A Qualitative Analysis of Push and Pull Models

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ABSTRACT

Those sub-systems corresponding to production processes are complex systems, due to information feedback, delays, and the nonlinearities present within the process of the company's decision making. The System Dynamics allows having a systemic vision of the processes, which leads to identifying some factors that might be generating behaviors not easily foreseen. With the theory of the Qualitative Mathematical Analysis it is possible to find the stability and instability zones of the systems beside the variation of some parameters and the starting values of the condition variables. The attraction basin might present behaviors such as the point ones, the cyclic ones, the odd ones, or the chaotic ones. The above mentioned concepts and techniques are used to compare two simple models of production systems: Push and Pull; the results show different behaviors in both systems when changes in the values of some parameters being common in both models are made.

Key Words: Qualitative Analysis, Attractors, Simulation, System Dynamics, Inventory Systems, Push Systems, Pull Systems, Stability, Modeling.

1. INTRODUCTION

The manufacturing systems have a hard mathematical representation of their behavior, and therefore, every time a study of them is required, it is necessary to make use of the ability, knowledge and criteria of those who have the responsibility of making decisions in such systems. A very important manufacturing sub-system is the one regarding the process of production, which demands a great effort from the company in order to know in great detail some factors such as the availability of production assets, personnel, material to be transformed, product demand, production costs, transport and production time, inventory of finished goods, and many other factors that intervene in the production process.

The union of the factors of productions and the productive processes management techniques generates very complex systems of production that many times are not easily understood. That's why there's the need to make use of diverse techniques allowing to analyze the system, such as the qualitative mathematical analysis and the simulation. The system dynamics is a simulation technique with which the relations between the factors intervening in a system and its influence in its dynamic behavior can be seen and understood. This quality of the system dynamics gives the analyst, manager or director using it the chance to have a systemic vision and therefore a greater understanding of the production process.

The present research has a main goal to describe, study, analyze, simulate and present policies for a simple production system having two stages: one corresponding to the production process of any given goods, and another one corresponding to the level of the inventory of finished goods. To achieve this, relations between other variables are established. Variables such as demand, prevision of demand, coverage extent of the expected inventory, inventory adjustment time and production process time. Additionally, it is intended to find the attraction basins and its respective balances.

In order to achieve the above mentioned goals, the general production model is studied, which includes a production process and an inventory of finished goods. The model is made starting from the System Dynamics. The differential equations representing the system are established and the qualitative mathematical analysis is made from the differential equations in state of balance, which leads to obtaining the Jacobian Matrix, the characteristic equation and the eigenvalues; this late ones indicate the kind of attractor that can be presented. A later study leads to making a qualitative analysis of the trajectories and behaviors of the results of the simulations, which are carried out starting the sensitivity analysis of the initial conditions of the state variables; and finally, the conclusions about the system stability are given.

2. THEORETICAL APPROACH

For the elaboration of the model it is required to know about three specific areas: direction of operations, qualitative mathematical analysis and simulation. The direction of operations allows the understanding of the production system and its relations; the qualitative mathematical analysis provides the basis to make the stability study, and the simulation shows the behaviors emerging when stating different policies for direction as well as varying the parameters for the model.

2.1. Direction of Operations

The industrial systems, and more concretely the manufacturing sub-systems are known to be complex systems where the required day after day decisions in order to carry out an operation system for manufacture, involves the need to understand and direct the dynamic behavior associated to it. Without understanding the dynamic behavior, it is very difficult to design proper control systems to decide when it might be necessary to intervene and what optimal policies must be given. The natural dynamics of system plays a crucial role in deciding how the direction control system must be chosen (BICKLE and MCGARVEY,1996). The behavior of complex systems depends on their structure, understanding as structure the group of variables of the system and their inter-relations. In (FORRESTER, 1961; FORRESTER, 1987; STERMAN, 1994; MACHUCA, 1998; SENGE et AL., 1995; PRIGOGINE, 1987) some of the factors and the relations between the effects and the factors that make up the complex systems, can be read. Such features increase the difficulty in the direction of businesses and increase the undesirable effects as well.

2.2. System Dynamics

The behavior of a system is observable through the patterns of its components such as levels, rates or other variables. With different variables, the behavior of the system must be different and the dominant curl of every pattern in every period of time should not be the same in every instant (TU, CHEN, and TSENG, 1997). The success of the research in system dynamics depend on a clear initial identification of a purpose and an important objetive; a system dynamics model must organize, clarify and unify the knowledge.

The simulation of the model must lead to a process of validation through which the stability of the model, the fluctuation periods, the relations of time among the variables, and other factors, are judged. The final concept of the industrial dynamic models will depend on the utility they have for the manager in the design of better industrial systems. The simulation, validation, and sensibility analysis of the parameters and the structures allow the proposal of scenarios, these ones understood as a hypothetical situation through which "an imaginative jump towards future" takes place. Its objective is not to predict what will happen but to propose several potential future scenarios. It is quite probable none of them will come to terms, but all of them might have more awareness about the forces acting on the present, which might also act on the future.

2.3. Qualitative Mathematical Analysis of Dynamic Systems

The joint application of the theory of Qualitative Mathematical Analysis and the one of System Dynamics is owed to Ilya Prigogine who, altogether with his collaborators, focuses his work on Mathematical Analysis of the bifurcations, chaos, and other forms of instabilities that occur even in non linear models relatively simple. The Qualitative

Analysis might be considered as a complement of the results obtained with the simulations of the models of System Dynamics and it helps understand the relations or interactions between the mental models and all the ways of behaviors they might generate.

The linear dynamic systems present a unique point attractor, or so to speak, a unique balance. The non-linear and the delay times in the feedback control systems might cause complex behaviors, such as those giving origin to the periodical attractors and the odd attractors; the non-linear systems might present multiple attractors, and every single one of them might be different. Over the last 300 years the behavior of the systems had been thought to be predicted if the systems behavior laws and their initial conditions were known. However, due to the discovery of the deterministic chaos, randomness in the behavior of the systems has been introduced, and therefore, it is understood that the deterministic processes might generate unforeseen situations when there are nonlinearities among them. (PRIGOGINE And STENGERS, 1984; CHEN, 1988; BRIGGS and PEAT, 1999; FORRESTER, 1972; STURIS and MOSEKILDE, 1988; ARACIL and TORO, 1993; ARACIL and GORDILLO, 1997).

The term *attractor* refers to the fact that the trajectories that start in different points within the space of phase, or in an attraction basin, all of them will approach a stationary movement when the transitory period has disappeared (ANDERSEN and STURIS, 1988). Aracil and Gordillo (1997) say that the first step towards the development of qualitative analysis of a dynamic system is the determination of the number and kind of its attractors, and their corresponding attraction basins. Altogether with the attractors, the theory of bifurcation shows up. There are three main kinds of attractors:

- Point Attractor: In the Point Attractor, all the trajectories converge towards the balance trajectory represented by repose. Figure 1 shows a point stable temporary behavior. In the phase, the point stability is represented by an asymptotically stable spiral.
- Cyclic Attractor: The cyclic behavior might be classified in: a simple attractor, which is a closed curve in the state space, and it is a non-cushioned wave in the temporary drawing (Figure 2); A periodical attractor, which has double original revolution period, that is a cycle-2.
- A quasi-periodical attractor, which gets to have a strange shape such as, for instance, the one known as Bull; the strange or chaotic attractor, which does not show any repetitive pattern but it has an unpredictable direction every time.

3. MODEL OF THE PRODUCTION SYSTEM

Figure 3 shows the model of a general, non-linear production system, in which the policy of the production order varies in three ways: first, when the production policy does not take into account the product in process; second, when the production policy does take into account the product in process; and third, when besides the product in process, a delay in the production order is present.
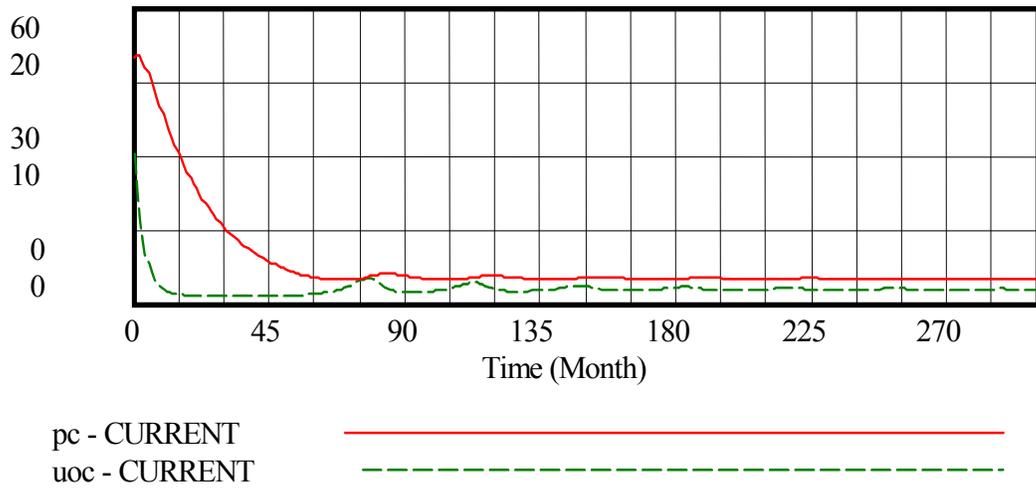
FIGURE 1 – Temporary behavior of the point attractor.
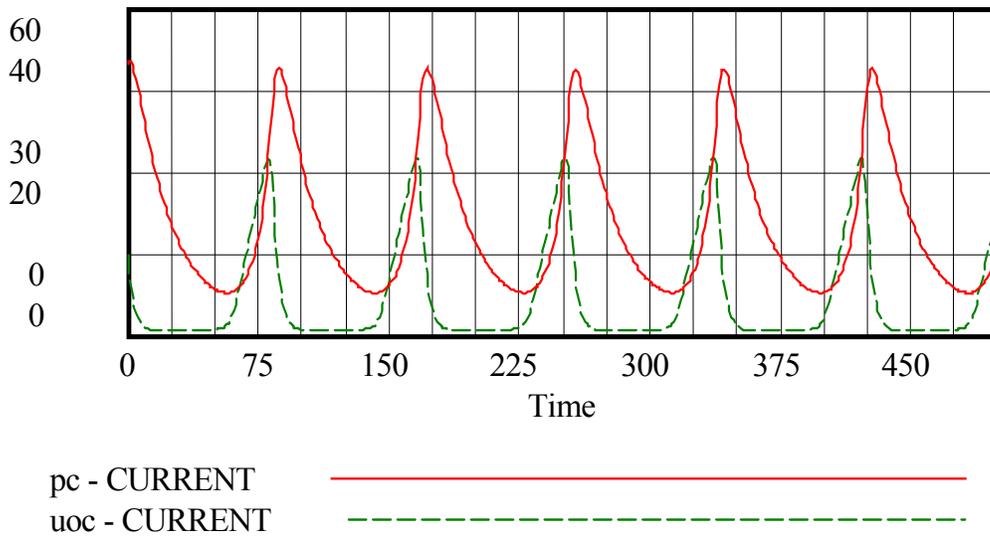
FIGURE 2 – Temporary behavior of the cyclic attractor.

3.1. Case 1. Non Linear Dynamic Model; the product in process is not taken into account.
This case was worked upon the feature of the "work in process" (WIP) variable not affecting the production order. There is a simple production system – inventory, represented by two processes: first, a production process that will provide information about the inventory in process every moment; second, the storage process of the finished goods (Figure 3). A general fabrication process is then analyzed, starting from a lower level of the inventory of finished goods. When the real inventory of finished goods lies under the level of expected inventory, production orders for the difference the level of expected inventory and the level of real inventory are launched, in addition to the demand; the production order is:
Order = demand + discrepancy/tai, where:

discrepancy = expected inventory-INV.
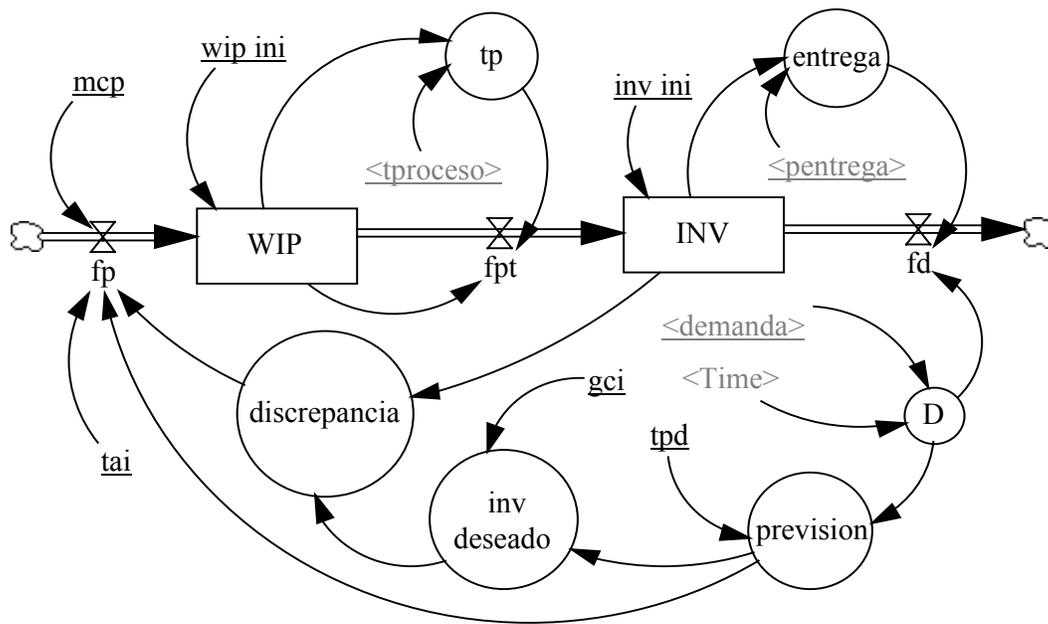
FIGURE 3. Flow diagram of a general production-inventory model.

The process time of the product depends on the quantity of product in process. The quantity of finished product that is delivered to the customer depends on the level of the inventory. Initially, the demand is constant through time, and that one not satisfied, does not remain pending.

Within a linear system there will never be a long term cyclic behavior, a situation that does happen within a non-linear system, and that in addition, might be considered as a normal

situation of the non-linear systems. In the theory of differential equations, this cyclic behavior is called *Center*, and in the theory of qualitative analysis is *called Limit Cycle Attractor*. The non-linear systems might show other strange behaviors in their trajectories, and some of them might be chaotic. The instable points of balance lead to the behavior of the system being directed towards one or another stable balance zone, or it leads to catastrophe. The non-linear system being analyzed in this case, includes nonlinearities represented by means of tables, for which it is somehow difficult to make a mathematical analysis; then it is necessary to make use of a graphical analysis, for both the non-linear and the results of the simulations. The simulation software used here is VENSIM. Figure 3 shows a Forrester diagram, also known as Flows and Levels diagram.

The parameters used in the model are: *gci* = coverage degree or period of security of the inventory; *tai* = time of adjustment for the inventory (constant); *tpd* = period of forecast for the demand; *mcp*: top production capacity (workmanship, capital equipment, raw materials, and other resources); *D*: The *D* Variable (demand) is a table that might take the values that range between 100 and 120, and it depends on time; *delivery(inv): "delivery"*, is a non-linear function that depends on the level of the inventory for finished product (INV); *tp(wip):* The "*tp*" variable is a table of a non-linear function that depends on the amount of product in process (WIP).

Procedure to find the balance regions:

Analytically, the stability and instability regions of the models might be found. Starting from the level equations, in state of balance, the balance points for such variables are found. With the derivatives of the expressions of the balance points, the Jacobian matrix is found, the characteristic polynomial is stated, and with this it is defined whether there is stability or instability. Below, the analytical development carried out until finding the characteristic polynomial of the case study is presented.

$$\frac{d(inv)}{d(t)} = fpt - fd \; , \quad \frac{d(wip)}{d(t)} = fp - fpt \tag{1}$$

In a situation of balance, there is:

$$\frac{d(inv)}{d(t)} = \left(\frac{wip}{tp(wip)}\right) - \left(D * entrega(inv)\right) = 0 \tag{2}$$

if there is to be supposed an unlimited top capacity (mcp), then:

$$\frac{d(wip)}{d(t)} = D + \left(\frac{D * gci - inv}{tai}\right) - \left(\frac{wip}{tp(wip)}\right) = 0 \tag{3}$$

$$inv = (tai * D) + (gci * D) - (tai * D * entrega(inv))$$
$$wip = D * entrega(inv) * tp(wip) \tag{4}$$

Being: $D = $ constant, $x = inv,$ $y = wip,$ $g_1(y) = tp(wip),$ $g_2(x) = entrega(inv)$

Then: $a_1 = D + \dfrac{D * gci}{tai}$, $h(y) = \dfrac{y}{g_1(y)}$,   therefore, the Jacobean matrix is:

$$J = \begin{bmatrix} -D*g_2' & h' \\ -\dfrac{1}{tai} & -h' \end{bmatrix} \quad \therefore \quad |J - \lambda| = \begin{vmatrix} -D*g_2' - \lambda & h' \\ -\dfrac{1}{tai} & -h' - \lambda \end{vmatrix} \tag{5}$$

Starting from the previous matrix, it is possible to obtain the characteristic equation, which allows calculating the characteristic roots, or the following eingenvalues:

$$\lambda = \frac{-\left(D*g_2'+h'\right)}{2} \pm \frac{1}{2} * \sqrt{\left(D*g_2'+h'\right)^2 - 4*\left(D*g_2'*h'+\frac{h'}{tai}\right)} \tag{6}$$

There are two conjugated eigenvalues that might be complex. However, it is somehow difficult to conclude on the form of the stability starting from these eigenvalues, due to the fact that the point or points of balance should be known accurately, in order to be able to know the sign of the derivative of h (*h'*), and with this, to determine whether the value of the root is positive or negative, and if the real part is negative or positive. What can be surely identified are the parameters that will affect the form of the trajectories.

The points of balance might be found when graphing the intersections of the balance equations. Figure 4 shows the three intersection points of the balance lines; points 1 and 5 correspond to point attractors, point 3 corresponds to a rejecter or separator, and the points 2 and 4 correspond to a top and lower respectively, which represent changes of attraction basins.

Figure 5 represents the behavior in the portrait of phase of the point attractors. Figure 6 presents the temporary behavior of the two point attractors, with trajectories being originated in different values of initial conditions; the space where the separator is, can also be seen; how the trajectories head towards the attractor of the upper part of the graphic and other trajectories head towards the attractor of the lower part of the graphic is seen. The two points of balance are in WIP = 107 and in WIP = 52.5.

Stability: Y/G1 y D*G2, con D = 105

Y/G1						
D*G2						
D = 105						
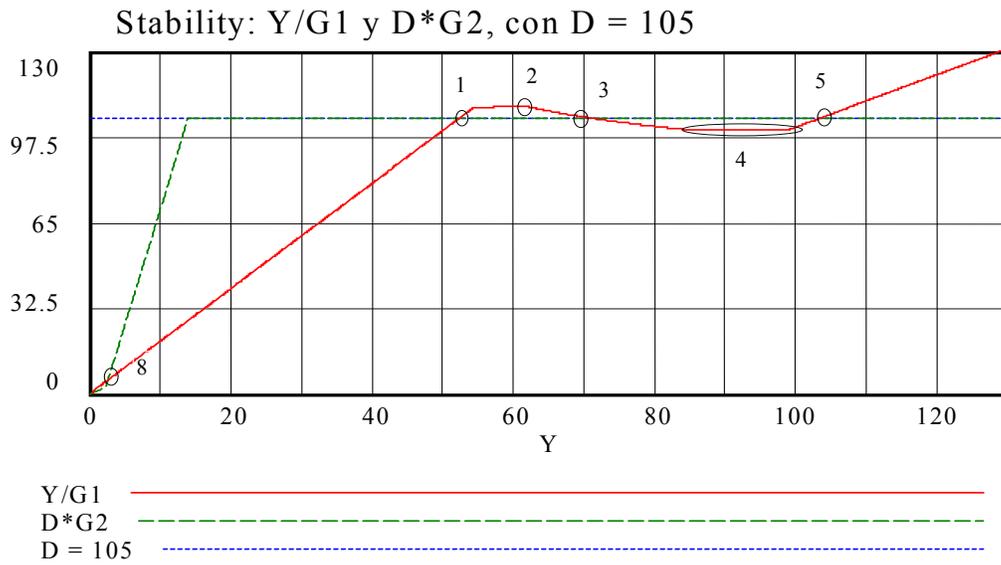
FIGURE 4. Points of stability

Phase drawing. WIP = 69

INV - W-INI-60
INV - W-INI-70
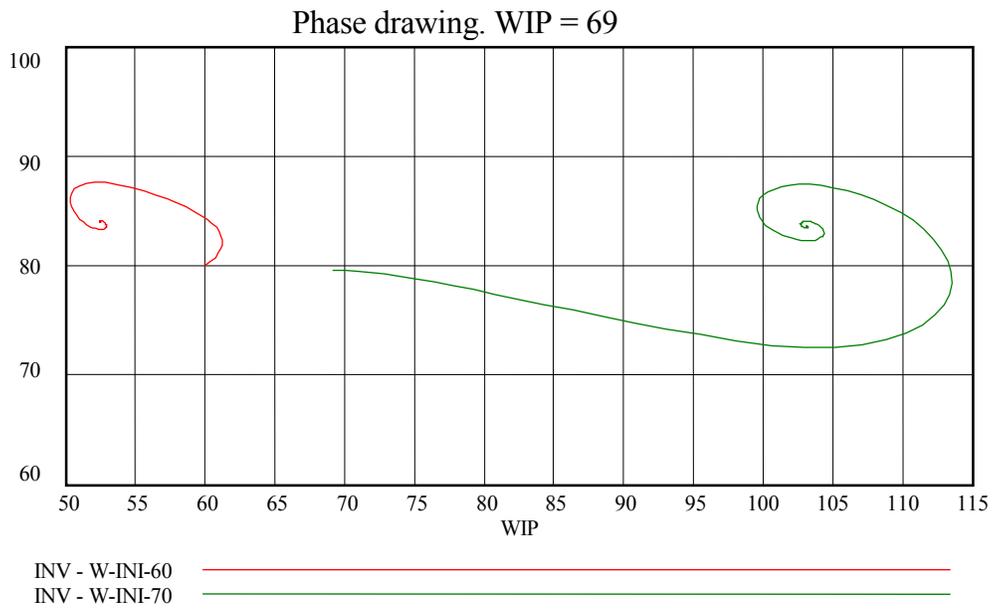
FIGURE 5. Phase Drawing of two point attractors.
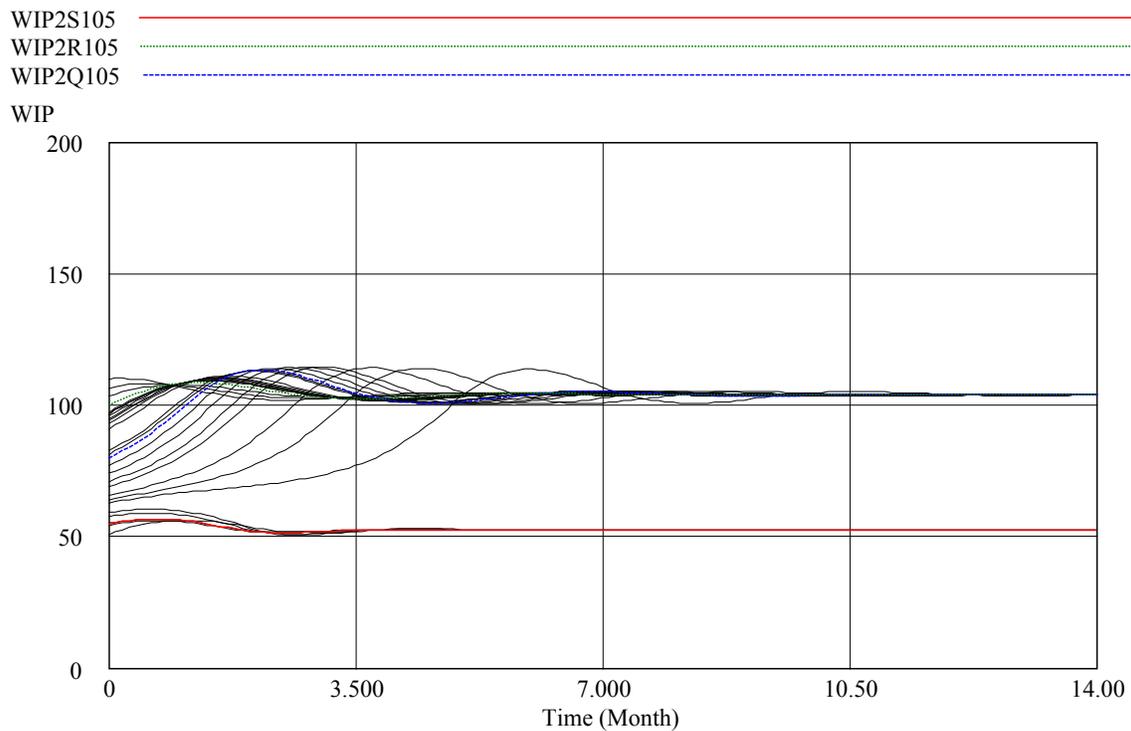
FIGURE 6. Two attraction basins and the bifurcation separator

3.2. Case 2. Non-linear Dynamic Model; the product in process is taken into account

Figure 7 represents the same system of case 1, but with a new policy of production orders; in this new policy, the WIP makes part of the production order, a situation that is considered in the diagram of Figure 3. Although with this new model the policy of production orders has been improved, which leads to having a situation of balance faster than it did before, two attraction basins still happen for the WIP variable, each one of them with a point attractor. The production order has the following expression:

Order = demand + discrepancy/tai, where:
discrepancy = expected inventory-INV-WIP.

The INV variable that is shown in Figure 8 presents two stable points of balance, a point attractor, in INV = 31.5, and another point attractor in INV = 12.5. The WIP variable, even though is not shown here, also presented two point attractors: one in the WIP = 74 value, and another in the WIP = 52.5 value. In the simulation of the model of CASE 1, the INV variable only presents a point attractor, that is, only one attraction basin, regardless of the initial values of the INV variable.
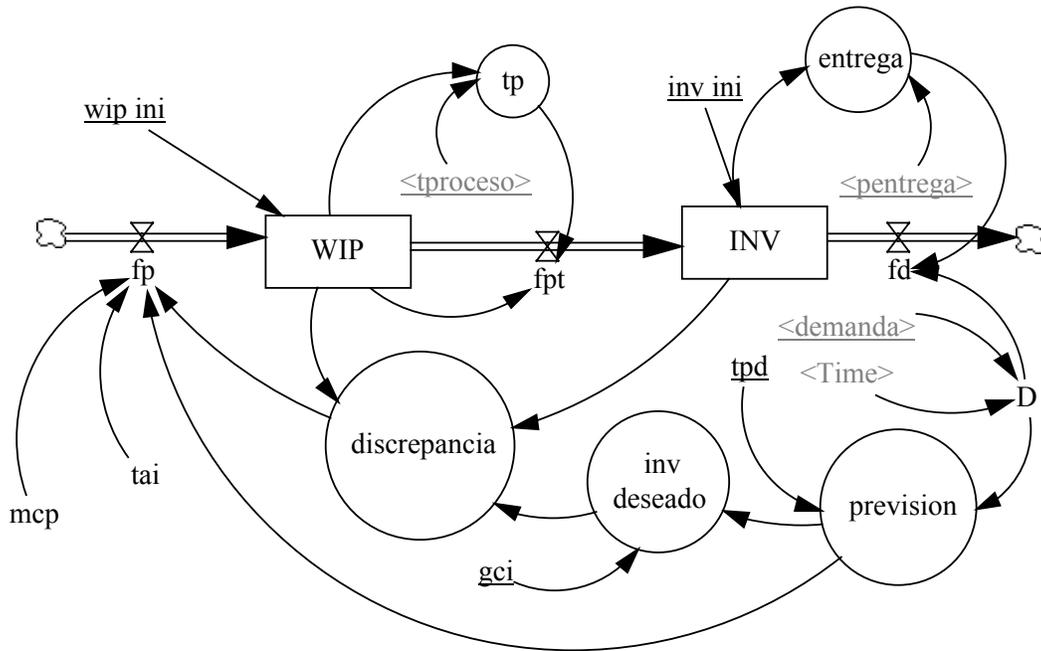
FIGURE 7. The WIP variable affects the production order.
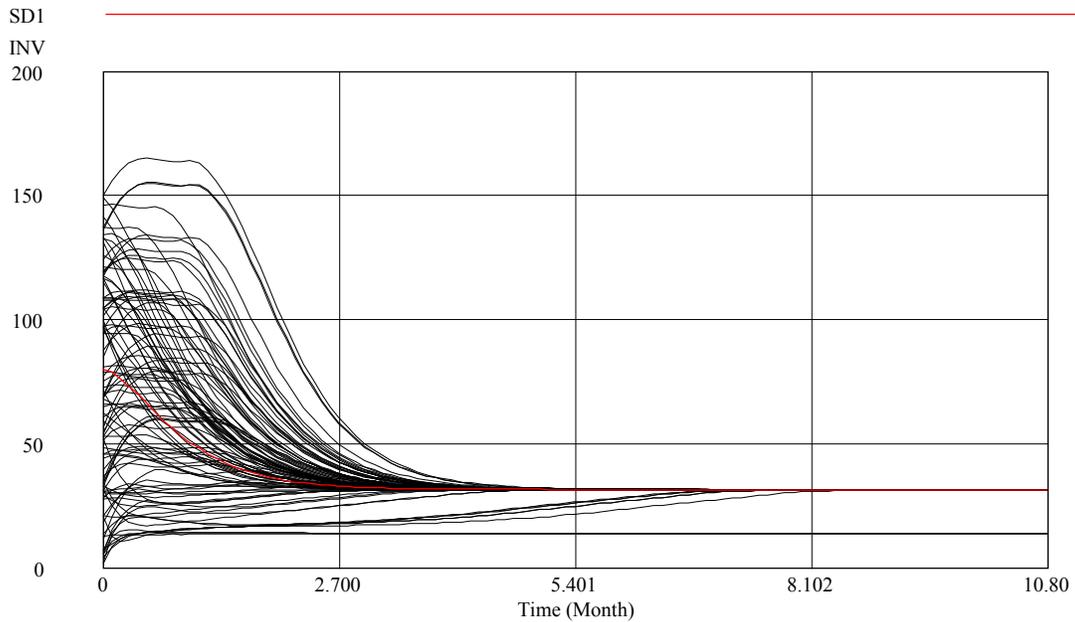
FIGURE 8. Case 2. Possibility of two point attractors: inv = 31.5; inv = 12.5

3.3. Case 3. Non-linear Dynamic Model; there is a delay in the production order.

This Case 3 is a system equal to that one of Figure 7, in which the Work in Progress (WIP) affects the Production Order variable, and this at the same time, has a delay of third order, with delay time equal to 1.5 units of time. With this production order, the trajectories of the WIP variable, each one with different initial values, present a cyclic attractor. Figure 9 shows the cyclic stable behavior of a trajectory of the WIP variable with an initial value equal to 50 units. The production order has the following form:

 Order = DELAY3(prevision + discrepancy/tai,1.5), where discrepancy = expected inv - INV-WIP.

Figure 10 shows the behaviors of several trajectories of the INV variable, facing different initial values; it can be seen that during the first 150 units of time there are two attractors: a periodic one around the INV = 12 point, and another chaotic one that might be described as an instable point attractor in form of an every time wider spiral, to turn into a periodic attractor around the time equal to 150. Starting from this time, the system will only have a periodic attractor. During the first 50 units of time it can be seen that the trajectories range pretty much up to reaching a stability, which may produce critical situations in the productive system if the possibility of these behaviors taking place, is not considered.
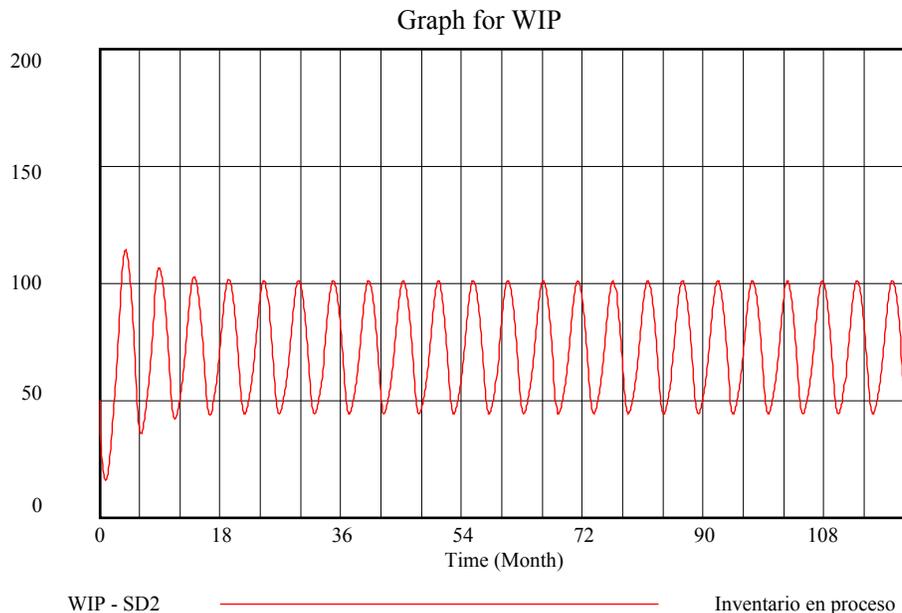
FIGURE 9. The WIP variable ranges cyclically between 44.27 and 100.92
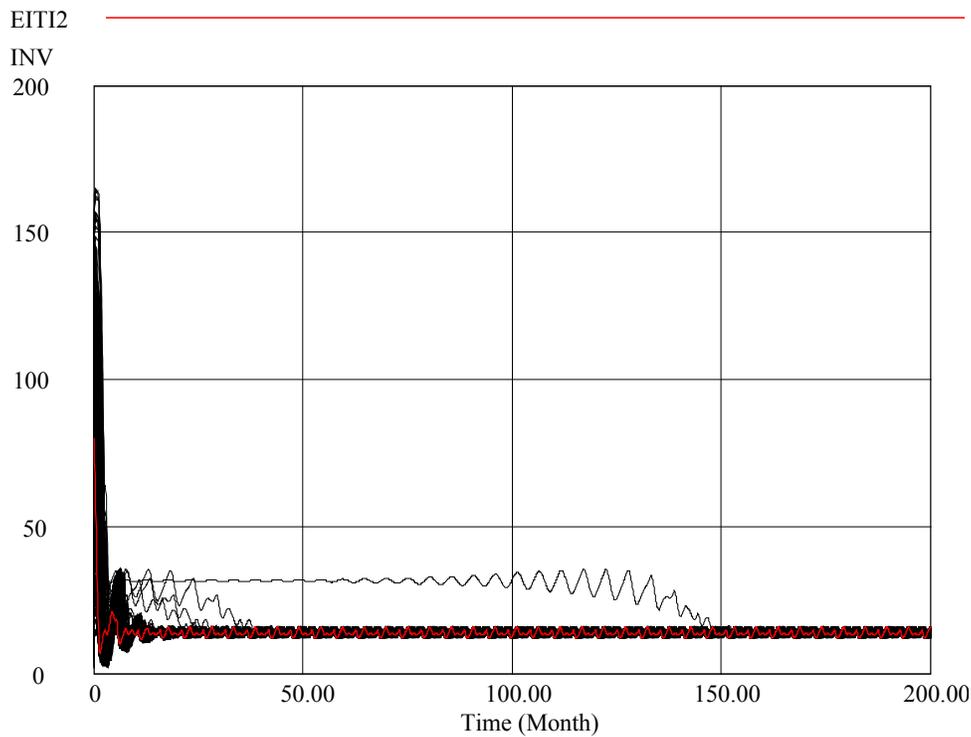
FIGURE 10. Case 3. Two attractors up to the time equal to 150: chaotic and periodic.

3.4. Other Cases. Non-linear Dynamic Model; delay times higher than 1.

Other cases may be the ones having delay times higher than 1 unit in the production order, which give as a result, for all the cases, trajectories with cyclic behavior in the WIP variable and N-periodic behavior in the INV variable; the higher the day time might be the higher width both for the cycle and the N-period. Delay times lower than the unit provide point stable behaviors.

4. SUMMARY OF THE THREE FIRST CASES

Figure 11 shows the temporary behaviors of the WIP and INV variables, according to four policies of the production order variable, such as: the three policies that are presented in this article, and an additional one that was simulated with the delay policy, where the delay has an average time of one unit and adjustment time of the inventory equal to the double of that one used in the three mentioned cases. For this later policy, the behaviors of the trajectories are stable point attractors, very similar to the behaviors presented in the model of Case 2. Although it is not possible to see in the graphic the accurate point of the bifurcation, it is indeed possible, for the naked eye, to observe that the critical zone where the bifurcation takes place has a WIP value equal to 50. This means that giving initial

actions in this point should be a very careful action, as there might be bifurcations that lead to different stabilities.

Simulations of four models

WIP - EITI2	———————————	WIP
WIP - EITI0	———————————	WIP
WIP - EITI1	- - - - - - - - - - - - -	WIP
WIP - EITI3	———————————	WIP
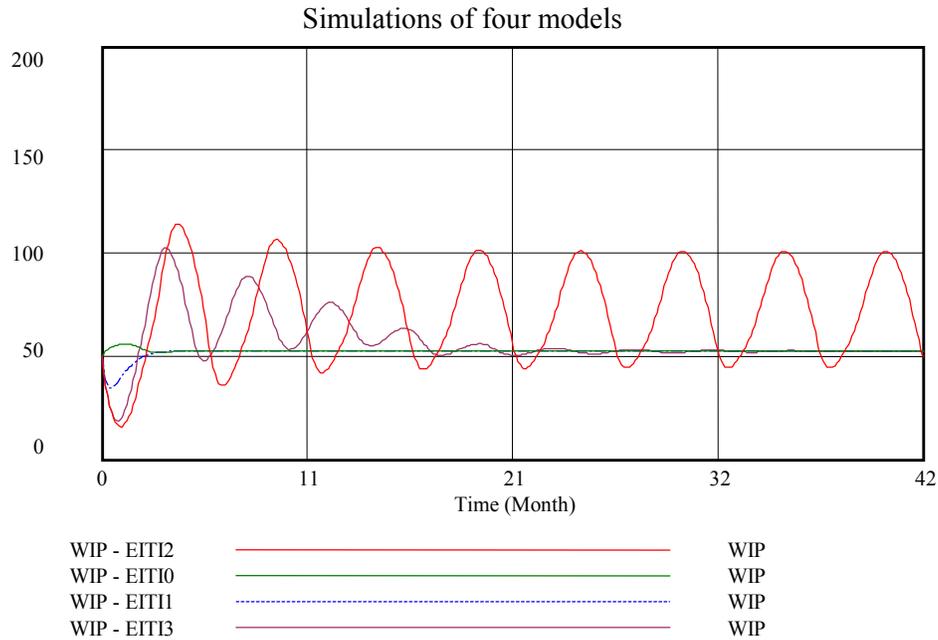
FIGURE 11. Four different cases of production orders.

5. PUSH AND PULL MODELS

In the present research two models corresponding to the PUSH and PULL systems have been analyzed, which are represented in an added form, and taking from every single one of them the main feature such the added planning in the PUSH system, and the concept of lower times and lower inventories for the PULL system, through the KANBANS concept. The two analyzed models have: three variables of level in the PUSH system, and four variables of level in the PULL system; but only starting from the variations of some parameters the behaviors of the variables of the Product in Process (WIP) level and Finished Product Inventory (INV) are analyzed. The other two variables of the PULL system level are: Pending orders portfolio (CTRA) and Production Planning (PP); the third variable of level in the PUSH system is the one corresponding to the pending orders portfolio.

The analysis of these two models was made for two cases: in the first one the relation among the variables Product in Process (WIP) and discrepancy of inventories is not considered. In the second case this relation is indeed considered.

5.1. PUSH AND PULL MODELS WITHOUT WIP CONTROL

The following five situations are analyzed because of considering that they affect somehow the behaviors of the two systems: PUSH and PULL.

(a) To analyze graphics obtained with a non-linear production time table.
(b) To modify the production times table slightly.
(c) To keep the first times table and modify the values of the security period SS in the PULL system and the coverage degree GCI in the PUSH system.
(d) To increase the demand in the horizon of the simulation. The other parameters are kept with the same values used in the first situation.
(e) To convert into non-linear tables the adjustment times for the TAI inventory and the IT Kanban cycle time. The other data remain as in those of the first situation.

The parameters that are modified in the five situations are:

TAI: Time of adjustment for the inventory; IT: Kanban Cycle; GCI: Coverage degree for the inventory; SS: Security Period; TP: Time of process (Lead Time); D: Demand.

Figure 12 shows the PUSH system with the relation between the INV PUSH variables and the expected inventory SS, starting from which the gross needs NN are calculated. Figure 13 shows the PULL system with the relation between the INV PULL variables and the KANBAN number, starting from which the expected production FPD is calculated.
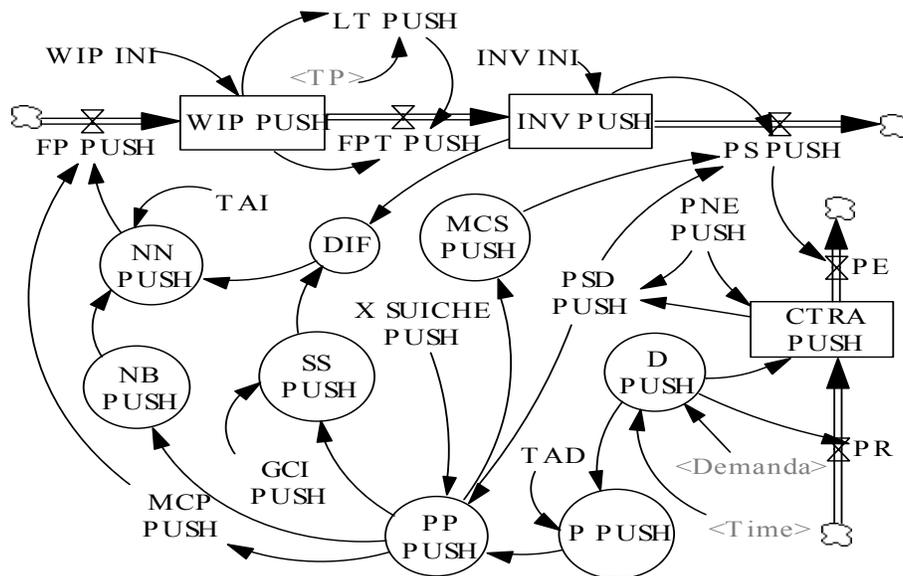
FIGURE 12. Diagram of a PUSH system. The WIP variable makes no part of the policy of the production order.
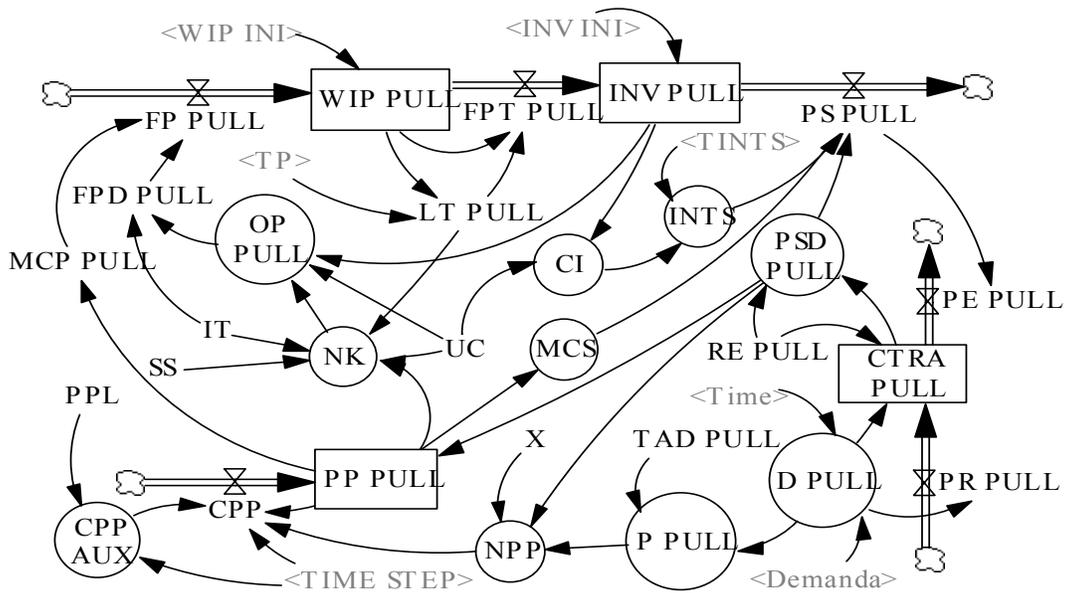
FIGURE 13. Diagram of a PULL system. The WIP variable makes no part of the policy of the production order.

In figures 12 and 13 it is seen that none of the two models, neither the PUSH nor the PULL, have related the WIP PUSH – DIF nor WIP PULL – OP PULL, respectively.

Table 1 shows the initial values of the model with which the situation (a) is simulated. In this situation the time of process (TP) is a non-linear function, specifically, logarithmic.

TABLE 1. Parameters used in the simulation. TP non-linear.

CASE	TAI	GCI	IT	SS	TP	D	WIP initial	INV initial	Result: Attractor
PUSH	0.5	0.8			Non- linear	105	50	80	Point
PULL			0.5	0.3	Non- linear	105	50	80	Limit Cycle

Figures 14 and 15 show the trajectories of WIP and INV of the PUSH system. In Figure 14 three attraction basins are observed, with their respective point attractors, whose approximate values are 50, 110 and 210 units. Figure 15 only shows an attraction basin with its respective point attractor, whose approximated value is 105.

Figures 16 and 17 show the trajectories of WIP and INV of the PULL system. Figure 16 shows an attraction basin with an attractor of limit cycle having an approximate range between 0 and 1,400 units. Figure 15 only shows an attraction basin with an attractor of limit cycle having an approximate range between 60 and 1,200 units.
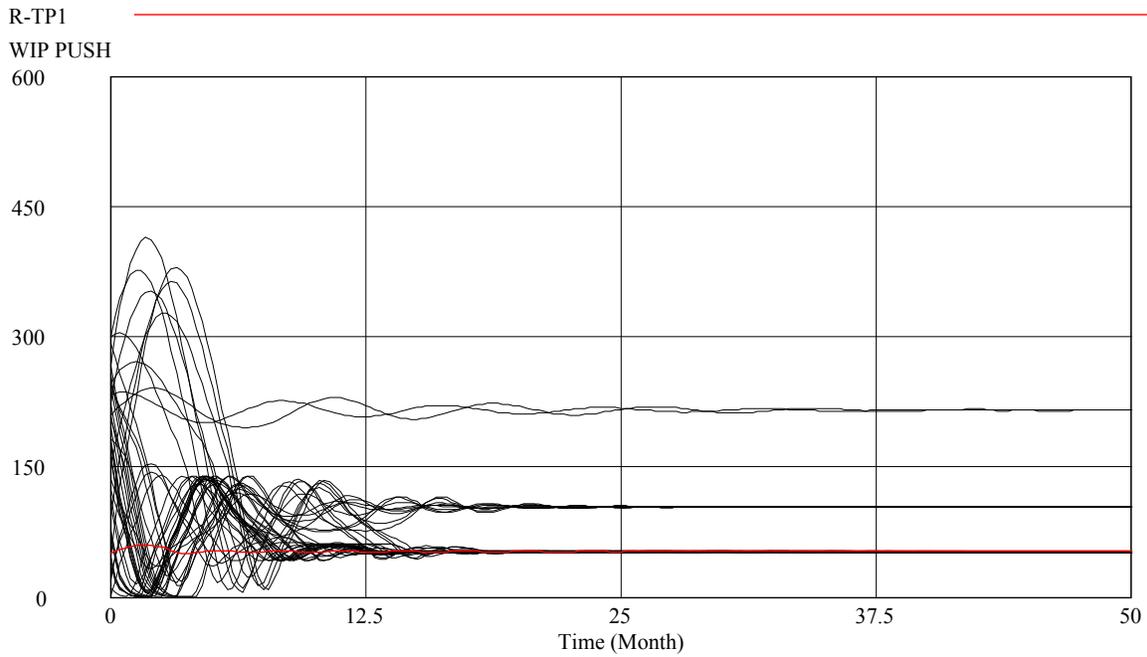
FIGURE 14. Temporary drawing of the work in process, WIP. Three point attractors.
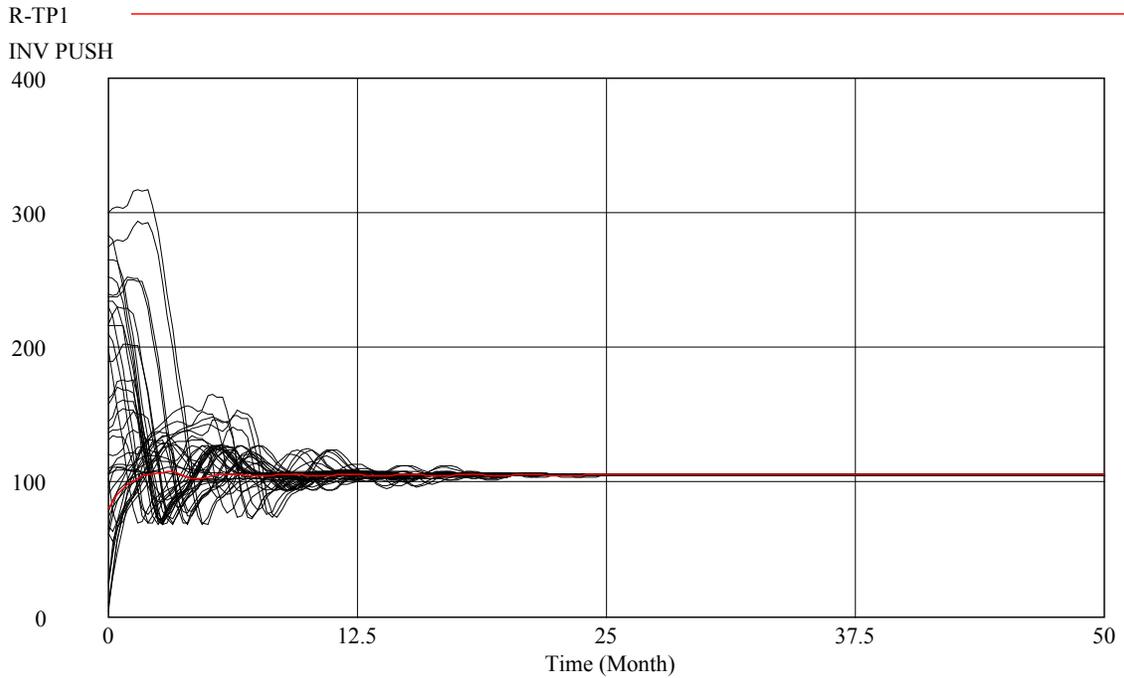
FIGURE 15. Temporary drawing of the finished inventory, INV. One point attractor.
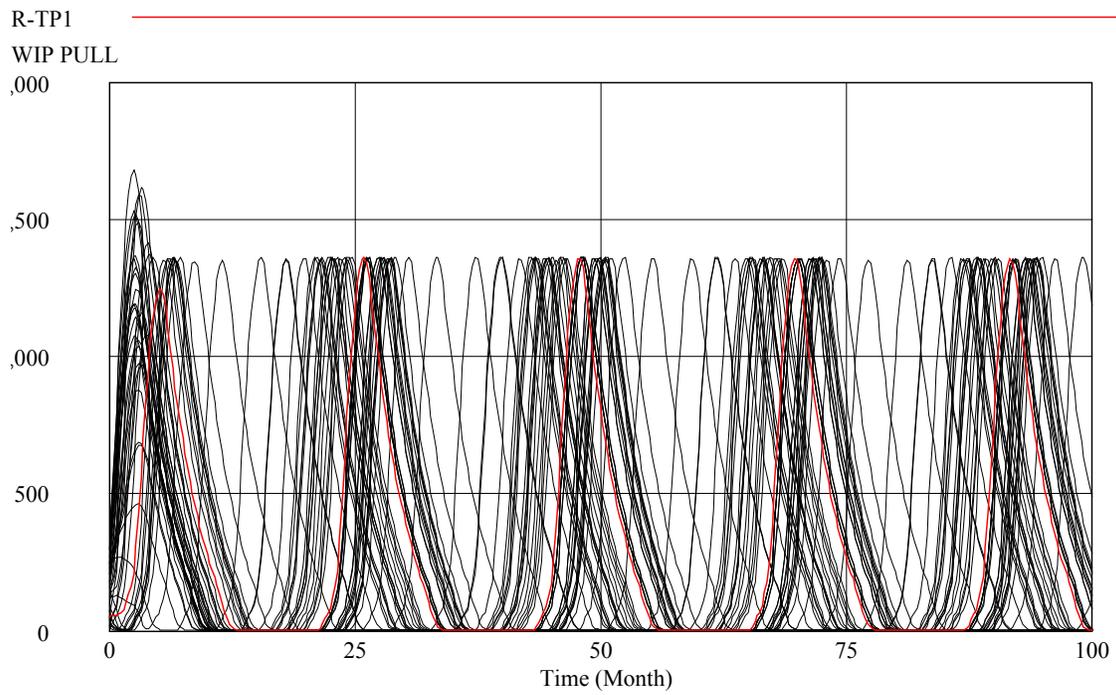
FIGURE 16. Temporary drawing of the work in process, WIP. Limit cycle attractor.
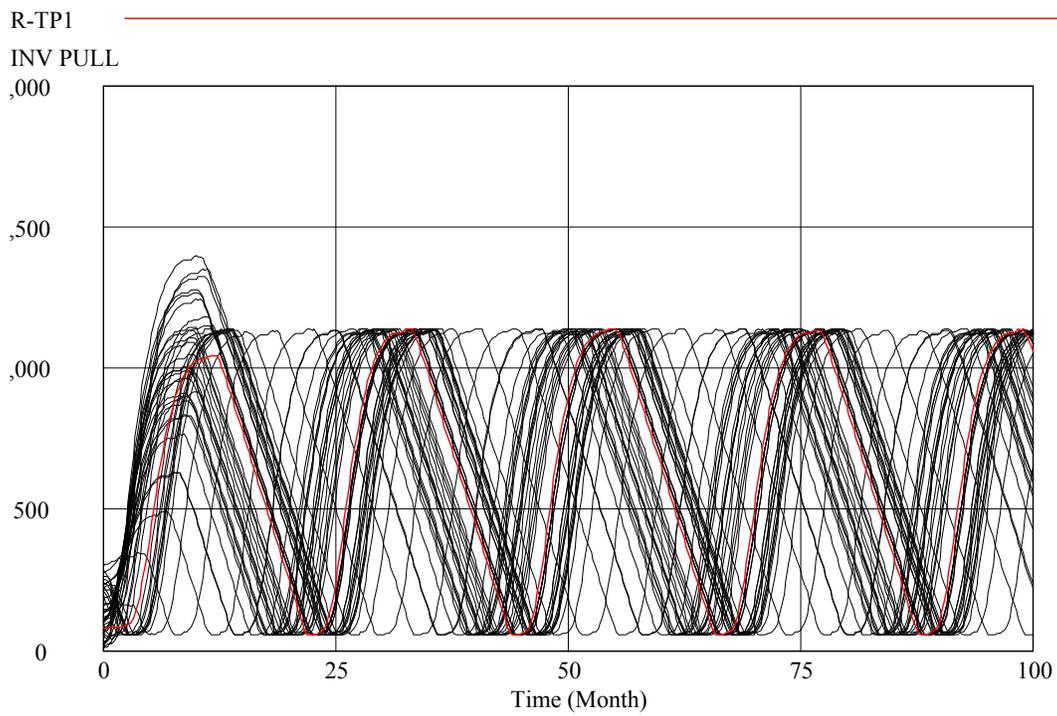
FIGURE 17. Temporary drawing of the finished inventory, INV. Limit cycle attractor.

From these four figures (14, 15, 16 and 17), it can be seen that for the variation of the time of process used here and for the policy of the production order in which the amount of the product in process is not taken into account, the PUSH system seems to be better than the PULL system, due to the fact that it has much lower inventories and besides they reach a point stable condition. According to what was expressed by Damodarna and Malouk (2002), most of the times the PUSH system provides better results than the PULL system.

Figure 18 shows the phase drawing for the WIP and INV variables, in the PULL system. The trajectory has a transitory condition that starts in the initial conditions (wip, inv)= (50, 80) and it moves until reaching the cyclic stable condition.

The results obtained up to this moment with the models of figures 12 and 13, show one, two and up to three attraction basins with point attractors whose trajectories are asymptotically stable spirals; limit cycle attractors are also present, specifically in the PULL system. The most common behaviors shown by the results are the ones of point attractors, specifically for the PUSH systems. In the PULL systems the limit cycle attractors are more common.
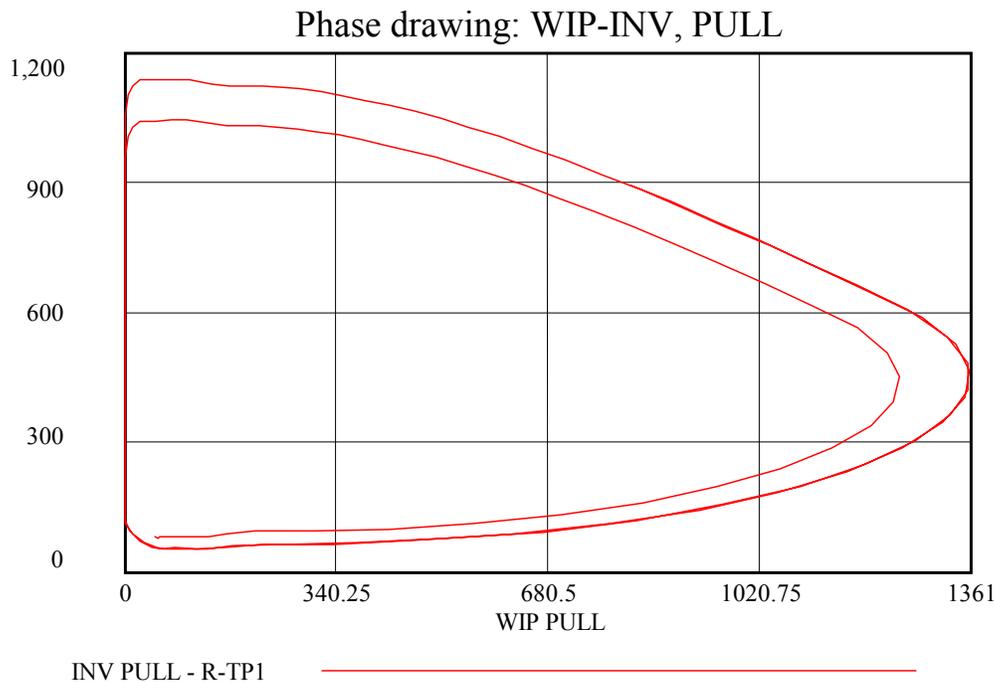
FIGURE 18. Phase drawing of the WIP and INV. Limit cycle attractor. One period.

5.2. PUSH AND PULL MODELS WITH WIP CONTROL

The analysis made from now on represents both systems, PUSH and PULL, and it has as its main feature the fact that for the calculation of the production orders, the inventory of the product in process is taken into account; such calculation is made with information related to the inventory of product in process, with the one of finished product INV and the one of expected inventory (SS in the PUSH system and OP in the PULL system).

Figure 19 shows the PUSH system with the relation between the WIP PUSH and DIF (difference between the expected inventory and the existing inventory, for both the product in process and the finished product) variables. Figure 20 shows the PULL system with the relation between the WIP PULL and OP PULL (production order) variables.

In this section the same five situations of the 5.1. section were analyzed, but only the results of the (a) situation are shown, that is, non-linear Time of Process (TP). Table 2 shows the values with which the models PUSH and PULL are simulated. The values are the same ones used in section 5.1.
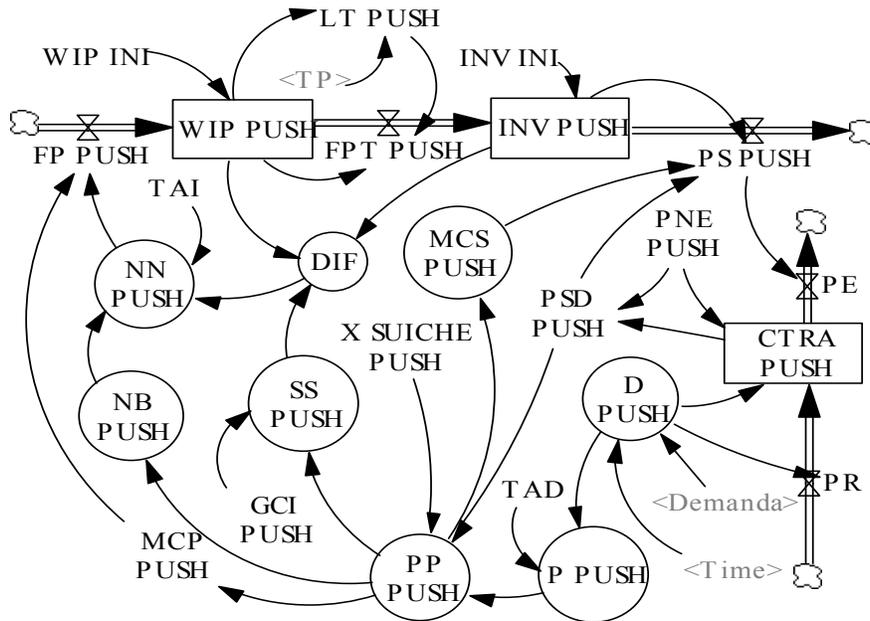
FIGURE 19. Diagram of a PUSH system. The WIP variable makes part of the production order policy.
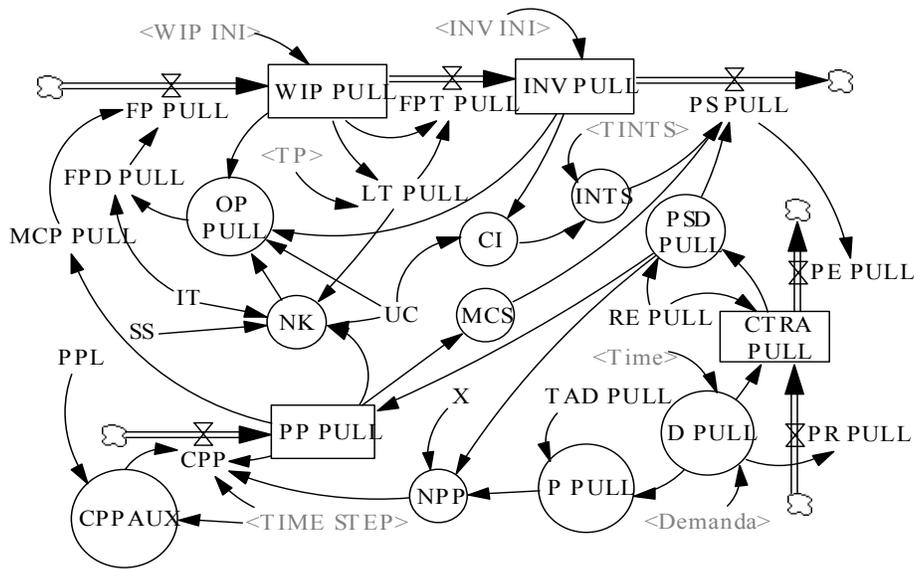
FIGURE 20. Diagram of a PULL system. The WIP variable makes part of the production order policy.

TABLE 2. Parameters of the simulation. Non-linear time of production.

CASE	TAI	GCI	IT	SS	TP	D	WIP initial	INV initial	Attractor
PUSH	0.5	0.8			Non- linear	105	50	80	Point
PULL			0.5	0.3	Non- linear	105	50	80	Point

Figure 21 keeps two of the three point attractors that it had in Figure 14 when there was no control for the WIP. The new balance points are about 52 and 110, as well as two of the attractors in Figure 14. The third point balance that was in Figure 14 is not shown in Figure 21. It might be thought that the control of the WIP balances the PUSH system a little more when a relatively high balance such as that one of Figure 14 disappears, whose approximate value is 230 units. Besides, the trajectories of Figure 21 reach balance faster, almost at half the time in which the trajectories of Figure 14 do it, and they do not present that many fluctuations. The trajectories of Figure 14 are asymptotically stable spirals, whereas the trajectories of Figure 21 are more approximate to improper asymptotically stable nodes, which means the stability is reached after a single oscillation that does not even complete a cycle.

Figure 22 presents the trajectory of the INV variable in the PUSH system when there is control of the WIP variable in the production order. The balance point, a point attractor, is exactly the same one of Figure 15, even though in this last Figure there is no control of the WIP. The balance is reached approximately in 105 units. The attractor of Figure 15 is an asymptotically stable spiral whereas in Figure 22 it tends to be an improper asymptotically stable node.
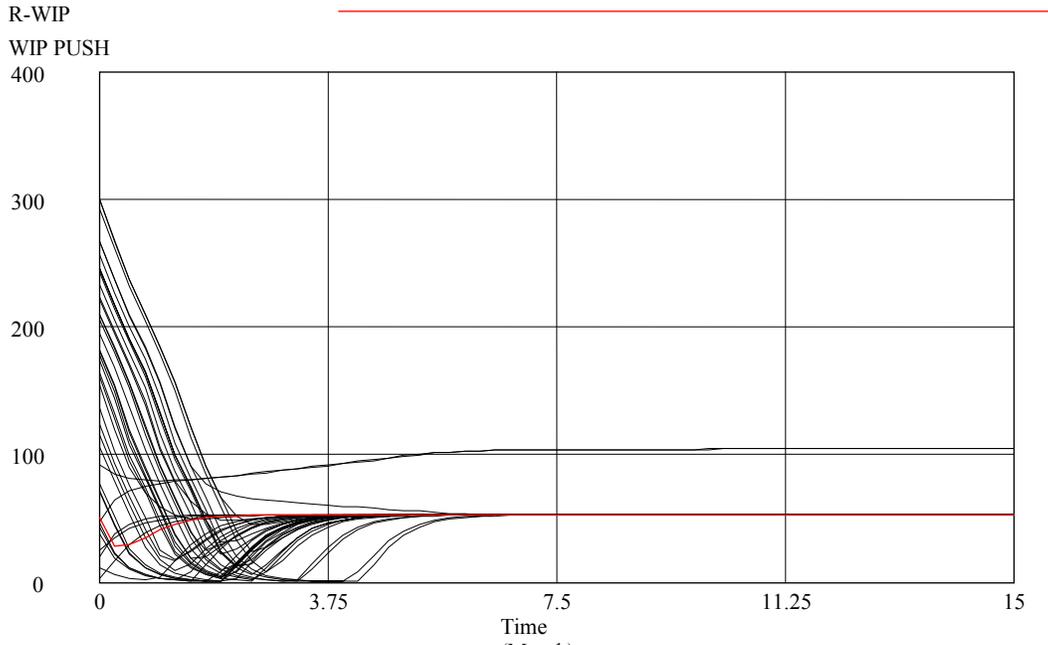
FIGURE 21. Temporary behavior of the WIP in the PUSH system with control of the WIP in the production order policy. Two attractors.
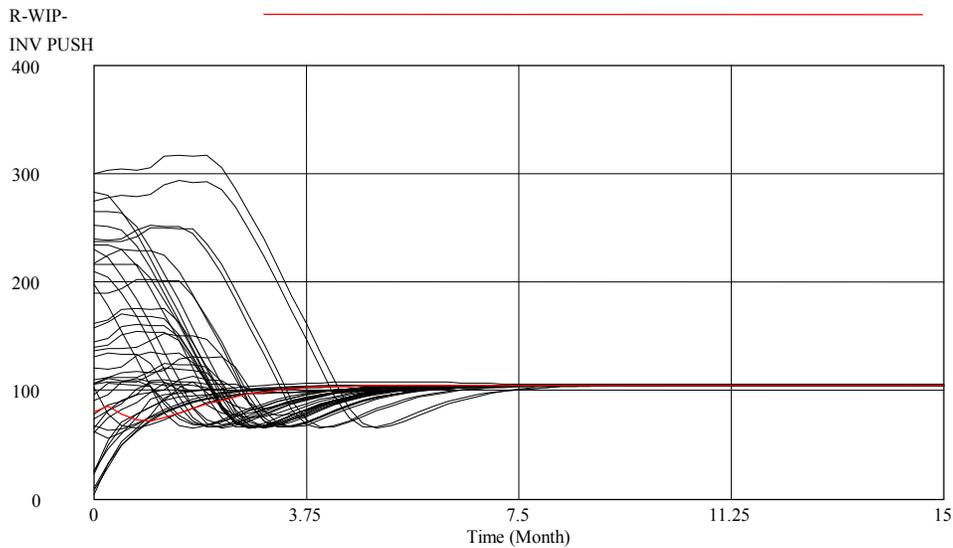
FIGURE 22.   Temporary behavior of the INV in the PUSH system with control of the WIP in the policy of production order. One attractor.

According to the graphics presented in Figures 14, 15, 21 and 22, it might be said that the PUSH system with control of the WIP variable in the production order policy, leads to having a more stable system due to the fact that there are fewer attractors; the balances are

reached almost in half the time of the system without control of the WIP; and moreover, the trajectories fluctuate less when passing the asymptotically stable spirals in the system with the control of the WIP, to improper asymptotically stable nodes.

The limit cycle attractor of the PULL system in Figure 16 turns into a system that may have three point attractors, which are shown in Figure 23, and these last ones are exactly equal to the attractors of the PUSH system in Figure 14. The difference between the PUSH system in Figure 14 (without control of the WIP variable in the production order) and the PULL system (with control of the WIP variable in the production order) is that the trajectories of Figure 14 are asymptotically stable spirals, whereas the trajectories in Figure 23 are improper asymptotically stable nodes. The two systems PUSH (without control of the WIP) and PULL (with control of the WIP) show that depending on the initial conditions of the level variables, three balances in both systems might be present, and regardless of the system, the points of balance are approximately 52, 110 and 220. The balances in Figure 23 are reached in a third of the time they are reached in Figure 14. It might be said that between the PULL system of Figure 16 (without control of the WIP in the production order) and the PULL system of Figure 23 (with control of the WIP variable), the PULL system of Figure 23 should be preferred as it shows stability in form of point attractors, whereas the system shown in Figure 16 shows a stability of limit cycle with seemingly large sizes as they range between zero and about 1,400 units.
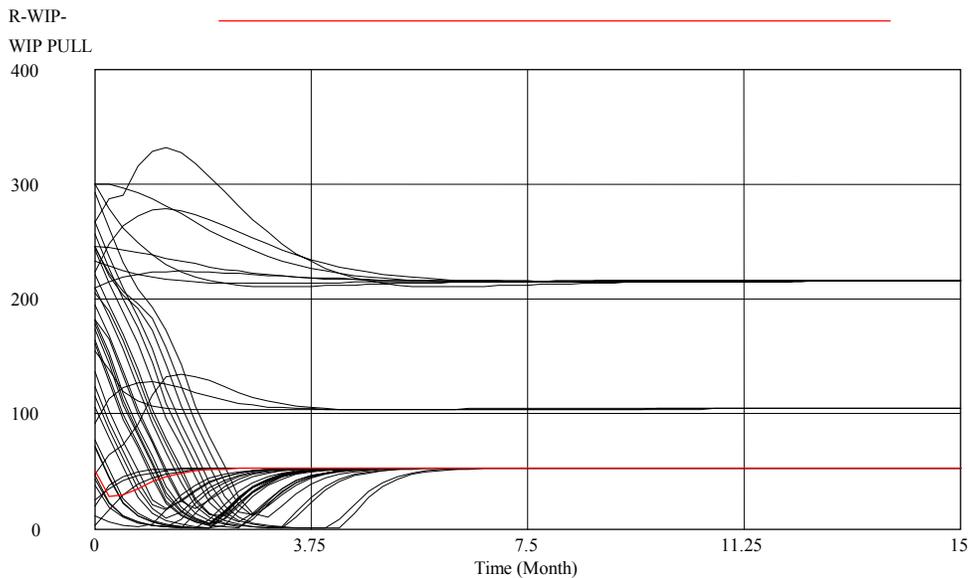
FIGURE 23. Temporary behavior of the WIP in the PULL system. Three attractors.

It is important to note that being in the same PULL system, when including the WIP variable as a control in the policy of production order, the behavior of the system goes from some trajectories with stability of limit cycle, Figure 16, to some trajectories that might

start from three attraction basins that are stabilized each one in point attractors, Figure 23. It is also remarkable to remember that the time of the process is a non-linear function.

Figure 24 shows the trajectories of the INV variable in the PULL system with control of the WIP variable in the production order. Three balance points are seen, point attractors, with approximate values of 90, 52 and 30 units. The corresponding PUSH system is the one that does not have the WIP variable as control in the policy of production order, and this is seen in Figure 17, which presents stable trajectories of limit cycle with approximate sizes between 15 and 1,200 units, which seemingly are pretty large. When comparing the trajectories of this PULL system, with and without the WIP variable as control in the production order, it is possible to say that is preferable to choose the PULL system with control of the WIP variable in the production order, due to the fact that it does not present such wide cycles as those of Figure 17, but balances of point attractor that might be present in the range (0,100). However, when comparing the INV variable in the PUSH system (with and without control of the WIP; Figures 17 and 22) and PULL (with control of the WIP; Figure 24), it might be preferable the PUSH system, as for any initial condition there is always going to be a single attraction basin and therefore a single point attractor reaching a value of about 105 units; unless that those who make decisions might consider that even though in the PULL system with control of the WIP three point attractors might be present, depending on the initial conditions, this system might be preferable to that one shown in Figures 17 and 22 (both of them PUSH), due to the fact that any of the three point attractors are always below the point if balance of the PULL system shown in Figure 24.

The attractor of Figure 17 (PULL system) is an asymptotically stable spiral, and the ones of Figures 22 and 24 (PUSH and PULL systems respectively) tend to be improper asymptotically stable nodes. As in Figures 16 and 23, in the case of the INV variable, it is important to note that even being in the same PULL system, when including the WIP variable as a control in the policy of production order, the behavior of the system goes from some trajectories with stability of limit cycle, Figure 17, to a system that might have three attraction basins with trajectories that are stabilized each one in point attractors, Figure 24. From the graphics presented in Figures 16, 17, 23 and 24, it might be said that the PULL with control of the WIP variable in the policy of production order, leads to having a more stable system due to the fact it goes from a system with stability of limit cycle with pretty large sizes, to a system of point stability with attractors of relatively low values.

R-WIP
INV PULL
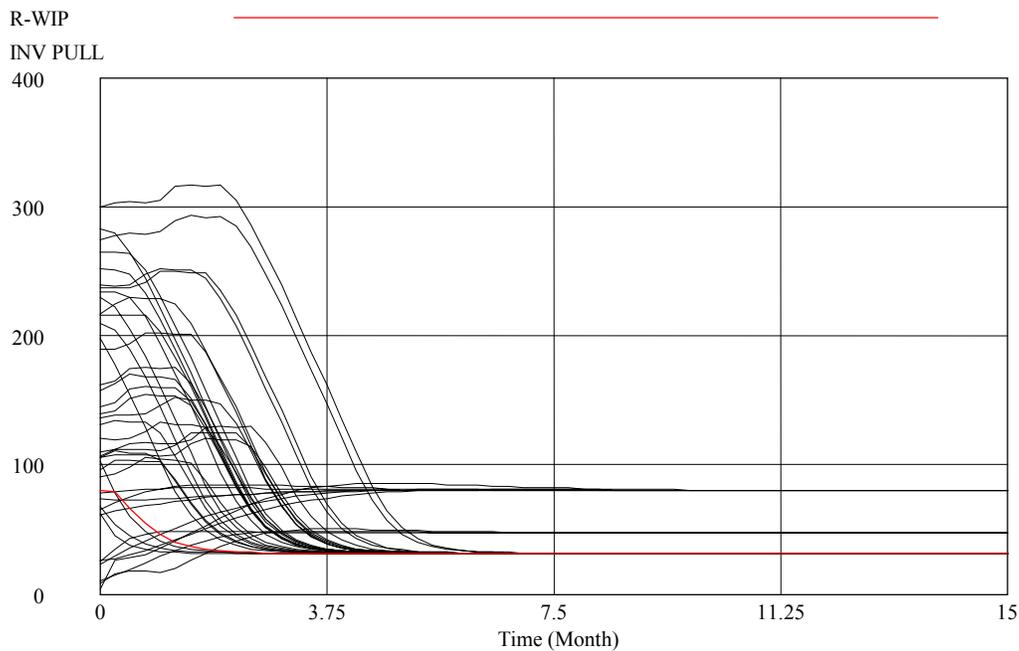
FIGURE 24. Temporary behavior of the INV in the PULL system. Three attractors.

6. CONCLUSIONS

The qualitative mathematical analysis and the simulation, allow seeing the critical values of some parameters that might lead the system to a collapse. A collapse might be produced both for excess of production and for lack of it. The excess of production might lead to the use of physical resources that might result very expensive for the company. The lack of production might lead to not having enough production and therefore, not enough sales to keep the company going.

Initial intermediate production values also might present strange behaviors, for instance, that after a period of stability, it goes to a new point of balance completely different to that one they used to have.

The qualitative analysis and the simulation, specifically with System Dynamics, might get those people in charge of decision making in an organization, either a manufacture, an agricultural, a health, a financial, or an educational one, and in general, in every kind of systems, to have a better clarity on the effects that might result from the process of decision making; it is therefore achieved, a greater capacity to foresee the emerging behavior of every conglomerate, calling a conglomerate a group of people, companies, environments, resources.

To make the analysis of the systems the different values of the parameters and the initial conditions must be analyzed. The fact that a system might be controlled by some initial

conditions and parameters, does not guarantee the global stability, as any change, no matter how insignificant, might change it.

The analysis of sensitivity of the systems that do not involve the inventories of product in process, WIP, shows one, two and up to three attraction basins that present point attractors with trajectories that are asymptotically stable spirals; there are also limit cycle attractors, specifically in the PULL system. The more common behaviors that show the results are those of point attractors, especially for the PUSH systems. In the PULL systems the limit cycle attractors are more common.

 The analysis of sensitivity of the systems that do involve the inventories of product in process, WIP, shows one, two and up to three attraction basins both for the PUSH and the PULL systems, but in this case the attractors are all point attractors and their trajectories are improper asymptotically stable nodes; only in one figure it is seen a proper asymptotically stable node.

The inventories of product in process and finished product, when the WIP variable is not included in the policy of production order, have levels higher than the inventories in the systems that do include the WIP variable in the production order.

The stability of the inventories, either with high or low values, is reached in a faster and smoother way in the systems including the WIP variable in the production order, than it is in those systems not including it. As seen in the figures of section 5.1, which does not include the WIP variable in the production order, the point attractors have trajectories in the form of asymptotically stable spirals, whereas the figures in section 5.2 show point attractors with trajectories in the form of improper asymptotically stable nodes.

Starting from this analysis it is not easy to identify the preference for a PUSH system or a PULL system, due to the fact that, even though both systems are pretty simple, the nonlinearities cause the variations in the parameters to lead the simulations to sometimes show the PUSH system being better, and sometimes the simulations show the opposite.

Another situation that is not easy to define, is the one considering if a system with cyclic stability might be preferable to a system with trajectories in asymptotically stable spiral, or a system with improper asymptotically stable trajectories. The selection of one or another situation remain to the choice of a decision maker; in some occasions a limit cycle attractor of low size, might be preferable to a point attractor with a pretty high level of inventory.

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