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UNKNOWN PARAMETERS AND STATE ESTIMATION OF MANUFACTURING AND SELLING FIRM UNDER UNCERTAINTY

Shiryaev V.I., Smolin V.V.
Southern-Ural State University, Department of Applied Mathematics
454080, Russia, Chelyabinsk, pr.Lenina, 78
+7 (3512) 39-91-74
smolin@prima.susu.ac.ru

Abstract. *For the built model of a manufacturing and selling firm problem of unknown parameters and state estimation is solved. The estimation is necessary to find an optimal control of the firm and is solved under incomplete and inaccurate information about parameters, state and market situation. The problem is solved during firm functioning as new data come from information system of the firm. The data of the information system is considered to be incomplete and inaccurate. Solution of the estimation problem is described when disturbances and information system errors are casual or have uncertain nature.*

Key words. Manufacturing and selling firm, model, estimation, unknown parameters and sta te.

Introduction. Nowadays there is a necessity in decision-making support tools, systems which could allow to estimate current state of firm under changing market situation, to "play" possible consequences of making some decisions, to understand better the processes taking place inside the firm. All this requires an approach to modeling, considering inner firm structure.

Under changing market situation static models cannot be used for the purposes as many economic processes have transitional character and could be describe by dynamic models only. Various approaches to building of dynamic firm models could be found in [1-3].

In this paper a dynamic economic-mathematical model of firm is build using Jay W. Forrester's approach [4]. In the model unknown parameters of firm subdivisions and structure of interaction between the firm subdivisions are considered. The model is described by system of nonlinear finite-difference equations.

For the model built, unknown parameters and state estimation problem is solved under incomplete and inaccurate information about parameters and state of firm and about market situation. The problem is solved during firm functioning (in real time) as new data come from information system of the firm. The data of the information sys tem is considered to be incomplete and inaccurate. To solve the estimation problem, well-known methods of estimation theory are used [5-7,15]. It is shown, that decomposition of the model can provide more accurate estimates.

Actuality of the paper is confirmed by increasing number of publications considering similar problems [2,8]. The paper continues research [8,9] and develops approaches [4-7,10,11].

1. Dynamic economic-mathematical model of firm. The used approach [4] to manufacturing and selling firm modeling considers the firm as an abstract system with various flows circulating between its elements.

There are such types of flows as flows of information, materials, orders, money, labor. The basic elements of the system are abstract levels and delays, which can be represented as levels.

Each level is characterized by quantity of some type (e.g. materials, money) storing in it. Each delay element contains a variable amount of the quantity in transit. Rate of the flow circulating between elements of the system is a characteristic of influence of one element to another. It is considered, that rates of all flows are constant during modeling step. Hence, dependences between states of the system elements and rates of the corresponding flows can be represented as finite-difference equations of the sufficiently simple form. Such models are relatively simple and compact and can reproduce functioning of the rather complex real firms.

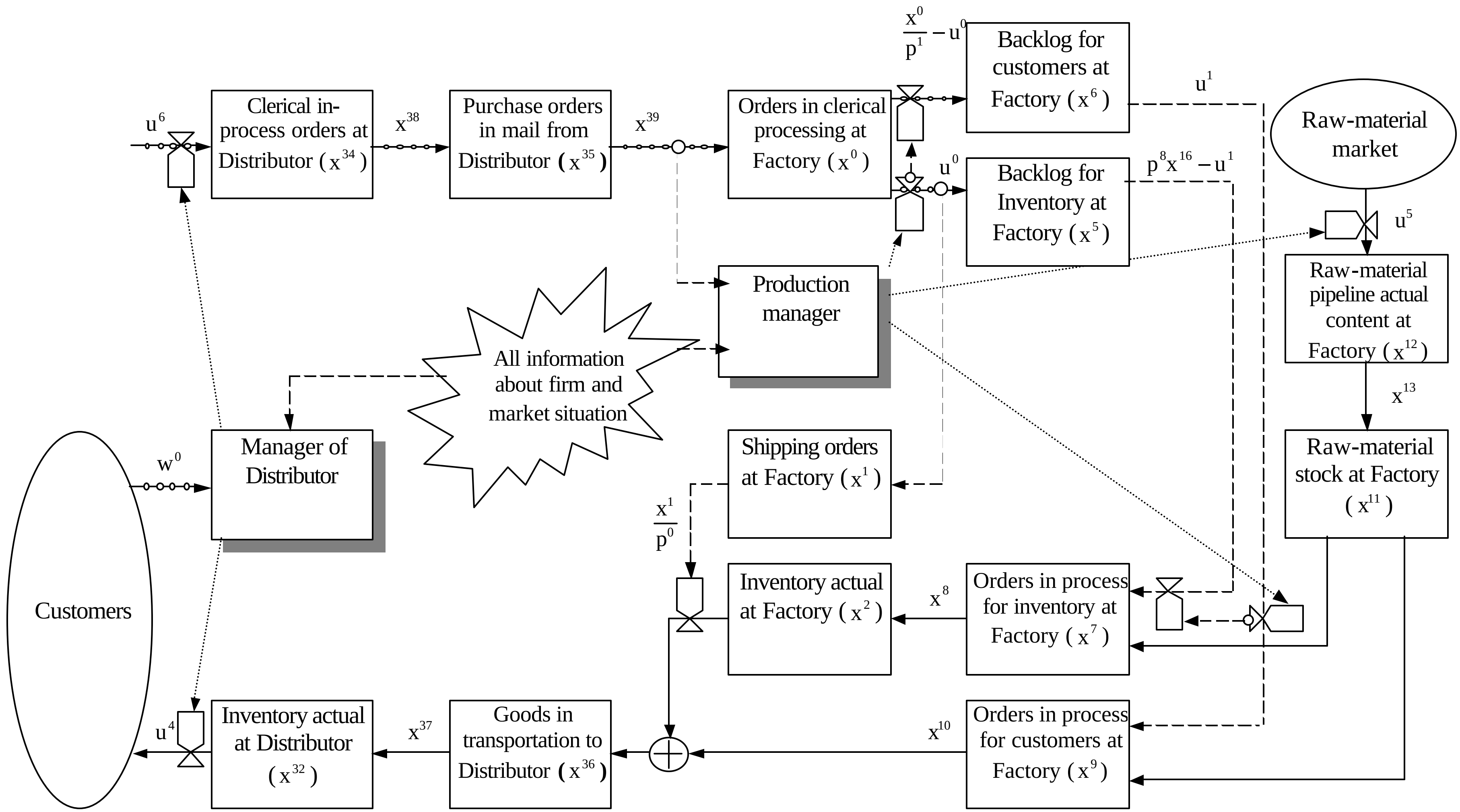


Figure 1. Structure of manufacture and selling firm

In the model built flows of orders, materials, labor, information and financial flows, structure of interactions between firm subdivisions (distributor, production) and parameters of the subdivisions (e.g. productivity of labor, delay in clerical processing at factory) are considered. Structure of the firm considered is shown in Figure 1.

Distributor receives orders from customers, delivers finished products accordingly to demand and sends orders to production. Production sends part of the finished products to store at factory warehouse; another part is shipped to distributor.

E.g. equations characterizing the change in store at factory warehouse are of the form

$$\left.\begin{aligned} x_{k+1}^{3} &= x_{k}^{3} + (T / p_{k}^{15})\left(\frac{x_{k}^{1}}{p_{k}^{0}} - x_{k}^{3}\right), \\ x_{k+1}^{4} &= x_{k}^{4} + (T / p_{k}^{16})(x_{k}^{40} - x_{k}^{4}), \end{aligned}\right\} \tag{1.1}$$

where x_{k+1}^{3} is average shipments from factory warehouse (units per week); p_{k}^{15} is time to average shipments from factory warehouse (weeks); x_{k}^{1} is shipping orders at factory (units); x_{k+1}^{4} is requisition rate smoothed at factory (units per week); p_{k}^{16} is time to requisition smoothing at factory (weeks); x_{k}^{40} is requisition rate received at factory (units per week); T is the time interval between solutions of the equations; $k = 0,1,\ldots$.

The structure considered is typical for many manufacturing and selling firms. Thus, the model built is described by a system of the following finite-difference equations

$$x_{k+1} = F(x_k, u_k, w_k, p_k) + Bu_k + \tilde{A}\; x_k , \tag{1.2}$$

$$y_{k+1} = Gx_{k+1} + h_{k+1} , \tag{1.3}$$

where $x_k \in R^n$ is state vector, describing firm behavior; $u_k \in R^m$ is control vector; $w_k \in R^r$ is vector, describing current market situation; $x_k \in R^t$ is vector, characterizing inaccuracy of information about the behavior of firm; $p_k \in R^s$ is vector of unknown parameters of firm; $F(x_k, u_k, w_k, p_k)$ – nonlinear vector-function; $y_k \in R^h$ – measurement vector; $B, \tilde{A}, G$ are constant matrices; $h_k \in R^h$ is vector of measurement.

Here, parameters is understood as some constants (e.g. quantity of production equipment, delay in clerical order processing at distributor), defining firm functioning, and state is set of variables reflecting the current state of the firm.

Control vector u_k includes such variables as rate of raw material purchases at production, labor hiring and dismissal rates, i.e. variables, which values could be changed by firm management.

Vector w_k includes factors, influencing on firm functioning, which values could not be changed by the firm management (e.g. demand, raw material price). Vector x_k has to consider not only inaccuracy of information about firm functioning, but possible deviation from normal functioning as well.

It is assumed that there is an information system at firm, measuring its state x_k with error h_k. Matrix G defines, what information about firm (what part of vector x_k and, maybe, parameters p_k) is available for measurement. Its size will vary significantly if firm management is interested in receiving this information or there is an external observer trying to receive more detailed information than available for public access. Obviously, an inner observer, estimating firm state for the firm has more detailed information about its current state. In the model it leads to increasing of vector

y_k dimension in (1.3) in comparison with the external observer and consequently to improving of estimation quality.

Thus, the problem of estimation of unknown parameters and state of firm can be reduced to estimation of parameters p_k and state x_k of the system (1.2) when measurements (1.3) are incomplete and inaccurate.

The whole vector x_k or its part can be available for measurement. The fact, whether a variable is measured or not, can be determined not only from its economic nature (e.g. it is simply to organize measurement of size of inventory at factory x_k^2, then delay in quoted delivery at factory x_k^{21}), but depends on the funds available for information system too. Moreover, the system (1.2)-(1.3) should be observable. If several separate subsystems of this system are to be estimated, it is necessary to check if all subsystems are observable separately. All equations of the model can be found in [8].

2. Solution of the estimation problem. Solution of the estimation problem is necessary not only to get more accurate information about state of firm when measurements are incomplete and inaccurate; it precedes building of firm control u_k under changing market situation [8,9].

To the moment the current estimate of state vector and unknown parameters is computing, the current market situation w_k is known. So, system (1.2), (1.3) can be rewritten for the problem of estimation in the form

$$x_{k+1} = f(x_k, u_k, p_k) + Bu_k \tilde{A}+ \; x_k, \tag{2.1}$$

$$y_{k+1} = Gx_{k+1} + h_{k+1}. \tag{2.2}$$

If it is possible to point out statistical characteristics of disturbances and errors in system (2.1), (2.2), it is reasonable to use kalman filtering methods [5,13]. The situation could take place e.g. when there is enough of statistical information about the firm to determine the statistical characteristics. Using of kalman filtering theory to solve macroeconomic problems is considered e.g. in [12].

Equation (2.1) is reduced to linear one using linearization procedure:

$$x_{k+1} = A_k x_k + Bu_k \tilde{A}+ \; x_k, \tag{2.3}$$

where x_k is a vector to be estimated.

For simultaneous estimation of the state vector x_k and vector of unknown parameters p_k the following equations are added to system (2.3)

$$p_{k+1} = p_k + x_k^p, \tag{2.4}$$

where x_k^p is a noise vector, characterizing level of confidence, that elements of vector p_k are constant. If vector p_k is known to be constant, x_k^p could be omitted.

Then, for simultaneous estimation of state and unknown parameters of firm, vector x_k designates the extended vector $[x_k, p_k]'$, where $'$ is the transposition operator. In case of non-measurable parameters, s nonzero columns are added to matrix G.

A peculiarity of the approach to modeling, used in the paper, is relative simplicity of decomposition of the model built to several separate subsystems of smaller dimension. Such subdivisions of the firm as distributor and production or the whole flows (e.g. information, products, money) could be considered as subsystems. Then equations of each i-th subsystem could be written in the following form

$$x_{k+1}^i = f_i(x_k^i, u_k^i, p_k^i, z_k^i) \tilde{A}+ \; {}^i x_k^i, \tag{2.5}$$

$$y_{k+1}^{i} = G^{i} x_{k+1}^{i} + h_{k+1}^{i}, \qquad (2.6)$$

where z_k^i is vector (known) of variables from other subsystems. E.g. for the system (1.1) the extended vector $x_k = \left[x_k^3, \quad x_k^4, \quad p_k^0, \quad p_k^{15}, \quad p_k^{16}\right]'$, first two elements of matrix G main diagonal is equal to 1, the other elements are zero, $z_k = \left[x_k^1, \quad x_k^{40}\right]'$ (upper index is omitted).. Values of the measurement vector y_k correspond to values of variables x_k^3 and x_k^4, received from information system of firm.

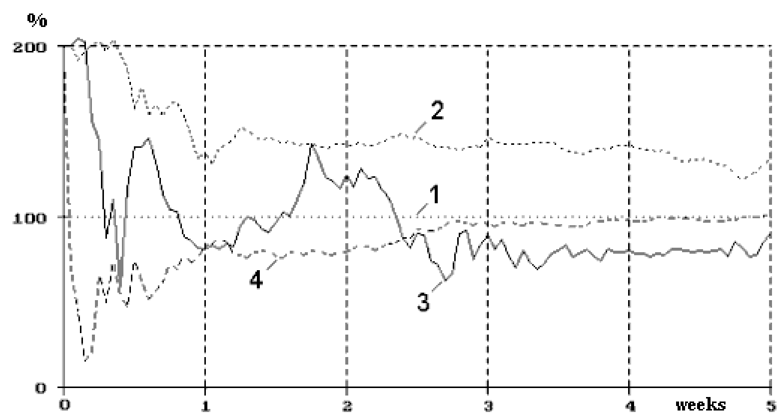


Figure 2. Estimate of parameter p^{26} (proportionality constant for inventory at distributor)
1 – real parameter value, 2 – estimation of the whole system, 3 – estimation of the model under decomposition, 4 – measurable parameter estimation

Then the estimation problem is solved separately for each subsystem. Dimension of state vector of a subsystem, as usual, significantly higher then dimension of variable vector of other subsystems. In practice, it leads to decreasing of linearization errors and, consequently, to higher quality of estimation (Fig. 2).

Additionally, it facilitates possible changing and addition of the model. If it is necessary to estimate parameters of some part of subsystems only, it is possible to not spend time and resources for estimating of parameters of the whole system.

If for some reason estimate of a parameter is considered unsatisfactory it is reasonable to organize measurement of the parameter by information system of the firm. In this case estimates are most accurate (Fig. 2).

This estimation method is rather simple and do not require high computational resources. But it has well known disadvantages: it requires prior information about statistical characteristics of disturbances and errors, and about initial value of estimated vector. Besides it, to prevent divergence of the estimation algorithm, a special modification of the method was used [13].

An alternative to estimation is a guaranteed approach, when disturbances and errors are considered as sets of some type [6-7,10,11]. In this case it is most convenient to use rectangular parallelepipeds, allowing representing of restrictions in the most simple and clear for non-experts form.

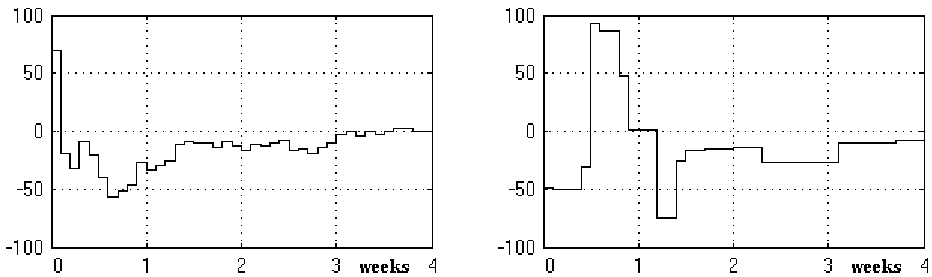
The basic estimation equations in this case are of the form

$$X_{k/k-1} = A_{k-1}\hat{X}_{k-1} + \Gamma W_{k-1}, \qquad (2.7)$$

$$X[y_k] = M\left[x / y_k,\ h \in V_k\right] = \{x \mid G_k x + h = y_k, h \in V_k\}, \qquad (2.8)$$

$$\hat{X}_k = X_{k/k-1} \cap X[y_k], \qquad (2.9)$$

where W_k and V_k are sets describing possible values of disturbances h_k and measurement errors x_k respectively. As an estimate $\hat{x}_k$ Chebyshev center is taken.



a) b)

Figure. 3. Estimation of variable x^4 (requisition rate smoothed at factory)
a – casual disturbances and measurement errors; b – guaranteed approach

Here, as in the previous method, estimates are received in real time and are constantly improving. Though, in this case the estimates are more pessimistic (Fig. 3).

Conclusion. The estimation is necessary to find an optimal control of the firm and is solved under incomplete and inaccurate information about parameters, state and market situation.

The problem is solved during firm functioning (in real time) as new data come from information system of the firm. The data of the information system is considered to be incomplete and inaccurate. Solution of the estimation problem is described when disturbances and information system errors are casual or have uncertain nature. It is shown, that decomposition of the model can provide more accurate estimates.

Results of the work can be used in automated control systems, in expert systems of decision-making support and by businessmen for analysis of decisions made.

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