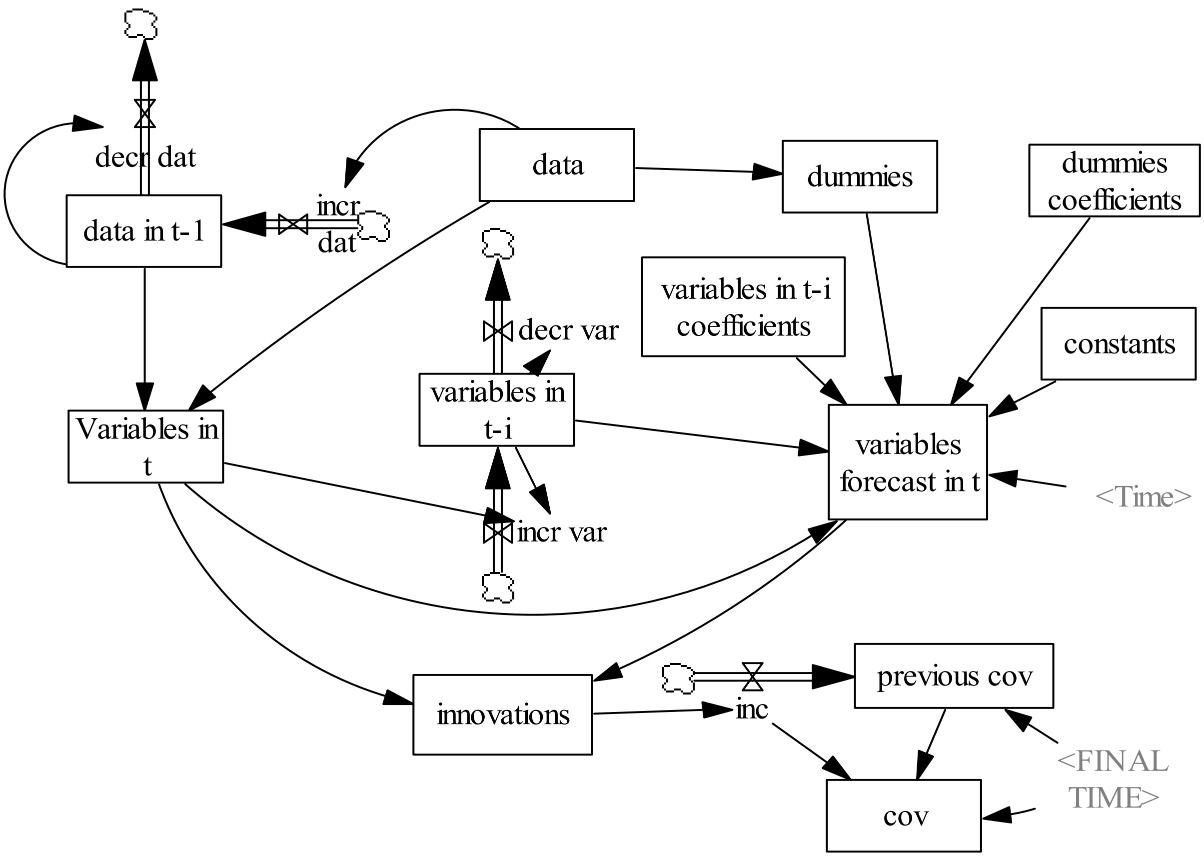


Figure 3. Stock and flow diagram of labor 1

First, the model reads the "data" imported from an external file: quarterly series[12] of values for vacancies (V), unemployment (U) and labor force (L), besides the quarterly dummy variables.

Later, several data initial transformations are made, obtaining "variables in t" and the "dummies". The variables obtained are v1=Δ(v-u), v2=Δu and v3=Δl, composing the vector $\mathbf{y}_t$. In order to calculate these differences, the level variables "data in t-1" need to be calculated first, and are updated at the end of every period with an inflow "incr dat" that stores the data of this period as lagged information for the following period, and an outflow "decr dat" that eliminates the data stored previously. Moreover, the four seasonal dummies (d1, d2, d3, d4) are reduced to three (t1, t2, t3), in order to avoid the problem of perfect collinearity among them, defining:

[11] We have replicated the analysis developed in Dolado and Gómez (1997) using the econometric software Eviews 3.1.

[12] In appendix B we show the data that we have used in our analysis.

$$
\begin{aligned}
t1 &= d1-d4 \\
t2 &= d2-d4 \\
t3 &= d3-d4
\end{aligned}
\qquad [10]
$$

As we said previously, the core of labor 1 is the forecast of the variables for every period "variables forecast in t" from their values in the previous periods. The model parameters to estimate are "variable coefficients in t-i", "dummies coefficients" and the "constants". Using the “calibration” option of Vensim, with model 1, the following values are obtained for the parameters:

$$
\begin{vmatrix} \textbf{v1 "t"} \\ \textbf{v2 "t"} \\ \textbf{v3 "t"} \end{vmatrix}
=
\begin{vmatrix} -0{,}0227 \\ 0{,}0154 \\ 0{,}0023 \end{vmatrix}
+
\begin{vmatrix} 0{,}1266 & -1{,}3112 & 7{,}5704 \\ -0{,}0174 & 0{,}5078 & -0{,}6107 \\ -0{,}0026 & -0{,}0215 & 0{,}1264 \end{vmatrix}
\begin{vmatrix} \textbf{v1 "t-1"} \\ \textbf{v2 "t-1"} \\ \textbf{v3 "t-1"} \end{vmatrix}
+
$$

$$
+
\begin{vmatrix} -0{,}2134 & 0{,}3097 & 0{,}9093 \\ -0{,}0073 & 0{,}1930 & -2{,}1762 \\ 0{,}0002 & -0{,}0439 & -0{,}0313 \end{vmatrix}
\begin{vmatrix} \textbf{v1 "t-2"} \\ \textbf{v2 "t-2"} \\ \textbf{v3 "t-2"} \end{vmatrix}
+
$$

$$
+
\begin{vmatrix} 0{,}2090 & -0{,}1916 & -6{,}1599 \\ -0{,}0103 & -0{,}1398 & 1{,}1385 \\ 0{,}0007 & 0{,}0039 & 0{,}1545 \end{vmatrix}
\begin{vmatrix} \textbf{v1 "t-3"} \\ \textbf{v2 "t-3"} \\ \textbf{v3 "t-3"} \end{vmatrix}
+
$$

$$
+
\begin{vmatrix} 0{,}0646 & 0{,}9359 & 7{,}8355 \\ 0{,}0368 & 0{,}1115 & -2{,}4865 \\ 0{,}0009 & 0{,}0313 & 0{,}0049 \end{vmatrix}
\begin{vmatrix} \textbf{v1 "t-4"} \\ \textbf{v2 "t-4"} \\ \textbf{v3 "t-4"} \end{vmatrix}
+
$$

$$
+
\begin{vmatrix} -0{,}0562 & 0{,}3256 & -0{,}2187 \\ 0{,}0095 & -0{,}0570 & 0{,}0275 \\ -0{,}0008 & -0{,}0005 & 0{,}0048 \end{vmatrix}
\begin{vmatrix} \textbf{t1} \\ \textbf{t2} \\ \textbf{t3} \end{vmatrix}
$$

With the estimated values of the parameters, we obtain the estimated “innovations” and the estimate of the variance–covariance matrix "cov":

	v1	**v2**	**v3**
v1	0,020200	-0,000163	0,000091
v2	-0,000163	0,000386	0,000017
v3	0,000091	0,000017	0,000010

Sub-model labor 2

The stock and flow diagram corresponding to the sub-model labor 2 is presented in figure 4.

1) Obtaining the polynomial matrix $\mathbf{\Psi}(L)$ *corresponding to the reduced moving average form, and the impulse-response functions (non orthogonalized)*

In the case of our application to the labor market, the terms of $\mathbf{\Psi}(L)$ are 3x3 matrices and their elements correspond to the response of each one of the three variables (v1, v2, v3) to each one of the three simulated unitary impulses[13].

2) Obtaining the matrix $\boldsymbol{S} = \boldsymbol{B}_0^{-1}$, *the structural autoregressive form, the structural moving average form, and the structural shocks*

[13] The numerical results obtained at this step are not relevant since impulse-response functions are non orthogonalized.

As in our example **Ω** is a 3 x 3 symmetrical matrix, the equation [7] provides six conditions in order to identify the nine elements of **S**. The other three conditions are obtained as implications of the theoretical model. These conditions are that a labor force shock does not have permanent effects on unemployment and vacancies, and that a reallocation shock does not have permanent effects on the vacancies/unemployment ratio.

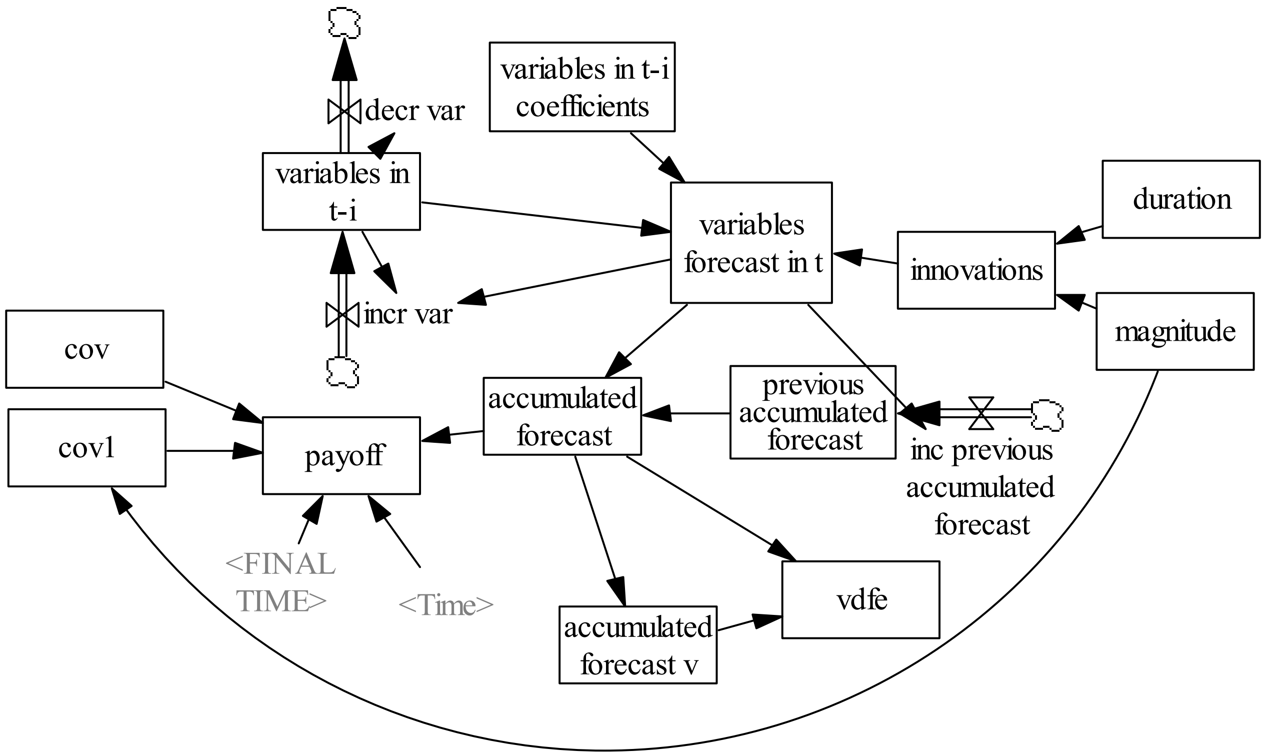


Figure 4. Stock and flow diagram of labor 2

The numerical optimization is guided by the fulfillment of the aforementioned conditions, by means of the maximization of a vector of nine variables "payoff", giving the same weight to all these variables. The first three ([po1], [po2], [po3]) are the square of the forecast in the final period (long term) of u and v, responding to an unitary labor force shock, and the square of the forecast in the final period of v-u responding to an unitary reallocation shock, all of them with negative sign. The other six ([po4], [po5], [po6], [po7], [po8], [po9]) are the square of the differences among all six non identical elements of the symmetrical matrices **S S** ' and **Ω,** also with negative sign. That value of **S** is reached when **Ω** = **S S**´, and the forecast of u and v in the final period responding to an unitary labor force shock, and the forecast of v-u responding to an unitary reallocation shock in that final period are all of them zero. Remember that the matrix **S** that we look for will be composed by the values that the variable "magnitude" takes in our model (of each innovation in each of the three simulations). The result obtained after the simulation is as follows:

$$S = \begin{vmatrix} 0{,}1161 & 0{,}0745 & -0{,}0343 \\ -0{,}0054 & 0{,}0127 & 0{,}014 \\ 0{,}0018 & -0{,}0006 & 0{,}0025 \end{vmatrix}$$

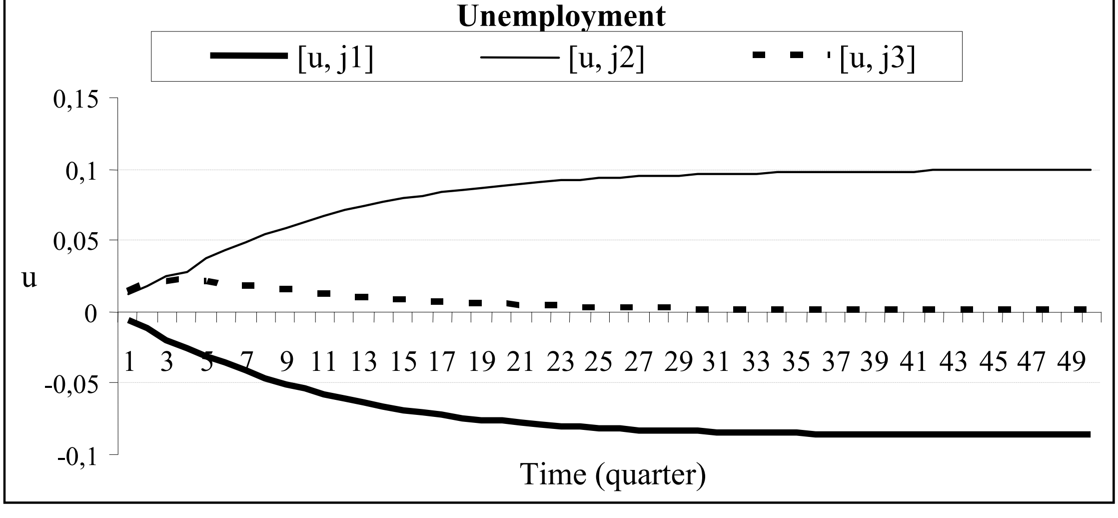
The properties of the theoretical model refer to the values of u, v and v-u in the long term. As the variable "variables forecast in t" corresponds to the first differences Δ(v-u), Δu y Δl, the model recovers v-u, u and l accumulating the forecast in the variable "accumulated forecast". The level "previous accumulated forecast" is updated at the end of every period with the inflow "inc previous accumulated forecast", that is equal to the forecast of the variables obtained in this period, and "accumulated forecast" is obtained by adding this forecast to the one accumulated previously (notice that the updating of the level variables at the end of every period is not registered in the output of the model until the following period). "Accumulated forecast v" is then obtained adding the levels v-u and u.

3) Obtaining the orthogonalized impulse-response functions

In the third version of labor 2, the elements of "magnitude" are made equal to the values obtained for S in the previous step. Therefore, the values obtained in the simulations of the variables u and l in "accumulated forecast", and of v in "accumulated forecast v", represent the orthogonalized impulse-response functions for these variables, recovered from their first differences. The specification "orthogonalized" refers to the fact that the structural shocks are not contemporaneously correlated among each other.

As an example, we represent in figure 5 the impulse-response functions obtained for the variable unemployment [u = ln(U)]. A positive shock of aggregate activity (j1) permanently reduces unemployment, whereas a positive reallocation shock (j2) provokes a permanent increase of the same variable. Finally, a positive labor force shock increases unemployment in the short term; however, in the long term, as job creation and destruction adjust to the decrease of the real wages associated with the increase of unemployment, the effects on unemployment will tend to disappear.

Figure 5. Orthogonalized impulse-response functions. Unemployment

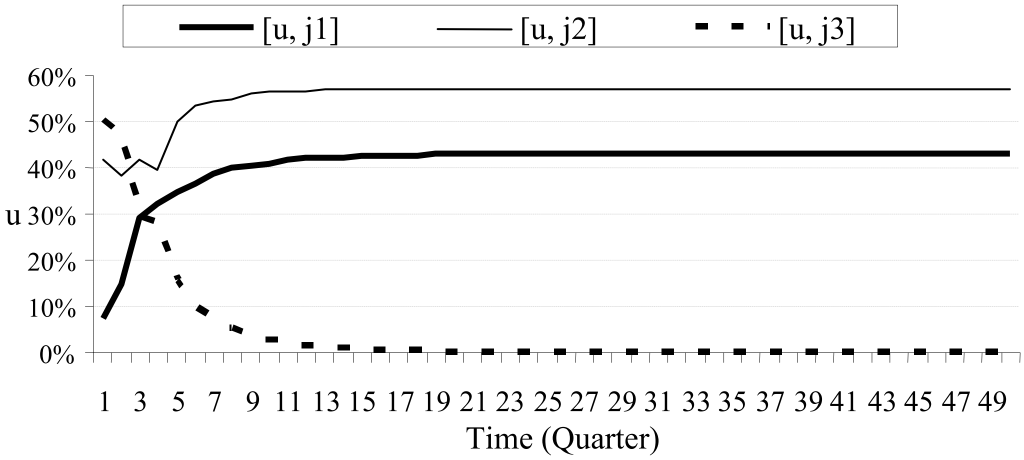


4) Variance decomposition of the forecasting error

From the values, in each period, of u and l in "accumulated forecast" and v in "accumulated forecast v" in the orthogonalized impulse-response functions of the previous step, we can obtain the variance decomposition of the forecasting error "vdfe" in the same period for u, l and v. This forecasting error is originated by the responses to each one of the three structural shocks.

In figure 6 we represent the variance decomposition of the forecasting error for unemployment [u = ln(U)]. In the very short term the structural shocks with more weight in unemployment variability are labor force (j3) and reallocation (j2). However, in the medium and long term the shocks of aggregate activity (j1) and reallocation (j2) explain completely the unemployment variability[14].

Figure 6. Variance decomposition of the forecasting error. Unemployment



5. Conclusion

Our main goal has been the creation of a tool or "macro" to be used with Vensim simulation environment which, applied on a given SD model, provides autoregressive endogenous structure and short term forecasting capabilities. The main effort has been done searching for the correspondence, in stock-flow modeling, of the main concepts and procedures that appear in the SVAR model.

In order to develop this stock-flow version of the SVAR model, we have built two sub-models using the Vensim simulation environment. Each of these sub-models corresponds to different phases of the process of the SVAR analysis. The lagged variables, essential in the SVAR analysis, are now treated as level variables. The calculation procedures have been similar to those of the original econometric SVAR analysis, although the analytical resolution of some of the steps of the problem has been done through simulation within the stock-flow sub-models.

The core of the stock-flow sub-models built is the reduced autoregressive form of the SVAR analysis. The responses to the structural shocks have been obtained transforming them into non orthogonalized innovations, by means of the corresponding matrix, which also has been estimated with the second one of these sub-models.

As an illustration, we present an application to the study of the Spanish labor market. The results obtained (estimates of the parameters, impulse-response functions, decomposition of the variance of the forecasting error) with the stock-flow sub-models reproduce faithfully those of the original application of the SVAR analysis.

[14] An important feature of the SVAR methodology is that the results obtained from the analysis of the impulse-response functions and the variance decomposition of the forecasting error are related to the identification restrictions adopted.

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APPENDIX A

SUB-MODEL LABOR 1 VENSIM CODE:

Data input:
data[x]
x: u,v,pa,d1,d2,d3,d4

Obtaining the data lagged one period:
"data in t-1"[x]= INTEG (incr dat[x]-decr dat[x], 1)
incr dat[x]= data[x]
decr dat[x]= "data in t-1"[x]

Obtaining the model variables:
Variables in t[v1]= LN(data[v])-LN("data in t-1"[v])-LN(data[u])+LN("data in t-1" [u])
Variables in t[v2]= LN(data[u])-LN("data in t-1"[u])
Variables in t[v3]= LN(data[pa])-LN("data in t-1"[pa])
variables: v1,v2,v3

Autoregressive Structure:
"variables in t-i"[variables,lag]= INTEG (incr var[variables,lag]-decr var [variables,lag], 0)
incr var[variables,"t-1"]= Variables in t[variables]
incr var[variables,"t-2"]= "variables in t-i"[variables,"t-1"]
incr var[variables,"t-3"]= "variables in t-i"[variables,"t-2"]
incr var[variables,"t-4"]= "variables in t-i"[variables,"t-3"]
decr var[variables,lag]= "variables in t-i"[variables,lag]
lag: "t-1","t-2","t-3","t-4"

Dummies:
dummies[t1]= data[d1]-data[d4]
dummies[t2]= data[d2]-data[d4]
dummies[t3]= data[d3]-data[d4]
quarters: t1, t2, t3

Variables forecast:
variables forecast in t[variables]= IF THEN ELSE (Time>=6, SUM("variables in t-i"
[variables!,lag!]*"variables in t-i coefficients" [variables,variables!,lag!]) + constants[variables]+
SUM(dummies[quarter!]*Dummies coefficients[variables,quarter!]),
Variables in t[variables])

Coefficients to estimate:
"variables in t-i coefficients"[variables,variables,lag]= 0
Dummies coefficients[variables,quarter]= 0
constants[variables]= 0

Obtaining the innovations and the variance-covariance matrix:
innovations[variables]= Variables in t[variables]-variables forecast in t[variables]
inc[v1v1]= innovations[v1]*innovations[v1]
inc[v2v2]= innovations[v2]*innovations[v2]
inc[v3v3]= innovations[v3]*innovations[v3]
inc[v1v2]= innovations[v1]*innovations[v2]
inc[v1v3]= innovations[v1]*innovations[v3]

inc[v2v3]= innovations[v2]*innovations[v3]
previous cov[cova]= INTEG (inc[cova]/(FINAL TIME-5-16),0)
cov[cova]= previous cov[cova]+inc[cova]/(FINAL TIME-5-16)
cova: v1v1,v2v2,v3v3,v1v2,v1v3,v2v3

Simulation control parameters:
FINAL TIME = 72
~ Quarter
~ The final time for the simulation.
INITIAL TIME = 1
~ Quarter
~ The initial time for the simulation.
SAVEPER = 1
~ Quarter
~ The frequency with which output is stored.
TIME STEP = 1
~ Quarter
~ The time step for the simulation.

SUB-MODEL LABOR 2 VENSIM CODE:

Autoregressive Structure:
"variables in t-i"[variables,lag,j]= INTEG (incr var[variables,lag,j]-decr var [variables,lag,j], 0)
incr var[variables,"t-1",j]= variables forecast in t[variables,j]
incr var[variables,"t-2",j]= "variables in t-i"[variables,"t-1",j]
incr var[variables,"t-3",j]= "variables in t-i"[variables,"t-2",j]
incr var[variables,"t-4",j]= "variables in t-i"[variables,"t-3",j]
decr var[variables,lag,j]= "variables in t-i"[variables,lag,j]
lag: "t-1","t-2","t-3","t-4"
variables : v1, v2, v3

Variables forecast:
variables forecast in t[variables,j]= SUM("variables in t-i"[variables!,lag!,j] * "variables in t-i coefficients"[variables,variables!,lag!]) + innovations[variables,j]
j: j1,j2,j3

Estimated Coefficients:
"variables in t-i coefficients"[v1,v1,lag] = 0.12661, -0.213419, 0.209034, 0.0646055
"variables in t-i coefficients"[v1,v2,lag] = - 1.31123,0.30969,-0.19163,0.935878
"variables in t-i coefficients"[v1,v3,lag] = 7.57044,0.909342,- 6.15989,7.8355
"variables in t-i coefficients"[v2,v1,lag] = - 0.0174353,- 0.00726969,- 0.0102628,0.0367592
"variables in t-i coefficients"[v2,v2,lag] = 0.507797,0.193024,- 0.13983,0.11152
"variables in t-i coefficients"[v2,v3,lag] = - 0.610707,- 2.17617,1.13846,- 2.48651
"variables in t-i coefficients"[v3,v1,lag] = - 0.00261106,0.000206853,0.000728788,0.0009468
"variables in t-i coefficients"[v3,v2,lag] = - 0.0214941,- 0.0438737,0.00386141,0.0312963
"variables in t-i coefficients"[v3,v3,lag] = 0.12642,- 0.0313129,0.154495,0.00487324

Obtaining the variables in levels:
accumulated forecast v[j]= accumulated forecast[v1,j]+accumulated forecast[v2,j]
accumulated forecast[variables,j]= previous accumulated forecast[variables,j] + variables forecast in t[variables,j]
inc previous accumulated forecast[variables,j]= variables forecast in t[variables,j]

previous accumulated forecast[variables,j]=INTEG(inc previous accumulated forecast[variables,j], 0)

Obtaining the matrix S (only in version 2):
payoff[po1]= IF THEN ELSE(Time = FINAL TIME, -accumulated forecast[v1,j2]^2 , 0)
payoff[po2]= IF THEN ELSE(Time = FINAL TIME, -accumulated forecast[v1,j3]^2 , 0)
payoff[po3]= IF THEN ELSE(Time = FINAL TIME, -accumulated forecast[v2,j3]^2 , 0)
payoff[po4]= IF THEN ELSE(Time = FINAL TIME, -(1-(cov1[v1,j1]/cov[v1,j1]))^2 , 0)
payoff[po5]= IF THEN ELSE(Time = FINAL TIME, -(1-(cov1[v1,j2]/cov[v1,j2]))^2 , 0)
payoff[po6]= IF THEN ELSE(Time = FINAL TIME, -(1-(cov1[v1,j3]/cov[v1,j3]))^2 , 0)
payoff[po7]= IF THEN ELSE(Time = FINAL TIME, -(1-(cov1[v2,j2]/cov[v2,j2]))^2 , 0)
payoff[po8]= IF THEN ELSE(Time = FINAL TIME, -(1-(cov1[v2,j3]/cov[v2,j3]))^2 , 0)
payoff[po9]= IF THEN ELSE(Time = FINAL TIME, -(1-(cov1[v3,j3]/cov[v3,j3]))^2 , 0)
po: po1,po2,po3,po4,po5,po6,po7,po8,po9
cov1[v1,j1] = magnitude[po1]^2+magnitude[po4]^2+magnitude[po5]^2
cov1[v1,j2] =
magnitude[po1]*magnitude[po7]+magnitude[po4]*magnitude[po2]+magnitude[po5]*magnitude[po6]
cov1[v1,j3] =
magnitude[po1]*magnitude[po8]+magnitude[po4]*magnitude[po9]+magnitude[po5]*magnitude[po3]
cov1[v2,j2] = magnitude[po7]^2+magnitude[po2]^2+magnitude[po6]^2
cov1[v2,j3] =
magnitude[po7]*magnitude[po8]+magnitude[po2]*magnitude[po9]+magnitude[po6]*magnitude[po3]
cov1[v3,j3] = magnitude[po8]^2+magnitude[po9]^2+magnitude[po3]^2
cov1[v2,j1] =
magnitude[po1]*magnitude[po7]+magnitude[po4]*magnitude[po2]+magnitude[po5]*magnitude[po6]
cov1[v3,j1] =
magnitude[po1]*magnitude[po8]+magnitude[po4]*magnitude[po9]+magnitude[po5]*magnitude[po3]
cov1[v3,j2] =
magnitude[po7]*magnitude[po8]+magnitude[po2]*magnitude[po9]+magnitude[po6]*magnitude[po3]
cov[v1,j] = 0.0202,-0.000162531,9.0526e-005
cov[v2,j] = -0.000162531,0.000385541,1.74239e-005
cov[v3,j] = 9.0526e-005,1.74239e-005,1.02585e-005

Innovations:
Version 1:
innovations[variables,j]= duration*magnitude[variables,j]
magnitude[v1,j]= 1,0,0
magnitude[v2,j]= 0,1,0
magnitude[v3,j]= 0,0,1
duration = PULSE(1,1)

Version 2:
innovations[v1,j1] = duration*magnitude[po1]
innovations[v1,j2] = duration*magnitude[po4]
innovations[v1,j3] = duration*magnitude[po5]
innovations[v2,j1] = duration*magnitude[po7]

innovations[v2,j2] = duration*magnitude[po2]
innovations[v2,j3] = duration*magnitude[po6]
innovations[v3,j1] = duration*magnitude[po8]
innovations[v3,j2] = duration*magnitude[po9]
innovations[v3,j3] = duration*magnitude[po3]
magnitude[po1]= 1
magnitude[po2]= 1
magnitude[po3]= 1
magnitude[po4]= 1
magnitude[po5]= 1
magnitude[po6]= 1
magnitude[po7]= (cov[v1,j2]-magnitude[po4]*magnitude[po2]-magnitude[po5] *magnitude[po6]) / magnitude[po1]
magnitude[po8]= (((cov[v1,j3]-magnitude[po5]*magnitude[po3])*magnitude[po2])-(cov[v2,j3]-magnitude[po6] *magnitude[po3])*magnitude[po4])/(magnitude[po2]*magnitude[po1]-magnitude[po7] * magnitude[po4])
magnitude[po9]= (cov[v1,j3]-magnitude[po5]*magnitude[po3]-magnitude[po1]*magnitude[po8]) / magnitude[po4]
duration = PULSE(1,1)

Version 3:
innovations[variables,j] = duration*magnitude[variables,j]
magnitude[v1,j] = 0.116078,0.0745076,-0.0342579
magnitude[v2,j] = -0.0054,0.0127004,0.0139544
magnitude[v3,j] = 0.0018,-0.000581161,0.00251562
duration = PULSE(1,1)

Obtaining the variance decomposition of the forecasting error “vdfe” (only in versión 3):
vdfe[v1,j]= accumulated forecast[v2,j]^2 / (accumulated forecast[v2,j1]^2+accumulated forecast[v2,j2]^2+accumulated forecast[v2,j3]^2)
vdfe[v2,j]= accumulated forecast[v3,j]^2 / (accumulated forecast[v3,j1]^2+accumulated forecast[v3,j2]^2+accumulated forecast[v3,j3]^2)
vdfe[v3,j]= accumulated forecast v[j]^2 / (accumulated forecast v[j1]^2+accumulated forecast v[j2]^2+accumulated forecast v[j3]^2)

Simulation control parameters:
FINAL TIME = 100
~ Quarter
~ The final time for the simulation.
INITIAL TIME = 1
~ Quarter
~ The initial time for the simulation.
SAVEPER = 1
~ Quarter
~ The frequency with which output is stored.
TIME STEP = 1
~ Quarter
~ The time step for the simulation.

APPENDIX B

QUARTERLY SERIES OF VALUES FOR VACANCIES, UNEMPLOYMENT, LABOR FORCE AND DUMMIES

Source: Labor Force Survey (published by INE), Employment Statistics (published by INEM)

Quarters 1977:01-1994:04	Vacancies (Thnds)	Unemployment (Thnds)	Labor Force (Thnds)	1: 1st quarter 0: rest	1: 2nd quarter 0: rest	1: 3rd quarter 0: rest	1: 4th quarter 0: rest
Time	Data[V]	Data[U]	Data[L]	Data[d1]	Data[d2]	Data[d3]	Data[d4]
1	22,8	644	13265	1	0	0	0
2	28,7	634	13285	0	1	0	0
3	23,3	704	13337	0	0	1	0
4	22,3	750	13380	0	0	0	1
5	22,4	846	13377	1	0	0	0
6	30,8	864	13287	0	1	0	0
7	28	934	13302	0	0	1	0
8	25,4	996	13306	0	0	0	1
9	28,8	1061	13294	1	0	0	0
10	33,6	1061	13257	0	1	0	0
11	26,8	1137	13327	0	0	1	0
12	27,1	1241	13337	0	0	0	1
13	30,3	1384	13340	1	0	0	0
14	25,2	1449	13261	0	1	0	0
15	18,2	1504	13279	0	0	1	0
16	17,4	1631	13273	0	0	0	1
17	16,4	1755	13300	1	0	0	0
18	16,9	1798	13251	0	1	0	0
19	21,2	1891	13354	0	0	1	0
20	26,6	2002	13375	0	0	0	1
21	29	2077	13423	1	0	0	0
22	33,9	2052	13419	0	1	0	0
23	30,6	2148	13493	0	0	1	0
24	33,7	2248	13572	0	0	0	1
25	42,4	2336	13550	1	0	0	0
26	60,2	2275	13561	0	1	0	0
27	57,3	2352	13648	0	0	1	0
28	47,9	2453	13703	0	0	0	1
29	51,5	2670	13679	1	0	0	0
30	64,9	2681	13623	0	1	0	0
31	53,9	2745	13691	0	0	1	0
32	50,7	2907	13719	0	0	0	1
33	59,7	2963	13739	1	0	0	0
34	98,3	2934	13705	0	1	0	0
35	82,8	2931	13800	0	0	1	0
36	80,5	2981	13853	0	0	0	1

Quarters 1977:01-1994:04	Vacancies (Thnds)	Unemployment (Thnds)	Labor Force (Thnds)	1: 1st quarter 0: rest	1: 2nd quarter 0: rest	1: 3rd quarter 0: rest	1: 4th quarter 0: rest
Time	Data[V]	Data[U]	Data[L]	Data[d1]	Data[d2]	Data[d3]	Data[d4]
37	103,6	3017	13916	1	0	0	0
38	126,7	2932	13983	0	1	0	0
39	82,6	2894	14042	0	0	1	0
40	75,4	2925	14145	0	0	0	1
41	96,4	2992	14232	1	0	0	0
42	113,8	2936	14266	0	1	0	0
43	101	2918	14440	0	0	1	0
44	97,5	2904	14498	0	0	0	1
45	114,6	2941	14553	1	0	0	0
46	136,7	2899	14608	0	1	0	0
47	120,3	2850	14701	0	0	1	0
48	105,9	2701	14621	0	0	0	1
49	111,3	2698	14702	1	0	0	0
50	151,6	2555	14750	0	1	0	0
51	138,7	2468	14895	0	0	1	0
52	124,5	2522	14930	0	0	0	1
53	128,5	2511	14993	1	0	0	0
54	183,6	2438	14996	0	1	0	0
55	129	2392	15048	0	0	1	0
56	125,8	2424	15045	0	0	0	1
57	114,4	2421	15000	1	0	0	0
58	140,4	2388	15011	0	1	0	0
59	119,2	2480	15157	0	0	1	0
60	105,5	2566	15126	0	0	0	1
61	100,4	2632	15082	1	0	0	0
62	109,8	2686	15144	0	1	0	0
63	107	2789	15202	0	0	1	0
64	109	3047	15193	0	0	0	1
65	79,1	3300	15181	1	0	0	0
66	101,9	3397	15265	0	1	0	0
67	81,7	3546	15423	0	0	1	0
68	72,1	3682	15406	0	0	0	1
69	65,4	3793	15428	1	0	0	0
70	90,5	3763	15491	0	1	0	0
71	86,9	3698	15486	0	0	1	0
72	61,9	3698	15468	0	0	0	1

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