Loan Portfolio Dynamics

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Abstract: *This paper presents an approach to credit risk modeling that builds on standard techniques to develop a system dynamics model. To this author's knowledge, this is the first attempt to analyze a loan portfolio using transition matrices within a system dynamics framework. The paper shows how a simple model considering the stock and flow structure of a loan portfolio can give valuable information about the performance of the portfolio over time and analyzes the steady state equilibrium. The simulation results of a more detailed model indicate that misperception of the dynamic structure and the use of decision heuristics to determine growth strategies and pricing may generate significant oscillations. This is true even in this simplified framework, in which a single and isolated bank is modeled in an environment of fixed funding rate, and the response to a single discrete change in parameters is considered.*

Key words: Loan Portfolio, Credit Risk, Transition Matrix, Return on Equity.

1. Introduction

Credit risk modeling has become increasingly popular among academics and financial intermediaries. Several models have been developed to price the risk of default (see Madan, 2000 for a survey). Also some commercial risk models, namely CreditRisk of Credit Suisse First Boston (1997) and Credit Metrics of JP Morgan (1999), which focus on determining the distribution of losses for a portfolio due to credit risk, are widely used in practice. These are statistical models that derive estimated losses for a portfolio from individual exposures' default probabilities, volatilities and recovery rates. Many new portfolio models rely on credit rating migration matrices. For valuation purposes, the matrix is used to estimate the probability of being in, and the market spread of, each possible credit rating in the future during the tenor of the asset. A present value of this contingent state payoffs determine the value of the asset.

However, as explained by Aguais and Santomero (1997), in spite of the sophisticated pricing techniques, banks frequently fail to price their loans correctly and therefore returns do not cover the true costs. Furthermore, this imbalance is repeated as credit spreads go through the trough of their cycles. Our hypothesis is that the misperception of the dynamic structure of a loan portfolio is the main cause of the regular tight spreads. Drawing on the extensive work done on credit risk in the finance literature, we build a system dynamics model which analyzes the structure of a loan portfolio. We focus our analysis on the dynamics the structure generates over time, in particular looking at portfolio returns and the connection with growth and pricing decisions. The main contributions of the model presented here are the following:

- It analyzes the determinants of return on equity for a loan portfolio in steady state equilibrium, based on a simplified Markov structure.
- It studies how growth and contraction affect the instantaneous return on the portfolio. In particular, we focus on the consequences of credit losses materializing with a lag after loan underwriting while revenues are generated immediately. To this author's knowledge, no work has been done on this issue.
- We also make interest spreads endogenous by introducing a growth policy based on performance, which uses the credit spread as transmission mechanism.
- Finally, the model simulates the effect of changing economic conditions from the standpoint of a single bank.

Jobst and Zenios (2001), develop a pricing model for a portfolio of bonds that incorporates both credit migration and stochastic interest rates. However, in their analysis simulated economic scenarios determine the interest rate and credit spreads, but the transition probabilities in the migration matrix are constant. On the other hand, Bangia, Diebold, Kronimus, Schagen and Schuermann (2002) analyze a long series of migration data from S&P and note that migration probabilities are sensitive to economic activity. Our model makes pricing an endogenous decision, and also considers changes in the economic environment.

The stock and flow structure of a portfolio plays a key role, as the relative weight of performing and non-performing assets is what determines the profitability of the portfolio. The composition changes according to the migration probabilities, determined by the economic scenario. In turn, profitability determines growth plans, and this is the base for pricing decisions. Since changes in the size of the portfolio feed back on the portfolio composition and profitability with a significant lag, the effects of changing economic conditions are propagated over time.

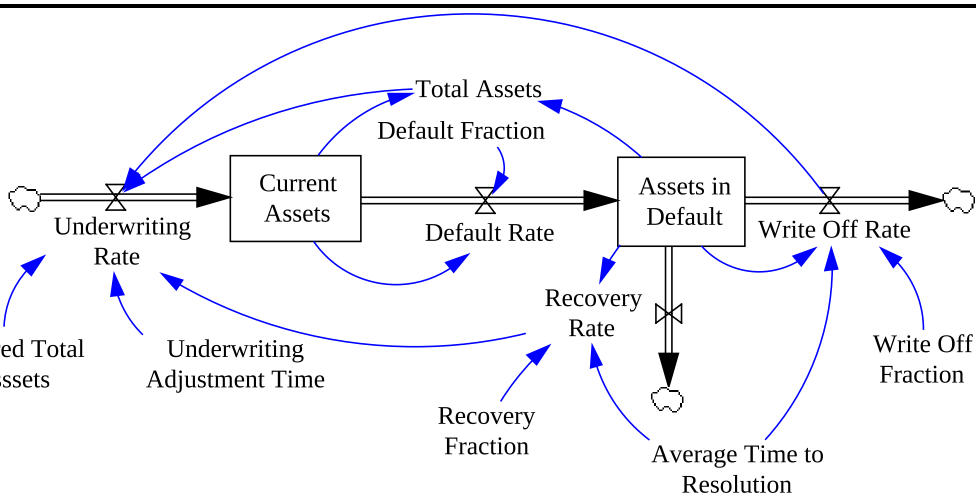
Section 2 presents a simplified version of the model, which analyzes the determinants of portfolio returns. In section 3 the model is expanded to include several credit quality states, endogenous determination of growth and a credit demand function. The simulation results of the complete model are presented in section 4. The richer model shows the oscillatory response of the system to macroeconomic shocks. Section 5 covers the conclusions and some directions of future research in the subject.

2. The basic model

2.1 The core structure of a loan portfolio

The basic structure of the model refers to the different possible states of each asset in a portfolio from a credit quality standpoint, being the state of default the one of lowest quality. Following the pioneering work by Merton (1974), default events can be modeled as a drop in the firm's assets below a certain threshold. Although this approach has provided the basis for a vast literature, it is difficult to use in practical applications. Instead, we follow Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997) modeling approach, which introduces transition probabilities as exogenous processes and specifies the default event as the transition to this particular state. However, in this simplest version of the model, we only separate the assets in two categories: "current" comprising all non-default states (ratings AAA to CCC in S&P terminology) and "in default". New assets enter the portfolio as current, and therefore generate revenues for the lender. Eventually, some of the current clients fail to pay their obligations and fall into the "in default" category. After some time, assets in default are either written off causing a loss or recovered. The default state is assumed to be absorbing. However, we consider that for each recovered or written off asset, a new loan is underwritten, so that the size of the portfolio is maintained. Figure 1 below shows this core stock and flow structure.

Figure 1: Clients stock and flow structure



There are four hazard rates that determine the dynamics of the structure shown above. The *Underwriting Rate* is modeled as a typical adjustment to goal structure building a negative feedback loop and giving the rate a goal-seeking behavior. Following the standard formulation for default rates of Jarrow et al (1997), the *Default Rate* is modeled as a fractional decrease rate. The *Recovery Rate* and the *Write Off Rate* are also modeled as a fractional decrease. Our model assumes that the size of the portfolio is a goal and therefore each written off or recovered loan is replaced with a new one. The equations for the four rates are the following:

$$Underwriting\ Rate = \frac{(Desired\ Total\ Assets - Total\ Assets)}{Underwriting\ Adjustment\ Time} + Recovery\ Rate + Write\ Off\ Rate \tag{1}$$

$$Default\ Rate = Current\ Assets \times Default\ Fraction \tag{2}$$

$$Recovery\ Rate = Assets\ in\ Default \times \frac{Recovery\ Fraction}{Average\ Time\ to\ Resolution} \tag{3}$$

$$Write\ Off\ Rate = Assets\ in\ Default \times \frac{Write\ Off\ Fraction}{Average\ Time\ to\ Resolution} \tag{4}$$

It follows from the stock and flow structure and the equations above that the system has a steady state equilibrium when the *Default Rate* equals the *Recovery Rate* plus the *Write Off Rate*.

$$Default\ Rate = Recovery\ Rate + Write\ Off\ Rate \tag{5}$$

This is an obvious result for those who have studied system dynamics. All feedback loops are balancing in this structure, and the only possible equilibrium is when the flow into each stock equals the flow out of the stock. Intuitively, if the *Underwriting Rate* is greater than the net *Default Rate*, then the stock of *Current Assets* increases pushing the *Default Rate* up. Similarly, if the *Default Rate* is higher than the *Write Off Rate* plus the *Recovery Rate*, then the level of *Assets in Default* grows, increasing the *Write Off Rate* and the *Recovery Rate*. Therefore, starting from any portfolio composition, the system reaches an equilibrium with a determined structure of current assets and assets in default. In particular, using the steady state equilibrium equation, the ratio of assets in default to current assets is the following[1]:

$$\frac{Assets\ in\ Default}{Current\ Assets} = Default\ Fraction \times Avg.\ Time\ to\ Resolution \tag{6}$$

Rearranging, this can be written as

$$Assets\ in\ Default = Current\ Assets \times Default\ Fraction \times Avg.\ Time\ to\ Resolution \tag{7}$$

This is equivalent to Little's Law for queuing systems (see Winston 1994 for an overview on queuing theory), in which *Assets in Default* is the average number in the system, *Current Assets * Default Fraction* is the average arrivals entering the system and *Average Time to Resolution* is the average time spent in the system.

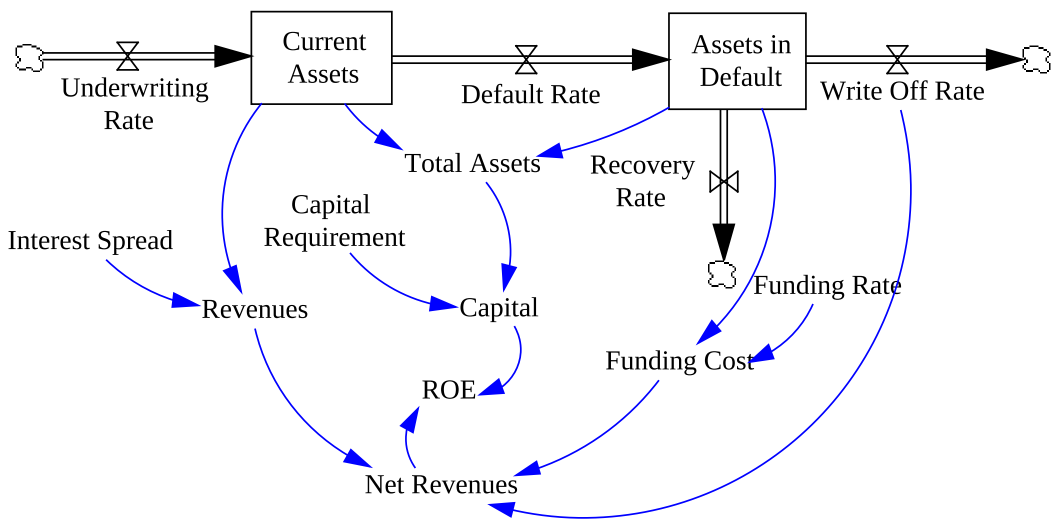
If there is a steady state equilibrium and, by definition, assets are underwritten as current, then the ratio of *Assets in Default* to *Current Assets* must be below the equilibrium value when the portfolio is growing. Also, the inverse is true for a shrinking portfolio, since also by definition, only current assets can be collected. To see this clearly, we can consider the case of a newly created portfolio. Immediately after the new loans are underwritten, 100% of the assets are current and therefore the ratio is 0, but when enough time has passed and the system has reached the steady state equilibrium, the ratio must be positive. Since current assets generate revenues and assets in default generate losses, this dynamic behaviour in the portfolio structure indicates that the return on the portfolio varies as the size of the portfolio changes.

[1] Note that *Recovery Fraction* + *Write Off Fraction* = 1

2.2 Revenues, Funding Cost and Write Off

So far we have concentrated on the flow of assets through the system. We will now introduce revenue generation and costs of bad loans explicitly. *Current Assets* and *Interest Spread* (the mark-up over the Funding Rate) determine *Revenues*, while *Assets in Default* together with the *Funding Rate* determine the *Funding Cost*. In addition, the *Write Off Rate* is the cost of assets deemed unrecoverable. From an accounting point of view, interests are not accrued for assets in default, therefore the funding cost has to be accrued (note that only the spread is registered as revenue in financial businesses). Figure 2 shows net revenue generation from the portfolio composition of assets.

Figure 2: Revenues and costs generation



The equations are shown below:

$$Revenues = Current\ Assets \times Interest\ Spread \quad (8)$$

$$Funding\ Cost = Assets\ in\ Default \times Funding\ Rate \quad (9)$$

$$Net\ Revenues = Revenues\text{-}Funding\ Cost\text{-}Write\ Off \quad (10)$$

From equations (6), and (8) – (10), we can determine the ratio of *Net Revenues* to *Revenues*:

$$\frac{Net\ Revenues}{Revenues} = 1 - \frac{Default\ Frn. \times \left(Write\ Off\ Frn. + Funding\ Rate \times Avg.\ Time\ to\ Resolution\right)}{Interest\ Spread} \quad (11)$$

The implications of this equation are compatible with the intuition. The *Default Fraction*, the *Funding Rate*, the *Average Time to Resolution* and the *Write Off Fraction* have a negative impact on the *Net Revenues* ratio in steady state, while the *Interest Spread* has a positive impact.

2.3 Return on equity

The results obtained can be used to determine long-term return on equity (*ROE*) for the portfolio. The *ROE* can be expressed as the product of three different ratios as shown in equation 12 below. Note that no expenses other than funding costs and write off are included. For a simple modeling of other expenses the Revenue/Expense Ratio (*RE*), a commonly measured ratio in the industry, could be introduced and then all results shown in this section should be corrected adjusting *Revenues* by (*1-1/RE*).

$$ROE = \left(\frac{Net\ Revenues}{Current\ Assets}\right)\times\left(\frac{Current\ Assets}{Total\ Assets}\right)\times\left(\frac{Total\ Assets}{Equity}\right) \tag{12}$$

We can determine each of these three components of *ROE* separately. The first term can be in turn expressed as the product of *Net Revenues/Revenues*, for which we have found an equation already, and the *Interest Spread*, or *Revenues/Current Assets*, therefore

$$\frac{Net\ Revenues}{Current\ Assets} = Interest\ Spread\text{-}Default\ Frn.\times\left(Write\ Off\ Frn. + Funding\ Rate\times Avg.\ Time\ to\ Resolustion\right) \tag{13}$$

Since *Total Assets* is the sum of *Current Assets* and *Assets in Default*, and using equation (6), the second term can be expressed as

$$\frac{Current\ Assets}{Total\ Assets} = \frac{1}{1 + Default\ Frn.\times Avg.\ Time\ to\ Resolution} \tag{14}$$

Finally, the third term is simply

$$\frac{Total\ Assets}{Equity} = \frac{1}{Capital\ Requirement} \tag{15}$$

Putting the three elements together, the expression is

$$ROE = \frac{Interest\ Spread\text{-}Default\ Frn.\times\left(Write\ Off\ Frn. + Funding\ Rate\times Avg.\ Time\ to\ Resolution\right)}{Capital\ Requirement\times\left(1 + Default\ Frn.\times Avg.\ Time\ to\ Resolution\right)} \tag{16}$$

This equation gives therefore the long-term *ROE* for a portfolio, based on six variables that can be easily estimated at any moment in time. It can also be used as a pricing tool, giving directly the minimum *Interest Spread* for profitability (the term after the minus in the numerator) and, rearranging to make it the dependent variable, determining the required *Interest Spread* for a given *ROE* goal. Of the variables involved in the equation, the *Capital Requirement* is determined by regulation, the *Interest Spread* and the *Funding Rate* can be related to policy decisions, but are significantly constrained by market conditions. Finally, the *Default Fraction*, the *Average Time to Resolution* and the *Write Off Fraction* depend primarily on credit risk policies and recovery management, although related to market situation. The impact of each of the variables on *ROE* has been analyzed.

However, as the only difference between *Return on Assets* (*ROA*) and *ROE* is that the second is divided by *Capital Requirement*, we show the impact on *ROA* to avoid repetition, which can be transformed into *ROE* by dividing by *Capital Requirement*.

Interest Spread and Funding Rate

Interest Spread has always a positive impact on *ROA*, but this is always lower than one. Similarly, the *Funding Rate* has a marginal impact between minus one and zero.

$$\frac{\partial ROA}{\partial (Interest\ Spread)} = \frac{1}{(1 + Default\ Frn. \times Avg.Time\ to\ Resolution)} \tag{17}$$

$$\frac{\partial ROA}{\partial (Funding\ Rate)} = -\frac{Default\ Frn.\ \times Avg.\ Time\ to\ Resolution}{(1 + Default\ Frn. \times Avg.Time\ to\ Resolution)} \tag{18}$$

The question of whether it is better to increase interest spread or to lower the funding cost depends on the ratio of *Assets in Default* to *Current Assets*, as shown in equation 18. This would normally be less than 1 and therefore indicates that it is always better to increase *Interest Spread*. The lower the *Default Fraction* and the shorter the *Average Time to Resolution*, the higher the effect of *Interest Spread* relative to *Funding Rate*.

$$\frac{-\dfrac{\partial ROA}{\partial \left(Funding\ Rate\right)}}{\dfrac{\partial ROA}{\partial \left(Interest\ Spread\right)}} = Default\ Frn.\ \times Avg.\ Time\ to\ Resolution = \frac{Assets\ in\ Default}{Current\ Assets} \tag{19}$$

However, if the *Interest Spread* is increased by reducing the funding cost, i.e. the funding cost is reduced while the active rate is maintained, then the impact on *ROA* is proportional to the spread increase and this is always better than increasing the interest spread by rising the active rate.

$$\frac{\partial ROA}{\partial (Interest\ Spread)} - \frac{\partial ROA}{\partial (Funding\ Rate)} = 1 \tag{20}$$

Default Fraction

The effect of increasing the *Default Fraction* on equilibrium *ROA* is shown below. As expected, the *Default Fraction* always has a negative impact on *ROA*.

$$\frac{\partial ROA}{\partial (Default\ Fraction)} = -\frac{Write\ Off\ Fn. + Avg.\ Time\ to\ Resolution \times \left(Funding\ Rate + Interest\ Spread\right)}{\left(1 + Default\ Fn. \times Avg.\ Time\ to\ Resolution\right)^2} \tag{21}$$

Average Time to Resolution and Write Off Fraction

$$\frac{\partial ROA}{\partial (Avg.\ Time\ to\ Resolution)} = \frac{Default\ Fn. \times \left[Default\ Fn. \times Write\ Off\ Fn. \text{-} \left(Funding\ Rate + Insterest\ Spread\right)\right]}{\left(1 + Default\ Fn. \times Avg.\ Time\ to\ Resolution\right)^2} \tag{22}$$

We show below that for all positive values of ROE, the Average Time to Resolution has a negative impact on ROE.

$$\begin{aligned}&ROE > 0 \quad \text{iff}\\ &\textit{Interest Spread} > \textit{Default Frn.}\times\left(\textit{Write Off Frn.} + \textit{Funding Rate}\times \textit{Avg. Time to Resolution}\right)\\ &\textit{Interest Spread} - \textit{Default Frn.}\times \textit{Funding Rate}\times \textit{Avg. Time to Resolution} > \textit{Default Frn.}\times \textit{Write Off Frn.}\end{aligned}$$

And since *Default Fraction*, *Funding Rate* and *Average Time to Resolution* are all positive,

$$\textit{Interest Spread} + \textit{Funding Rate} > \textit{Default Frn.}\times \textit{Write Off Frn.} \tag{23}$$

As expected, the impact of the *Write Off Fraction* is always negative and less than 1 in absolute value.

$$\frac{\partial ROA}{\partial\left(\textit{Write Off Fraction}\right)} = -\frac{\text{Default Frn.}}{\left(1+\text{Default Frn.}\times\text{Avg. Time to Resolution}\right)} \tag{24}$$

Another interesting issue that can be explored analytically is the relative convenience of an early recovery against a higher recovery. For a single loan, it is simply a matter of present value, but for the steady state equilibrium of a portfolio the answer is more complex. We show below the impact of a 1% increase in *Average Time to Resolution* compared with a 1% decrease in the *Write Off Fraction*. The sign of the difference in the semi-elasticities, i.e. the difference between the positive impact of lowering the write off and the negative impact of extending the time to resolution, depends on the parameters. For high *Write Off Fraction*, high *Default Fraction* and low *Average Time to Resolution*, reducing the Write Off Fraction is relatively more convenient. On the other hand, when the *Funding Rate* and the *Interest Spread* are relatively high, early recoveries are better.

$$\begin{aligned}&\frac{\partial ROA\times\text{Avg. Time to Resolution}}{\partial\left(\textit{Avg.}\,\text{Time to Resolution}\right)} - \frac{\partial ROA\times\text{Write Off Frn.}}{\partial\left(\textit{Write Off Fraction}\right)} = \text{Default Frn.}\times\text{Avg. Time to Res.}\times\\ &\qquad\left[\text{Write Off Frn.}\times\left(2\times\text{Defualt Frn.}+\frac{1}{\text{Avg. Time to Res.}}\right)-\left(\text{Funding rate}+\text{Interest Spread}\right)\right]\end{aligned} \tag{25}$$

3. The complete model

In this section we extend our model to include several states within the current assets, we define a growth policy based on performance and we also introduce a demand function. This puts together a richer dynamical structure of a loan portfolio, with a decision-making mechanism. In each sub-section we present a part of the model.

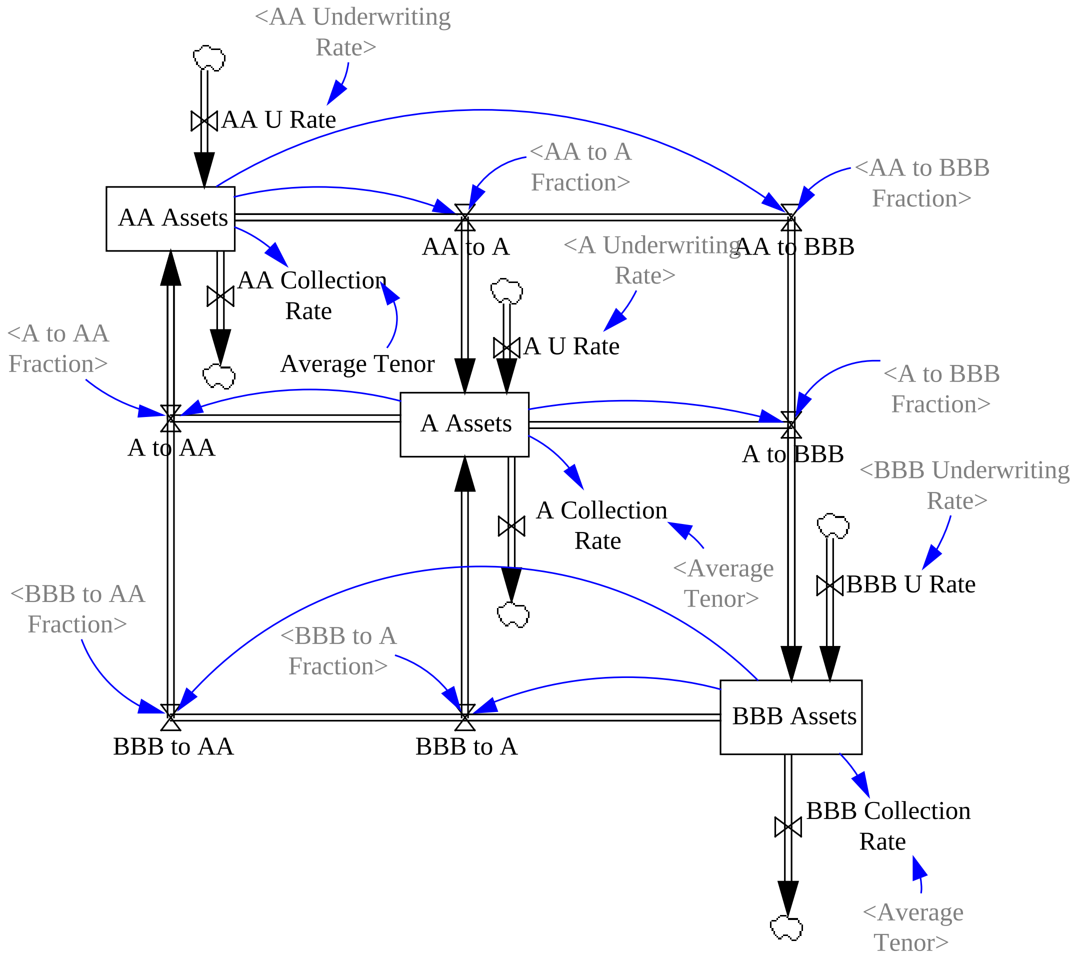
3.1 The transition matrix

Transition matrices have been widely used for modeling credit risk and pricing of credit derivatives. As we introduced in our simpler model for the transition from current to default states, transition probabilities can be related to the probability of the value of and asset falling below a certain threshold. Extending this reasoning, thresholds can be determined for transitions between other states. The result is a finite state space Markov chain (Jarrow et al, 1997), which is commonly denominated transition matrix. The empirically determined transition matrix can be used to calculate the probability of being in each state, including default, in the future, given an initial rating. This methodology allows for a dynamic approach to pricing risky securities. However, we give the transition matrix an alternative use, which is to actually simulate the dynamics of a portfolio of loans over time, and to calculate the returns on such portfolio during the simulation.

We base our work on the data presented in Bangia, Diebold et al. (2002), which uses data from Standard & Poor's CreditPro™ 3.0 database. Modifiers +/- are eliminated in this work and therefore the seven resulting categories are AAA, AA, A, BBB, BB, B and CCC. In addition there is an absorbing state of default. The sample contains 7,328 obligors between 1981 and 1998, 166,000 obligor quarters of data. This work estimates the unconditional quarterly transition matrix and also the transition matrices considering only the quarters of economic expansion or contraction, for what they label each quarter in their sample according to the National Bureau of Economic Research classification. This serves our purpose of analyzing the dynamics of a loan portfolio when economic conditions change.

Each rate I to J, where I and J are different ratings, is modeled as the default rate in the previous section. For example the rate *AAA to AA* is equal to *AAA Assets* * *AAA to AA Fraction*. In addition, we introduce here an *Average Tenor* for loans that determines the collection level in each quarter. On the other hand, the *Underwriting Rate* includes *Total Collection* in addition to the *Write Off Rate* and the *Recovery Rate*, to ensure that the size of the portfolio is maintained. Figures 3 shows the expanded stock and flow structure of assets in the portfolio for AA, A and BB as a sample of the structure. To analyze the dynamics of this structure, we assume that only loans are underwritten according to som *Underwriting Share* of total underwriting for each rating. This structure has also a stable steady state equilibrium as in the simpler case. The revenues and costs are generated as in the simpler model.

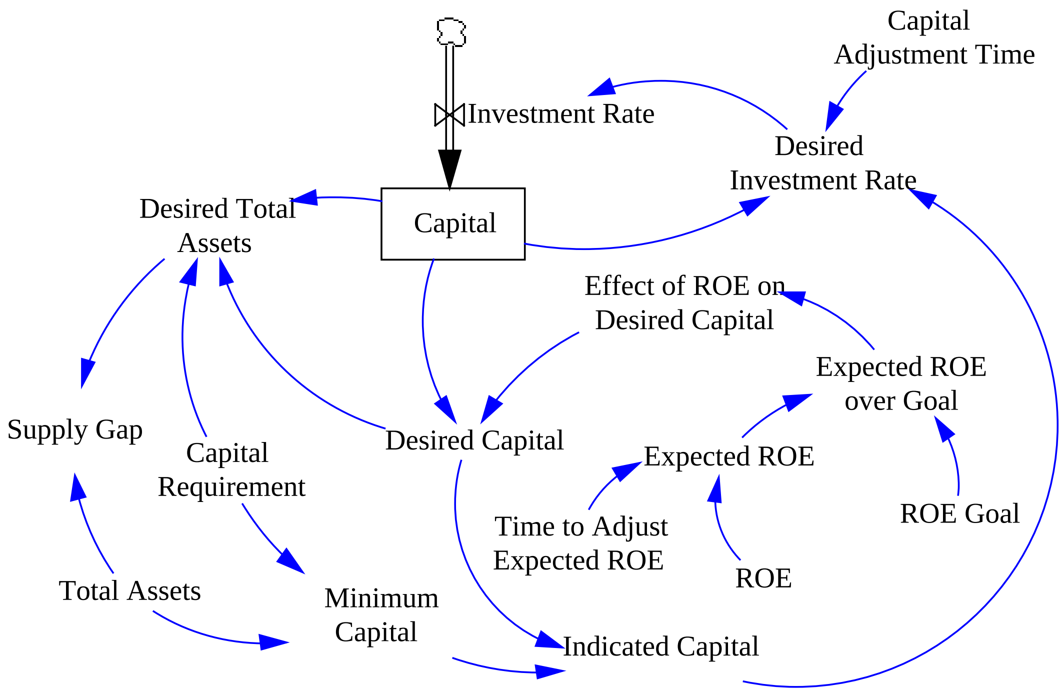
Figure 3: The transition matrix



3.2 Investment decisions

Investment decisions are based on behavioral decision theory. The bank does not decide its investment level using optimization techniques in this model. We rather introduce a simpler mechanism in which if *Expected ROE* is above the *ROE Goal*, then more resources are allocated to the portfolio, but if it is below this reference value, the desired size of the portfolio is reduced. *Expected ROE* is based on past *ROE,* which therefore determines the desired exposure. This type of anchoring and adjustment mechanism is intended to be rational, as it directs more resources to the portfolio when returns are high. The capital could be thought of being provided by a corporation headquarters or by the capital markets. In any of the cases it is reasonable to assume that a *ROE* above the expected one, which could be based on history or industry averages, attracts more resources. Figure 4 shows the investment decision process.

Figure 4: Investment decisions



Capital is increased or decreased by the *Investment Rate*, which can be negative, and is determined by the gap between *Indicated Capital* and actual *Capital*.

Capital = INTEGRAL(Investment Rate, Capital$_0$) (26)

Investment Rate = Desired Investment Rate (27)

Desired Investment Rate = (Indicated Capital – Capital)/Capital Adjustment Time (28)

Since there is usually a regulatory requirement on *Capital* level for financial institutions, *Indicated Capital* must be the maximum of the *Minimum Capital* required and the *Desired Capital*.

Indicated Capital = Max(Desired Capital, Minimum Capital) (29)

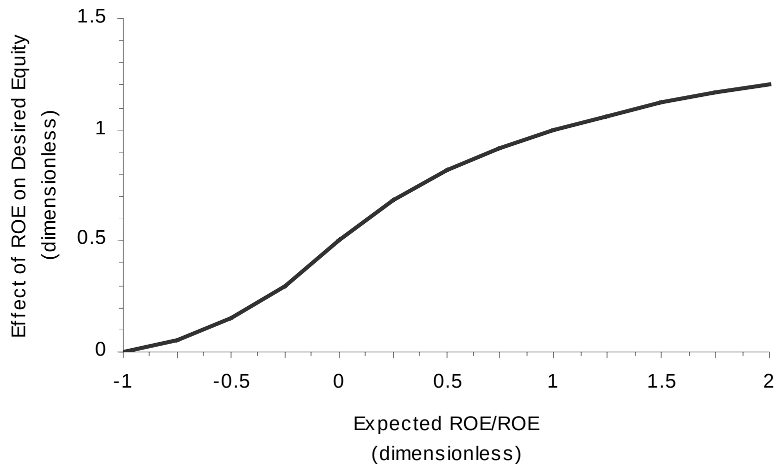
*Minimum Capital = Total Assets * Capital Requirement* (30)

The *Desired Capital* is determined by the actual level of *Capital* and the *Effect of ROE on Desired Capital*. *Expected ROE* is compared to the *ROE Goal* and *Capital* is adjusted accordingly. The table function used in shown in Figure 5. When *Expected ROE* equals *ROE Goal*, *Desired Capital* equals actual *Capital*. For any value of *Expected ROE* below the *ROE Goal*, the size of the portfolio is reduced. However, note that *Desired Capital* is 0 only when *Expected Capital* is significantly negative. Although a negative expected return on investment should trigger the liquidation of the portfolio, managers can blame market conditions or other external factors to reduce exposure only partially. As the expected loss is more important, managers become less confident in the business and eventually decide to quit and write down the investment.

*Desired Capital = Capital * Effect of ROE on Desired Capital* (31)

Effect of ROE on Desired Capital = FUNCTION(Expected ROE/ROE) (32)

Figure 5: Effect of ROE on Desired Equity Function



Expected ROE is adjusted to actual *ROE* with a time lag. There is a reporting lag for the latest figures, then there is also an averaging time to determine the *ROE* of the portfolio and finally there is a time period required for manager to adapt their expectations of future *ROE* to the average past value. A SMOOTH3 function seems a reasonable approach for this case.

Expected ROE = SMOOTH3(ROE, Time to Adjust Expected ROE) (33)

The level of *Desired Total Assets* is determined by the minimum of *Capital* and *Desired Capital*. If the portfolio is growing, *Capital* is the constraining resource, but if it is declining the *Capital* limitation is not active and *Desired Total Assets* are adjusted to *Desired Capital*.

Desired Total Assets = MIN(Desired Capital, Capital)/Capital Requirement (34)

The *Desired Total Assets* compared to the actual level of *Total Assets* determine the *Supply Gap*, which will be the basis for adjusting interest spreads.

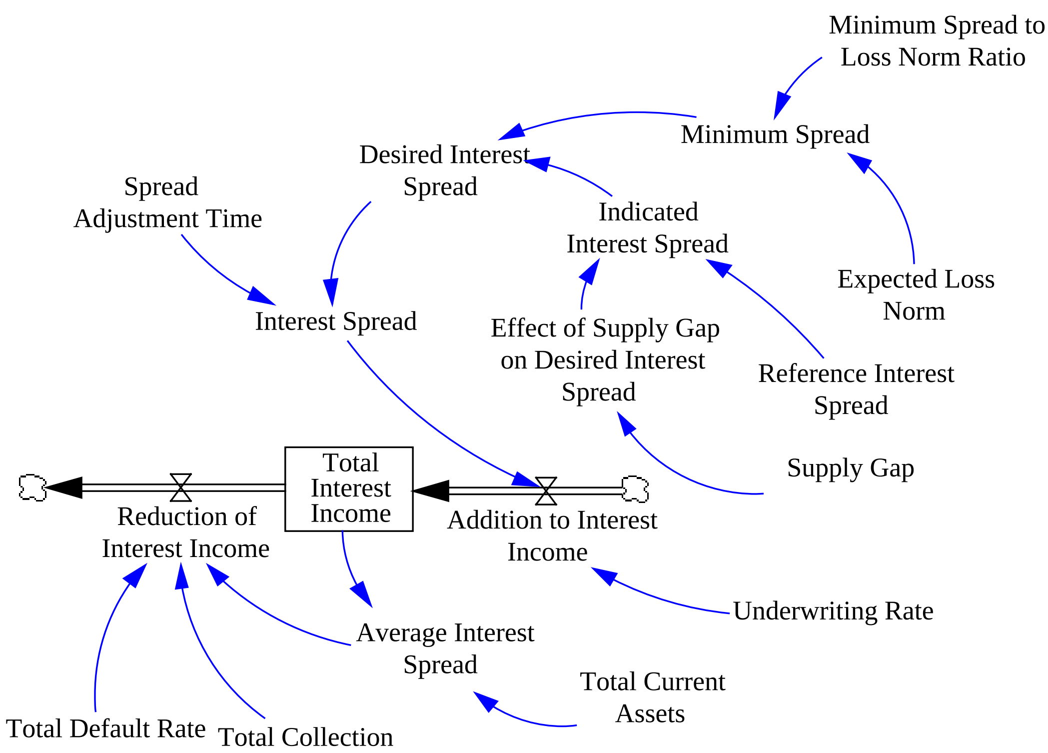
Supply Gap = Desired Total Assets/Total Assets (35)

The *Desired Total Assets* is therefore positively related to the *Interest Spread* level. This would indicate an upward sloping supply function. However, the relationship is circular, as the interest spread is changed to close the exposure gap. Regarding the decision making process, it is in fact precisely the gap between the desired exposure and its actual level, that determines the interest spread and not the other way round. The model assumes therefore that the banks act in oligopolistic competition, changing interest spreads according to their desired exposure, while the borrowers are price takers, deciding on the amount borrowed depending on the interest rate level. The pricing mechanism to close the supply gap is described in the next section.

3.3 Pricing mechanism

In a typical credit business the interest spread is the main instrument to manage the level of exposure. The interest rate is lowered to expand the portfolio and increased to reduce the exposure. As in the case of investment decisions, the interest spread adjustment is modeled as anchoring and adjustment process. This mechanism is equivalent to the underlying reasoning in Betrand-type models of oligopolistic competition. In these models, each player has incentives to lower the price (interest spread in our model) if profits are positive and to raise them if they are negative. This gives a unique equilibrium in which no firm has positive profits[2] (Mas-Colell, Whinston and Green 1995 provide a clear analysis of oligopolistic competition models). Figure 6 shows the mechanism.

Figure 6: Price adjustment



[2] Profits in our model are equivalent to a ROE above the expected value.

Since the interest rate at which loans are given changes over time, an average has to be tracked to determine the revenues of the portfolio. The co-flow formulation is used.

Average Interest Spread = Total Interest Income/Total Current Assets (36)

Interest Income = (Addition to Interest Income – Reduction of Interest Income, Interest Income$_0$) (37)

*Addition to Interest Income = Total Underwriting Rate * Interest Spread* (38)

*Reduction of Interest Income = (Total Collection + Total Default Rate) * Average Interest Spread* (39)

The *Interest Spread* is actually adjusted with exponential smoothing to the desired level, which is constrained to be above the expected loss norm, itself a smoothing on historical loss norm.

Interest Spread = SMOOTH(Desired Interest Spread, Spread Adjustment Time) (41)

Desired Interest Spread = Max(Indicated Interest Spread, Minimum Spread) (42)

*Minimum Spread = Expected Loss Norm * Minimum Spread to Loss Norm Ratio* (43)

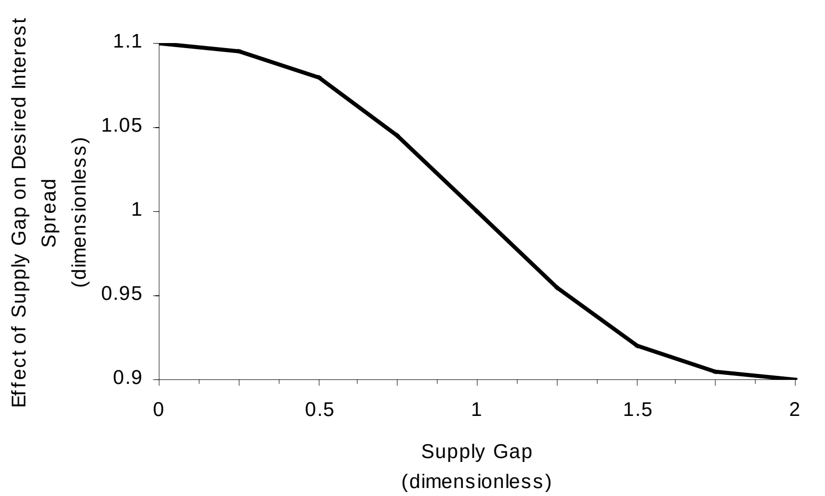
Expected Loss Norm = SMOOTH3(Indicated Loss Norm, Loss Norm Adjustment Time) (44)

Indicated Loss Norm = (Funding Cost + WO)/Total Assets (45)

*Indicated Interest Spread = Interest Spread * Effect of Supply Gap on Desired Interest Spread* (46)

Effect of Supply Gap on Desired Spread = FUNCTION(Supply Gap) (47)

Figure 7: Effect of Supply Gap on Desired Interest Spread Function



Finally, the *Interest Rate* is the total rate charged to customers and is the sum of the *Funding Rate* and the *Interest Spread*, which is determined by the growth plans reflected in the *Supply Gap*.

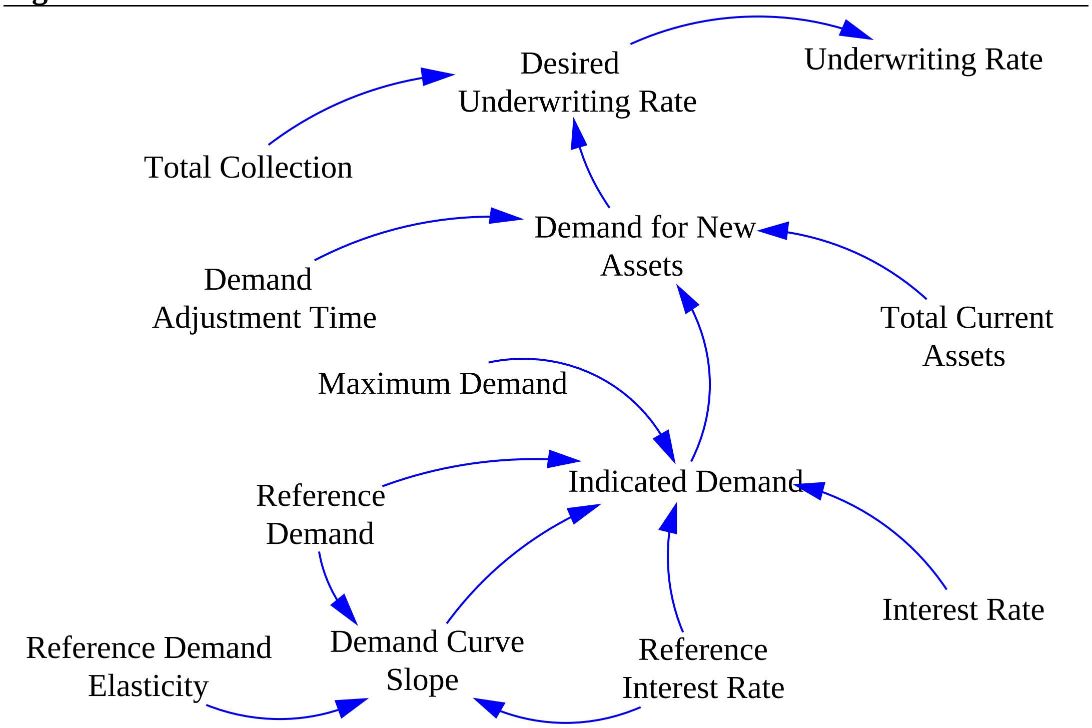
Interest Rate = Funding Rate + Interest Spread (48)

The funding rate is exogenous and constant in the model. This assumes that the portfolio is sufficiently small for the funding available in the market for financial institutions, and therefore that the bank can borrow as much as needed at the market interest rate. Note that, conversely, on the assets side we assume that there is a limited market.

3.4 Demand

Although demand for credit is affected by several factors, the model focuses on the impact of interest rate. The underlying assumption is that companies have several investment opportunities with different rates of return, and that the interest rate level determines which of these possible projects are executed, and therefore how much debt is needed by the corporate sector. This type of reasoning was already masterly explained by John Maynard Keynes in the General Theory of Employment, Interest and Money (1936). In his book, Keynes explains how it is the interest rate that sets the limit to the production of new investments. Since we are considering an oligopoly set up, we assume the elasticity is finite but high, as the possibility for differentiation is weak. Banks may seek to gain more clients by offering longer tenor, more flexibility, reliability or related services, but price is usually the main driver for demand, especially in the corporate segment. Following the generic commodity model in and Sterman (2000), a linear demand function is assumed. Figure 7 shows how the *Total Underwriting Rate* is determined in the model.

Figure 8: Demand



The *Total Underwriting Rate* is determined by the renewal of maturing loans and debtors desired change in their debt level. Loans are considered to be non pre-payable so a non negativity condition is imposed on the *Underwriting Rate*.

Total Underwriting Rate = Max(Desired Underwriting Rate,0) (49)

Desired Underwriting Rate = Total Collection + Recovery Rate + WO + Demand for New Assets (50)

Demand for New Assets = (Indicated Demand-Total Current Assets)/Demand Adjustment Time (51)

*Indicated Demand = MIN(Maximum Demand, Reference Demand * Max(0,*
1 + Demand Curve Slope(Interest Rate-Reference Interest Rate)/Reference Demand))* (52)

Demand Curve Slope =
*- (Reference Demand * Reference Demand Elasticity)/Reference Interest Rate* (53)

Funding is not considered explicitly and the funding rate is completely exogenous. This implies that the portfolio is sufficiently small for the funding available in the market for financial institutions, and therefore that the bank can borrow as much as needed at the market interest rate. Note that, conversely, on the assets side we assume that there is a limited market, in which the organization modeled has some degree of price setting power. That is to say, we assume that funding elasticity of supply is infinite, while the elasticity of demand for loans is not.

4. Simulation results and analysis

The model was evaluated for a set of parameters based on different sources. The transition matrix data is based on Bangia, et al (2002). The unconditional transition probabilities are shown in Table 1 below. The last column in Table 1 shows also the Underwriting Share for each rating. The shares used are the ones that generates a rating distribution in steady state consistent with the one presented in Lucas, Klaasen, Spreij and Straetmans (2001) for a typical bank portfolio of average quality.

Table 1: Quarterly Transition Probabilities - Unconditional

	AAA	AA	A	BBB	BB	B	CCC	D	U. Share
AAA	97.92%	1.95%	0.10%	0.02%	0.01%	-	-	-	3%
AA	0.16%	97.95%	1.75%	0.10%	0.01%	0.02%	-	-	4%
A	0.02%	0.57%	97.91%	1.34%	0.10%	0.06%	-	-	9%
BBB	0.01%	0.07%	1.37%	96.90%	1.38%	0.23%	0.02%	0.03%	29%
BB	0.01%	0.03%	0.17%	1.87%	95.35%	2.26%	0.18%	0.13%	45%
B	-	0.02%	0.07%	0.11%	1.66%	95.72%	1.46%	0.96%	7%
CCC	0.04%	-	0.16%	0.20%	0.41%	3.28%	87.18%	8.72%	3%

Table 2 shows the parameters used in the simulation. The Reference Interest Spread and Funding Rate are based on Lucas et al. (2001). The spread is the equivalent of the reported spreads for 5-year loans for each rating weighted by the initial rating distribution. The ROE Goal is the resulting ROE of the initial configuration and is consistent with ROE in commercial banking (see Damodaran 1996 for typical ROE of large US commercial banks). The capital requirement is the 8% rule of the Basle Committee on Bank Supervision (1988). The Recovery Rate and the Average Time to Resolution are from a report by Moody's (2000) on bank loans loss given default for senior unsecured loans. The Reference Demand Elasticiy is based on Martin (1990). The remaining values are judgemental. These parameters initiate the model in equilibrium.

Table 2: Parameters

Variable	Units	Value
Reference Interest Spread	Dimensionless	2%
Funding Rate	Dimensionless	4%
ROE Goal	Dimensionless	14%
Capital Requirement	Dimensionless	8%
Recovery Rate	Dimensionless	52%
Reference Demand Elasticity	Dimensionless	1.5
Average Time to Resolution	Years	1.5
Average Tenor	Years	3
Capital Adjustment Time	Years	1
Spread Adjustment Time	Months	1
Demand Adjustment Time	Months	1
Reference Demand	$	100
Maximum Demand	$	200

4.1 Changing Economic Conditions

To study the response of the system to changes in the underlying economic conditions, we first analyse the response to a step change in transition probabilities. The test consists of replacing the the unconditional transition matrix with expansion or recession ones estimated by Bangia et al. (2002) based on quarterly data for the US from the National Bureau of Economic Research.

Table 3: Quarterly Transition Probabilities - Expansion

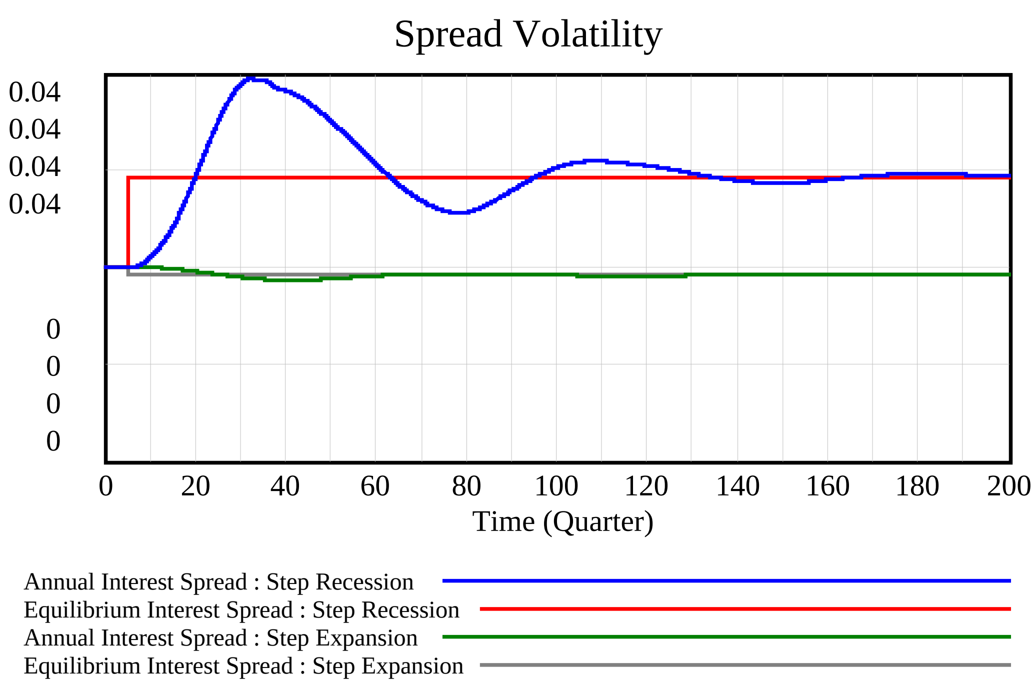
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	98.21%	1.66%	0.11%	0.02%	0.02%	-	-	-
AA	0.15%	98.08%	1.61%	0.12%	0.01%	0.03%	-	-
A	0.02%	0.53%	98.06%	1.21%	0.11%	0.06%	-	-
BBB	0.01%	0.07%	1.47%	96.94%	1.25%	0.22%	0.02%	0.02%
BB	0.01%	0.03%	0.19%	1.93%	95.31%	2.25%	0.16%	0.12%
B	-	0.02%	0.07%	0.10%	1.70%	95.91%	1.31%	0.88%
CCC	0.05%	-	0.19%	0.23%	0.47%	3.57%	87.32%	8.17%

Table 4: Quarterly Transition Probabilities - Recession

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.99%	1.76%	0.25%	-	-	-	-	-
AA	0.18%	96.89%	2.79%	0.05%	0.09%	-	-	-
A	0.02%	0.88%	96.44%	2.59%	0.07%	-	-	-
BBB	0.04%	0.04%	1.11%	96.31%	2.33%	0.07%	-	0.11%
BB	-	0.06%	0.06%	1.39%	94.98%	2.72%	0.42%	0.36%
B	-	0.06%	0.06%	0.11%	0.72%	95.02%	2.27%	1.77%
CCC	-	-	-	-	-	1.20%	85.60%	13.20%

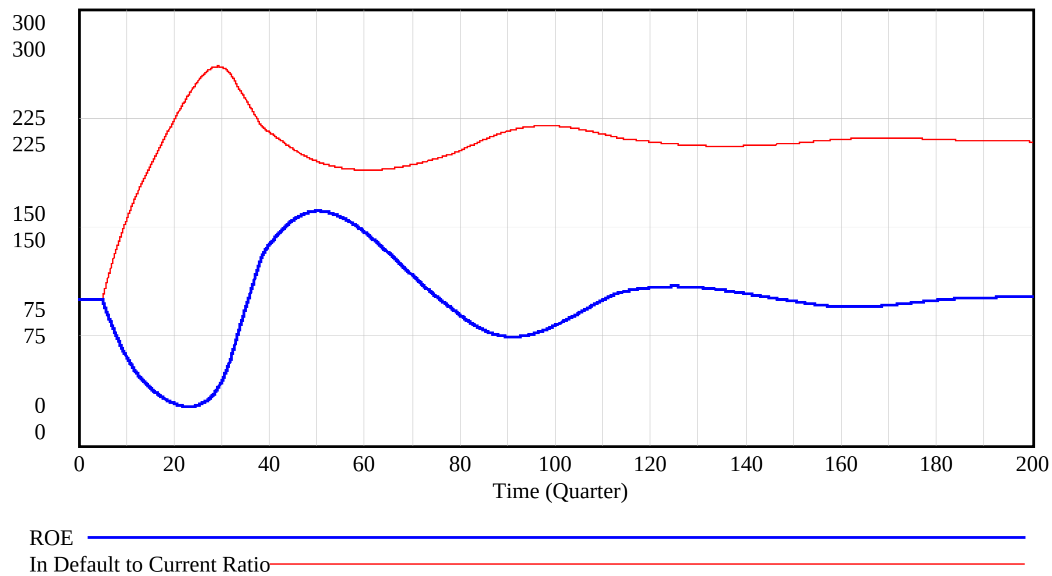
While the contraction matrix exhibits significantly higher downgrading and default probabilities, the changes in the expansion matrix are less dramatic. We show below the outcome of a 50-year simulation in which the unconditional transition matrix is replaced by the expansion and recession ones in year 2. Figures 9 shows the equilibrium interest spread, i.e. the interest spread that makes *ROE* equal to the goal, and the actual interest spread. While the equilibrium interest rate is almost 50% higher for the contraction matrix, in the case of the expansion matrix the equilibrium spread is slightly lower. In both cases, actual interest spread oscillates significantly around the new equilibrium, only approaching the new steady state level by the end of the simulation.

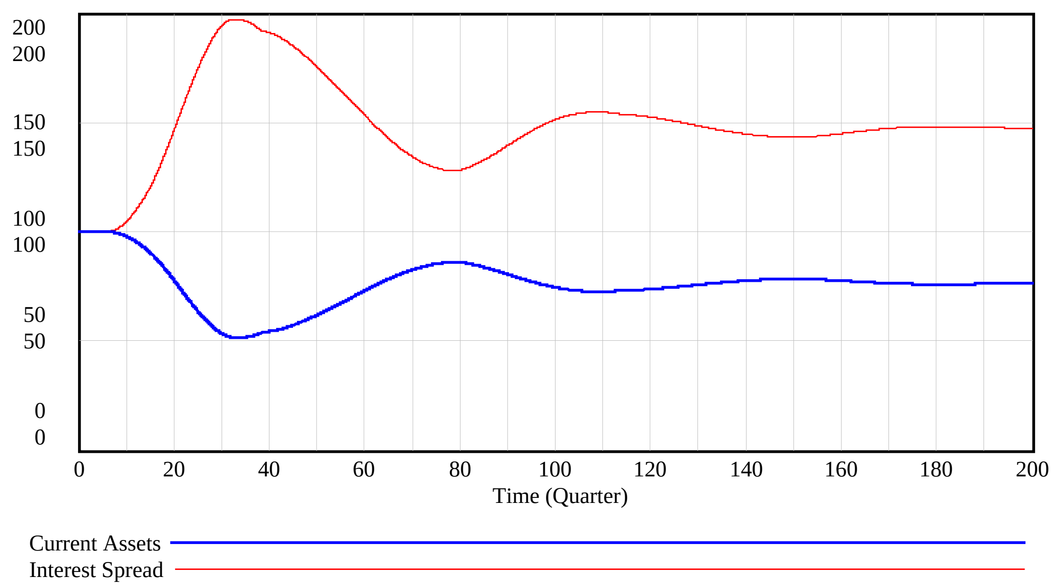
Figure 9: Interest spread – Recession and Expansion



We analyse the oscillatory response to the shock because dynamics are clearer, but similar dynamics apply to the expansion case. As shown in Figure 10, immediately after the change of transition matrix, the ratio of *Assets in Default* to *Current Assets* starts to increase, and consequently the *ROE* starts declining. As *ROE* lowers, the desired capital is reduced and interest rate is increased, effectively making total assets decrease. The key to the oscillatory dynamics is that the contraction of the portfolio makes the ratio of assets in default to current assets grows even more. This happens until the interest spread has gown enough to stop *ROE's* fall. Once *ROE* starts to recover, the cycle works in the opposite direction: interest spread falls, the underwriting rate grows and the ratio of in default to current drops.

Figure 10: Dynamics - Recession





5. Conclusions

With a very simple stock and flow formulation, we find an expression for steady state ROE and show some analytic results. We also indicate that due to the delays in the materialization of losses, the returns on a portfolio increase as the portfolio grows and diminish as it shrinks. To this author's knowledge, this is also the first attempt to analyze the dynamics of a loan portfolio using transition matrices within a system dynamics model. Our model shows that the misperception of the dynamics of this structure, and the use of decision heuristics such as anchoring and adjustment to determine growth strategies and pricing, generate significant volatility. This is true even in this simplified framework, in which a single and isolated bank is modeled in an environment of fixed funding rate and the response to a single discrete change in parameters is analyzed.

From a portfolio manager standpoint, the main lesson is that to avoid the costly oscillations it is necessary to understand the structure and to track the transition matrix continuously. The equilibrium ROE for a given set of parameters, or the required parameters for a given expected ROE, can be easily estimated with a spreadsheet. The growth decisions feedback on performance must be explicitly considered for strategic decisions if overshooting is to be prevented. The model could also be a basis for for a credit cycle model. We would expect that expanding our model to include all players in the lending industry, making the funding rate endogenous and linking it with a macroeconomic model, would probably reinforce the cyclical behaviour.

References

Aguais, S.D. and Santomero A.M. (1997). Incorporating New Fixed Income Approaches Into Commercial Loan Valuation. *The Wharton Financial Institutions Center Working Papers*. (http://fic.wharton.upenn.edu/fic/papers/98/9806.pdf)

Bangia A., F.X. Diebold, A. Kronimus, C. Schagen and T. Schuermann. (2002). Rating Migration and the Business Cycle, With Applications to Credit Portfolio Stress Testing. *Journal of Banking and Finance*, 26, 445-474.

Bank of International Settlements. Basle Committee on Bank Supervision (1988). International Convergence of Capital Measurement and Capital Standards. *Report 4*.

Credit Suisse First Boston (1997). *Credit Risk+: A Credit Risk Management Framework*. (http://www.csfb.com/creditrisk/assets/creditrisk.pdf)

Damodaran, A. (1996). *Investment Valuation*. New York: John Wiley & Sons.

Gupton, G.M. D. Gates and L.V. Carty (2000). Bank Loss Given Default. *Moody's Investor Service, Special Comment*.
(http://riskcalc.moodysrms.com/us/research/lied/61679.pdf)

J.P. Morgan (1999). *Credit Metrics 4th ed.* (http://www.creditmetrics.com)

Jarrow, R., D. Lando and S. Turnbull (1997). A Markov Model for the Term Structure of Credit Risk Spreads. *Review of Financial Studies*, 10:2, 481-523.

Jarrow, R. and S. Turnbull (1995). Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance*, 50, 53-85.

Jobst, N.J. and S.A. Zenios (2001). Extending Credit Risk (Pricing) Models for the Simulation of Portfolios of Interest Rate and Credit Risk Sensitive Securities. *The Wharton Financial Institutions Center Working Papers*.
(http://fic.wharton.upenn.edu/fic/papers/01/0125.pdf)

Keynes, J.M (1992). *The General Theory of Employment*, Interest and Money (Spanish translation). Buenos Aires: Fondo de Cultura Economica.

Lucas A., P. Klaasen, P Spreij and S. Straetmans (2001). An Analytic Approach to Credit Risk of Large Corporate Bond and Loan Portfolios. *Journal of Banking and Finance*, 25, 1635-1664.

Madan, D.B. (2000). *Pricing the Risks of Defult*.
(http://www.mbs.umd.edu/Finance/dmadan/drskicbi.pdf)

Martin, C. (1990). Corporate Borrowing and Credit Constraints:Structural Disequilibrium Estimates for the U.K. *The Review of Economics and Statistics*, 72:1, 78-86.

Mas-Colell, A., M.D. Whinston and J.R. Green (1995). *Microeconomic Theory*. New York: Oxford University Press.

Merton, R. (1974). On the Pricing of Corporate Debt: the Risk Structure of Interest Rate. *Journal of Finance*, 29, 449-470.

Sterman, J.D. (2000). *Business Dynamics: System Thinking and Modeling for a Complex World*. Boston: McGraw-Hill.

Winston W. L. (1994). *Operations Research: Applications and Algorithms*, 3rd ed. Belmont, California: International Thomson Publishing.

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