# Modeling the Mobile Service Market of the Region and Control Problems Solution

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#### Abstract

This paper presents a system dynamics approach for the analysis of the mobile service competitive market model. The model evaluates dynamic competition between major operators: the dominant operator and the others. Their market share, as per the number of subscribers, is influenced by the pricing policy, service quality, subscriber base, potential subscribers, marketing, etc. The method for the identification of the market carrying capacity is considered and the problem of the optimal price management is formulated. The obtained results can be used to forecast and improve decision-making within the real dynamic systems.

**Keywords:** system dynamics, modeling, forecasting, mobile service market, price strategy, management.

# Introduction

The task of analyzing existing experimental data and building growth models to illustrate the sales volume has a wide range of implications to be considered in various areas [4]. The results of defining the market share carrying capacity and the synthesis of the optimal price strategy for the competitive market can be used to understand the managerial decision-making in complex dynamic systems.

The fist part of the paper describes a system dynamics model of the mobile service market. The model displays a process of adoption of the mobile services on the competitive market (Figure 1). It includes information about the popularity of the mobile services, which provides the growth of the subscriber base at the initial stage, when the number of subscribers is significantly smaller than the potential. Such popularity is generated by marketing, price strategy, dumping, etc.

In the second part we present the method for constructing the growth model. The approach taken is suitable for various initial conditions (when the number of people, who have taken an interest in the new services on offer is small). Parts three and four focus on the actual mobile service market model. The numerical results of the modeling process are established and the comparison with the real experimental data is made. We also describe different scenarios of the market development and formulate the problem of the synthesis of the optimal price strategy for the company.

The paper continues the research [1] and develops approaches [4, 6].

#### **Distribution Model of Mobile Services on a Competitive Market**

Figure 1 shows a system dynamics model illustrating the distribution of cellular communications company services in the regional competitive market. This model is based on the John D. Sterman's approach [4]. The total adoption rate is the sum of adoptions resulting from the "word of mouth" and adoptions resulting from the company's marketing and communication services, advertising and other external influences. Adoptions from the "word of mouth" are formulated exactly as in the logistic innovation diffusion model or the Bass diffusion model. The probability of adoption by potential users as a result of an exposure to a given amount of advertising, the volume of advertising and other external influences at each period remain constant.

The initial growth is driven by feedbacks outside the boundary of the simple logistics models. Existing subscribers of the company, having found out about satisfactory services available, spread the information among the unaware potential customers. As a result of such contact, there is a probability that new subscriptions will be made. This probability (the efficiency of the distribution of the "word of mouth") depends on the important principle of estimating appropriate costs for the services of the cellular communications operator. For example, the cost of calls made within the network of the same operator is cheaper than those calls made to destinations outside of the network (i.e. for calls between relatives, friends and immediate family it is cheaper to use one communications company).

An important factor influencing the rate at which customers subscribe is to inform potential subscribers of the services available which is usually done with the help of advertising.

It is important to clarify what is meant under "potential subscribers." Consumers or other customers typically move through a chain of development [5]. The precise character of customers in each of these stages and their detection vary from case to case. Rigorous market analysis methods should be applied to determine these customer characteristics accurately.

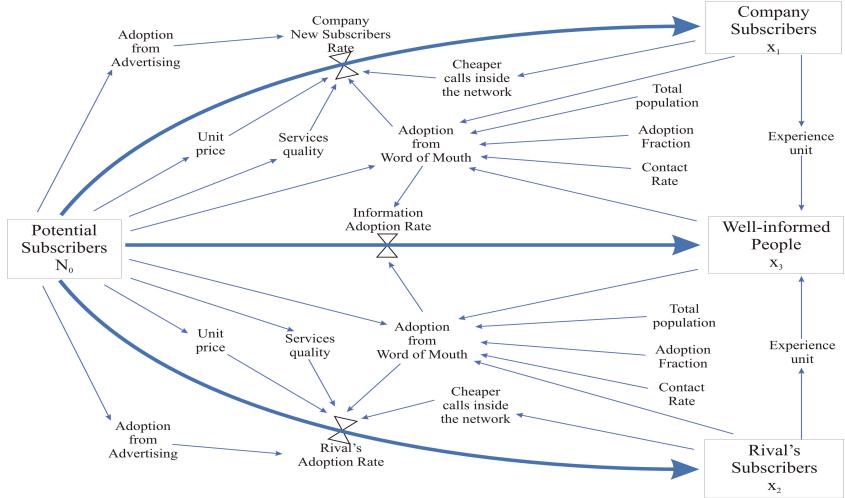


Figure 1. Mobile Service Competitive Market Model

### **Dynamic Growth Models**

The method of the growth model construction using the differential equation has been proposed in [1]. This approach is suitable for describing the sales process where mobile services are distributed into account for different initial conditions (when the number of people who have taken an interest in new services is very small compared to the actual market share carrying capacity). Different growth models are illustrated in [1].

One of the possible market development models for a single service can be described by the equation:

$$\Delta P = \frac{P_{\text{max}}}{A \cdot t^2} \cdot \exp\left(\frac{-1}{A \cdot t}\right),\tag{1}$$

where P(t) – cumulative sales of the mobile services;  $P_{\text{max}}$  – estimation of the potential market share carrying capacity, taking into account issues, specific to the socioeconomic factors of the region; A – parameter, which depends on the fractional growth rate; t – time (days, months, years, etc.).

The next growth model of cumulative sales volumes can be described by the differential equation:

$$\frac{dP}{dt} = \left[\alpha_1 + \alpha_2 P(t)\right] \cdot \left(P_{\max} - P(t)\right),\tag{2}$$

where  $\alpha_1$ ,  $\alpha_2$  – parameters, depending on company marketing activity and the "word of mouth" effect.

Figure 2 illustrates the real company statistical data (real subscribers) and the result of modeling the mobile service market of the region with the help of different models. Experimental data can be approximated with a high degree of accuracy through the model identification.

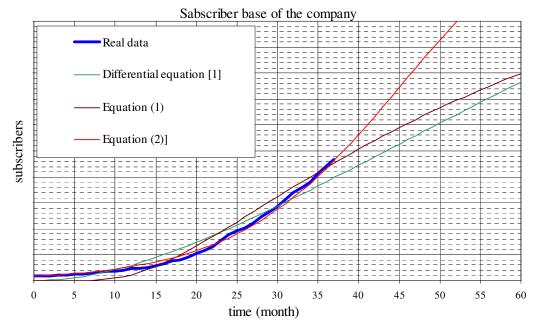


Figure 2. Growth Models and Experimental Data

Such modeling process allows estimating the market share carrying capacity  $P_{\text{max}}$ . This method also enables us to forecast the demand for services. The receipt of new sales volumes allows updating the model parameters continuously.

The next part of the paper is based on the system dynamics model (Figure 1) and the equation (2).

# **Competitive Market Model**

Consider conditions when the major operator is determined. This company has the largest subscriber base. The model (Figure 1) includes dynamic competition between major operators: the leader and the others. The pricing policy, service quality, subscriber base, potential subscribers, marketing, etc influence their market share in regards to the number of subscribers.

The system dynamic model from Figure 1 can be described by the set of equations:

$$\frac{dx_{1}}{dt} = \alpha \left[ N_{0} - \lambda (1 - K_{1})x_{1} - \sum_{i=1}^{2} x_{i} - \gamma_{1}x_{3} \right] * [a_{11}x_{1} + bu_{1}];$$

$$\frac{dx_{2}}{dt} = \alpha \left[ N_{0} - \lambda (1 - K_{2})x_{2} - \sum_{i=1}^{2} x_{i} \right] * [a_{21}x_{1} + a_{22}x_{2} + bu_{2}];$$

$$\frac{dx_{3}}{dt} = \alpha (N_{0} - \gamma_{0}x_{3})\sum_{j=1}^{3} a_{3j}x_{j},$$
(3)

where  $x_i$ , i = 1,2 – subscriber base of the *i*-operator,  $x_3$  – number of people, who know the actual situation (price, service quality, etc.);  $N_0$  – estimation of the potential market share carrying capacity, under a certain service price  $u_i$ , i = 1,2;  $K_i$ , i = 1,2 – service quality function for *i*-operator;  $\alpha, \gamma_0, \gamma_1, \lambda; a_{ij}, b_i$  i = 1,2, j = 1,2,3 – parameters, depending on the company's marketing, price strategy, advertising, customers' activity, etc.

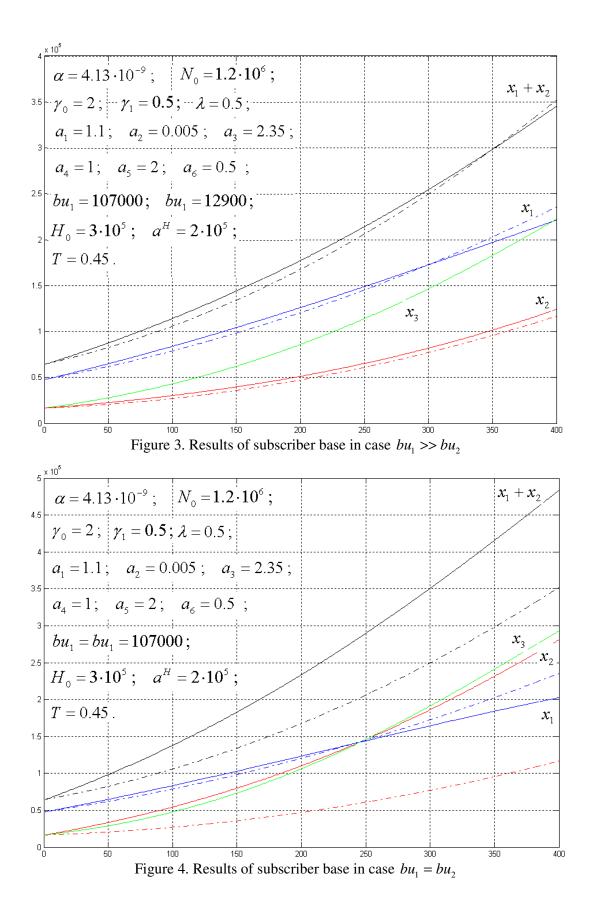
The service quality function can be described by the equations:

$$K_{i} = \frac{1}{\left(1 + e^{\left(-z_{i}/T_{i}^{H}\right)}\right)}; \quad T_{i}^{H} = 0.2H_{i}/3; \quad z_{i} = H_{i} - x_{i}^{k}; \quad H_{i+1} = H_{i} + a^{H}/365 + H_{0}.$$
(4)

The system usually dedicates a single radio channel to a local group of subscribers who share it. There is an optimal number of subscribers  $H_i$  using an *i* network with a normal connection quality  $K_i$ . If more subscribers use one channel of an *i* network, the connection cannot be guaranteed.

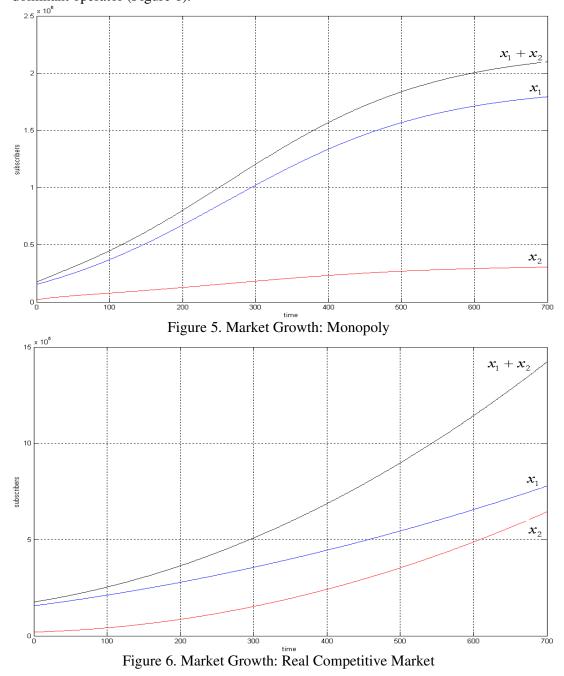
Different scenarios of the market development are illustrated in Figures 3 and 4. Experimental data are displayed by the dotted line. The parameters of the model (3) and equations (4) are described in the top left corner of the Figures 3 and 4.

A competitive market development for the condition  $bu_1 >> bu_2$  in the long run is illustrated in Figure 3 - the market share of the major operator demonstrates an increase. If  $bu_1 = bu_2$  and service quality of the major operator is decreased, the market share of other operators increases (Figure 4).



### **Scenarios of Regional Market Development**

Here we present the result of a model simulation under two scenarios: monopoly and real competitive market. Any regional mobile service market at the initial stage had a dominant operator with a subscriber base of  $x_1 >> x_2$ . Suppose, that other initial conditions (such as price, service quality, marketing, etc.) are equal and the oldest operator is the monopolist in the long run (Figure 5). The service price reduction allows new operators to attract the subscribers. In this case, the market actively develops. The distribution of new subscribers depends on the existing subscriber base of a less dominant operator (Figure 6).



There is a possibility of an industry-wide "word-of-mouth" and other growth effects (Figure 1). Simply put, the more subscribers already exist, the more new customers become aware and actively consider using its services. Similarly, the increasing awareness converts people who haven't represented likely subscribers into a group of those who are at least not being excluded. The influence of exogenous forces (political, economic, social, and technological [1, 5]) must be added to this internal market growth mechanism.

# **Control problems**

Consider the problem of optimal management for a mobile operator on a competitive market. The nonlinear model (3) includes control parameters  $bu_1$ ,  $bu_2$  (in Figure 1 it is called "Unit Price"). Let's formulate control problems [1–3, 6]. Control purposes for each mobile operator on different stages vary by: the subscriber base, revenue, market share, minimized churn rate, increased revenue per user, net income, improvement in financial efficiency, etc.

The obtained market share carrying capacity should be used for the analysis of the market situation and the determination of the price strategy, which is optimal in the sense of the firm development criterion [1]. Consider the control problem using the nonlinear model (3). The model built is described by the following finite-difference equations:

$$X_{k+1} = F(X_k, u_k, w_k) + B_k u_k + \xi_k , \qquad (5)$$

where  $X_k \in \mathbb{R}^3$  is a state vector, describing the market behavior;  $u_k \in \mathbb{R}^2$  is a control vector;  $w_k \in \mathbb{R}^3$  is a vector, describing current market situation;  $\xi_k \in \mathbb{R}^3$  is a vector, characterizing inaccuracy of information about the parameters of the market;  $F(\cdot)$ - a nonlinear vector-function;  $B_k$  – a constant matrix. It is assumed that all information about operators is available for measurement.

Equation (5) is reduced to the linear one using a linearization procedure:

$$X_{k+1} = A_k X_k + B_k u_k + \xi_k , (6)$$

where  $A_k$  is a constant matrix.

The use of control theory to solve management problems is considered e.g. in [6].

#### Conclusions

We presented the system dynamics approach for the analysis of the mobile service competitive market. The model includes the description of the dynamic competition between major operators: the leader and the others. The pricing policy, service quality, subscriber base, potential subscribers, marketing, etc influence their market share with regards to the number of subscribers.

In this paper, the method for the identification of the market carrying capacity is described and the problem of the optimal price management is formulated. The obtained results can be used in research to forecast and improve decision-making in real dynamic systems. We also established the numerical results of the modeling process and made comparisons with the real experimental data on the volume of sales of communications services in the competitive market.

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